To be presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 27, 1918.

THE OXIDE FILM LIGHTNING ARRESTER

BY CROSBY FIELD

ABSTRACT OF PAPER

The oxide film arrester is a new type of lightning arrester made up of a film of insulation in contact with a conducting powder. Upon the application of over-voltage, the insulation will be pierced, but the powder will very rapidly turn into insulation and plug any holes punctured in the original insulation by the over-voltage, thus forming in substance a resealing insulation. A brief description of this arrester is given together with the principles underlying its action and a comparison with other types of lightning arresters. Mention is also made of other characteristics of this combination which are not used in the present arrester, but which are being applied in other developments. A few notes of tests on the commercial arrester complete this paper. With the exception of the basic patents issued to the author this is the first time any disclosure has been made of this arrester.

THIS paper will be confined to a brief statement of the scientific principles underlying a new type of lightning arrester called the "oxide film arrester."* The functioning of this arrester depends upon the fact that certain dry chemical compounds can be changed with extreme rapidity from very good conductors of electricity to almost perfect non-conductors by the application of a slight degree of heat. Lead peroxide is a good example of such a substance. It has a specific resistance of the order of one ohm per inch cube. The resistance varies with the pressure to which it has been compressed. At a temperature of about 150 deg. cent. the lead peroxide (PbO_2) will be reduced to red lead, commercially known as minimum (Pb_3O_4). This has a specific resistance of about 24 million ohms per inch cube. At slightly higher temperatures this minimum will be reduced through the sesquioxide (Pb_2O_3) to litharge (PbO), which last named is practically an insulator. [A megger reading of infinity is obtained on a column 3 millimeters long (0.11 in.) and 5 square millimeters area (0.2 sq. in.)].

Again the oxides of bismuth give similar characteristics. There

*U. S. Patent No. 1,238,660—Crosby Field.

Manuscript of this paper was received April 24, 1918.

are, furthermore, several other compounds and mixtures of compounds that will give these same results.

Lead peroxide is normally in the physical state of a powder If this powder be placed between two electrodes and a current passed, the temperature due to the resistance at the contact of the peroxide and the metal will cause heat to be generated locally at the surface. When this heat is sufficient to create a temperature of about 150 deg. cent. a film of the lower oxides of lead

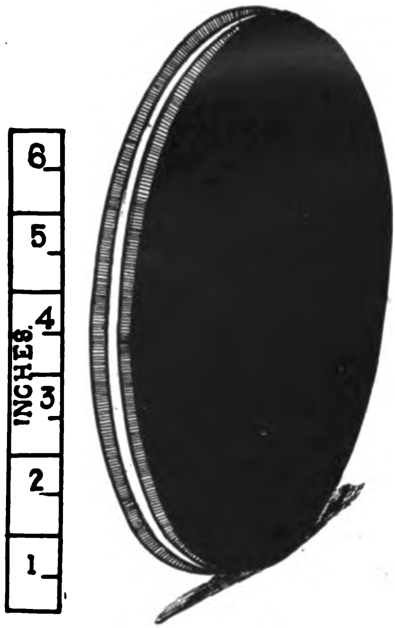


FIG. 1

A single cell of the oxide film lightning arrester consisting of a ring of porcelain with two circular steel disks, one spun on each side of the porcelain and insulated by the porcelain. The inside of the ring of porcelain between the plates is filled with peroxide of lead.

forms, producing a film of insulation which stops the current. This method of film formation over any large area is rather irregular, and of course the oxide is not used in such a fashion in the commercial arrester. Instead of this formation of litharge film any insulating film may be put on the electrodes initially. As insulating film spread on the metal plates there have been used thin layers of the following; glass, water glass, halowax, cloth, balsam, shellac, oil, paints, lead paints, varnishes, and lacquers of all available kinds. In all cases the results are sim-

ilar, varying only with the voltage at which puncture of the film of insulation occurs.

The foregoing statements define the principle of the commer-

FIG. 2

Shows the disassembled parts of a single cell of the oxide film lightning arrester. From left to right are a steel disk spun on a ring of porcelain, a pile of brown peroxide of lead, the other steel disk and an asbestos washer.

cial oxide film arrester. Fig. 1. It comprises two sheet metal electrodes, set about 0.5 in. (12.7 mm.) apart, one or both covered with a thin insulating film and the space between the plates

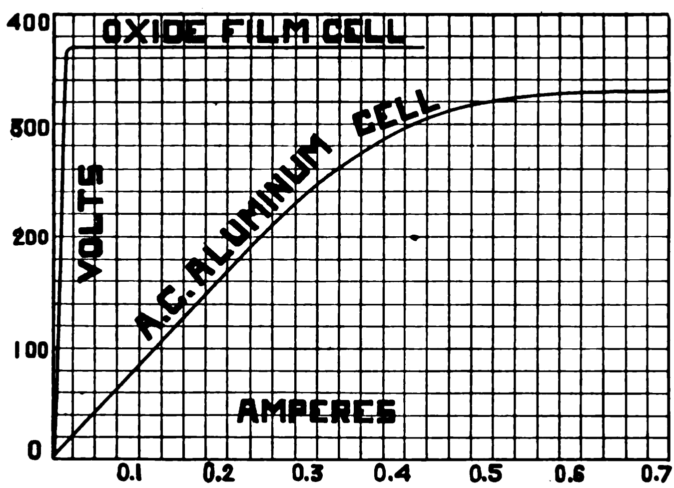


FIG. 3

The comparative volt-ampere characteristics of the oxide film cell and the a-c. aluminum arrester cell. The oxide film has only a few milliamperes of leakage current up to the critical film voltage when the film gives way more suddenly than the film in the aluminum cell. The critical voltage of the oxide film can be made approximately as low as the hydroxide film on the aluminum cell.

filled with some such substance as that described above, as, for example, lead peroxide. Fig. 2 shows the disassembled parts of a single cell. At a permissible voltage of 300 volts per cell the insulating film prevents any appreciable current flowing under

normal conditions. As soon as the voltage rises slightly above normal the film punctures in one or more microscopic points, the lightning charge meets with practically no resistance and flows to earth. Fig. 3. The dynamic current starts to follow but because of the fact that the insulation was punctured in such

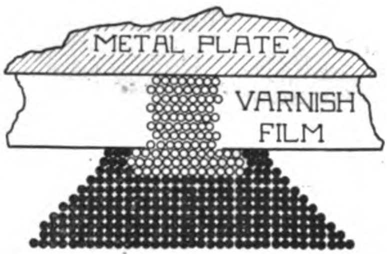


FIG. 4

A magnified, imaginary representation of one of the films on one metal plate. As shown the film is punctured by a spark and filled with a litharge plug which is represented by the open circles. The cross-section in the discharge path, a short distance away from the metal electrode, is sufficiently expanded to make the current of low enough density not to heat the peroxide to a temperature of reduction to litharge. The peroxide is represented by the solid dots and only those in the path of the discharge are shown. At the other electrode, not shown in this magnified diagram, a similar effect may be taking place although there is a difference between the positive and negative crators.

fine points, the current density near these points is exceedingly great. This results in a localized heating which speedily raises the temperature to a value sufficient to change to insulating litharge all the conducting peroxide in this minute path of the current flow in contact with the electrodes. The film conse-

FIG. 5

New electrode and also one from a cell having had passed through it several thousand discharges. The light colored plugs of litharge are plainly visible in the many spots on the surface of the plate.

quently reseals, stopping the further flow of dynamic current. This action is so rapid that its duration cannot be measured on an oscillograph giving two thousand cycles per second, that is to say, the action of resealing occurs in less than one four thousandth part of a second after the excess of lightning voltage has ceased. Fig. 5 shows the visible spots of insulating litharge plugs on the surface of an electrode.

This film can be made of litharge itself, as well as any of the insulating materials above named. For example, metal plates may be inserted in any of the well-known lead electroplating solutions, and thus a very thin lead peroxide film (measuring a few hundred thousandths of an inch) formed. By proper heating this will be changed to litharge and this form of electrode can be used. Peroxide may also be sprinkled over any metal plate and the plate heated, which will reduce the peroxide to litharge. Again, the metal chosen for the electrode itself may be lead and if heated in the air a thin film of litharge will be formed on the surface. Again, an aluminum electrode may be put in any of the common electrolytes, and a thin aluminum film be built up. This may be used with the peroxide powder. Of these methods of forming the film the most preferable is by dipping in varnish or lacquer highly burnished surfaces of brass, steel, or copper, and is consequently used in the commercial arrester. The ohmic resistance of the arrester during discharge is quite low (less than 1 ohm per cell). Thus, when the insulating film is punctured the arrester offers very slight impedance to the flow of energy at abnormal voltages.

There is a certain range of voltage necessary to pierce any given insulation. The exact voltage depends not only upon the thickness of the insulation and its dielectric strength but also on the relation of the dielectric spark lag to the duration of the super-spark potential and the frequency of alternations of the transient surge.

If an arrester is to give protection of insulation in shunt with it, the arrester must relieve the abnormal electric pressure before damage is done to the insulation. Although tests are frequently made with the arrester and the insulation it is to protect in parallel, a more convenient method has been standardized and is known as the equivalent sphere gap test. Both the insulation and the arrester are compared by comparing each to the equivalent sphere gap.

The equivalent sphere gap of the oxide film arrester may be analyzed, as in other cases, into separate and distinct parts. First, there is the equivalent sphere gap of the main gap in series with the cells. Second, the equivalent sphere gap to initiate a discharge through the insulating film on the plate surface of the cell. Third, there is the equivalent sphere gap of the resistance drop of the current discharging through the powdered peroxide in its path. Fourth, there is the equivalent sphere gap of the inductance of the arrester.

Commenting on these factors in their relation to this arrester, the main gap is itself a sphere gap which has the fastest spark of any practical gap. The gap setting, like that of the aluminum arrester, is only slightly above that of the normal voltage of the circuit.

The equivalent sphere gap of the film is several times greater than the thickness of the film because solid material has a greater dielectric spark lag than air, but with this multiple of the thickness of the film the equivalent sphere gap is still low. Since peroxide is a good conductor, the series resistance in the path of the discharge is insufficient to give an undesirable voltage drop. As to the inductance of the arrester, it has a minimum value due to the fact that each cell is only 0.5 in. long, as shown in Fig. 1, and these cells are placed one on top of another. In other words, the total length of the arrester (which constitutes the inductance) is short as compared to the necessary length of conductor from line to earth.

One of the obstacles that had to be overcome in the making of this arrester was the increase in the resistance after a great many heavy discharges had passed through it. The predominant reason for the increase seems to be explained by the following theory. The current passing through this small puncture in the film heats up very rapidly not only the powder but also the air contained within the interstices of the powder. The particles are thereby thrown out of contact with each other, thus producing a fluffiness. The decrease in the number of contacts decreases the actual cross sectional area of conduction' hence increases the resistance. This raises the equivalent sphere gap. This action is accelerated, of course, by the giving off of the oxygen itself evolved in the reduction from lead peroxide to the lower oxide. If, however, this same arrester be violently jarred or the filling powder be compressed, or any other method utilized to restore the particles to their previous intimate contact, the equivalent needle gap will fall again. While increased fluffiness appears to be the predominant cause of change of the equivalent sphere gap, the increased thickness of the film of litharge at the point of puncture of the film is finally a factor of moment. The total area of the film must be sufficient to give a reasonable number of years of life to the arrester. There are other factors relating to the details of manufacture which give a limited degree of control over this change in equivalent sphere gap.

In all the commercial oxide film arresters used for alternating

current the power factor is nearly unity. For special purposes however, the power factor can be made anything desired from 10 per cent. to unity. This is obtained by combining with the conducting oxide other non-conducting materials. This principle has been made use of for condensers but it has not been found desirable to incorporate it in the arrester.

To summarize—an arrester operating under a new principle has been made which comprises in essence one or more metal electrodes covered with an insulating film, and separated by a conducting powder, which has the peculiar characteristic of becoming a non-conducting powder upon the application of heat. Voltage higher than that which can be withstood by the insulating film punctures it in one or more points of about 0.005 cm. diameter. Dynamic current flowing gives a high current density in the conducting powder adjacent to these punctures which in turn heats it up rapidly, reducing the powder to a non-conductor, and sealing the holes in the film. The powder being a poor heat conductor localizes this action, so that very little more powder is reduced than is actually necessary to seal up these minute punctures.

The critical spark voltage and that part of the equivalent sphere gap controlled thereby is a function of the thickness and kind of material used for the film.

Comparison of the "Oxide Film" with Well Known Arresters

The earliest form of non-electrolytic film arrester was known as the dry aluminum arrester.* It was a direct attempt to utilize the dry film which forms on the surfaces of pure aluminum immediately after it comes in contact with the oxygen of the air. The hydroxide film is easily formed in electrolyte and on drying becomes a dry film which gives sufficient action to prevent a discharge up to a given critical voltage, depending upon the thickness of the film. The film can also be formed by a spark or arc of a conductor in contact with a plate. Naturally this conductor should be of a non-metallic nature. In the earliest form tried powdered carbon was used mixed with dioxide of manganese which gives a liberal supply of oxygen at the heated point.

One of the objects of the development of this arrester was to decrease the cost of manufacture and it was found with the new principle involved in the oxide film arrester, where the powder

* U. S. Patent, E. E. F. Creighton.

furnishes the film rather than the plate, that the aluminum could be replaced by a cheaper metal, such as steel, and, as already described, the initial film known in the early stages of development as the "paint skin" type could be furnished by a ayer of varnish. On first sight, knowing the extreme thinness of the hydroxide film on wet aluminum cell it might not seem that the dry cell would give the same general characteristics as the wet cell. But a comparison of the volt-ampere curves shows the same general characteristics. For a-c. voltages of 300 volts average per cell the current in the dry cell is of negligibly small value up to 40 milliamperes. The power factor is nearly unity and the current flow is due to very slight leakages through the films. In the case of the aluminum electrolytic cells there is an equivalent condition, the d-c. leakage current of the order of one milliampere being due to leaks through the hydroroxide film. In the a-c. aluminum arrester the leakage current on the plate area used is much greater due to the destructive action of the alternating current on the hydroxide film. Furthermore, the wet cell with its thinner film is a condenser of appreciable capacitance which takes a charging current of about 0.5 ampere at 60 cycles. When the voltage reaches a certain critical value which is between 300 and 400 volts for the wet aluminum cell and between 300 and 500 for the oxide film cell (or higher if the paint film is made thicker) the current is allowed to pass freely through the cells, limited only by the ohmic resistance of the cell independent of the film. Since the oxide film arrester has no dissolution of the film, as occurs in the wet aluminum cell, charging is not only unnecessary but undesirable. This extends the use of the oxide film arrester to localities where there are no attendants.

Although the wet aluminum plate becomes frosted to an appreciable thickness by the passage of current in long use, the actual thickness of the film, as represented by the critical voltage, is not changed. In the oxide film arrester, however, the film less than one 1 mil thick (0.025 mm.) initially thickens up by the addition of successive spots of litharge for each successive discharge. This represents the wear on the arrester and limits its total life. Fig. 3 shows comparative volt-ampere characteristics of the oxide film arrester and the a-c. aluminum arrester. Since both of these arresters have a leakage current which wears the plates of the cells when alternating current is supplied, it is necessary, as previously stated, to place a spark gap in series with the cells. This spark gap is set at a value

slightly above the normal potential of the circuit so that nothing but abnormal voltages will cause a discharge.

The foregoing data show that the oxide film arrester has general characteristics closely like the standard aluminum electrolytic arrester. It has the obvious advantage which comes from being dry rather than wet, it will not congeal and needs no daily charging.

In making the characteristic volt-ampere discharge curves the oxide film arrester does not lend itself as readily to the test as the wet aluminum cell. While its critical film voltage is evident, the change from no conduction to full conduction is more sudden. Therefore, the discharge rate at double potential is best shown by throwing double potential on the cell and subsequently reducing the voltage to its normal value per cell.

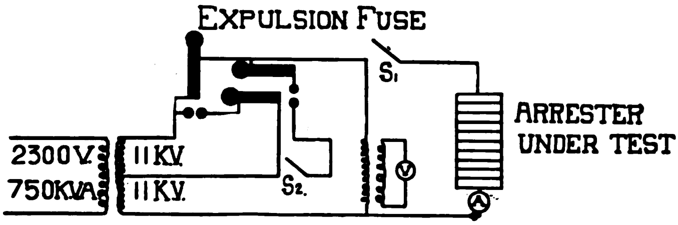


FIG. 6

Arrangement of three expulsion fuses which short-circuit gaps in the path of discharge to first throw double voltage on an arrester under test and then reduce the voltage to normal value by shifting the contacts on a transformer from full coil winding to half coil winding.

This gives a very considerable quantity of electricity through the cells and is a severe test.

In order to conveniently alter the voltage from double value to normal value the circuit is arranged as shown in Fig. 6. The object of the connections may be briefly stated: A transformer with two coils in series impresses voltage on the arrester and then the contacts of the arrester are automatically shifted to one coil giving half the voltage. A heavy pendulum closes switch *S*-1 and sets the oscillograph into operation. The full voltage on the transformer is thrown on to the oxide film arrester under test, marked *O-F*, which has a number of cells sufficient only for half the voltage of the transformer. In other words this throws double voltage on to the arrester and the heavy current passing through the cells causes fuse *F*-1 to blow. The operation of this fuse short-circuits half the transformer and throws the other half across the arrester. This is done by

means of gaps and fuses as follows: When the expulsion fuse *F*-1 blows, the conducting gases are shot into the open gap *G*-1 which closes the circuit through fuse *F*-2 to the mid-point of the transformer. This short circuit on half the transformer causes fuse *F*-2 to blow and the hot gases discharging from fuse *F*-2 close the gap *G*-2 which throws the mid-point of the transformer on to the arrester through the switch *S*-2 which is closed just previous to starting the test. These several operations occur with a rapidity depending upon the size of fuses used. It is possible

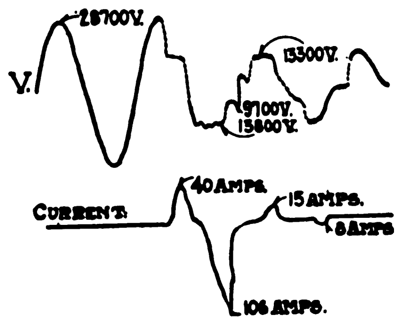


FIG. 7

Operation of an oxide film arrester on double dynamic voltage initially and its recovery on normal voltage. The upper record with a peak voltage of 28,700 volts shows the arrester connected to the circuit as the voltage wave starts to decrease on its third peak. The voltage immediately drops to the critical value of the cells—about 13,500. On the lower record the current is seen to rise to 40 amperes. On reversal of the voltage to 13,800 in the negative direction as shown by the upper record, the current as shown by the lower record rises to 106 amperes. The switching operations produce several electro-magnetic kicks as shown by the irregular voltage wave as it rises in the next cycle to 13,300 peak value. At this lower applied voltage the current in the series of cells, as shown by the lower record, is 15 amperes. In the subsequent half cycle the current rises only to 8 amperes. In the last half cycle of voltage shown, where the switching operation is complete, and the wave assumes its normal smooth form, the current in the cells is too small to be registered by the oscillograph. Its value is of the order of milliamperes. This figure is a copy of an oscillogram. The copy was made desirable by overlapping of the discharge on the two ends of the film.

by this means to throw momentarily 22,000 volts on an 11,000 volt arrester and note the character attending its discharge and recovery. Fig. 7 shows such an operation on an oxide film arrester. The initial discharge current is 40 amperes during the first half-cycle due to the point it strikes in the descending wave during the second half-cycle it is 106 amperes. After the third half-cycle the litharge film has so completely sealed up the path of discharge that the current is too small to show on the oscillogram. The leakage current with no series gap is of the order of a few milliamperes.

To be presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 27, 1918.

THE OXIDE FILM LIGHTNING ARRESTER

BY CHARLES P. STEINMETZ

ABSTRACT OF PAPER

A short history of lightning protection of electric systems is given, as relating to the three successive types of electric circuits; the communication circuits, the power circuits of negligible electrostatic capacity, and the high power circuits containing distributed capacity and inductance and capable of electric oscillation, leading to the three problems of discharging over-voltage to ground, opening the power current which follows the discharge and discharging so that no power current follows even for a fraction of a half wave. It is shown that these problems are solved by the spark gap to ground, by the use of non-arcing metals in the multigap arrester, which opens the circuit at the end of a half wave of current, and by the so-called "counter e.m.f." type of arrester, represented by the aluminum cell and the oxide film arrester.

It is shown that the necessity of taking care of recurrent discharges in high-power systems had led to the universal adoption of the aluminum cell arrester in such systems, in spite of its disadvantage of requiring daily attendance in charging, and of containing an electrolyte and oil.

In the oxide film arrester a type of arrester is presented which has the same characteristics and therefore the same advantages as the aluminum cell arrester, but does not require daily attendance and contains no liquids.

Its method of operation is explained, and its difference from the aluminum cell arrester; the dielectric film, which punctures under the discharge, and reseals after the discharge, is formed from the solid materials between the terminal plates, compressed PbO_2, and therefore no spontaneous chemical action occurs which dissolves the film, as in the aluminum cell, in which the film forms from the aluminum electrode, gradually dissolves, and therefore requires daily charging.

A short description of the construction of the oxide film arrester is given, a record of its operation in industrial service for over three years, and oscillograms showing the performance of this arrester under recurrents, oscillations and under high-power impulses.

THREE periods can be distinguished in the development of lightning arresters, corresponding to the three periods of the use of electricity:

(1) Electric circuits of negligible power, telegraph and telephone.

(2) Electric power circuits of negligible electrostatic capacity;

Manuscript of this paper was received April 25, 1918.

d-c. lightning and railway, a-c. secondary and 2300-volt primary distribution.

(3) High-voltage electric power circuits, transmission lines, etc.

(1) In electric circuits of negligible power, such as telegraph and telephone circuits, a simple minute spark gap to ground afforded protection by discharging lightning to ground, and was sufficient until the recent years, when with the general introduction of electric power circuits the problem arose, in electric communication circuits to take care of crosses with power circuits.

(2) In electric power circuits, a simple spark gap to ground became insufficient for protection, since the power current, following the lightning discharge as arc, short-circuited the system and burned up the arrester. The problem then arose, to safely open the short circuit of the machine current to ground, through the lightning arrester, after the lightning discharge has passed, and to leave the arrester in operative condition to receive following lightning discharges.

Of the various devices developed heretofore, the magnetic blow-out lightning arrester still is used in direct-current railway circuits.

The first scientific investigation of this problem, is recorded in the paper[1] by A. J. Wurts, presented before the A. I. E. E. at the meeting of May 1894. Since that time all lightning arresters for alternating-current power circuits of negligible electrostatic capacity are based on the multigap principle between non-arcing metals, whatever constructive forms the arrester may assume—as the present compression chamber lightning arrester. The multigap arrester operates on the principle, that the lightning discharge over the multigaps closes the circuit to ground, but the power arc following the discharge extinguishes at the end of the half wave of the alternating current, as the non-arcing character of the gaps does not permit the reverse current of the next half wave to start. The multigap arrester thus short-circuits for a part of a half wave. It obviously is suited only for alternating currents.

For years difficulties were met with the question of resistance; without series resistance, in large systems the short-circuit power

1. TRANS. A. I. E. E., 1894, Vol. XI, p. 337, "*Discriminating Lightning Arresters and Recent Progress in Means for Protection Against Lightning*", by A. J. Wurts.

even of a part of a half wave may be sufficient to disable and destroy the arrester, while the use of a series resistance, while limiting the power current and thereby protecting the arrester, also limited the discharge capacity and thereby reduced the protection. This problem was solved by the use of multigaps shunting the series resistance, so that moderate discharges passed over the resistance, while high power lightning discharges found a path without series resistance, over the shunted gaps, and at the same time, the shunting resistance made the power arc at the shunted gaps unstable and thus assisted in the extinction of the short circuit at the end of the half wave.[2]

(3) As soon however, as circuits came into use, which had considerable electrostatic capacity, such as high-voltage transmission circuits, extended underground cable systems, or lower voltage circuits (including generator circuits) inductively connected with such circuits, the multigap arrester failed by frequently, or even usually destroying itself by the discharge, burning up.

In such circuits, oscillations between capacity and inductance may occur, started by a lightning discharge or any internal disturbance such as switching etc., resulting in recurrent high frequency oscillations, of which the arcing ground on a transmission line is typical and probably best known. With such continual discharges, often several per half wave, the multigap arrester short-circuits at the first oscillation, for the remainder of the half wave, and while the multigap functions properly and opens the short circuit at the end of the half wave, the oscillation of the next half wave again short-circuits, and so on, so that the effect is that of a continuous short circuit, and no lightning arrester, no matter how large, can dissipate the short-circuit power of a big system for any appreciable time.

For such systems, in which recurrent high frequency oscillations, as arcing grounds, may occur, a lightning arrester is necessary, which does not short-circuit the machine current even for a fraction of a half wave, but merely discharges the over voltage, the oscillation which, however high in voltage it may be, is small in energy compared with the short-circuit power of

2. See A. I. E. E. TRANS., 1907, Vol. XXVI, p. 425, "*Protection Against Lightning, and the Multigap Lightning Arrester*" by D. B. Rushmore and D. Dubois.

the system, as it represents only the stored energy of capacity and inductance. The only arrester of this character heretofore was the electrolytic, or aluminum cell lightning arrester, developed by E. E. F. Creighton, J. L. R. Hayden, F. W. Peek and others. It acts towards an over-voltage discharge like a counter e. m. f. equal to the normal circuit voltage, and the discharge current passing through the arrester thus is the short-circuit current of the over voltage, while the normal machine voltage does not discharge, is held back and not disturbed. The aluminum cell arrester thus can discharge continual disturbances, over-voltage oscillations occurring at every half wave, for a considerable time, half an hour to several hours, before it is endangered by the temperature rise due to the accumulated energy of these discharges.

The aluminum cell arrester comprises a series of cells—usually conical and stacked into each other—of aluminum electrodes with an electrolyte, of which neither the salt nor its ions appreciably dissolve alumina. In "forming" the cell, by an alternating current passing through it, the electrodes are coated by a thin non-conducting film of alumina, which grows in thickness until it holds back the impressed voltage. Any over-voltage punctures this film, but the current passing through the puncture holes, again forms alumina and closes the holes. Thus the aluminum cell acts like a self repairing electrostatic condenser of a disruptive strength equal to the impressed voltage: about 250 to 300 volts per cell.

The practical experience of the last ten years has proven the aluminum cell arrester as the only type capable of affording protection in modern high power circuits, and proven this so conclusively, as to lead to its universal adoption in such circuits in spite of the inconveniences incident to the need of daily attention in charging, the use of a liquid electrolyte, and the difficulty of testing the arrester without taking it apart, except by watching the appearance of the charging arc, or measuring the charging current.

These inconveniences incident to the aluminum cell arrester were well realized however, and as soon as the minor troubles met with the aluminum cell arrester in the early years had been overcome engineers went energetically to work on the problem of developing a lightning arrester of the characteristics of the aluminum cell arrester, but which does not require any attention

beyond that given to every apparatus in a well managed system, that is, an occasional inspection, at least once or twice a year.

Numerous researches were made by the engineers whose splendid work I here acknowledge: Messrs. H. D. Brown, E. E. F. Creighton, Crosby Field, V. E. Goodwin, J. L. R. Hayden, N. A. Lougee, and G. B. Phillips. Some of these researches have not yet led to results ready for communication, but I am glad to present to you here as the result of the work of these men, and more particularly of E. E. F. Creighton, Crosby Field and N. A. Lougee, in the *Oxide Film Lightning Arrester* a new type of lightning arrester, which has all the characteristics and advantages of the aluminum cell arrester, but does not require any charging and thus requires no special attention, contains no liquid electrolyte, no inflammable material, and like the aluminum cell arrester, can be located outdoors as well as indoors.

The oxide film arrester, like the aluminum arrester, acts like a counter e. m. f. equal to the normal circuit voltage, freely discharging any over voltage, but holds back the normal machine voltage. Thus the discharge is limited to the energy of the over voltage, as in the aluminum arrester, and like the latter, the oxide film arrester can continuously discharge recurrent surges, such as arcing grounds etc., without endangering itself, for a considerable time, sufficiently long to notice and eliminate the disturbance.

Compared with the almost entire absence of knowledge of lightning phenomena in electric circuits, under which Mr. Wurts had to work in developing the non-arcing metal multigap lightning arrester, our present knowledge of lightning phenomena is very great. Nevertheless, there are so many disturbances in large electric systems, which we cannot or only incompletely reproduce in our laboratories, that the final decision on the success, that is, the effectiveness and permanence of a lightning arrester, still is best given by the experience in industrial systems.

Therefore, after extensive laboratory tests had been completed and had proven the oxide film arrester as of the same characteristic as the aluminum arrester, but requiring no special attention, a number of industrial installations were made, and more added the next year and the third year. Now, however, when a considerable number of installations of these arresters, for voltages from 110 to 33,000, have been in successful operation, some for over three years, and have proven their protective value and

their permanence, I consider it desirable to bring the arrester to your attention.

Of the numerous tests made on the performance of the arrester, it may be sufficient here to give only two, by the oscillograms Figs. 1 to 4, showing the action on a recurrent oscillation, in Figs. 1 and 2, and on a single high power impulse, Figs. 3 and 4.

The tests, oscillograms Figs. 1 and 2, were made in the usual manner: a surge or continual oscillation was produced by a large condenser, connected to an alternating-current supply and discharging over a spark gap through an inductance. The latter was chosen so as to give a frequency of 1200 cycles to the oscillation, and thereby bring it well within the range of the oscillograph. This surge was impressed upon the apparatus to be protected, a transformer energized by another alternating-current circuit. Fig. 1 shows the oscillogram without protection of the transformer, and Fig. 2 the oscillogram with an oxide film cell shunting the transformer and thereby protecting it.

In Fig. 1, the lowest curve shows the voltage of the 320-volt 47-cycle power supply circuit of the transformer, with the oscillations superimposed on it, rising to surge peaks of 2800 volts. The upper curve shows the oscillating currents passing through the transformer, rising to current peaks of 40 amperes. The middle curve is absent, as no arrester is used in this test. In Fig. 2 however, where an oxide film cell is shunted across the transformer, the middle oscillogram shows the current oscillations passing through the arrester, with peaks of 35 to 41 amperes. The lower curve in Fig. 2 then shows again the circuit voltage wave impressed upon the transformer, with the oscillations cut down by the oxide film cell. This voltage wave, the lower curve in oscillogram Fig. 2, well illustrates the characteristic action of this type of "counter-e. m. f. arrester"—to which the oxide film arrester and the aluminum cell belong. The oscillation peaks are sharply cut off at a maximum voltage of 60 per cent above circuit voltage: the value for which the spark gap was set. As the result, the oscillations are very greatly cut down from the high values which they have in the unprotected circuit Fig. 1 (2800 volts), and become unsymmetrical. The half waves of oscillation in the same direction as the circuit voltage are greatly reduced, by the limitation of the voltage to 60 per cent above-normal, while the reverse half waves—which lower the instantaneous circuit voltage— are less affected. Corresponding thereto the discharge current through the arrester, the middle oscil-

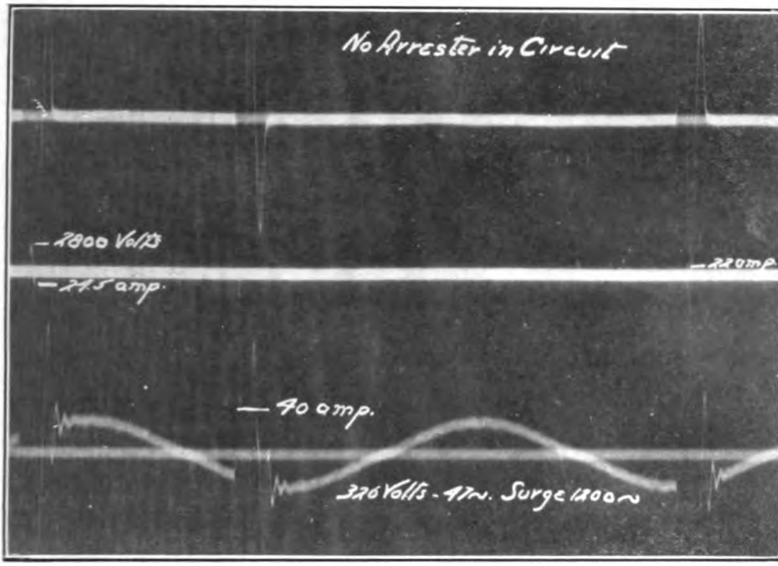
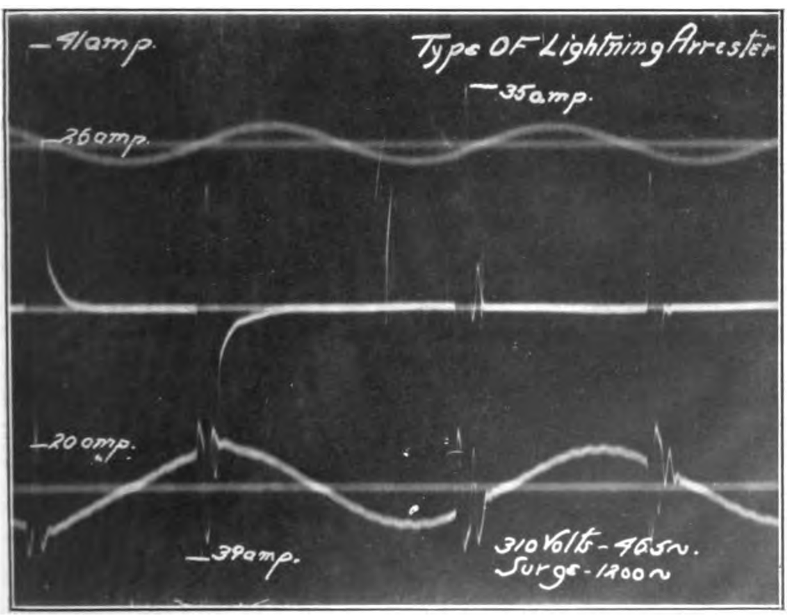


FIG. 1—SURGE DISCHARGE AS IN FIG. 2—NO ARRESTER IN CIRCUIT

Top vibrator—current through transformer.
Bottom vibrator—circuit voltage.



[STEINMETZ]

FIG. 2—SURGE DISCHARGE THROUGH ARRESTER CELL

Top vibrator—60-cycle timing wave.
Middle vibrator—current through arrester.
Bottom vibrator—voltage across arrester.

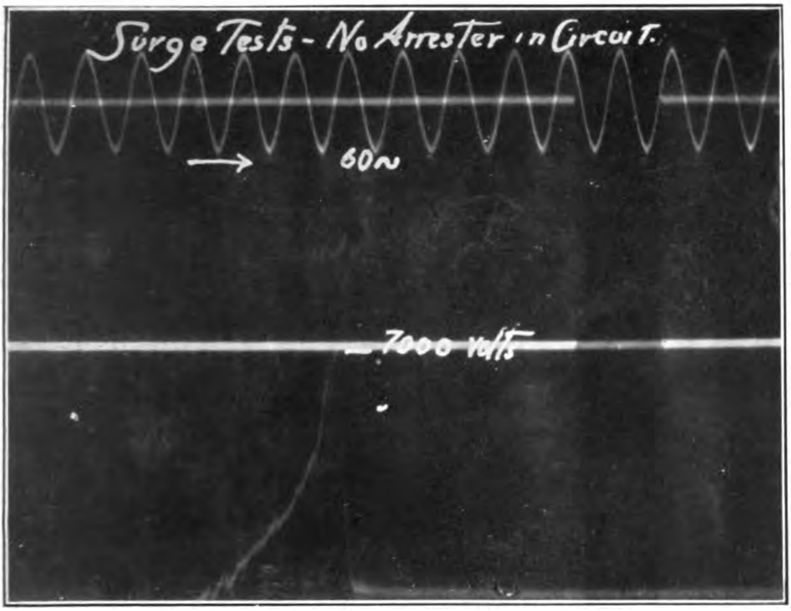
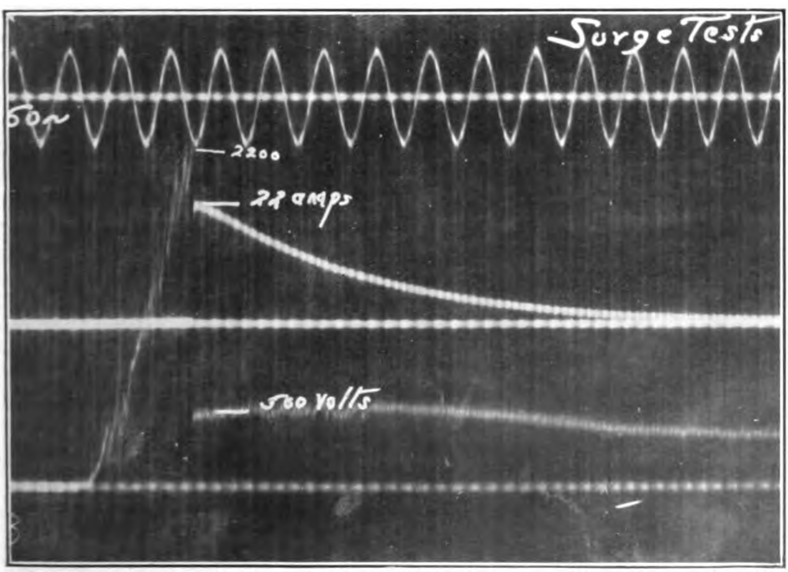


FIG. 3—SINGLE IMPULSE DISCHARGE—NO ARRESTER IN CIRCUIT

Top vibrator—60-cycle timing wave.
Bottom vibrator—circuit voltage.



[STEINMETZ]

FIG. 4—SINGLE IMPULSE DISCHARGE AS IN FIG. 3—ARRESTER IN CIRCUIT

Top vibrator—60-cycle tirring wave.
Middle vibrator—currnt through arrester.
Bottom vibrator—voltage across arrester.

logram in Fig. 2, is unsymmetrical also: in the first and second oscillation of Fig. 2, the first and third half wave of oscillating voltage is cut off, and the first and third half wave of the oscillating discharge current therefore higher than the second half wave of the oscillating discharge, which latter corresponds to a half wave of oscillating voltage in opposition to the circuit voltage, therefore not raising the circuit voltage. The third oscillation of Fig. 2 happens to start with a half wave of oscillating voltage in opposition to the circuit voltage, and the first half wave oscillating discharge current through the arrester, in the middle curve, thus is smaller than the second half wave; the second half wave of the oscillation is cut down in the voltage and therefore gives the maximum discharge current, 35 amperes in this case.

This feature is well brought out by the oscillograms Figs. 1 and 2, due to the use of a different frequency, 60 cycles, for the power supplying the oscillator. This caused the successive oscillations to occur at different parts of the half waves of the 47-cycles circuit which was to be protected.

Fig. 3 and 4 then show the protective action of the oxide film arrester on a 550-volt direct-current circuit, against a single (non-oscillatory) impulse produced by opening a highly inductive circuit (railway motor). Fig. 3 shows on the lower curve the oscillogram of the impulse in the 550-volt circuit, rising to 7000 volts. The upper curve merely is a 60-cycle timing wave, to enable measuring the duration of the impulse. Fig. 4 shows the same circuit, with an arrester shunting it. The impulse voltage rises to the value for which the discharge gap of the arrester is set, in this case 2200 volts. Then the arrester discharges, and the voltage instantly drops back to normal, while a slowly decreasing discharge current through the arrester dissipates the magnetic energy of the impulse.

The cell of the oxide film arrester, shown in Fig. 5, consists of two circular metal plates as electrodes, which are kept apart by a porcelain ring, as shown in the figure. The space between the electrodes inside of the porcelain ring, is filled with the active material, lead peroxide PbO_2, which is put in under moderate pressure. This active material is a good conductor, but has the characteristic, that by the action of an electric discharge, it is converted in the path of the discharge, into a lower oxide, which is an insulator. Thus when an alternating current is passed through such a cell, the active material at the electrodes grad-

ually converts into a non-conductor, and forms a thin insulating film at the electrode. This grows in thickness, until it cuts off the further flow of current and holds back the voltage, about 250 to 300 volts per cell. Then only a small leakage current, of a few milliamperes, passes at normal voltage, but if an over voltage of any kind appears at the cell, the insulating film of lead oxide punctures and freely discharges through the lead peroxide, but in doing so, converts the surface of the lead peroxide in the path of the discharge into the lower non-conducting oxide, and thereby closes the puncture holes, repairs or reseals the film.

In manufacture, naturally, just as in the aluminum cell arrester, the insulating film is not produced after assembly of the cell by the slow process of passing a current through, but the film is put on before assembly, in the oxide film arrester, by dipping the plates in a suitable insulating varnish, which gives them a coating just thick enough to hold back the circuit voltage. Then after assembly, voltage is put on the cell for testing it and sealing any holes or defects which may exist in the varnish film.

In the oxide film arrester, the electrodes have nothing to do with the arrester action, and any suitable material can be used. First we used brass, but now use sherardized iron, the latter having a higher melting point and thus standing high power discharges, which would melt holes in the brass electrodes.

In this arrester, the action, which holds back the normal voltage but passes freely an over voltage, thus resides in the active material between the electrodes, and it is this material which forms and reforms the film. As this material is a solid, no chemical action occurs such as the gradual dissolution of the alumina film in the aluminum cell arrester, but the film remains intact permanently, and thus no daily "charging", that is, repairing of the film, is required.

A number of such cells, depending on the voltage of the circuit, are piled on top of each other, with a spark gap in series, and, for low and moderate voltages, incased as shown in Fig. 6.

As the cells are hermetically sealed, by the metal of the electrodes being spun over the porcelain separating ring, the cells can be installed outdoors as well as indoors, requiring in outdoor installation merely some protection by petticoats, as shown in Fig. 7, to keep the rain from short-circuiting the cells. Fig. 8 shows such an outdoor installation of a 33-kv. arrester, with three-phase stacks and the ground stack of cells, protected against the

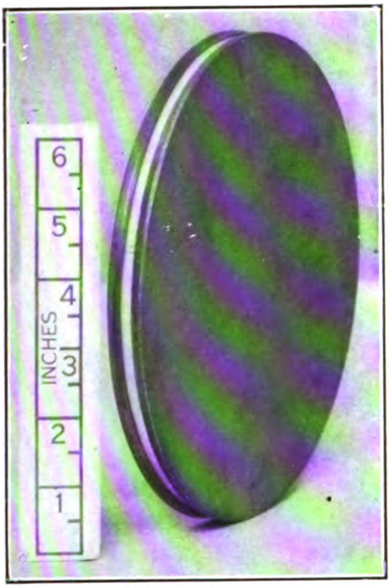
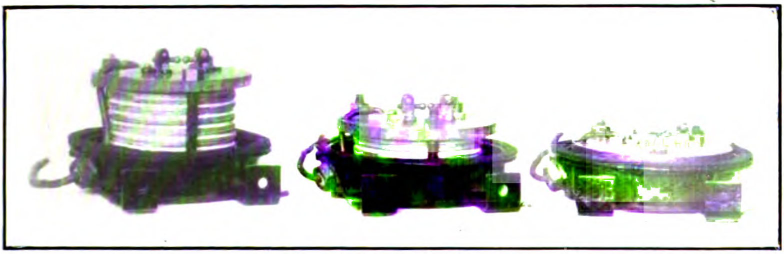


FIG. 5—TYPE "O F" LIGHTNING ARRESTER CELL



[STEINMETZ]

FIG. 6—TYPE "O F" LIGHTNING ARRESTERS WITH COVERS REMOVED RATINGS 325-650, 900-1350, 2100-2600-VOLT ALTERNATING CURRENT OR DIRECT CURRENT RESPECTIVELY

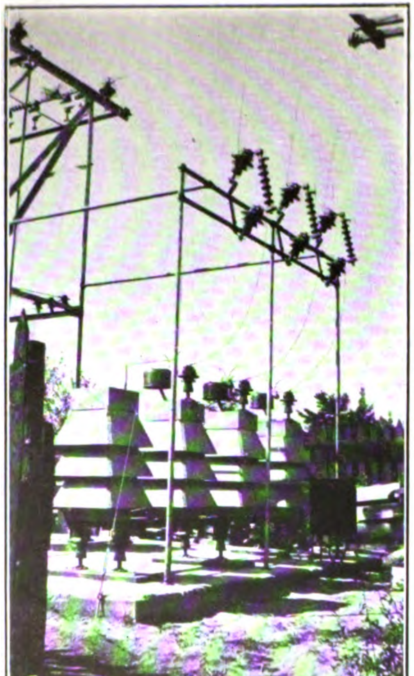
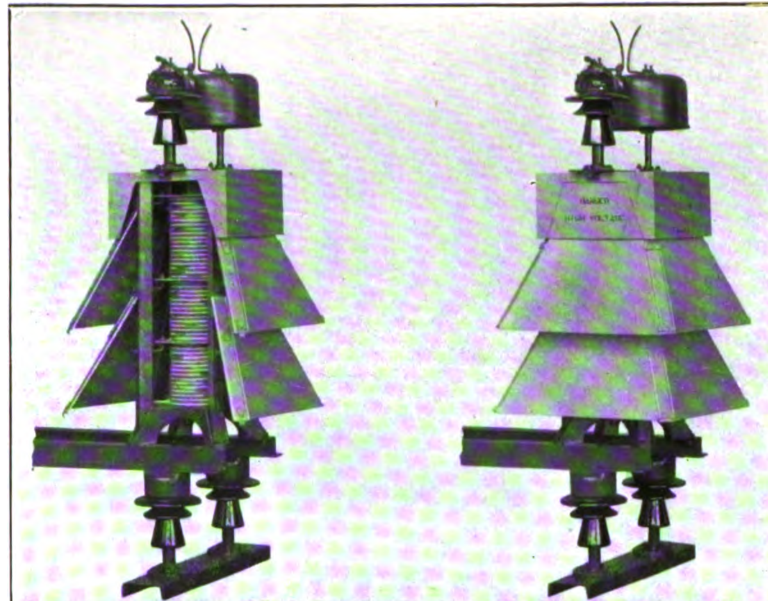


FIG. 8—TYPE "O F" LIGHTNING ARRESTER FOR OUTDOOR SERVICE INSTALLED ON A 33,000-VOLT CIRCUIT



[STEINMETZ]

FIG. 7—PHASE SECTION OF 15,000-25,000-VOLT OUTDOOR TYPE "O F" LIGHTNING ARRESTER WITH SIDE OF HOUSING REMOVED TO SHOW INTERIOR ARRANGEMENT

weather by metal petticoats. Fig. 8 shows also the spark gaps on the line side of the arresters. They are protected sphere gaps, to give instantaneous discharge, with a horn attachment to allow the arc to flare up and thereby help in its extinction.

As well known, the plain horn gap has the disadvantage of requiring an appreciable —though a very short (microseconds) —time for discharge, and an extremely sudden high voltage, as a very steep wave front, thus may pass the horn gap and flash over elsewhere. Therefore in modern high voltage lightning arresters the horn gap is shunted by a properly proportioned sphere gap, the latter being "instantaneous" in its action. In outdoor use however, rain lowers the discharge voltage of the sphere gap, and thus requires a setting which gives a higher discharge voltage in dry weather than necessary. Therefore a protected sphere gap has been designed, which overcomes this disadvantage in the open sphere gap, and is shown in Fig. 8.

The need of using this spark gap in series with the arrester, is the only still remaining undesirable feature which the oxide film arrester shares with the aluminum cell arrester, the multigap arrester and other types. While by the work of Mr. F. W. Peek, on the time lag of electric discharges[3], the means have been given to make the discharge gap "instantaneous", that is, faster than any other discharge path over gaps or through insulation in the system, so that the arrester takes care effectively of any over voltage above its discharge voltage, it does not discharge voltages lower than the discharge voltage of its spark gap, even if these lower voltages may involve some danger to the system by their high frequency. Such low voltages, while they cannot endanger the main insulation between circuit and ground, may, if of sufficiently high frequency, lead to local accumulations of voltage across inductive parts of the circuit, as regulators, current transformers, end turns and coils of generators and transformers, and there cause damage by puncturing insulation between turns and causing internal short circuits.

Against these high frequency disturbances of moderate voltage, the only existing protection is the addition to the arrester of a capacity discharge path permanently connected from the circuit to ground. Such capacity path should be without resistance to flatten steep wave fronts, and contain a moderate series

3. A. I. E. E. Trans. 1915, Vol. XXXIV, Part II, p. 1857, *The Effect of Transient Voltages on Dielectrics* by F. W. Peek, Jr.

resistance, to dissipate high frequency energy and stop cumulative oscillations in their beginning. Before I leave the field of electrical engineering, I hope still to see an arrester, of the type of the oxide film or the aluminum cell, which has no spark gap, but is permanently shunted across the circuit, and thus capable of taking care not only of over voltages, but equally well of steep wave fronts and high frequency oscillations, even if of lower than the circuit voltage. Such an arrester then would give universal protection.

To be presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 26, 1918.

AERIAL CABLE CONSTRUCTION FOR ELECTRIC POWER TRANSMISSION

BY E. B. MEYER

ABSTRACT OF PAPER

This paper deals with the problem of supplying high-tension electric service where conditions do not permit of the use of open wire or underground circuits.

The methods of overcoming difficulties incidental to providing for high-tension service are discussed in detail, together with a description of the types of cable used and methods of installation.

The experience of a large central station company operating several hundred miles of overhead and underground cable is given and the paper brings out the fact that the type of construction described may be used advantageously for both 13,200- and 26,400-volt service.

CENTRAL station companies have had to meet a number of difficult problems during the past three years but the most important has been that of supplying enormous power demands imposed upon them, particularly since the United States has become one of the allies in the European war.

On account of the rapidity with which most of the materials covered by war contracts must be delivered, industrial companies found that the building of isolated plants was out of the question, not only because of the time necessary for erection, but because of the low rates and excellent service furnished by utility companies.

At the present time the central station engineer in dealing with the customer has to provide for thousands of kilowatts rather than hundreds, which were the usual demands previous to the war.

These large demands have made it necessary to solve numerous operating problems in connection with the transmission system and to devise special methods of construction in order to serve the industries upon which the Government is depending to help win the war.

The Public Service Electric Company, which operates in 200

Manuscript of this paper was received April 24, 1918.

municipalities throughout the State of New Jersey, supplies light and power to approximately 170 manufacturing plants engaged directly or indirectly on Government contracts. The material furnished under such contracts consists of high explosives, chemicals, shells, textiles, rubber goods, motor cars, castings, wire and cable, wireless apparatus, ships of various types, and many other accessories. In addition to this, Public Service Electric Company has contracts with the United States Government for furnishing light and power to Camp Dix, Camp Merritt, Raritan Ordnance Depot, Colonia Base Hospital, and the Quartermaster's Department located at the Port Newark Terminal. The supplying of these industries has made enormous demands on the generating system of the company so that at the present time about 80 per cent of the company's output in commercial power is for war work.

One of the special methods adopted by the Public Service Electric Company in meeting war time demands, was that of furnishing the customer with primary service by the use of aerial cable run on poles and supported by messenger wire, a type of construction similar to that used in telephone work.

This type of construction was first used by the company about seven years ago when it was found necessary to connect two large generating stations through tie feeders.

The matter of running overhead wire was considered but found impracticable because the line in several places would have to cross freight yards, trestles, and bridges, and the owners of these structures objected to open-wire high-tension lines.

Most of the section between these two stations was soil of a marshy character, through which it would have been impossible to run a duct line without the use of foundation piling.

It was therefore concluded that the use of aerial cable furnished the most satisfactory solution of the problem. In this installation ordinary lead-covered cable of the same type as that used for underground work was run on a pole line with 50-ft. (15.2-m.) pole spacing. To protect the sheath from mechanical injury there was applied a covering consisting of several layers of jute and marlin with an outer armor of soft steel tape.

The use of lead-covered cable for aerial work was found undesirable, however, because of the excessive weight of the cable and the fact that it could not be installed on standard pole line construction, and a special form of cable was developed to overcome these objections.

In Fig. 1 is shown the modified form of cable for 13,200-volt operation, which is made up with 7/32-in. (5.5-mm.) paper conductor insulation, a 3/32-in. (2.3-mm.) paper jacket and a 4/32-in. (3.17-mm.) reinforced rubber covering over the paper jacket. The reinforced rubber covering is similar in construction to that of the ordinary garden hose, being made up of several plies of fabric and rubber. The entire cable is saturated with rubber compound and covered with tape and a weatherproof braid, thoroughly impregnated with a waterproofing compound. For mechanical protection, the whole core is encased in an armor made up of galvanized steel tape. The use of this form of construction reduces the weight of the cable approximately 50 per cent and permits the use of lighter pole line construction.

The process of manufacture of the reinforced rubber covering

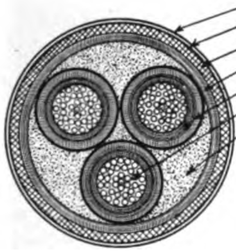


Fig. 1—Reinforced Rubber Covered Aerial Cable

consists in calendering both sides of the cotton fabric, previously dried and waterproofed, with a 30 per cent Para rubber compound, so as to obtain a thorough filling of rubber, which, under the process of calendering, becomes partially vulcanized. The prepared fabric is then cut into tapes. These are applied to the electrical conductor in the usual manner, all contact surfaces and interstices being filled with a rubber cement. The insulated conductor is then dried under moderate heat. According to whether the reinforced rubber covering is applied over an insulating layer of rubber compound, or a layer of cambric or paper, the finished cable may or may not be subjected to vulcanization. In the latter case, the partial vulcanization of the rubber in the reinforced rubber is further advanced during the drying process and during leading in the case of leaded cables; otherwise further vulcanization takes place with aging and under service.

The finished material is perfectly homogeneous. Its specific insulating and dielectric constants are lower than those of rubber, paper and varnished cambric insulation, and for that reason, among others, it is preferable to combine a thickness of reinforced rubber with one of the other materials. By placing the reinforced rubber outside a thickness of a higher dielectric compound near the copper wire, the potential gradient is reduced so that the lower dielectric strength of the reinforced rubber does not materially decrease the total dielectric strength of the cable.

Many engineers have been of the opinion that paper insulated cable with the reinforced rubber jacket would not give satisfactory service when subjected to the heat of the summer sun, but in spite of the fact that the cable is exposed to the elements throughout the year the Public Service Electric Company has never experienced a service interruption through the failure of any of the aerial cable in use in the transmission system.

In Fig. 2 are shown several types of reinforced rubber multiconductor cable with and without lead covering.

The following table gives approximate weights and outside diameters of three-conductor cables, insulated for 13,200 volt operation:

APPROXIMATE WEIGHT AND DIAMETER
OF THREE-CONDUCTOR, 13,200-VOLT AERIAL CABLE

Size	Weight per foot-lbs.	Dia. inches
No. 4	3.50	2.25
No. 2	4.05	2.41
No. 1	4.45	2.50
1/0	4.80	2.57
2/0	5.70	2.66
4/0	6.70	2.91
250,000 cm.	7.05	3.00
350,000 cm.	8.50	3.22

The principal advantage of aerial cable for tie feeder installations is that it makes little difference how many working lines are carried on a single pole line. Additional cable may be run, existing construction changed, transferred or repaired without taking out of service any line except the one on which the actual work is being done. Lightning discharges seem to have little effect because the messenger wire which carries the aerial cable is permanently grounded.

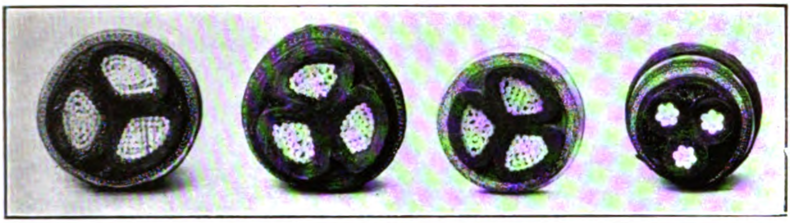


Fig. 2—Reinforced Rubber Multi-Conductor Cable

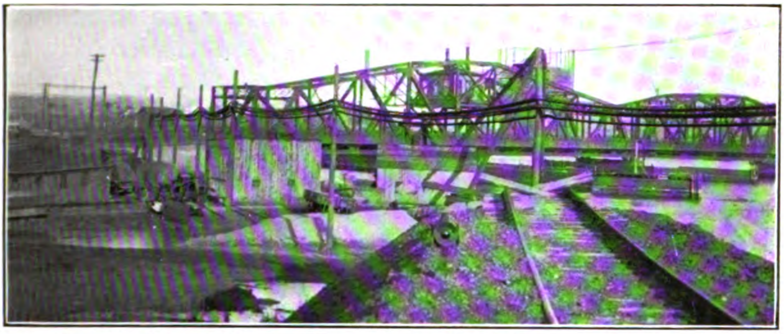
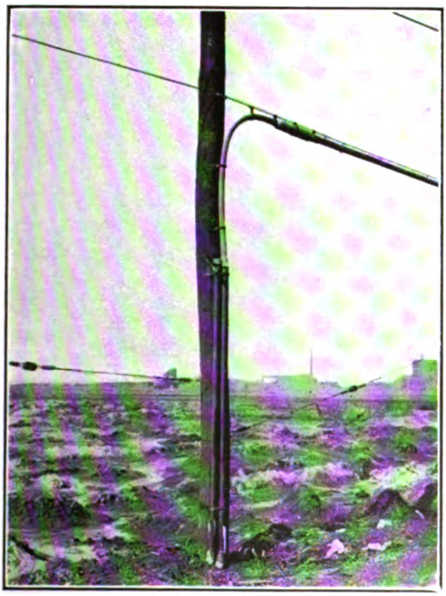


Fig. 3—Aerial Cable Line with Five 13,200-Volt Circuits



[MEYER]

Fig. 7—Connection Between Aerial and Underground Cable.

Fig. 3 shows a pole line carrying five 13,200-volt feeders. This line has been in operation for a period of over seven years without the occurrence of a single failure.

The usual aerial cable installation requires the use of Class B chestnut poles, with a normal spacing of from 90 to 100 ft. (27 to 30 m.). Where conditions make it necessary, sections as long as 150 ft. (45 m.) are permissible, but in such cases the adjacent sections should not exceed 130 ft. (39 m.). Sections longer than 150 ft. (45 m.) should receive special attention, and Class A poles should be used on long sections and at points of special strain. The location and frequency of guys is largely dependent on local conditions and can, in most cases, be decided upon by a competent line superintendent.

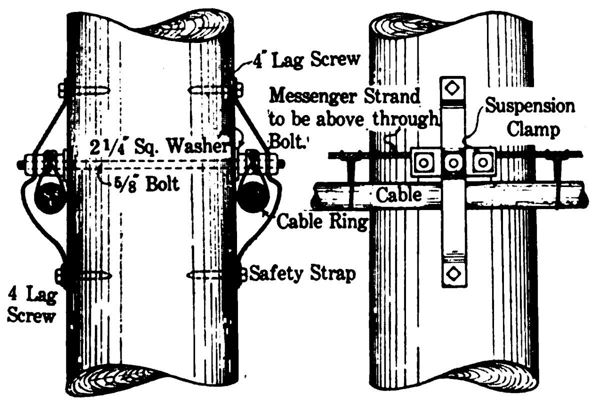


FIG. 4—METHOD OF SUSPENDING MESSENGER CABLE

Attention is called, however, to the fact that the stress at dead ends and corners is very great, frequently being as much as 25,000 lb. (11,339 kg.). These points of special stress need to be well guyed. Both the anchors and the guys should be designed with a factor of safety so high that the messenger will fail before the pole will pull over. In all cases it will be necessary for guy stubs to be reinforced by an anchor guy.

For the suspension of the messenger a double ended 5/8-in. (15.8-mm.) through bolt is recommended, as illustrated in Fig. 4. The use of a safety clamp is also desirable. This clamp serves the double purpose of reinforcing the through bolt and preventing the cable from falling to the ground in case the rings fail. Careful tests made on the method of suspension show that it will withstand the maximum loads to which it will be subjected.

The type of clamp used is similar to that used by the American Telephone and Telegraph Company, the size depending on the diameter of the messenger strand adopted. The clamp is designed expressly for construction of this character and is not built like a guy clamp which is designed to grip two strands instead of one. It affords a greater lever arm for the bolts to work upon in grasping the messenger and supports the messenger strand closer to the bolt, decreasing the bending moment on the bolt due to the weight of the cable.

The messenger strand should always be placed above the bolt in order that the weight of the cable will not be supported by the clamp. Various forms of cable rings may be used in supporting the cable on the messenger wire.

Where two or more cables are to be installed on a pole line,

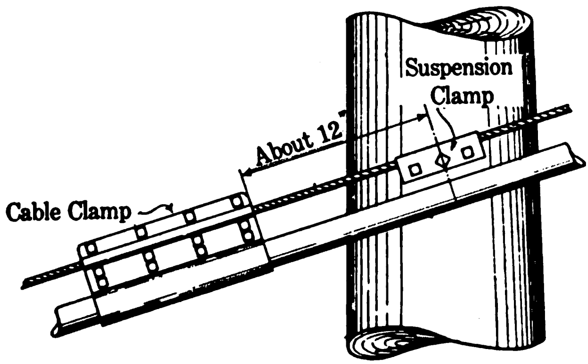


Fig. 5—Clamping Cable to Messenger on Steep Grades

they are usually suspended in pairs, two from each through bolt. The messenger wire is extra strength 5/8-in. (15.8-mm.), seven-strand, galvanized, steel wire. The wire composing the strand should be free from scale, inequalities, splints or other imperfections, not consistent with the best workmanship. It is usual in purchasing galvanized steel wire of this character to have it conform to a specification covering the galvanizing. This is necessary as otherwise inferior grade wire might be obtained.

It is very important that the messenger wire be drawn as tight as possible, in order to prevent sagging when subjected to the weight of the transmission cable. If this is not done, an unsightly installation will result. After the messenger wire has been given its final pull and properly dead-ended, the placing of the aerial cable is the next step. In pulling the cable up to the

messenger wire it is very important that precautions be exercised to prevent mechanical damage or excessive strains which would tend to weaken or damage the insulation.

It is customary in aerial installations to ground the messenger strand. Where the soil is dry or soil conditions unfavorable for grounding, a ground connection should be installed at every second pole. Where the earth is damp and soil conditions are favorable, a ground should be installed at every fourth pole. In marshy ground and in places where conditions are particularly favorable, a ground at every eighth pole will be sufficient. Where possible, this ground connection should be well bonded to some metallic subsurface structure. If this is not possible, the standard artificial pipe ground should be installed.

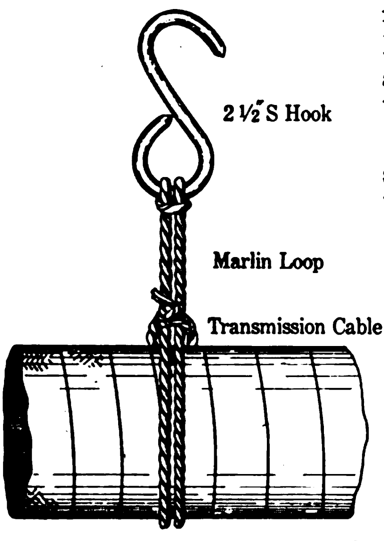


Fig. 6—Method of Fastening *S* Hooks to Aerial Cable

It is also desirable that the steel tape on the cable be bonded to the messenger strand with bonding wire at every cable joint, as proper bonding is necessary in order to furnish the required protection against lightning. Where cable is run through trees and likely to be damaged by abrasion it should be protected by several layers of galvanized tape similar to that later described for use in protecting the joint.

In Fig. 5 is illustrated a method of clamping the cable to the messenger wire. On steep grades where the angle between the cable and the horizontal is greater than 30 deg. the use of such a cable clamp is recommended. This clamp can be made up as required and should be used on every fourth pole. It is designed to take the greater portion of the down hill pull on the cable, which otherwise would be carried by the cable rings.

In erecting the cable, the first reel is set up in the usual manner and the cable run off to the first pole, at which is placed a sheave of approximately 12 in. (30 cm.) diameter, the top of the sheave being located about 5 in. (12.7 cm.) below the messenger wire. On the four or five succeeding poles similar sheaves or cable

rollers are placed, and in feeding out the cable 2.5-in. (6.35 cm.) "*S*" hooks, spaced 18 in. (45.6 cm.) apart, are fastened to it. These hooks are fastened to the cable with a small piece of marlin, made up in a loop knot, as illustrated in Fig. 6.

A lineman is stationed at each pole to change the "*S*" hooks from one side of the pole to the other, which process is repeated until the entire length of cable has been installed in place.

The "*S*" hooks, which were used as a temporary support, are now removed and permanent rings put in place. This is done by a lineman supported on a boatswain's chair, which is moved along the section supported by the messenger wire.

In running the transmission cable, either a motor truck, horses, hand or power winch may be used.

In the splicing of aerial cable, no special means are employed, but the usual precautions observed in the installation of underground cable must be followed. The jointing of any cable is more or less a matter of individual experience and great care must be exercised in all cases to exclude moisture. The work should be carefully done by a reliable and experienced workman and no splicing should be undertaken when weather conditions are unfavorable.

Each conductor of the cable is insulated with black bias-cut varnished cambric tape of a thickness of about 30 per cent greater than the machine applied insulation. Between each layer of tape, varnished cambric insulating compound is applied. After the individual conductors have been insulated a jacket of bias-cut black cambric tape, well painted between layers with an insulating compound, is applied to a thickness of 4/32-in. (3.17 mm.). Over the jacket of cambric tape several layers of the best grade rubber tape, 5/32-in. (3.9 mm.) in thickness, are applied and painted between layers with a high grade rubber compound. The completed joint is then covered with three or four layers of friction tape well painted with rubber compound. The joint is then ready for the application of a soft steel galvanized tape over which is finally applied an outer covering consisting of three or four layers of friction tape painted between the layers with a good grade of waterproof compound.

Where it is necessary to make connection from an aerial cable to an underground system a standard form of lead covered cable is used and installed in a lateral pipe as shown in Fig. 7. The joint between the underground and aerial cable is made up in the manner just described, and there is slipped over the joint a

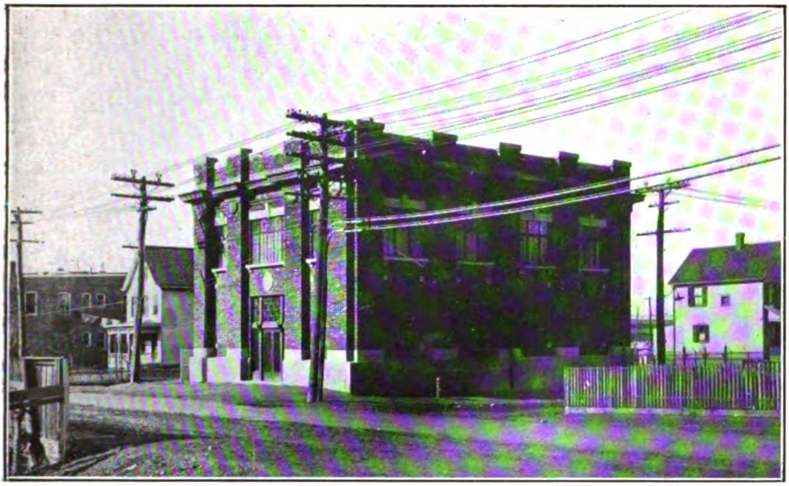


FIG. 8—AERIAL CABLE ENTRANCE TO SUBSTATION

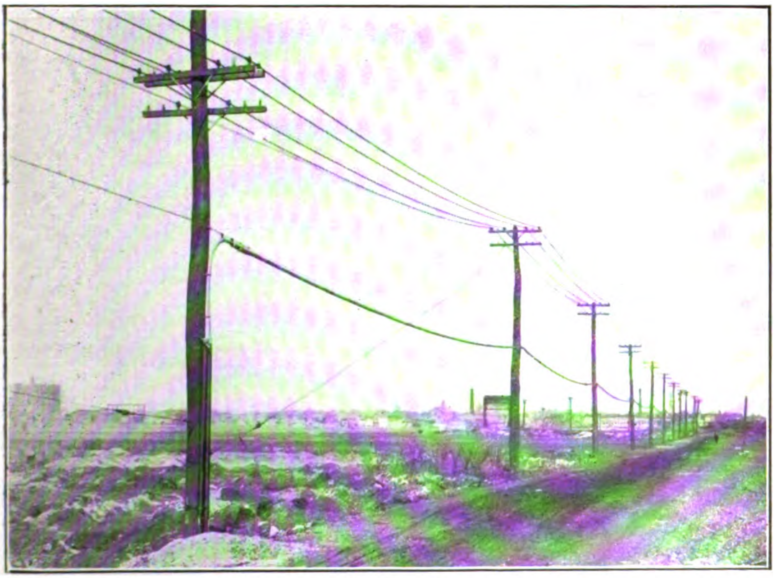


FIG. 9—26,000-VOLT AERIAL CABLE INSTALLATION [MEYER]

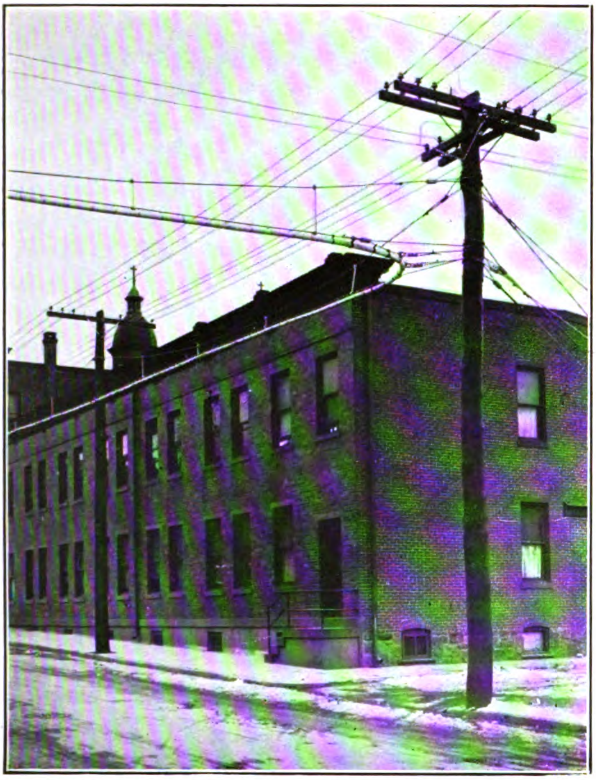
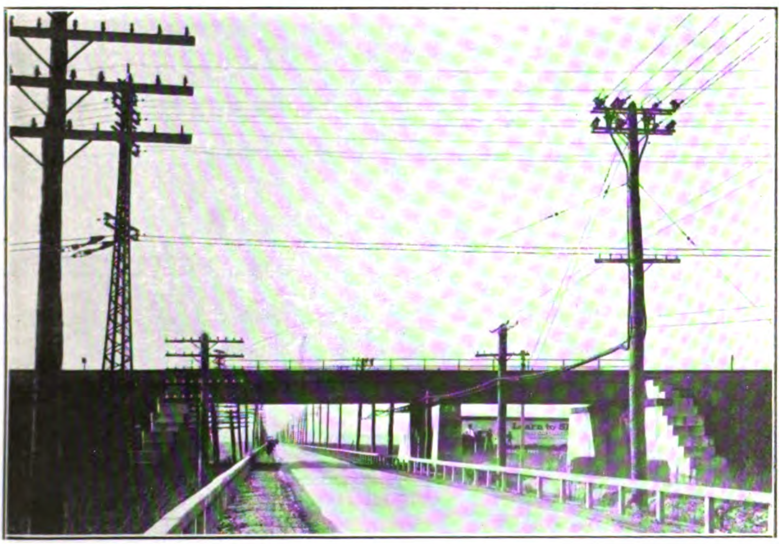


FIG. 10—CATENARY METHOD OF SUSPENDING AERIAL CABLE



[MEYER]

FIG. 11—26,000-VOLT CABLE CROSSING UNDER RAILROAD BRIDGE AND HIGH TENSION POWER CIRCUITS

lead sleeve, one end of which is wiped to the lead covered cable. The other end is well taped to prevent moisture from penetrating the cable.

Two aerial cables entering a power substation are shown in Fig. 8. In this installation iron pipe laterals are run up the pole and underground lead covered cable is spliced on to the aerial cable as just described.

While most of the existing circuits are operated at voltages under 15,000, the excellent results obtained with aerial cable has led the company to use this type of construction on all special work for operation at 26,000 volts.

In Fig. 9 is illustrated a completed 26,000-volt aerial cable installation. It should be noted that this method of construction permitted the erection of a high-voltage line without reconstructing the existing low-voltage open-wire distribution circuits.

To keep the cable in good condition it is necessary to paint it every four or five years with some form of insulating paint. This serves to keep the outside jacket from disintegrating and protects it from the action of the elements.

There is in service on the various transmission lines of the Company approximately 65,000 ft. (19,812 m.) of aerial cable operating at 13,200 volts and about 16,000 ft. (4,876 m.) either operating or in course of construction for 26,400-volt service.

Fig. 10 illustrates a catenary method of installing aerial cable. The line so erected was built to furnish 3,000 kw. to a customer who required service within a few weeks after the signing of the contract.

It was impossible within this short length of time to obtain the standard aerial cable with reinforced rubber insulation, and it was found necessary to take ordinary lead covered cable out of stock.

The erection of lead covered cable by the methods commonly used in installing aerial cable, on account of the weight and long pole spacing, would have resulted in throwing too great a stress on the messenger wire and lead cable. It was, therefore, decided to use the catenary form of construction so as to reduce the strain with the result that the transmission cable hangs perfectly level and without sag.

In Fig. 11 is shown an illustration of aerial cable crossing under a railroad bridge and electric railway power circuits. Open air potheads or line terminals are installed between the cable and the open wire transmission conductors. No protection in the

form of lightning arresters is provided as after a number of years of operation without failure from any source, the cost of the installation of arresters seemed to be unwarranted.

Aerial cable construction is somewhat more expensive than ordinary open wire construction but its cost is less than that of an underground conduit system. As the costs of the various types are so largely dependent on local conditions, no comparative estimates will be given here. In general, the cost of an aerial cable line is about midway between underground and open wire construction.

While this paper deals primarily with the use of reinforced rubber cables, there are numerous installations throughout the country where other forms of insulation have been used with satisfactory results.

It is not the intention of this paper to recommend exclusively the use of reinforced rubber insulation, but primarily to bring out the fact that aerial cable construction may be used advantageously for power transmission, where open-wire construction would be undesirable and the time and cost of underground construction would be prohibitive.

Where line extensions have to be made over marshy ground which would require the use of foundation piling for a conduit line, over private property such as railroad freight yards, trestles or bridges where very special overhead construction would be necessary, or on important streets on which open wires are not permitted, and subway construction would be expensive or impossible, aerial cable furnishes an ideal form of construction.

To be presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 26, 1918.

APPLICATION OF THEORY AND PRACTISE TO THE DESIGN OF TRANSMISSION LINE INSULATORS

BY G. I. GILCHREST

ABSTRACT OF PAPER

The paper first gives a summation of the items that are apparently the main causes of pin-type insulator failures in service. Each item is thereafter briefly discussed and the opinions of operating men are cited.

A brief description is given of the method used to determine the form of the dielectric field about porcelain insulators under normal line voltage. Diagrams of the dielectric field and photographs of flash over tests of theoretical designs are shown. Thereafter, the necessary modifications of the theoretical designs, in order to meet operating and manufacturing conditions, are discussed.

In the latter part of the paper, diagrams and illustrations are shown of a proposed type of commercial insulator design which has been evolved by linking together the theoretical and practical phases of the problem. A comparison is then made between the older types of design and the proposed type, as regards the resistance of each to the conditions that cause failure of the insulator in service.

In conclusion, a summary is made of the advantages it is believed that the new type of design has over the present commercial insulator designs.

INTRODUCTION

USUALLY any design problem of engineering may be quite easily separated into two rather distinct phases. The one phase is termed "theoretical" and infers that the service experience, processes of manufacture, cost of materials, cost of manufacture, etc., are placed secondary in importance in the search of an ideal design. The other phase is termed "practical" and infers that the design has been evolved mostly from a consideration of service experience. It is generally conceded that a design evolved by either method may have certain advantages. The object of the following investigations has been to link together these two phases in a specific application, namely; the design of pin-type transmission-line insulators.

In order to save repetition, the present article will deal, for

Manuscript of this paper was received April 20, 1918.

the most part, with laboratory tests of various new designs and the comparison of these designs with those now in commercial use. The theoretical elements of the problem which laid the foundation for the following developments are clearly given in two papers published in the 1913 A. I. E. E. TRANSACTIONS. One paper, by C. Fortescue, is subjected *The Application of Theorem of Electrostatics to Insulation Problems.* The other, by C. Fortescue and S. Farnsworth, is subjected *Air as an Insulator.*

Since an attempt will be made to deal with the practical and theoretical phases of the problem, two logical questions at once arise:

First, are the insulator designs installed in service at the present time satisfactory?

Second, can one type of design be developed that will be satisfactory in all localities?

The first question is answered by a resume of current engineering literature which offers convincing evidence that there is a field for improvement. A comparison of the flashover voltage versus overall dimensions and weight of the present insulator designs would seem to warrant an attempt toward uniformity. Furthermore, the divergence of certain characteristics of some designs from the average curves indicates that some of the designs must be far from efficient.

The causes of such chaos in the present day insulator designs are quite obvious and have been frequently stated. First of all, the progress in transmission engineering has been rapid. The expanding transmission companies demanded insulator designs which would offer a good factor of safety. There was no previous operating experience to use as a basis in new developments and consequently it was often necessary for the transmission engineer to propose his own design. Moreover, the majority of our present insulator types were designed when the electrical and mechanical characteristics of porcelain were less understood than at the present day. As a result, the type of design has fluctuated and various features were, at one time or another, accentuated as most important. That is, at one period a long leakage path was required, regardless of voltage distribution per shell from capacity current or leakage current; then again a high puncture voltage, than a high mechanical strength, and so on. Naturally, many mistakes were made and a large proportion of the older insulator designs have failed in service application.

Causes of Insulator Failures

The knowledge that certain insulator types have failed in service is of little value in the redesign of insulators unless actual conditions of service and cause of failure are known. Also, the cause of failure of a particular type in one locality should be compared to the cause of failure in other localities. Hence, before attempting to develop a new type of design, a study was made of available data on insulator deterioration and the opinions of operating engineers in various parts of the country were obtained.

From discussions with these engineers, from published data on insulator deterioration, and from observations of insulators that had failed in operation, it would seem that the following items are the main causes of pin type insulator failures:

1. Improper distribution of dielectric field.
2. Improper distribution of surface leakage.
3. Porosity.
4. Mechanical breakage.
   a. From handling.
   b. Mischievous shooting and stone throwing.
   c. Insufficient strength as a support.
   d. Brittle material.
5. Lightning.
6. Birds and animals short-circuiting line.
7. Unequal expansion of metal, cement and porcelain.
8. Internal stresses in material.
9. Defective batches.

Items 3, 4d and 9 are the problems in which the ceramic engineer is vitally concerned and will not be considered further in this paper. These items have doubtless been of great importance in the past but more scientific and painstaking factory control must minimize them in the future. ("Electrical Porcelain", *Electric Journal*, February and March, 1918, by T. A. Klinefelter and G. I. Gilchrest).

The manner in which these items have caused ultimate failure of certain designs are very briefly enumerated below:

1. *Improper Distribution of the Dielectric Field.* Failure to consider the electrostatic field distribution as regards every part of the unit has resulted in designs which have an unequal voltage distribution per shell even when the unit is dry and clean. When the rain sheds are closely spaced the air between them is ionized with the result that preliminary discharges take place before flashover. Parts of the unit in the dielectric field are,

thereby partially short-circuited and other parts overstressed. Flashover voltage will, therefore, be low in relation to overall dimensions. These conditions are usually augmented as the insulator becomes dirty and wet.

2. *Improper Distribution of Surface Leakage.* The most serious trouble from surface leakage is probably experienced on sections of line along the California coast, sections near Great Salt Lake, Utah, etc. Moreover, the climatic conditions of certain localities, especially where a "dry season" is followed by a "rainy season", augments the difficulty. For example, along the California coast the insulator surface gradually becomes coated with dirt during the "dry season" of the year. At night a strong breeze drives a fog containing more or less salt spray over the transmission lines. The dirt on the insulator surface then becomes moist and conducting and a rather high leakage current is often the result. Where wooden construction is used, charring of the wood at points of highest ohmic resistance may take place, resulting in the burning of pins and cross arms. The leakage resistance per shell of many of the older designs are such as to give a very uneven voltage distribution per shell under service conditions. Moreover, the voltage drop over the surface between closely spaced sheds often becomes sufficient to cause static discharges between them. The effective leakage surface of the insulator is thereby decreased and the arcing imposes an electrical impact on the insulator at the same time. Of course, this same trouble occurs on sections of lines near factories, steam railroads, cement mills, smelter plants, etc., to a more or less degree.

3. *Porosity.* The deterioration of porcelain insulators in service was given little consideration during the early days of transmission engineering. The majority of transmission engineers preferred an insulator having a porcelain body which offered a high resistance to mechanical breakage. As a consequence, the porosity of the material, which varies inversely to the mechanical strength as regards resistance to mechanical impact, was considered of secondary importance. The results that the condition has caused in service have been clearly presented before the Institute by Professor H. J. Ryan.[1]

4. *Mechanical Breakage.* Mechanical breakage has been a frequent source of annoyance, and has worked havoc in a number of ways.

1. "*Ceramics in Relation to the Durability of Porcelain Suspension Insulators.*" A. I. E. E. TRANSACTIONS, Vol. XXXV, 1916.

(a) The deep thin sectioned sheds are easily broken in handling. This results in a loss of insulator units. What is of more importance, the danger of installing defective units is considerable, since many fine cracks may pass the usual construction crews' inspection.

(b) Some of the operating engineers, especially those located in the middle West, or near mining camps, claim that 80 to 90 per cent of the defective insulators removed from the line were first injured by rifle shooting or stone throwing.

(c) Many designs are not sufficiently strong as a support due to the thin sections of porcelain and small area under mechanical stress, or to the fact that the deep center shed necessitates a high pin. Such designs fail in service when unusual stresses occur, such as are caused by sleet storms, by poles giving way during freezing and thawing of the ground, or heavy rains, etc.

5. *Lightning.* It is generally conceded that a direct stroke of lightning will destroy any insulator that comes within its wake. However, some of the older designs, especially those having deep nner shells and heads of large diameter, were very vulnerable to any sudden impact voltage. In the first place, the impulse ratio (flashover voltage at high frequency divided by flashover voltage at normal frequency) of such insulators is rather high and in the second place the ratio between flashover voltage in air and puncture voltage under oil is comparatively low.

6. *Birds and Animals Short-Circuiting Line.* Some transmission companies have found it necessary to place shields on the poles in certain sections in order to prevent squirrels climbing the poles and short-circuiting the lines. In other localities it has been necessary to increase the height of insulator pins or the wire spacing in order to prevent cranes, eagles, etc., from short-circuiting or grounding the line.

7. *Unequal Expansion of Metal, Cement and Porcelain.* In many cases solid metal pins or heavy cast thimbles have been cemented into the insulator. Apparently the equal expansion of the metal, cement and porcelain has caused the cracking of the porcelain and ultimate failure of the insulator.

8. *Internal Stresses in the Material.* Corners of small radii and non-uniform sections of the porcelain shells have possibly produced internal stresses in the material during the manufacturing processes and these have developed cracks later on in service. Also, the relation between the shape of the shells in

the cemented area and the shape of the cemented area itself has been such as to allow the full effect of unequal expansion of the porcelain and cement which is caused by temperature changes or absorption of moisture.

Investigation of Dielectric Field

In the papers of Fortescue and Farnsworth, several insulator forms were evolved mathematically, and the dielectric field explored by means of an electrolytic bath. It was believed that the data from which these papers were written in conjunction with the available data of other investigators, of both analytical and experimental nature, afforded sufficient basis from which to formulate preliminary designs.[2]

After a careful summation of the data at hand, it seemed that the logical method of attacking the problem would be to have several theoretical insulator designs produced out of a usual commercial porcelain body. The dielectric field of these should then be investigated under a voltage of approximately the same value that would be impressed in service. Thereafter practical considerations, such as deterioration of the various commercial units in service, manufacturing limitations, etc., should be taken into account with the intent of arriving at a compromise between the theoretical and practical features.

Method of Determining Form of Dielectric Field

The dielectric field was determined by the following procedure: The insulator was fastened rigidly in a position such that the plane of the field to be determined extended horizontally. A piece of fullerboard was fitted over a half section of the insulator in this plane. In all cases the apparatus was so arranged that the cross-arm supporting the insulator was grounded as in service where steel construction is used. Finely divided asbestos was then sifted evenly onto the sheet of fullerboard, voltage at 60 cycles of the desired value applied, and the sheet was gently tapped until the particles had adjusted themselves. Permanent records were obtained by placing a sheet of photographic printing paper over the fullerboard, obtaining the field as above, and exposing the paper after the particles had become arranged.

2. "Distribution of Potential about High Voltage Line Insulators," by C. T. Allcutt and W. K. Skolfield. *Journal of Electricity, Power and Gas*, June 17, 1916. *Electrostatic Problems*, by C. W. Rice. A. I. E. E. Transactions, Vol. XXXVI, 1917.

That the stronger portion of the field around an insulator was not disturbed materially by the presence of the fullerboard or the asbestos particles was proven by suspending a piece of finely drawn glass in parts of the field by means of a silk fibre supported by small insulated rods. As nearly as could be checked, the glass indicated the same direction of the field as the asbestos particles.

Theoretical Insulator Designs

The dielectric fields of five theoretical designs were determined. Wherever a customary transmission cross-arm and line-wire are used, there are two principal planes of the dielectric field which show the greatest difference, *i.e.*, the plane of the cross-arm and the plane of the line wire. These two planes are 90 degrees apart and in passing from one to the other the transition is gradual. During the investigation, records were taken of the dielectric field of these two principal planes and of a plane midway between the two. In this paper the diagram taken in one plane of the unit is usually shown. Diagrams of three planes of two designs are shown in order to illustrate the variations that occur. The plane in which the particular field was taken is indicated by the reduced top projection at the upper left portion of each figure.

Dielectric Field Forms and Illustrations of Theoretical Designs

Fig. 1 shows the field form of a bushing having dimensions of ring and rod chosen such as to give maximum breakdown voltage over the surface for the mean diameter of torus ring.

Fig. 2 shows a 60-cycle flashover on bushing of Fig. 1.

Fig. 3 gives the field form of a design using a confocal system of ellipsoids and hyperboloids of revolution.

Fig. 7A, 60-cycle flashover on shape shown in Fig. 3.

Fig. 4, field form between special metal cap and pin as might be used as terminals of a line insulator.

Fig. 7B, 60-cycle flashover between cap and pin as shown in Fig. 4.

Fig. 5, field form of line insulator without rain sheds.

Fig. 7C, 60-cycle flashover on shape shown in Fig. 5.

Fig. 6, field form of pin type insulator, the porcelain of which has a curvature similar to that of Fig. 5. However, the porcelain body is separated into three sections and metal rainsheds added to give wet arcing distance.

Fig. 7D, 60-cycle flashover on unit given in Fig. 6.

In Table I are given the length of path over the insulator surface between electrodes and the 60-cycle flashover voltage.

TABLE I

Shape in figure	Length of surface		Effective kilovolts flashover voltage		
	Inches	Centimeters	Total	Per inch	Per centimeter
1	4.25	10.8	87	20.4	8.1
7*a*	6.5	16.5	148	22.8	9.0
7*c*	8	20.3	115	14.4	5.7

From a consideration of Table I it is evident that a flashover value of between 20 and 23 kilovolts per inch (8 and 9 kilovolts per centimeter) of surface may reasonably be expected if the unit is designed with contours of the surfaces approximating the flow lines of the dielectric field. Of course, the flashover on the unit without rain sheds is somewhat lower, being 14.4 kilovolts per inch. The lower flashover on this unit is due to two conditions, *i.e.*, the porcelain surface does not follow the dielectric field in all planes and the small tie wire produces corona and subsequent static discharges at a relatively low voltage. Placing a static shield on the top of this unit increased the flashover voltage 18 per cent.

With the field form between cap and pin as given in Fig. 4, and the voltage values given above in Table I, theoretical insulator designs could be determined for such electrodes. Such designs should follow surfaces indicated on Fig. 4, as (*a*) and (*b*). The highest flashover voltage using a given weight of insulating material would thereby be obtained. Moreover, the flashover voltage of such a unit could be closely approximated if the electrodes have sufficient radius of curvature at points of contact with the insulating material and a good seal is made between the metal and the insulating material.

Modifications of Theoretical Design to Meet Operating and Manufacturing Conditions

Insulators based on such theoretical data would be excellent from the electrical and mechanical standpoints if they were to operate in clean, dry air. However, the commercial insulator must maintain the transmission system during the heaviest of

snow and rain storms. Moreover, it must have sufficient leakage distance to prevent flashover or even high power loss from surface leakage when the surface becomes dirty and wet.

The production of one-piece insulators for high-voltage service, although possible, would be costly. Also, the puncturing voltage of a one piece unit would be low for a given thickness, since the stress in an insulating material between metal electrodes of different potential varies as a logarithmic function. The separation of the unit into parts that are cemented together, more uniformly distributes the stress of the dielectric if the unit is properly designed. It also decreases the probability of complete failure of the insulator and facilitates factory production, lessening the cost of the commercial unit.

The use of a special cap would be desirable from a dielectric standpoint. However, the voltage characteristics under rain are the same whether the usual line and tie wire or a special cap are used. Moreover, the cost and ease of replacement, cost of construction, etc., favor the line and tie wire construction.

Proposed Commercial Insulator Design

With the above limitations of the theoretical designs and the causes of insulator failures in mind, the type of unit indicated in Fig. 8 was evolved.

Summed up briefly this type of design embodies the following features:

1. Surfaces *a* conform to the flow lines of the electrostatic field.
2. Surfaces *b* of the rain sheds conform to the equipotential surfaces.
3. Lines of mechanical stress are parallel to the electrostatic flow lines.
4. The leakage resistance per shell is about equal, being increased gradually from the head to the center shell.
5. Approximately equal capacity per shell.

Comparison With Older Designs

It is not possible to much more than indicate in the following discussion the methods employed to compare the proposed type of design given in Figs. 8A and 8B with the older commercial insulators. Samples of various commercial designs were produced and were subjected to rather thorough laboratory tests at the same time tests were made on insulators of the proposed design.

It should be noted that the insulators of the new type used in the comparative tests do not exactly correspond to the proportions of Fig. 8. In order to lessen the cost of investigation, insulator sheds of several diameters were obtained from one set of molds by trimming the individual shells before burning. This also accounts for the unfinished appearance of the edges of sheds, etc., in some of the experimental designs.

In the following comparison it is not assumed that the evolved design should be final in each detail. The main goal toward which work is being directed is uniformity of all the elements entering into the designs with the idea in view of arriving at a

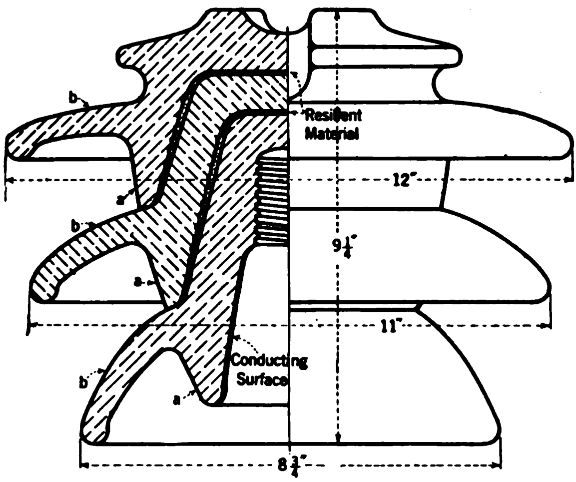


FIG. 8A—THREE-PIECE INSULATOR OF THE PROPOSED TYPE OF DESIGN—INSULATOR *A*

type of design which will be equally successful in resisting failure in service whatever the requirements are in that particular section. In the following comparisons the items causing failure in service are discussed in the order given at the beginning of the paper.

1. *Dielectric Field Distribution.* The shortest air path under electrostatic stress should be at least long enough to prevent overstressing of the air at any point. In the theoretical discussions referred to in the introduction it was proved that wherever porcelain and air are in series in a dielectric field the voltage gradient per unit distance through the porcelain will be ¼ to 1/5 the voltage gradient through the air. It is obvious that

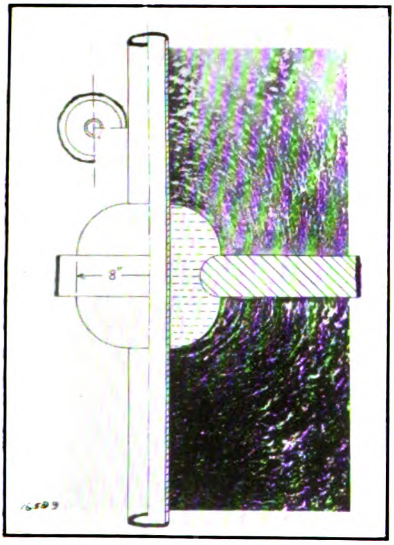


FIG. 1—DIELECTRIC FIELD ABOUT THEORETICAL BUSHING

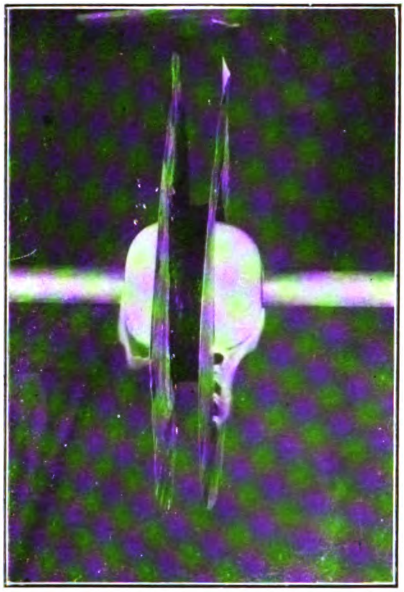


FIG. 2—60-CYCLE FLASHOVER BUSHING OF FIG. 1

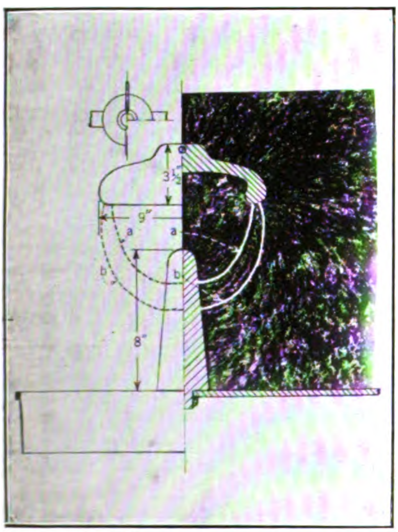
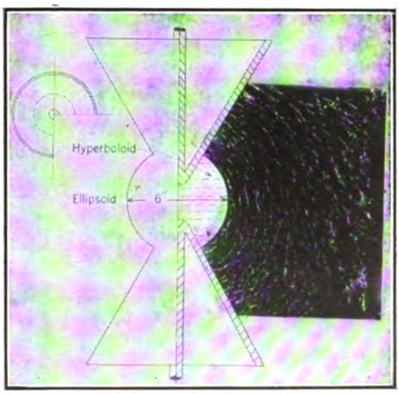


FIG. 4—DIELECTRIC FIELD BETWEEN METAL CAP AND PIN



[GILCHREST]

FIG. 3—DIELECTRIC FIELD ABOUT THEORETICAL DESIGN USING CONFOCAL SURFACES OF REVOLUTION

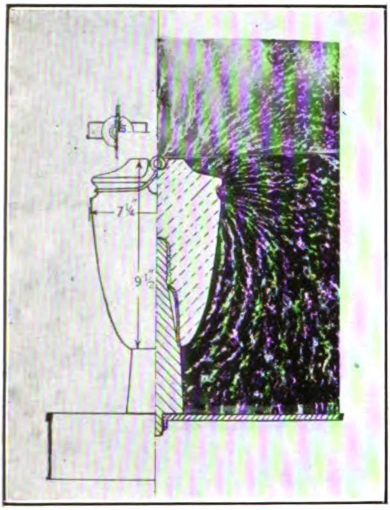


FIG. 5—DIELECTRIC FIELD ABOUT LINE INSULATOR WITHOUT RAIN SHEDS

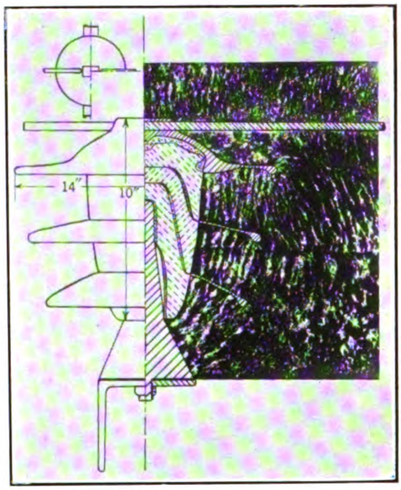
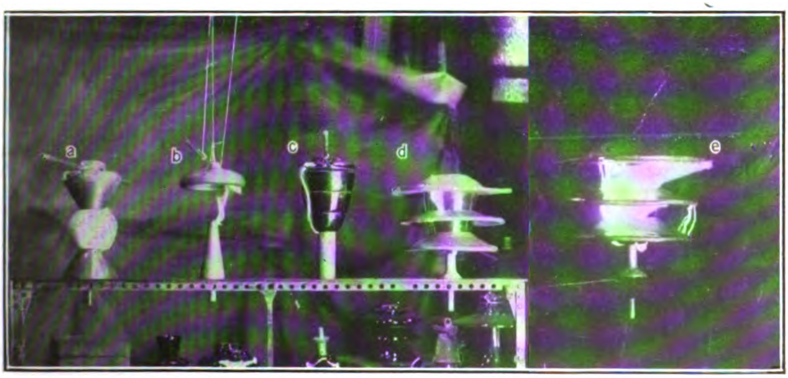


FIG. 6—DIELECTRIC FIELD ABOUT INSULATOR WITH METAL RAIN SHEDS



[GILCHREST]

FIG. 7—60-CYCLE FLASHOVERS ON THEORETICAL PORCELAIN SHAPES

any thin section of air between porcelain sheds of a customary line insulator will be over-stressed even at the normal line voltage of the insulator.

In order to make a comparison of the dielectric fields of various insulators, their field forms were determined as in the investigation of the theoretical designs. It is believed that the following field forms and illustrations sufficiently indicate that many present types have not been designed with a full appreciation of the advantages of shapes that conform to the electrostatic flow lines in obtaining the most efficient distribution of the stresses in the dielectric field.

Fig. 9 (insulator *C*) gives the dielectric field of a unit of the type used in the early developments of high-voltage trans-

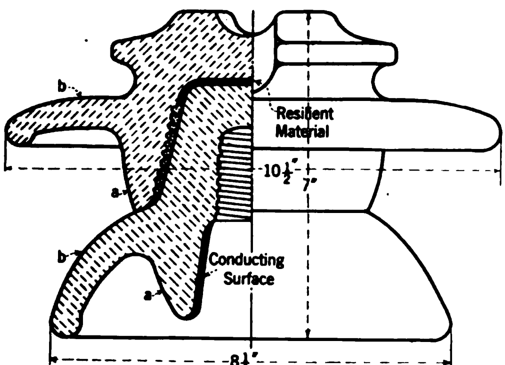


FIG. 8B—TWO-PIECE INSULATOR OF THE PROPOSED TYPE OF DESIGN —INSULATOR *B*

mission. The air between sheds just below the cement section is highly stressed. Because of the height of the pin in proportion to other dimensions of the unit the stress toward the base of the pin and the supporting cross arm is very low. Moreover, the third shell of the insulator is spaced so close to the insulator pin that it does not take its proportion of voltage stress when either dry or wet flashover occurs.

Fig. 10 (insulator *D*) shows the dielectric field of a three piece insulator of a somewhat more recent design. The center shed is better spaced than in insulator *C*. However, the air just below the cement sections is highly stressed and the short rain shed of the second shell gives an unequal voltage distribution at flashover, dry or wet.

Figs. 11, 12 and 13 (insulator *E*) show the dielectric field of a four-piece unit of comparatively recent design. The sheds of this design are more uniformly spaced, but the air between sheds just below the cement sections is highly stressed. The stress throughout the dielectric field of this unit is an improvement over the types *C* and *D*. However, the short second shed and protected fourth shed give unequal voltage distribution at flashover dry or wet.

Figs. 14, 15 and 16 (insulator *F*) show the dielectric field of a unit of the proposed design. The shortest air path between shells is sufficient so that the air is not overstressed at working voltage of the insulator or until flashover occurs. Moreover, the rain sheds are so spaced that each section of the unit takes its share of the stress at flashover, dry or wet.

Fig. 17 (insulator *G*) shows the dielectric field of a unit similar to insulator *F*, but having rain sheds of greater diameter. The diameter of the head of this unit is probably out of proportion and greater than would be most satisfactory for service. However, the stress in the dielectric is well proportioned and the voltage distribution per shell at flashover, dry or wet, is fairly well proportioned.

Fig. 18 (insulator *F*) shows the dielectric field of the insulator having upper surfaces of the rain sheds covered with a conducting paint. This field form which approximates the rain conditions indicates that the stress per shell on the unit during rain would be approximately equal. Moreover it indicates that the stress in the dielectric field is more uniform during rain.

Fig. 19 (insulator *F*) shows the dielectric field of the insulator when equipped with Nicholson Arcing Rings, and indicates that the most highly stressed portion of the field about the insulator is not changed. However, the most highly stressed portion of the field between the line wire and cross arm is now between arcing rings and flashovers would, therefore, occur between rings.

Fig. 20 (insulator *F*) shows the dielectric field when static shields are placed at the top and base of the insulator. This combination would give a very fine distribution of stresses in the dielectric but would be rather expensive commercially.

60-Cycle Flashover Tests

Flashover on most of the older insulator types is caused by the corona formation at the line and tie wires and the edges of the

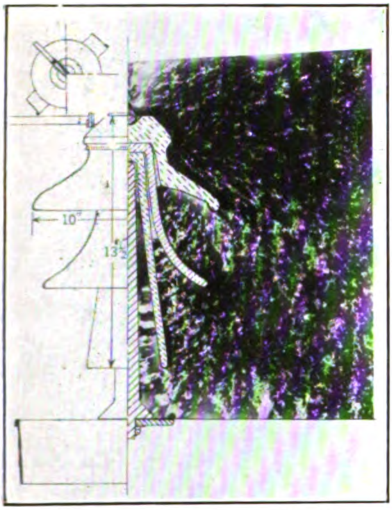


FIG. 9—INSULATOR *C*—DIELECTRIC FIELD ABOUT LINE INSULATOR OF EARLY DESIGN

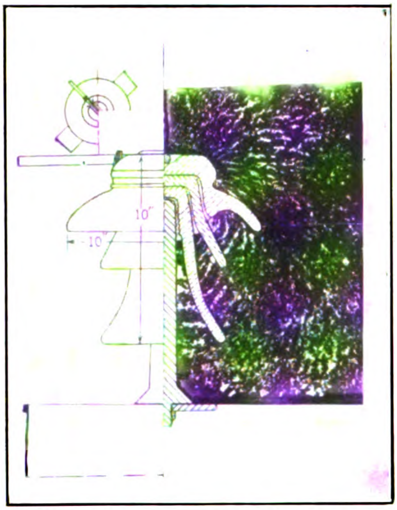


FIG. 10—INSULATOR *D*—DIELECTRIC FIELD OF INSULATOR OF FAIRLY RECENT DESIGN

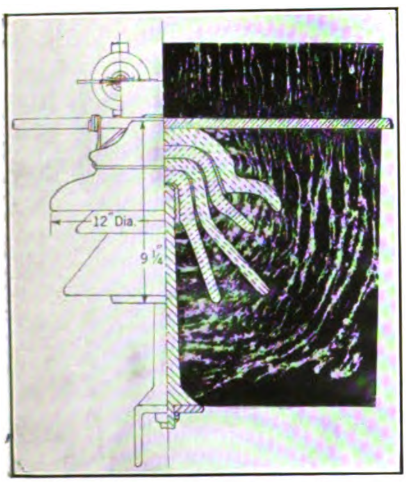
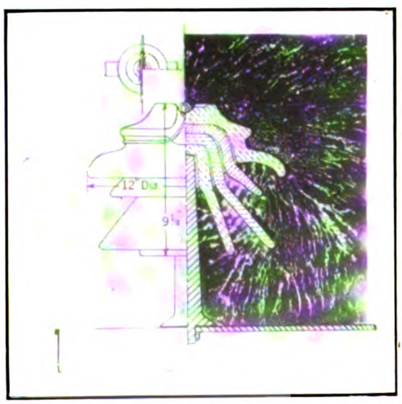


FIG. 11—INSULATOR *E*—DIELECTRIC FIELD ABOUT INSULATOR OF RECENT DESIGN



[GILCHREST]

FIG. 12—(SEE FIG. 24)

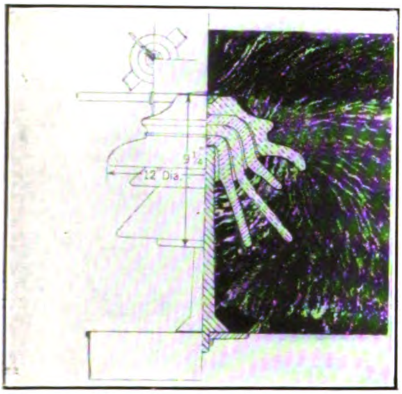


FIG. 13—(SEE FIG. 24)

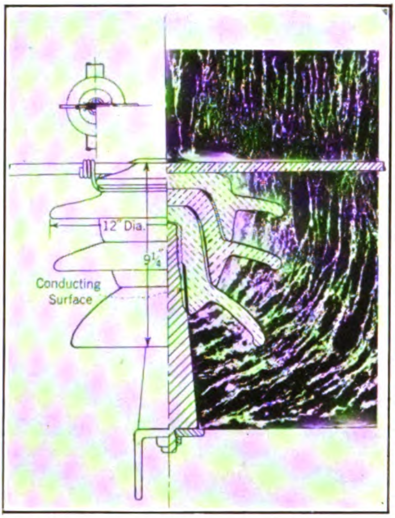


FIG. 14—INSULATOR *F*—DIELECTRIC FIELD ABOUT INSULATOR OF PROPOSED DESIGN

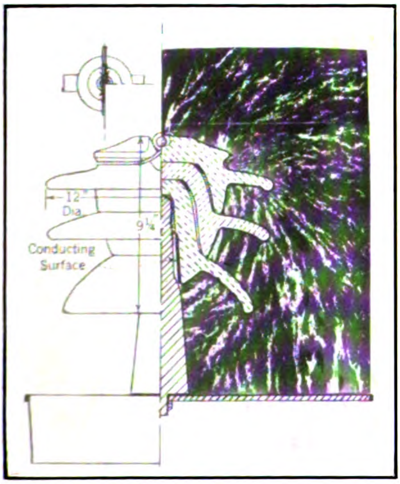
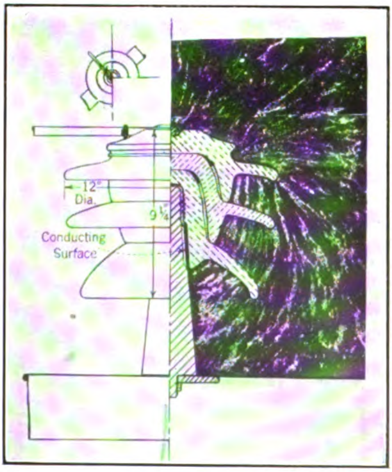


FIG. 15—(SEE FIG. 27)



[GILCHREST]

FIG. 16—(SEE FIG. 27)

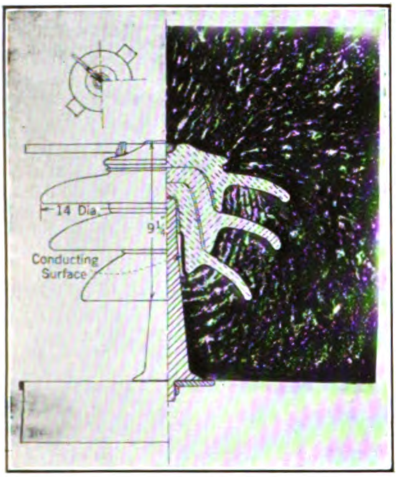


FIG. 17—INSULATOR G—DIELECTRIC FIELD ABOUT INSULATOR OF PROPOSED DESIGN

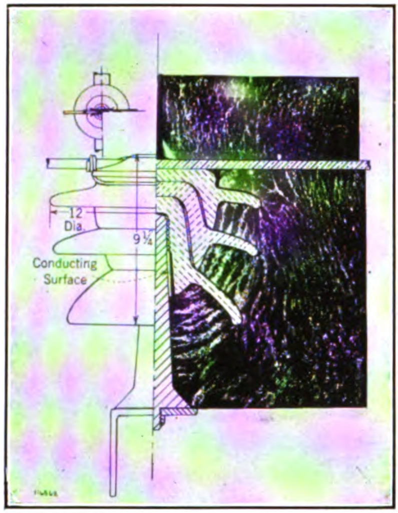


FIG. 18—INSULATOR F—DIELECTRIC FIELD ABOUT INSULATOR OF PROPOSED DESIGN UNDER CONDITIONS APPROXIMATING RAIN

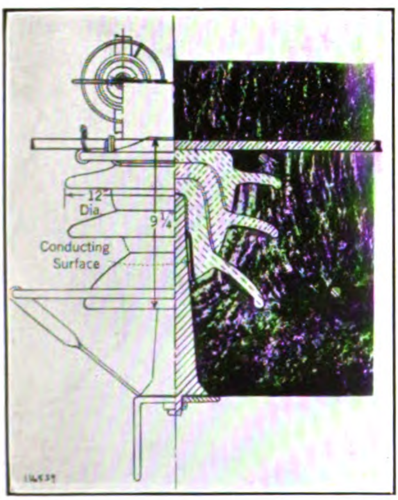
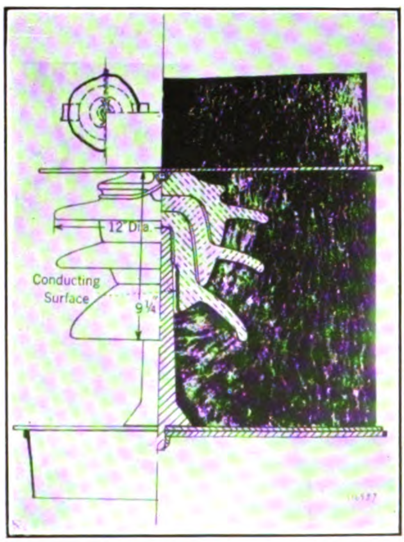


FIG. 19—INSULATOR F—DIELECTRIC FIELD ABOUT INSULATOR OF PROPOSED DESIGN INSTALLED WITH NICHOLSON ARCING RINGS



[GILCHREST]

FIG. 20—INSULATOR F—DIELECTRIC FIELD ABOUT INSULATOR HAVING STATIC SHIELDS AT TOP AND BASE

cement joints between shells. As the voltage applied to the insulator is increased, the area of the corona formation increases and static streamers gradually spread over the surface of the insulator sheds. The static streamers increase in length until the air insulation between them finally fails and flashover follows. Obviously, the path of the flashover will start along the path of these streamers and thus trouble may be caused by the intense heat of the power arc and rain sheds may be stripped from the insulator.

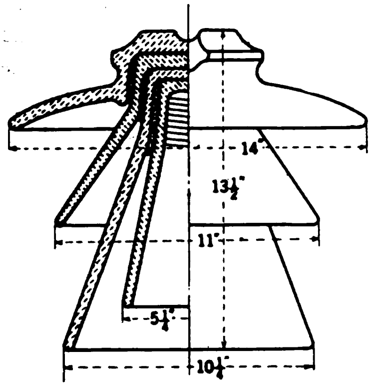


Fig. 21a—Insulator *I*

In the proposed type of design there are no static streamers from the edges of the cement section between shells up to flash-over voltage. The corona formation at the tie and line wires therefore, builds up until flashover occurs by breaking down an air path between the line and pin or cross arm. The proof of these statements may be seen in the following illustrations. The axes of the two units in each of the following figures giving comparative flashovers

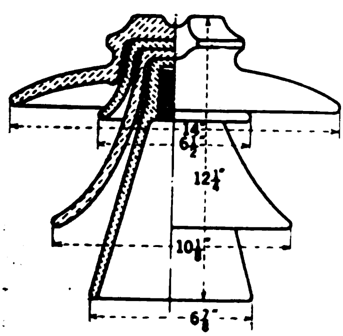


Fig. 21b—Insulator *K*

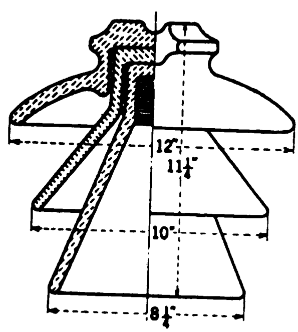


Fig. 21c—Insulator *H*

were at the same distance from the camera lens and hence the dimensions are directly comparable.

Figs. 22 and 23 show illustrations of dry and wet flashovers respectively, on insulators *G* and *H*. The design of insulator *H* is given in Fig. 21c.

Fig. 24 shows an illustration of wet flashover on insulators *J*

and *I*. The design of insulator *I* is given in Fig. 21A. Insulator *J* is of the proposed type similar to the unit given in Fig. 8.

Fig. 25 illustrates of one of the early types of high-voltage insulators and insulator *J* in parallel. The finer lines over the surface of the old type unit are preliminary static discharges. The final power arc passes from the left of the insulator head around in front of and finally to the pin at the back of the insulator.

Fig. 26 shows insulator *F* in parallel with the unit of early design shown in Fig. 25, and shows the corona formation and static discharge over the head and between the head and second shell of the old type unit. The camera exposure was $\frac{3}{4}$ of a minute at *F*-8.

The difference in the stress in the air around the insulators just below flashover voltage dry was very marked. Insulators *F*, *G* and *J* of the proposed type of design showed no appreciable corona except at line and tie wires until flashover occurred. Flashover occurred from tie wire or line wire to pin or cross arm, there being no tendency for the arc to start between the rain sheds. Considerable corona formation and static streamers could be dectected on insulators *E*, *H* and *I*. Static streamers began to spread out over the surfaces between shells of insulators *H* and *I* at 80 per cent of flashover voltage, and unless these units are mounted on rather low pins the power arc holds close to the insulator surfaces. Of course, the old type design of Figs. 25 and 26 has been entirely superseded but these two figures clearly indicate the entire neglect of a consideration of the dielectric field.

The difference of distribution of stress before wet flashover is even more noticable. In insulator *E*, *G*, *H* and *I* the unequal spacing of rain sheds and consequent unequal wet arcing distances, combined with a highly stressed air between the sheds below the cement sections produces preliminary discharges (marked *p*) between rain sheds. These preliminary discharges throw electrical impacts onto parts of the insulator and short circuit portions of the porcelain between the line and pin. Consequently, when a line surge occurs during a rain storm or when the unit is wet and dirty the factor of safety of these insulators in resisting puncture or flashover is actually no more, and sometimes is less, than it would be minus one of the shells.

Insulators *F*, *G* and *J* of the proposed design show no prelimi-

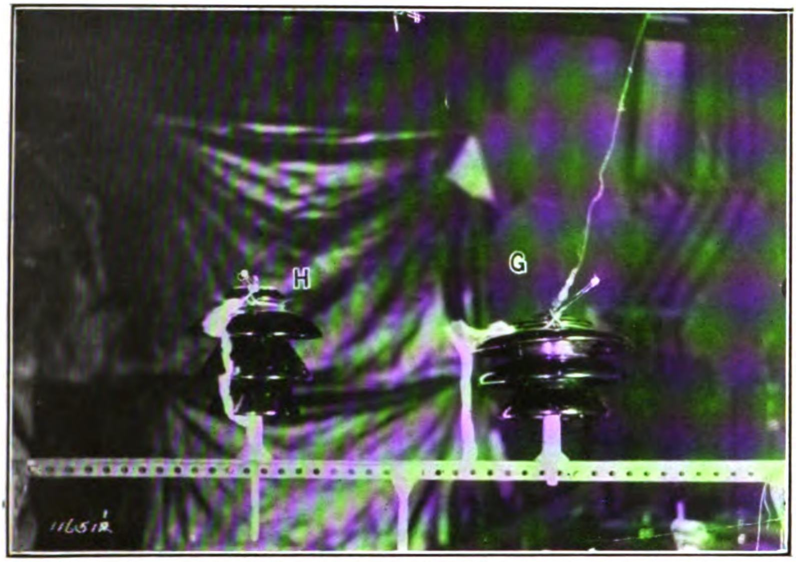


FIG. 22—60-CYCLE DRY FLASHOVER

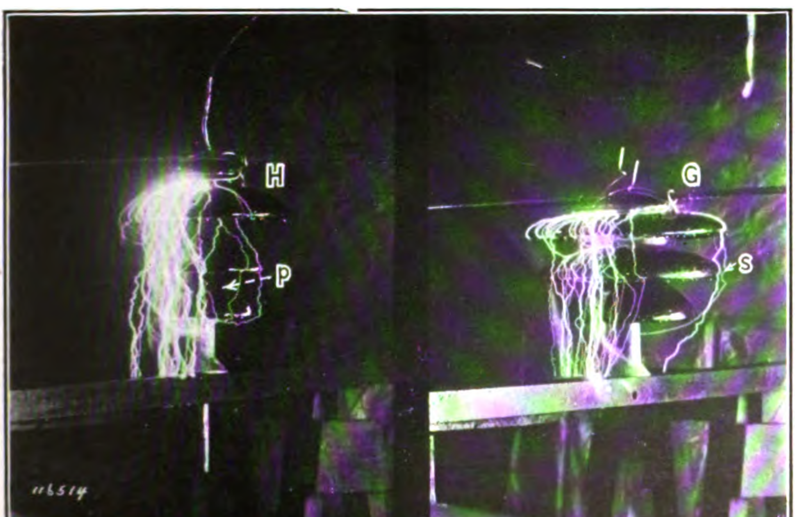


FIG. 23—60-CYCLE WET FLASHOVER

[GILCHREST

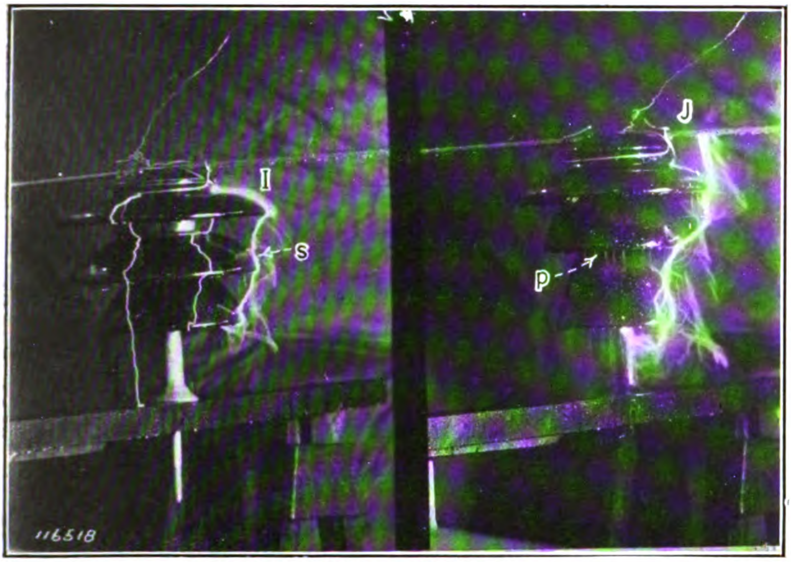


FIG. 24—60-CYCLE WET FLASHOVER

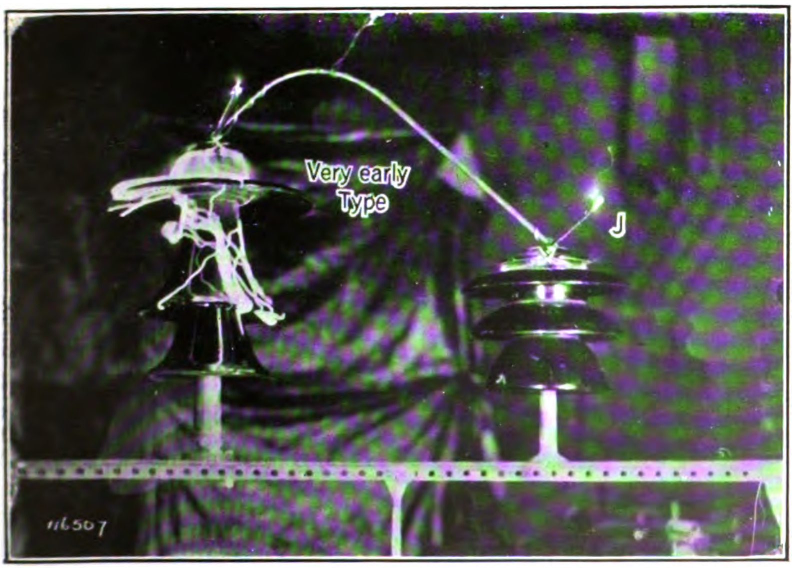


FIG. 25—60-CYCLE DRY FLASHOVER [GILCHREST]

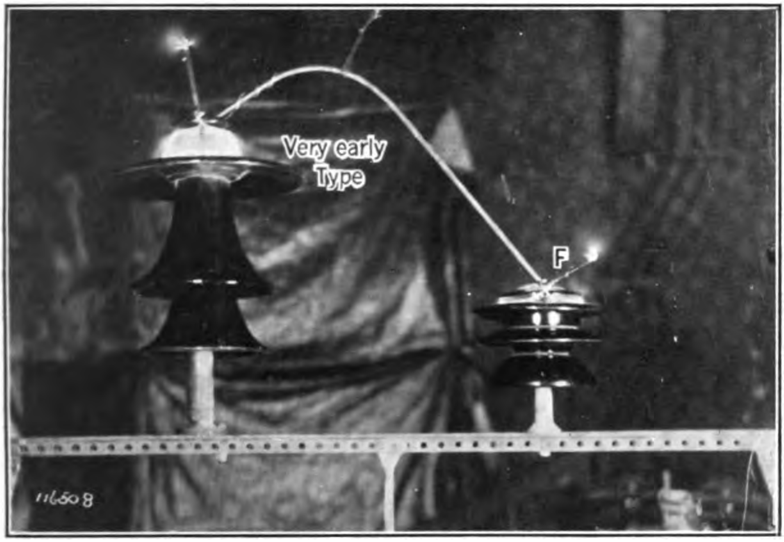


FIG. 26—60-CYCLE TEST—COMPARATIVE CORONA FORMATION

[GILCHREST]

FIG. 27—PLASTER PARIS DUST DEPOSITED WITH INSULATORS UNDER 60-CYCLE VOLTAGE

nary discharges except static from tie or line wires to pin or cross arm. Static discharges (marked *s*) are shown on each of these units. All of the leakage surface and thickness of porcelain between line and pin are, therefore, effective up to failure by flashover.

2—*Surface Leakage.* As previously stated, the leakage surface of many of the older designs gives a very uneven voltage distribution per shell. Table II gives the resistance per shell of various insulators tested during this investigation. The values were obtained by an integration of the surface, *i.e.*, surface resistance equals $S \frac{d\,s}{2\,\pi\,y}$ where $d\,s$ is an element of surface and y the radius of that element from the axis of the insulator.

It is obvious from this table that certain of the older designs especially those having a short second shell, long inner shells, etc., have a very unequal surface resistance per shell. If the insulator surface becomes dirty and wet so as to pass a leakage current of even a thousandth of an ampere the voltage distribution would depend upon this current and the capacity current could be neglected. The voltage gradient over the insulator surface thus often becomes sufficient to cause discharge between sheds and pin or cross arm or over the short sheds. An electrical impact is thereby applied to parts of the insulator and portions of the porcelain body between line and pin are short circuited. It is believed that the continued overstressing of parts has been the cause of many insulator failures in the past.

TABLE II

SURFACE RESISTANCE PER SHED IN PER CENT OF TOTAL RESISTANCE

Insulator	Number of shed			
	First	Second	Third	Fourth
A	28	30	42	..
B	45	55	..	..
E	14	13	32	41
F	26	29	45	..
G	26	31	43	..
H	18	29	48	..
I	12	16	32	40
K	15	11	30	44

The surface resistance of the proposed designs as typified by insulators *A* and *B* in Table II is gradually increased from the top

to center shells, the increase being considered as an advantage since the center sheds will usually become dirtiest.

A novel feature of the proposed design is illustrated in Fig. 27 showing insulators *D*, *E*, *F* and *H*. These units were set on a cross-arm line, and tie wire attached as in service, voltage applied and plaster of paris dust blown around them. The surfaces along the lines *a* of the proposed design (Fig. 8) are practically free of dust.

The reason for this is quite apparent. All the force acting in the dielectric field along this surface *a* is tangential and would tend to force the particles to the sheds above or below. The same action was noted when the units were subjected to atomized salt water. This feature would doubtless have some value in dust laden sections since the dust would tend to settle mostly on the lower shed and rain and wind would clean this to some extent.

It is necessary to clean the insulators in long portions of line in certain sections of country as the coast districts of California. It is very apparent that the proposed type of design may be cleaned much more readily and thoroughly than any of the older types.

3. *Porosity*. As denoted previously, the porosity of porcelain is a specific problem of the ceramic engineer rather than the designer. As clearly pointed out in a recent paper[3] by Prof. Ryan we apparently have no method of detecting the very slightly porous material which may cause trouble. Since porosity is a function of the body composition, manufacturing process and burning, even with the most careful production and testing, a small amount of this slightly porous material is not detected. The thicker portions of the porcelain in the cemented area of the proposed type of design should minimize the number of the pieces that will give trouble later in service.

4. *Mechanical Breakage*.

(a) From Handling: The increase of thickness of the rain sheds and addition of a drip edge will materially decrease the percentage loss from this cause.

(b) Mischievous Stone Throwing and Rifle Shooting: (1) The following photographs give comparative flashover voltages dry and wet on units having various rain sheds broken by throwing a small weight at the insulator. Figs. 22 and 23 show the dry and wet flashovers on insulators *G* and *H* respectively. Table III gives the flashover voltages of broken units in per

[3]loc. cit.

cent of flashover of the unit when unbroken. Reference to illustrations is made in the Table. Fig. 24 shows the wet flashover on units *J* and *I*. Table IV gives comparative flashovers of broken units, as in Table III.

TABLE III

Sheds broken....	Top		Second		Second and third	
Illustrated in Fig.		28	29	30	31	
Dry or wet......	dry	wet	dry	wet	dry	wet
Insulator *G*.....	85	80	100	100	68	74
Insulator *H*.....	79	77	82	97	54	70

TABLE IV

Sheds Broken.........	Top	Top and third	All sheds	
Photographs in Fig 32				
Dry or wet...........	dry	dry	dry	wet
Insulator *J*...........	87	78	59	30
Insulator *I*...........	85	70	34	15

As would be expected from a study of the dielectric field, diagrams the breaking of the second shed of the proposed type of design has practically no effect upon the flashover values of the insulator. In fact, as shown in the illustrations, the paths of the dry and wet flashovers did not follow over the broken shed. When sheds are broken, the corona formation and static streamers build out over the surface of the older type of design at a lower voltage than when the units are intact. The paths of flashover over these older types, therefore, follow the surface of the insulator. In the proposed type of design the absence of streamers from the porcelain surface causes the arc to keep clear of the insulator. A power arc will, therefore, be less liable to cause complete failure of a broken unit of the proposed design.

One of the most important features of the proposed design is that when the units are hit by stones, etc., the rain sheds will not crack or break beyond line *a* Fig. 8, due to the shape of the individual parts. The rain sheds of the older types of designs when hit are very likely to crack or break up into the cemented section. The first voltage surge or even normal line voltage will, therefore, often puncture the remaining shells. In fact, in the two series of tests photographed, both the older type of units

punctured during the dry arcover after sheds were broken. Static streamers shot over surface of insulator *H* in Fig. 31 and then puncture occurred. Insulator *I*, Fig. 32, flashed over and then punctured before the circuit breaker of the testing transformer operated.

(2) One each of units *H*, *I* and *K* and two of *J* were subjected to rifle shots. Twenty-two caliber long bullets were shot at the insulators from about 30 yards distance and in a line at 45 deg. to their axes. The following photographs show the comparative breakage and the ability of the broken units to thereafter withstand electrical test. The shooting was done by men disinterested in the design of the insulators and they were requested to do as much damage as possible.

Fig. 33 shows insulators *I* and *J* after 15 shots were fired at each. The top, second and third shells of *I* were broken, the second shell being cracked into the cemented section. The second shell of *J* was chipped in two places, the rest of the insulator being intact. Fig. 34 shows insulators *H*, *K* and *I* after 14 shots were fired at *H*, 12 at *K* and 28 at *I*. The second and center shells of *H* were cracked and the center of *K*. The sheds of *J* were chipped off in a few places but the shells were not cracked. These five units were than set with their axes at right angles to the line of fire. Not more than 5 or 10 shots were necessary to strip the main part of the remaining sheds from Insulators *H*, *K* and *I* while one unit of type *J* still retained a considerable portion of its sheds after approximately 100 shots had been fired at it. The sheds remaining on the two units *J* were then knocked off by a hammer, to illustrate to those present that the surface of the insulator that follows flow line *a* would not be cracked thereby.

Fig. 35 shows the first dry flashover test made on these units after the shooting. Units *H*, *K* and *I* punctured at voltage of 33, 43 and 56 kilovolts, respectively. Unit *J* flashed over at 105 kilovolts, the remaining porcelain body bounded by line *a* still being intact.

(*C*) *Insufficient Strength as a Support*: Two samples as per Fig. 36, were tested to determine the resistance to side pull. In each case load was applied at the wire groove which was one foot from the base of pin. The parts from which insulator *L* was formed were obtained by trimming off the rain sheds of individual shells of unit *J* before burning and the mechanical test should, therefore, be about the same as of unit *J*. The pin of

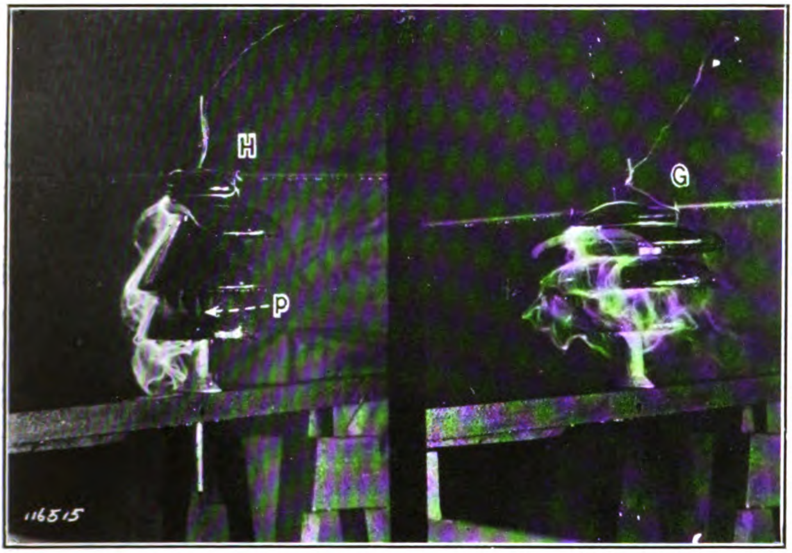
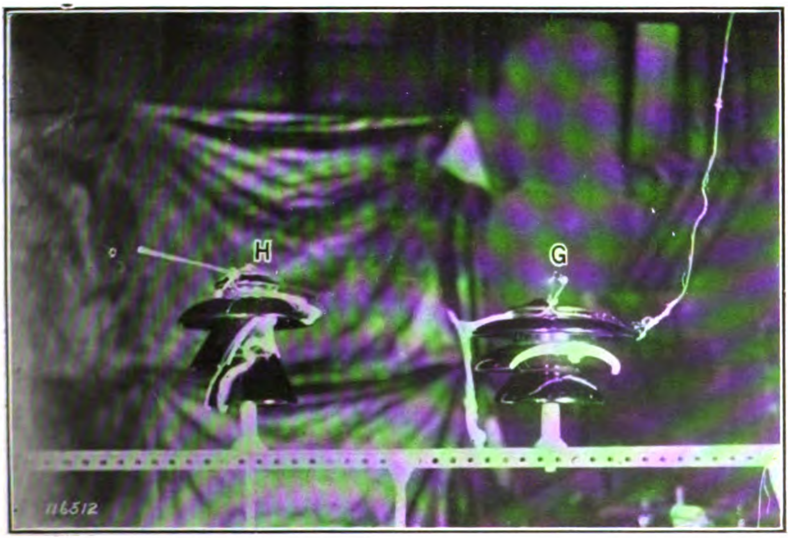


FIG. 28—TOP SHED BROKEN—60-CYCLE WET FLASHOVER



[GILCHREST]

FIG. 29—SECOND SHED BROKEN—60-CYCLE DRY FLASHOVER

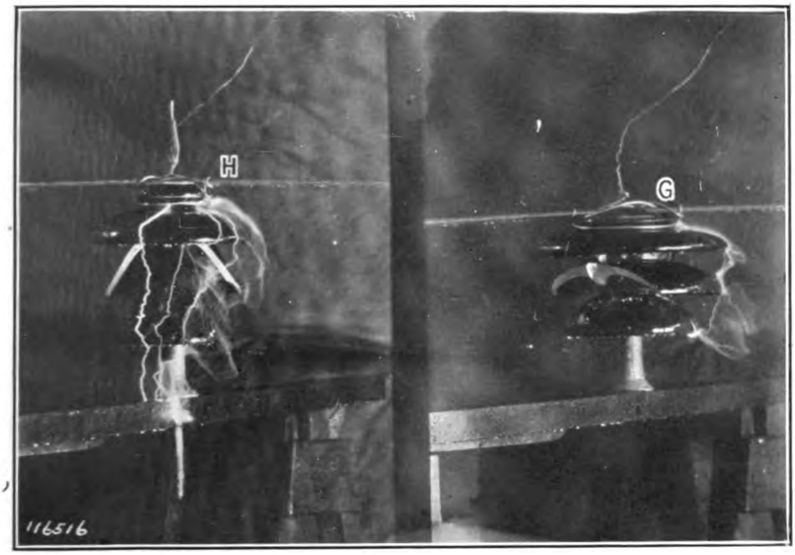
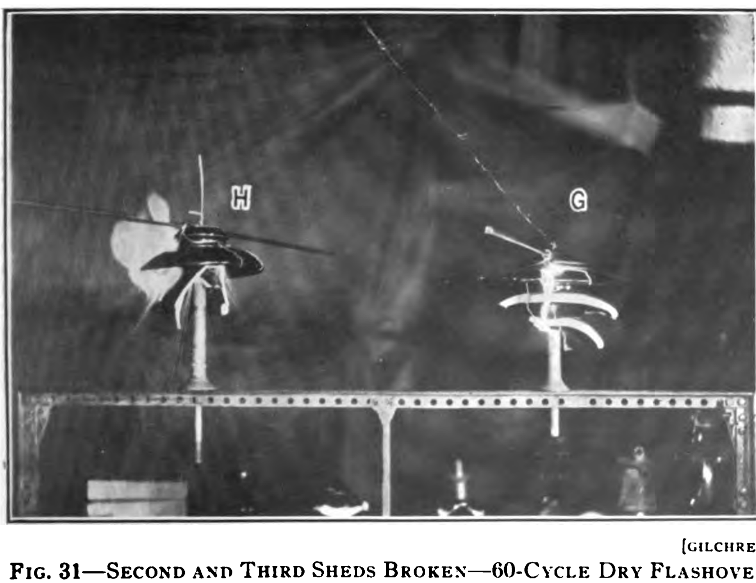


FIG. 30—SECOND SHED BROKEN—60-CYCLE WET FLASHOVER



[GILCHREST]

FIG. 31—SECOND AND THIRD SHEDS BROKEN—60-CYCLE DRY FLASHOVER

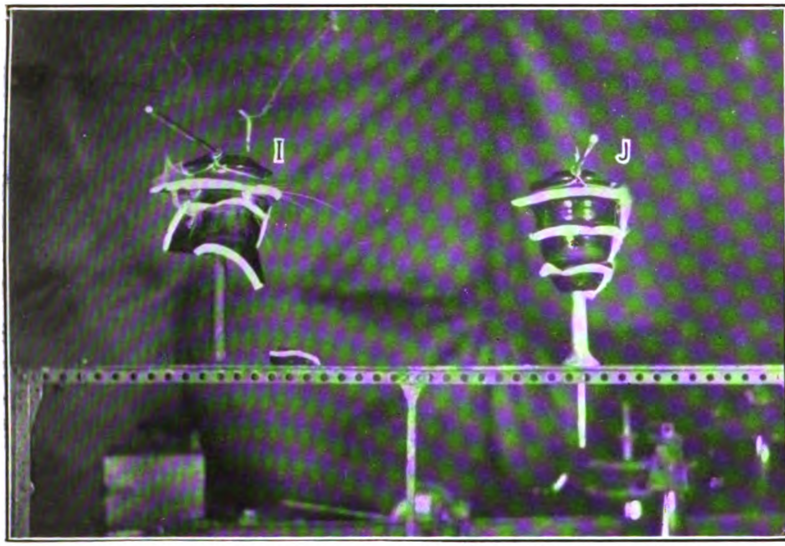
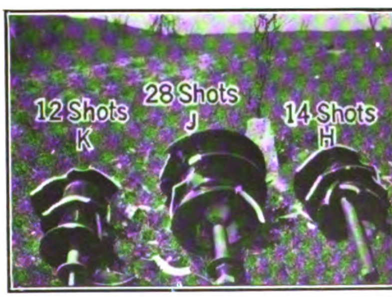


FIG. 32—ALL SHEDS BROKEN—60-CYCLE DRY FLASHOVER

FIG. 33—AFTER 15 SHOTS WERE FIRED AT EACH



[GILCHREST]

FIG. 34—AFTER NUMBER OF SHOTS AS INDICATED

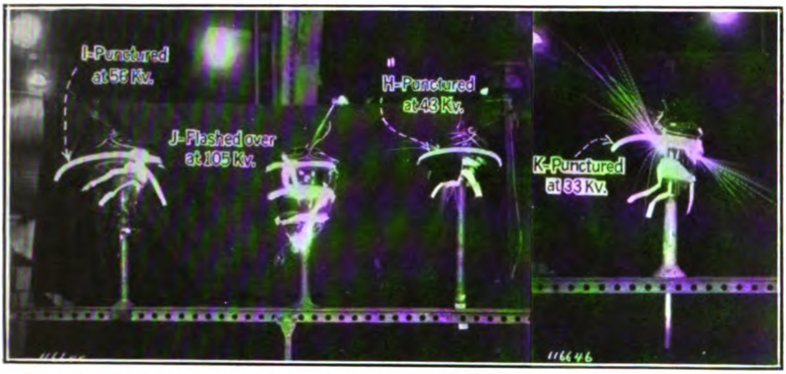


FIG. 35—60-CYCLE DRY FLASHOVERS ON INSULATORS *H*, *I* AND *J* AFTER BREAKAGE FROM RIFLE SHOTS

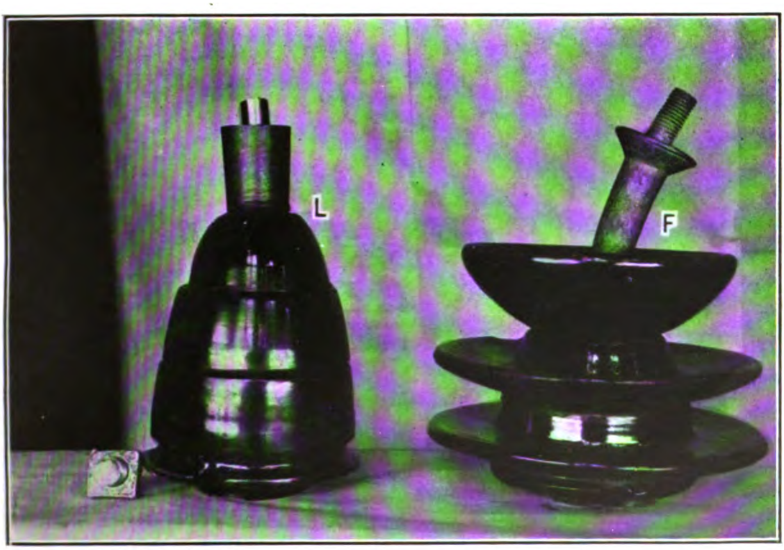


FIG. 36—RESISTANCE TO SIDE-PULL [GILCHREST]

unit L was cemented directly into the insulator. A separate pressed steel thimble was cemented into insulator F. The one-inch bolt of the pin cemented into insulator L failed at 4400 ft-lb., and 3100 ft-lb. bent the pin of insulator F as shown, the position of insulator being such that additional load could not be applied. Both units were electrically intact after these tests.

(d) *Brittle Material.* All units used in these comparative tests were made of the same porcelain body and hence the question of brittleness, which is a ceramic problem, does not enter.

5. *Lightning.* The impulse ratio of the proposed design is lower than that of most of the older types of design. The actual value has not been determined. This statement is based on tests of the proposed design in parallel with older types. The old types of design were set on a pin of such height above the cross arm as to cause the old type of unit to flash over when set up alone at a voltage slightly greater than that of the proposed type of design alone. When tested in parallel the static discharges over the porcelain surface of the older type would often cause the proposed type to flashover first.

Furthermore, the body of the porcelain bounded by the flow lines *a* should have an impulse ratio close to one. A very high impulse voltage might, therefore, puncture through the rain sheds of the insulator leaving this body of the unit intact. The thicker section of porcelain between line and pin will also materially increase the factor of safety of the unit.

(6) *Unequal Expanion of Metal, Cement and Porcelain.* The introduction of a resilient material between tops of shells should eliminate the tendency of certain older designs to split off. Greater radii of curvature at the tops of the insulator shells and a cement section sloped from the axis should tend to eliminate the trouble from any difference of coefficient of expansion of the porcelain and cement.

(7) *Internal Stresses in the Material.* Internal stresses set up in the insulator parts during manufacture should be very much decreased by the elimination of small radii in corners and sudden changes of cross section of the material.

Conclusions

Briefly stated, it is believed that the advantages of the proposed type over the older commercial types in resisting failure in service would be as follows:

1. When the insulator is dry, the corona and static forma-

tions are practically limited to the tie wire and line wire up to flashover voltage.

2. When the insulator is wet, no corona or static formation occurs up to flashover voltage. The flashover voltages for given overall dimensions are thereby increased.

3. The leakage resistance per shell is increased gradually from the head to the center shell. This takes into account the probability of the lower sheds becoming dirtier than the tops. The voltage distribution per shell is, therefore, equal when the insulator becomes dirty and wet and a heavy leakage current passes over the insulator.

4. Since the capacity per shell is about equal, the voltage distribution per shell will be equal when the insulator is clean and in dry air.

5. Since the distribution of voltage per shell depends upon the capacity current and leakage current, the distribution of voltage per shell in these designs should be approximately equal under all operating conditions.

6. The resistance of the insulator to side pull for a given weight and given electrical strength is relatively high. This is due to the feature of the design whereby the flow line *a* of the electrostatic field and the mechanical stress lines coincide.

7. The design of the individual shells is such that when they are tested before assembly the surface conforms to the electrostatic flow lines *a*. This allows testing of the individual parts to a higher percentage of service voltage than was possible in case of the individual shells of older designs.

8. Due to the shape of individual parts and of the assembled unit, the insulator sheds when hit by stones, riflle, balls, etc., do not break beyond surface *a*. The unit, therefore, offers a considerable percentage of its original resistance to flashover after the sheds are broken. The same feature tends to protect the insulator from complete failure during flashover in service.

9. Each characteristic of the insulator which would vitally affect durability in service has been treated uniformly throughout the line.

To be presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 27, 1918.

AMERICA'S ENERGY SUPPLY

BY CHARLES P. STEINMETZ

ABSTRACT OF PAPER

The gist of the paper is to demonstrate that the economical utilization of the country's energy supply requires generating electric power wherever hydraulic or fuel energy is available, and *collecting the power electrically, just as we distribute it electrically.*

In the first section a short review of the country's energy supply in fuel and water power is given, and it is shown that the total potential hydraulic energy of the country is about equal to the total utilized fuel energy.

In the second section it is shown that the modern synchronous station is necessary for large hydraulic powers, but the solution of the problem of the economic development of the far more numerous smaller waterpowers is the adoption of the induction generator. However, the simplicity of the induction generator station results from the relegation of all the functions of excitation, regulation and control to the main synchronous station. The economic advantage of the induction generator station is, that its simplicity permits elimination of most of the hydraulic development by using, instead of one large synchronous station, a number of induction generator stations and collecting their power electrically.

The third section considers the characteristics of the induction generator and the induction-generator station, and its method of operation, and discusses the condition of "dropping out of step of the induction generator" and its avoidance.

In the appendix the corresponding problem is pointed out with reference to fuel power, showing that many millions of kilowatts of potential power are wasted by burning fuel and thereby degrading its energy, that could be recovered by interposing simple steam turbine induction generators between the boiler and the steam heating systems, and collecting their power electrically. It is shown that the value of the recovered power would be an appreciable part of that of the fuel, and that organized and controlled by the central stations, this fuel power collection would improve the station load factor, give the advantages of the isolated plant without its disadvantages, and produce a saving of many millions of tons of coal.

I. The Available Sources of Energy

A. COAL

THE only two sources of energy, which are so plentiful as to come into consideration in supplying our modern industrial civilization, are coal, including oil, natural gas, etc., and water power.

*Manuscript of this paper was received April 9, 1918.

While it would be difficult to estimate the coal consumption directly, it is given fairly closely by the coal production, at least during the last decades, where wood as fuel had become negligible and export and import, besides more or less balancing each other, were small compared with the production. Coal has been mined since 1822, and in Fig. 1 is recorded the coal production of the United States, from the governmental reports. The annual production is marked by circles, the decennial average marked by crosses for every five years. Table I gives the decennial averages, in millions of tons per year.

TABLE I

AVERAGE COAL PRODUCTION OF THE UNITED STATES

(decennial average)

Year	Million tons per year	Per cent increase per year
1825	0.11	....
30	0.32	22.4
35	0.83	19.7
40	1.92	17.0
45	4.00	14.5
50	7.46	10.45
55	10.8	8.35
60	16.6	8.72
65	25.9	9.22
70	40.2	8.58
75	56.8	7.42
80	82.2	7.95
85	122	6.80
90	160	5.40
95	206	5.75
1900	281	6.96
05	404	6.60
10	532	....

In Fig. 1 the logarithms of the coal production in tons are used as ordinates. With this scale, a straight line means a constant proportional increase, that is, the same percentage increase per year, and in the third column of Table I are given the average percentage increase of coal production per year.

This Fig. 1 is extremely interesting by showing the great irregularity of production from year to year, and at the same time a very great regularity over a long period of time. Since 1870 the average production may be represented by a straight line, the values lying irregularly above and below the line, which

represents an annual increase of 6.35 per cent and thus represents the average coal production[1] C by the equation

$$C = 45.3 \times 10^{0.0267\,(y-1870)} \text{ million tons}$$

or

$$\log C = 0.0267\,(y-1870) + 7.656$$

where $y =$ year.

Before this time, from 1846 to 1884, the coal production could be represented by

$$C = 7.26 \times 10^{.0365\,(y-1850)} \text{ million tons}$$

or

$$\log C = 0.0365\,(y-1850) + 6.861$$

representing an average annual increase of 8.78 per cent.

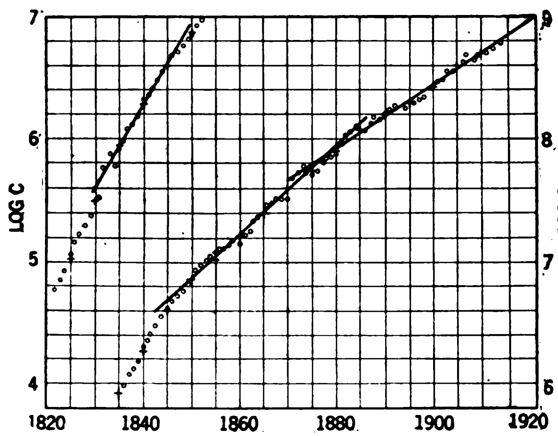


FIG. 1—COAL PRODUCTION OF THE UNITED STATES

It is startling to note how inappreciable, on the rising curve of coal production, is the effect of the most catastrophic political and industrial convulsions, such as the Civil War and the Industrial panic of the early 90's; they are indistinguishable from the constantly recurring annual fluctuations. It means, that the curve is the result of economic laws, which are laws of nature.

Extrapolating from the curve of Fig. 1, which is permissible, due to its regularity, gives 867 million tons as this year's coal consumption. As it is difficult to get a conception of such enormous amounts, I may be allowed to illustrate it. One of the great wonders of the world is the Chinese Wall, running

1. Soft coal and anthracite, and including oil reduced to coal by its fuel value.

across the country for hundreds of miles, by means of which China unsuccessfully tried to protect its nothern frontier against invasion. Using the coal produced in one year as building material, we could with it build a wall like the Chinese Wall, all around the United States, following the Canadian and Mexican frontier, the Atlantic, Gulf and Pacific Coast, and with the chemical energy contained in the next year's coal production, we could lift this entire wall up into space, 200 miles high. Or, with the coal produced in one year used as building material, we could build 400 pyramids, larger than the largest pyramid of Egypt.

It is interesting to note that 100 thousand tons of coal were produced in the United States in 1825; one million tons in 1836; 10 million tons in 1852 and 100 million tons in 1882. The production will reach about 1000 million tons in 1920, and, if it continues to increase at the same rate, it would reach 10,000 million tons in 1958.

Estimating the chemical energy of the average coal as a little above 7000 cal., *the chemical energy of one ton of coal equals approximately the electrical energy of one kilowatt year* (24 *hour service*). That is, one ton of coal is approximately equal in potential energy to one kilowatt-year.

Thus the annual consumption of 867 millions of tons of coal represents, in energy, 867 million kilowatt-years.

However, as the average efficiency of conversion of the chemical energy of fuel into electrical energy is probably about 10 per cent, the coal production, converted into electrical energy, would give about 87 million kilowatts.

Assuming however, that only one half of the coal is used for power, at 10 per cent efficiency, the other half as fuel, for metallurgical work etc., at efficiencies varying from 10 per cent to 80 per cent, with an average efficiency of 40 per cent, then we get 217 million kilowatts (24 hour service) as the total utilized energy of our present annual coal production of 867 million tons.

B. The Potential Water Powers of the United States

Without considering the present limitation in the development of water powers, which permits the use of only the largest and most concentrated powers, we may try to get a conception of the total amount of hydraulic energy which exists in our country, irrespective of whether means have yet been developed

or ever will be developed for its complete utilization. We therefore proceed to estimate the energy of the total rain fall.

Superimposing the map of rain fall in the United States, upon the map of elevation, we divide the entire territory into sections by rain fall and elevation. This is done in Table II, for the part of our continent between 30 and 50 degrees northern latitude.

TABLE II

TOTAL POTENTIAL WATER POWER OF UNITED STATES

In. rain fall	Ft. elevation	Area $m^2 10^{12} \times$	Avg. elevation m.	Avg. rainfall cm.	Kg-m. per m^2; $10^3 \times$	Kg-m total $10^{15} \times$
>10	>5000	0.54	2100	12.5	263	142
	1000–5000	0.29	900		112	32.5
10–20	>5000	1.18	2100	37.5	787	930
	1000–5000	1.96	900		338	660
20–30	1000–5000	0.32	900	62.5	563	183
	100–1000	0.97	150		94	91
30–40	1000–5000	0.35	900	87.5	786	275
	100–1000	1.40	150		131	184
40–60	1000–5000	0.27	900	125	1130	305
	100–1000	1.03	150		188	194
						Σ = 2996
						3000

As obviously only the general magnitude of the energy value is of interest, I have made only few sub-divisions: five of rain fall and four of elevation, as recorded in columns 1 and 2 of Table II[1]. The third column gives the area of each section, in millions of square kilometers, the fourth column the estimated average elevation, in meters, and the fifth column the average rain fall, in centimeters. The sixth column gives the energy, in kilogram-meters per square meter of area, and the last column the total energy of the section, in kilogram-meters, which would be represented by the rain fall, if the total hydraulic energy of every drop of rain were counted, from the elevation where it fell, down to sea level.

As seen from Table II, the total rain fall of the North American Continent between 30 deg. and 50 deg. latitude represents 3000×10^{15} kg-m. This equals 950 million kilowatt years (24 hour service). That is, the total potential water power of the United States, or the hydraulic energy of the total

1. The lowest elevation, < 100 ft., is not included, as having little potential energy.

rain fall, from the elevation where it fell, down to sea level, gives about 1000 million kilowatts.

However, this is not available, as it would leave no water for agriculture; and even if the entire country were one hydraulic development, there would be losses by seepage and evaporation.

An approximate estimate of the maximum potential power of the rain fall, after a minimum allowance for agriculture and for losses is made in Table III, allowing 12.5 cm. rain fall for wastage, and 37.5 and 25 cm. respectively for agriculture where such is feasible.

TABLE III

AVAILABLE POTENTIAL WATER POWER OF THE UNITED STATES

Avg. rainfall cm.	Avg. elevation m.	Area m^2 $10^{12} \times$	Wastage cm.	Agriculture, cm.	Available rainfall cm.	Kg. m. per m^2 $10^3 \times$	Kg-m. total $10^{15} \times$
12.5	2100	0.54	12.5	....	....	....	....
....	900	0.29	12.5	....	....	....	....
37.5	2100	0.39	12.5	25	....	....	....
....	2100	0.79	12.5	....	25	525	415
....	900	0.98	12.5	25	....	....	....
....	900	0.98	12.5	....	25	225	220
62.5	900	0.21	12.5	37.5	12.5	112	23
....	900	0.11	12.5	....	50	450	50
....	150	0.97	12.5	37.5	12.5	19	18
87.5	900	0.35	12.5	37.5	37.5	337	118
....	150	1.40	12.5	37.5	37.5	56	78
125	900	0.27	12.5	27.5	75	674	182
....	150	1.03	12.5	37.5	75	112	116
							$\Sigma = 1220$

This gives about 1200×10^{15} kg-m. as the total available potential energy, which is equal to 380 million kilowatts (24 hour service). Assuming now an efficiency of 60 per cent from the stream to the distribution center, gives 230 million kilowatts (24 hour service) as the maximum possible hydroelectric power, which could be produced, if every river, stream, brook or little creek throughout its entire length, from the Spring to the ocean, and during all seasons, including all the waters of the freshets, were used and could be used. It would mean that there would be no more running water in the country, but stagnant pools connected by pipe lines to turbines exhausting into the next lower pool. Obviously, we could never hope to develop more than a part of this power.

C. Discussion

It is interesting to note that the maximum possible hydraulic energy of 230 million kilowatts, is little more than the total energy which we now produce from coal, and is about equal to the present total energy consumption of the country, including all forms of energy.

This was rather startling to me. It means that the hope that when coal once begins to fail we may use the water powers of the country as the source of energy, is and must remain a dream, because if all the potential water powers of the country were now developed, and every rain drop used, it would not supply our present energy demand.

Thus hydraulic energy may and should supplement that of coal, but can never entirely replace it as a source of energy. This probably is the strongest argument for efforts to increase the efficiency of our methods of using coal.

A source of energy which is practically unlimited, if it could only be used, is solar radiation. The solar radiation at the earth's surface is estimated at 1.4 cal. per $cm.^2$ per min. Assuming 50 per cent cloudiness, this would give an average throughout the year (24 hours per day), of about 0.14 cal per $cm.^2$ horizontal surface per min., and on the total area considered in the preceding table, of 8.3 million square kilometers of North America between 30 and 50 latitude, a total of approximately 800,000 million kilowatts (24 hour service), or a thousand times as much as the total chemical energy of our coal consumption; 800 times as much as the potential energy of the total rainfall.

Considering that the potential energy of the rainfall from surface level to sea level, is a small part of the potential energy spent by solar radiation in raising the rain to the clouds, and that the latter is a small part of the total solar radiation, this is reasonable.

Considering only the 2.7 million square kilometers of Table III, which are assumed as unsuited for agriculture, and assuming that in some future time, and by inventions not yet made, half of the solar radiation could be collected, this would give an energy production of 130,000 million kilowatts.

Thus, even if only one-tenth of this could be realized, or 13,000 million kilowatts, it would be many times larger than all the potential energy of coal and water. Here then would be the great source of energy for the future.

II. Hydroelectric Station

A. The Modern Synchronous Generator Station

In developing the country's water powers, up to the present time only those of greatest energy concentration have been considered; that is, those where a large volume and a considerable head of water was available within a short distance.

This led to the present type of hydroelectric generating station, as best solving the problem. The equipment of such a station comprises the following apparatus:

Three-phase synchronous direct-connected generators.

Hydraulic turbines of the highest possible efficiency.

Hydraulic turbine speed governing mechanism.

An exciter plant comprising either exciters directly connected to the generators, or several separate exciter machines, connected to separate turbines.

Exciter bus bars.

Voltmeter, and ammeters in exciters and in alternator field circuits.

Field rheostats of the alternators.

Low-tension busbars, either in duplicate, or with transfer or synchronizing bus.

Circuit breakers between generators and busbars, usually non-automatic.

Circuit breakers between transformers and busbars, usually automatic, with time limit.

Voltmeters and potential transformers at the generators.

Synchronoscopes or other synchronizing devices.

Ammeters and current transformers at the generators.

Voltmeter and potential transformer at the busbars.

Ammeters and current transformers at the step-up transformers.

Totaling ammeter for the station output.

Integrating wattmeter.

Relays, interlocking devices etc., etc.

Step-up transformers.

High-tension busbars, possibly in duplicate.

High-tension circuit breakers between transformers and high-tension busbars.

High-tension circuit breakers between high-tension busbars and lines.

Lightning arresters in the transmission lines, with inductances etc.

Ground detectors, arcing-ground or short-circuit suppressors, voltage indicators etc.

Automatic recording devices (multi-recorder), rarely used though very desirable.

Due to the vast amount of energy controlled by modern stations, the auxiliary and controlling devices in these stations have become so numerous as to make the station a very complex structure, requiring high operating skill and involving high cost of installation. At the same time, not only are all these devices necessary for the safe operation of the station, but we must expect that with the further increase of capacity of our electric systems, additional devices will become necessary for safe and reliable operation. One such device I have already mentioned—automatic recording apparatus, such as the multi-recorder.

With this type of station, it is obviously impossible, in most cases, to develop water powers of small and moderate size. A generating station of a thousand horse power will rarely, and one of a hundred horse power will hardly ever be economical.

On the other hand, a hundred horse power motor installation is a good economical proposition, and the average size of all the motor installations is probably materially below one hundred horse power.

Looking over Tables II and III, especially the latter, in the preceding section, it is startling to see how large a part of the potential water power of the country is represented by comparatively small areas of high elevation, in spite of the relatively low rainfall of these areas. As most of these areas are at considerable distance from the ocean, most of the streams are small in volume. That is, it is the many thousands of small mountain streams and creeks, of relatively small volume of flow, but high gradients, affording fair heads, which apparently make up the bulk of the country's potential water power.

Only a small part of the country's hydraulic energy is found so concentrated locally as to make its development economically feasible with the present type of generating station.

Therefore, some different, and very much simpler type of generating station must be evolved, before we can attempt to develop economically these many thousands of small hydraulic powers, to collect the power of the mountain streams and creeks.

B. Simplification of Hydroelectric Station

In the following in discussing the simplification of the hydroelectric station to adapt it to the utilization of smaller powers, we limit ourselves to the case where the smaller hydraulic stations feed into a system containing some large hydraulic or steam turbine stations, to which the control of the system may be relegated.

1. We may eliminate the low tension bus bars, with generator circuit breakers and transformer low-tension circuit breakers and connect each generator directly to its corresponding transformer making one unit of generator and transformer, and do the switching on high-tension busbars, and locating high-tension busbars and circuit breakers outdoors. While it is dangerous to transformers to switch on the high-tension side, due to the possibility of cumulative oscillations, this danger is reduced by the permanent connection of the transformer with the generator circuit, and is less with the smaller units used in small power stations, and thus permissible in this case.

However, the simplification resulted therefrom is not so great, as ammeters, voltmeter and synchronizing devices with their transformers are still retained on the low-tension circuits.

2. As it is not economical to operate at partial load, proper operation of a hydraulic station on a general system is, to operate as many units fully loaded as there is water available, and increase or reduce the number of units (of turbine, generator and transformer, permanently joined together), with the changing amount of available water, thus using all the available energy of the water power.

In this case, the turbine governors, with their more or less complex hydraulic machinery, may be omitted. If then the generators are suddenly shut down by a short circuit which opens the circuit breakers, the turbines will race and run up to their free running speed, until the gates are shut by hand. However, generators, and turbines must stand this, as even with the use of governors, the turbines may momentarily run up to their free speed in case of a sudden opening of the load, before the governors can cut off the water. Where this is not desirable some simple excess speed cut-off may be used.

3. When dropping the governing of the turbines, and running continuously at full load, the question may be raised whether generator ammeters are necessary, as the load is constant, and is all the power the water can give, and it might appear, that am-

meters with their current transformers, etc. could be omitted. However, with synchronous generators, the current depends not only on the load, but also on the power factor of the load and with excessively low power factor due to wrong excitation, the generators may be overheated by excess current, while the power load is well within their capacity. Thus ammeters are necessary with synchronous generators. As soon, however, as we drop the use of synchronous generators, and adopt induction generators, the ammeters with their current transformers may be omitted, since the current and its power factor is definitely fixed by the load. At the same time, synchronizing devices become unnecessary, together with potential transformers, generator voltmeters, etc. A station voltmeter may be retained for general information, but it not necessary either, as the voltage and frequency of the induction-generator station are fixed by the controlling synchronous main station of the system.

4. With the adoption of the induction generator, the entire exciter plant is eliminated, as the induction generator is excited by lagging currents received from synchronous machines, transmission lines and cables existing in the system. This avoids the use of exciter machines, exciter busses, ammeters, voltmeters, alternator field rheostats, etc., in short, most of the auxiliaries of the present synchronous station become unnecessary.

The solution of the problem of the economic development of smaller water powers is the adoption of the induction generator.

Stripped of all unnecessary equipment, the smaller hydroelectric station thus would comprise:

Hydraulic turbines of simplest form, continuously operating at full load, without governors.

Low-voltage induction generators direct connected to the turbines.

Step-up transformers direct connected to the induction generators.

High-tension circuit breakers connecting the step-up transformers to the transmission line. In smaller stations, even these may be dispensed with and replaced by disconnecting switches and fuses.

Lightning arresters on the transmission line where the climatic or topographical location makes such necessary.

A station voltmeter, a totalling ammeter or integrating wattmeter and a frequency indicator may be added for the informa-

tion of the station attendant, but are not necessary, as voltage, current, output and frequency are not controlled from the induction generator station, but from the main station, or determined by the available water supply.

It is interesting to compare this induction generator station lay-out with that of the modern synchronous station given above. However, it must not be forgotten that *the simplicity of the induction generator station results from the relegation of all the functions of excitation, regulation and control, to the main synchronous stations* of the system, and the induction generator stations thus are feasible only as adjuncts to at least one large synchronous station, hydraulic or steam turbine, in the system, but can never replace the present synchronous generator stations in their present field of application.

C. Automatic Generating Stations

With the enormous simplification resulting from the use of the induction generator, it appears entirely feasible to make smaller hydroelectric generating stations entirely automatic, operating without attendance beyond occasional—weekly or daily—inspection.

Such an automatic generating station would comprise a turbine with low-voltage induction generator, housed under a shed, and a step-up transformer, outdoors, connecting into the transmission line with time fuses and disconnecting switches.

It is true that in the big synchronous generating stations of thousands of kilowatts, the cost of the auxiliaries, as exciter plant, regulating and controlling devices, etc., is only a small part of the total station cost, and little would therefore be saved by the use of induction generators. No induction generators would, however, be used for such stations. But the cost of auxiliaries and controlling devices, and the cost of the required skilled attendance, decreases far less with decreasing station size than that of the generators—whether synchronous or induction—or in other words, with decreasing size of the station, *per kilowatt output*, the cost of auxiliaries and controlling devices and of attendance increases at a far greater rate than that of the generators, and very soon makes the synchronous station of the present type uneconomical.

It is also true that in the big modern hydraulic power systems, the cost of the generating station usually is a small part of the cost of the hydraulic development. Therefore any saving in

the cost of the generating station would be of little influence in determining, whether the hydraulic development would be economical. With decreasing size of the water power the cost of the hydraulic development *per kilowatt output* usually increases so rapidly as very soon to make the development of the water power uneconomical, no matter how simple and cheap the station is.

However, the value of the induction generator is not so much in the reduction of the cost of the generating station, as in the reduction of the cost of the hydraulic development, by making it possible to apply to the electric generator the same principle, which has made the electric motor economically so successful: *Collect the power electrically, just as we distribute it electrically.*

We do not, as in the days of the steam engine, convert the electric power into mechanical power at one place, by one big motor, and distribute the power mechanically, by belts and shafts, but we distribute the power electrically, by wires, and convert the electric power to mechanical power, wherever mechanical power is needed, by individual motors throughout mill and factory.

In the same way we must convert the hydraulic, that is, mechanical power into electrical power by individual generators located along the streams or water courses within the territory, wherever power is available, and then collect this power electrically, by medium-voltage collecting lines and high-voltage transmission lines, and so eliminate most of the cost of the hydraulic development, to solve the problem of the economical utilization of the country's water powers. If we attempt to collect the power mechanically, that is, by a hydraulic development gathering the waters of all the streams and creeks of a territory together into one big station, and there convert it into electric power, the cost of the hydraulic development makes it economically hopeless except under unusually favorable conditions, where a very large amount of power is available within a limited territory, or where nature has done the work for us in gathering considerable power at a waterfall, etc.

It is the old problem, and the old solution: If you want to *do it economically, do it electrically.*

Naturally then, we would use induction generators in these small individual stations just as we use induction motors in individual motor installations; but where large power is available, there is the field of the synchronous generator, and the induction generator is undesirable, just as the synchronous motor

is preferable where large power is required—unless the synchronous motor is excluded by conditions of starting torque, etc.

At first, and for some time to come, we would not consider going to anywhere near as small sizes of induction generators, as we do in induction motors. However, there are undoubtedly many millions of kilowatts available in water powers throughout the country, which can be collected by induction-generator stations from 50 h.p. upwards, and that at fair heads, requiring no abnormal machine design (no very slow speed).

Consider an instance—a New England mill river with a descent, in its upper course, of about 1100 ft. (335 m.) within five miles (eight km.), of varying gradient. At three places, where the gradient is steepest, by a few hundred feet of cast iron pipe and a small dam of 20 to 30 ft. (6 to 9 m.) length and a few feet height—just enough to cover the pipe intake—an average head of 150 ft. (45 m.) can be secured, giving an average of 75 h.p. each, or a total of 225 h.p. or 170 kw. This would use somewhat less than half the total potential power. The development of the other half, requiring greater length of pipe line, or involving lower heads, would be left to meet future demands for additional power.

The installation of an electric system of 170 kw. would hardly be worth while, but there are numerous other creeks throughout the territory from which to collect power, and within a few miles passes a high potential transmission line, coming from a big synchronous station, into which the power collecting lines coming from the induction generator stations would be tied and from which they would be controlled.

Thus, the large modern synchronous station has its field, and is about as perfect as we know how to build for large concentrated powers; but beyond this, there is a vast field, and therefore *an economic necessity of the development of a different type of hydraulic generating station to collect the scattered water powers* of the country; and that is the *induction generator station*, to which I wish to draw the attention.

I must caution, however, not to mistake small power and low head power. There are on the lower courses of our streams some hydraulic powers, which are relatively small due to their low heads, and which can not be economically developed by the synchronous generator, due to the low head and correspondingly low speed. The designing characteristics of the induction generator, with regard to low-speed machines, are no better—if

anything rather worse—than those of the synchronous generator, and the problem of the economical utilization of the low-head water power still requires solution. It is not solved by the induction generator; the latter's characteristic is simplicity of the station, giving the possibility of numerous small automatic generating stations.

III. Induction Generator Station

A. Characteristics of Induction Generator

An induction motor at no load runs at, or rather very close to synchronism. If it is driven above synchronism by mechanical power, current and power again increase, but the electric power is outflowing, and the induction machine consumes mechanical power, and generates electrical power, as an induction generator.

The maximum electrical power, which an induction machine can generate as an induction generator is materially larger than the maximum mechanical power, which the same machine, at the same terminal voltage, can produce as an induction motor.

Resolving the current of the induction machine into an energy component and a wattless or reactive component, the energy current is inflowing, representing consumption of electric power (which is converted to mechanical power) below synchronism. It becomes zero near synchronism, and above synchronism the energy current is in the reverse direction, or outflowing, supplying electric power to the system (which is produced from the mechanical power input into the machine), and the induction machine then is a generator.

The wattless or reactive component is a minimum at synchronism, and increases with the slip from synchronism, and is in the same direction, whether the slip is below synchronism, as a motor, or above synchronism, as a generator. That is, the induction machine always consumes a lagging current (representing the exciting current and the reactance voltages), or, what amounts to the same, produces a leading current. The latter way of putting it is frequently used with induction generators, by saying that the current produced by the induction generator is leading, while the current consumed by the induction motor is lagging. Instead of saying however, that the reactive component of the current generated by the induction generator is leading, we may say, and this makes it often more intelligible, that the induction generator generates an energy current and consumes a lagging reactive current, while the induction motor consumes an energy current and consumes a reactive lagging current.

As with the increasing voltages and increasing extent of our transmission systems the leading currents taken by transmission lines and underground cables are becoming increasingly larger, the induction generator appears specially advantageous, as tending to offset the effect of line capacity. We may thus say that the induction generator (and induction motor) consumes a lagging reactive current, which is supplied by the synchronous generators, synchronous motors, converters and other synchronous apparatus in the system, and by the capacity of lines and cables. Or we may say that the lagging current consumed by the induction generator neutralizes the leading current consumed by the capacity of lines and cables. Or we may say that the leading current produced by the induction

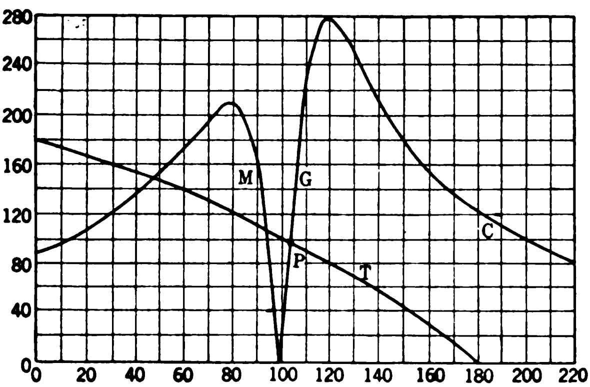


FIG. 2—SMALL HYDROELECTRIC INDUCTION GENERATOR PLANT CONSTANT TERMINAL VOLTAGE

generator supplies the capacity of lines and cables: these are merely three different ways of expressing the same facts.

In Fig. 2 are shown the torque curves, at constant terminal voltage, of a typical moderate size induction machine. *M* is the torque produced as an induction motor below synchronism, and *G* the torque consumed as an induction generator above synchronism, synchronism being chosen as 100 per cent. *T* is an assumed torque curve of a hydraulic turbine.

As seen, the point *P* where *G* and *T* intersect, is 4 per cent above synchronism, and this induction generator thus operates on full load at 4 per cent slip above synchronism or no-load. Assuming now, that the power goes off, by the circuit breakers opening. The turbine then speeds up to 80 per cent above

synchronism, where the curve T becomes zero. If at this free running turbine speed the circuit is closed and voltage put on the induction generator, the high torque consumed by the induction generator causes the turbine to slow down, and as at all speeds above 104, the torque consumed by the induction generator is very much higher than that given by the turbine, the machine slows down rapidly, to the speed where the induction generator torque has fallen to equality with the turbine torque, at speed 104, and stable condition is restored.

Inversely, if the flow of water should cease, the induction machine slows down to a little below synchronism, and there continues to revolve as induction motor.

In starting, the circuit may be closed before admitting the water, and the turbine started by the induction machine as a motor, on the torque curve M, running up to speed 100, and then, by admitting the water, the machine is speeded up 4 per cent more and thereby made to take the load as generator. Or the turbine may be started by opening the gates, running up to speed 180, and then, by closing the circuit, the induction machine in taking the power slows the speed down to normal.

With larger machines, the most satisfactory way of starting, as involving the least disturbance, probably would be, first to open the gates partly while the turbine speeds up, and when it has reached a speed in the neighborhood of synchronism, say between 95 and 105, the circuit is closed and the water gates opened fully.

B. Instability Conditions of Induction Generator

In Fig. 2, the torque consumed by the induction machine, at all turbine speeds above full load P, is much higher than the torque of the turbine. However, the induction generator torque curve has a concave range, marked by C, and if the induction generator should be such as to bring the generator torque curve at C below the turbine torque curve T, the speed, when once increased beyond the range C, would not spontaneously drop back to normal. While in Fig. 2, C is much higher than T, Fig. C represents the theoretical, but not real case of constant terminal voltage at the induction machine. The voltage however is kept constant at the controlling synchronous main station, and thus must vary with the load in the induction generator station. Assuming an extreme case, of 10 per cent resistance and 20 per cent reactance in the line from the induction machine station to

the next synchronism station, we get the modified torque curve shown in Fig. 3. As seen, at full load *P*, there is practically no change; about 4 per cent slip above synchronism. The maximum torque of generator *G* and motor *M*, and the torque at the concave part of the induction generator curve, *C*, have greatly

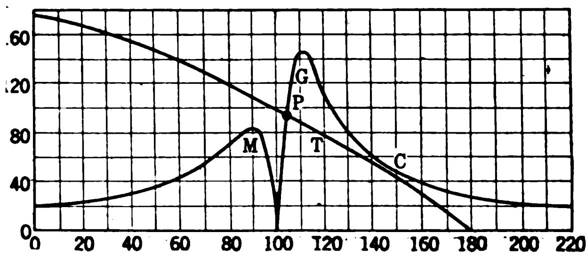


FIG. 3—SMALL HYDROELECTRIC INDUCTION GENERATOR PLANT—CONSTANT VOLTAGE IN SYNCHRONOUS STATION

decreased. However, *C* is still above *T*, that is, even under this extreme assumption, the induction generator would pull the turbine down from its racing speed of 180, to the normal full load speed of 104, though the margin has become narrow.

Assuming however an induction machine with much less slip,

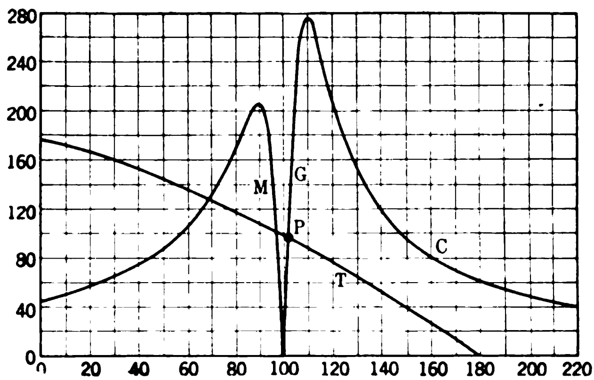


FIG. 4—LARGE HYDROELECTRIC INDUCTION GENERATOR PLANT—CONSTANT TERMINAL VOLTAGE

with only half the rotor resistance of Figs. 2 and 3. At constant terminal voltage, this gives the curves shown in Fig. 4. The full load *P* is at speed 102, or 2 per cent above synchronism, and while the curve branch *C* is much lower, the conditions are still perfectly stable. Assuming however, with this type of low

resistance rotor, a high line impedance, 10 per cent resistance and 20 per cent reactance, as in Fig. 3. We then get the condition shown in Fig. 5. The range C drops below T, and the induction generator torque curve G intersects the turbine torque curve T at three points: P, P_1 and P_2. Of these three theoretical running speeds, $P=102$, $P_1=169$ and $P_2=113.5$, two are stable, P and P_1; while the third one, P_2, is unstable, and from P_2, the speed must either decrease, reaching stability at the normal full load point P, or the machine speed up to P_1.

If with the conditions represented by Fig. 5, the turbine should—by an opening of the circuit for instance—have speeded up to its free running speed 180, closing the circuit does not bring the speed back to normal, P, but the machines slow down only to speed P_1, where stability is reached, at very little output and very large lagging currents in the induction generator. To

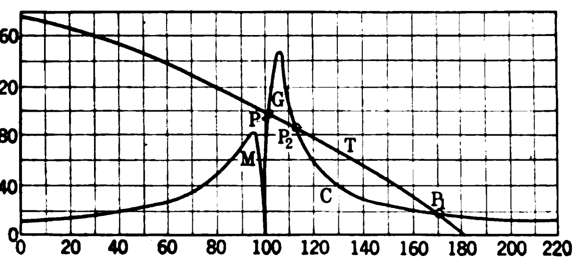


Fig. 5—Large Hydroelectric Induction Generator Plant—Constant Voltage in Synchronous Main Station

restore normal condition then would require shutting off the water, at least sufficiently to drop the turbine torque curve T below C, and then letting the machines slow down to synchronism. They would not go below synchronism, even with the water gates entirely closed, as the induction machine as a motor, on curve M, holds the speed.

A solution in the case Fig. 5 would be the use of a simple excess speed governor, which cuts off the water at 5 to 10 per cent above synchronism.

However, the possibility of difficulty due to the "dropping out of the induction generator" as we may call it in analogy to the dropping out of the induction motor, are rather less real than it appears theoretically. In smaller stations, such as would be operated without attendance, as automatic stations, the torque curve of the induction generator, as a small machine, would be

of the character of Figs. 2 and 3, and thus not liable to this difficulty. The low resistance type of induction machines, as represented in Figs. 4 and 5, may be expected only with the larger machines, used in larger stations. In those, some attendant would be present to close the water gates in case of the circuit breakers opening, or a simple cheap excess speed cut-off would be installed at the turbines, keeping them within 10 per cent of synchronism, and within this range, no dropping out of the induction generator can occur.

It is desirable however to realize this speed range of possible instability of the induction generator, so as to avoid it in the design of induction generators and stations.

APPENDIX

Collection of Fuel Power by Steam Turbine Induction Generator

A. The Automatic Steam Turbine Induction Generator Station

The same reason which in the preceding led to the conclusion that in the (automatic) induction generator station is to be found the solution of the problem of collecting the numerous small amounts of hydraulic energy, which are scattered throughout our country along creeks and mountain streams, also applies, and to the same extent, to the problem of collecting the innumerable small quantities of mechanical or electrical energy, which are, or can be made available wherever fuel is consumed for heating purposes. Of the hundred millions tons of coal, which are annually consumed for heating purposes, most is used as steam heat. Suppose then, we generate the steam at high pressure—as is done already now in many cases for reasons of heating economy—and interpose between steam boiler and heating system some simple form of high pressure steam turbine, directly connected to an induction generator, and tie the latter into the general electrical power distribution system. Whenever the heating system is in operation, electric power is generated, as we may say as "by-product" of the heating plant, and fed into the electric system.

The power would not be generated continuously, but mainly in winter, and largely during the day and especially the evening. That is, the maximum power generation by such fuel power collecting plant essentially coincides with the lighting peak of the central station, thus occurs at the time of the day, and the season when power is most valuable. The effect of such fuel

power collection on the central station should result in a material improvement of the station load factor, by cuttnig off the lighting peaks.

The only difference between such steam turbine induction generator stations, collecting the available fuel power scattered throughout the cities and towns, and the hydraulic induction generator stations collecting the powers of the streams throughout the country, is that in the steam turbine plant an excess speed cut-off must be provided, as the free running steam turbine speed is usually not limited to less than double speed, as is the case with the hydraulic turbine. Otherwise however, no speed governing is required. A further difference is, that the greater simplicity and therefore lower investment of the steam turbine plant would permit going down to smaller powers, a few kilowatts perhaps.

It is interesting to note, that even with a very inefficient steam turbine, the electric generation of such fuel power collecting plant interposed between boiler and heating system, takes place with practically 100 per cent efficiency, because whatever energy is wasted by the inefficiency of the steam turbine plant, remains as heat in the steam, and the only loss is the radiation from turbine and generator, and even this in most cases is useful in heating the place where the plant is located. The only advantage of a highly efficient turbine, is that larger amounts of electric power can be recovered from the fuel, and the question thus is that between the investment in the plant, and the value of the recovered power.

If then the total efficiency, from the chemical energy of the fuel to the electric power, were only 3 per cent, it would mean that 3 per cent more coal would have to be burned, to feed the same heat units into the heating system. At an average energy value of 30,000 kj. per kg. of coal, this would give per ton of coal, 900,000 kj. or 250 kw-hr. At a bulk value of ½ cent per kw-hr. it would represent a power recovery value of $1.25 per ton of coal. This is quite considerable, more than sufficient to pay the interest on the investment in the very simple plant required.

At first, the steam turbine induction generator plant, proposed for the collection of fuel power, would appear similar to the isolated plant which, though often proved uneconomical, still has successfully maintained its hold in our northern latitudes, where heating is necessary through a considerable part of the year. However, the difference between the steam turbine in

duction generator plant and the isolated steam electric plants in our cities, is the same as that between the automatic hydro-electric induction generator station, and the present standard synchronous generator station: by getting rid of all the complexity and complication of the latter, the induction generator station becomes economically feasible in small sizes; but it does so only by ceasing to be an independent station, by turning over the functions of regulation and control to the central main station and so becoming an adjunct to the latter. But by this very feature, the turbo induction generator plant might afford to the central station, the public utility corporation, a very effective means of combatting the installation of isolated plants, by relieving the prospective owner of the isolated plant of all trouble, care and expense and incidental unreliability thereof, supplying central station power for lighting, but at the same time utilizing the potential power of the fuel burned for heating purposes. The simplest arrangement probably would be, that the fuel power collecting plants scattered throughout the city would, as automatic stations, be taken care of by the public utility corporation, their power paid at its proper rates, those of uncontrolled bulk power, while the power used for lighting is bought from the central station at the proper lighting rates.

As this however means a new adjustment of the relation between customer and central station, and is not merely an engineering matter like the hydroelectric power collection, I have placed it in an appendix.

B. Discussion

We realize that our present method of using our coal resources is terribly inefficient. We know that in the conversion of the chemical energy of coal into mechanical or electrical energy, we have to pass through heat energy and thereby submit to the excessively low efficiency of transformation from the low grade heat energy to the high grade electrical energy. We get at best 10 to 20 per cent of the chemical energy of the coal as electrical energy; the remaining 80 to 90 per cent we throw away as heat in the condensing water, or worse still, have to pay for getting rid of it. At the same time we burn many millions of tons of coal to produce heat energy, and by degrading the chemical energy into heat, waste the potential high grade energy which those millions of tons of coal could supply us.

It is an economic crime to burn coal for mere heating without

first taking out as much high grade energy, mechanical or electrical, as is economically feasible. It is this feature, of using the available high grade energy of the coal, before using it for heating, which makes the isolated station successful, though it has every other feature against it. To a limited extent, combined electric and central steam heating plants have been installed, but their limitation is in the attempt to distribute heat energy, after producing it in bulk, from a central station. Here again we have the same rule; to do it efficiently, do it electrically. In the efficiency of distribution or its reverse, collection, no other form of energy can compete with electric energy, and the economic solution appears to be to burn the fuel wherever heating is required, but first take out its available high grade energy, and collect it electrically.

Assume we use 200 million tons of coal per year for power, at an average total efficiency of 12 per cent, giving us 24 million kw. (referred to 24-hr. service) and use 200 million tons of coal for heating purposes, wasting its potential power.

If then we could utilize the waste heat of the coal used for power generation, even if thereby the average total efficiency were reduced to 10 per cent, we would require only 240 million tons of coal, for producing the power, and would have left a heating equivalent of 216 million tons of coal, or more than required for heating. That is, the coal consumption would be reduced from 400 million to 240 million of tons, a saving of 160 million tons of coal annually.

Or, if from the 200 million tons of coal, which we degrade by burning it for fuel, we could first abstract the available high grade power, assuming even only 5 per cent efficiency, this would give us 10 million kw. (24-hr. rate), at an additional coal consumption of 10 million tons, while the production of the 10 million kw. now requires 100 million tons of coal, more or less, thus getting a saving of 90 million tons of coal; or putting it the other way, a gain of 9 million kw.—12 million horse power—24-hr. service, or 36 million horse power for an 8-hr. working day.

It is obvious that we never could completely accomplish this; but even if we recover only one-quarter, or even only one-tenth of this waste, it would be a vast increase in our national efficiency.

Thus the solution of the coal problem, that is, the more economic use of fuel energy, is not only the increase of the thermodynamic efficiency of the heat engine, in which a radical advance

Per ton of coal: Chemical energy, 30×10^6 kj: Heat energy of steam from boiler, at 75 per cent. boiler efficiency, 22.5×10^6 kj.

	Boiler press		Dist. press		Carnot-efficiency per cent.	Output at 50 per cent. efficiency		Value of power at ½c. per kw-hr. $..	Avg. kw., assuming 25% time of use	Tons of coal per kw.	Size of induction generator, per 100 tons coal annually
	atm:	lb.	atm:	lb.		kj. × 1000	kw-hr.				
Steam Heating	6	90	1.25	19	12.3	1380	385	1.92	0.176	5.7	25 kw.
	15	220	1.25	19	19.8	2230	620	3.10	0.283	3.5	45 kw.
Vacuum Heating	6	90	0.48	7	18.1	2030	565	2.82	0.258	3.9	40 kw.
	15	220	0.48	7	25.0	2820	810	4.05	0.37	2.7	55 kw.

is limited by formidable difficulties; but is the recovery of the potential energy of all the fuel, by electric collection.

C. Turbo Induction Generator

Assume then that wherever fuel is burned to produce steam for heating purposes, instead of a low-pressure boiler giving a few pounds over-pressure only, we generate the steam at high pressure, at six atmospheres (90 lbs.) or, in larger plants, even at 15 atmospheres (220 lb.) passing the steam through a high pressure turbine wheel directly connected to an induction generator tied into the electric supply system, and then exhaust the steam at 1.25 atmospheres (19 lb.) into the steam heating system, or at 0.48 atmospheres (7 lb.) into a vacuum heating system.

At a fuel value of the coal of 30,000 kj. per kg. we have (see table)

From this it would follow that the average magnitude of the steam turbine induction generator plant for power collection from fuel in heating plants, would be about one-quarter to one-half kw. per ton of coal burned annually, under the assumption, that the use of the heating plant is equivalent to full capacity during one quarter of the time, and the turbine induction generator plant 50 per cent larger, to take care of maximum loads.

As seen, the value of the recovered power would be a substantial percentage of the fuel cost.

With 100 million tons of coal used for heating purposes annually, assuming an average recovery of 600 kw-hr. per ton, this gives a total of 60,000 million kw-hr. per year. One-quarter of this is more electric power than is now produced at Niagara, Chicago, New York and a few other of the biggest electric systems together.

To be presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 28, 1918.

PROTECTION FROM FLASHING FOR DIRECT CURRENT APPARATUS

BY J. J. LINEBAUGH AND J. L. BURNHAM

ABSTRACT OF PAPER

The equipment developed for the protection of direct-current apparatus as described in this paper is applicable to all direct-current apparatus and all methods of operation. Special means of protection for use only with particular apparatus or conditions of operation have not been mentioned. The principal steps in the experimental development of high-speed circuit breakers and flash barriers are briefly given.

The protection afforded by the high-speed breaker or barriers is sufficient for most apparatus and service, but *complete* protection for *any* direct-current apparatus and service requires both the high-speed breaker and flash barriers. Attention is directed to the importance of arranging the connections to the brush rigging so that the magnetic action on the arc will be a minimum, and properly directed, so the flash will do the least damage.

THE problem of protection from flashing has for many years confronted engineers who build and operate direct-current machines. Numerous schemes and suggestions have been put forward which it was hoped would overcome the tendency to flashover on extra heavy overloads or short circuits. Some time ago it was felt that the subject of prevention and protection from flashing has not received the study and investigation justified by the trouble experienced and it was decided to make a comprehensive study of the entire subject.

Some form of barrier has been the most common protection suggested, and different forms have been tried with a slight degree of success on some machines and absolute failure on others. It was the opinion of many engineers that barriers could not be designed to take care of a short circuit and that their value was doubtful. However, a special form of barrier, which gives the required protection, will be described later.

It was realized that the means for prevention of flashing at the commutator and brushes of direct-current machines must operate to remove the cause very quickly. The use of some form of high-speed device, which would open the circuit or insert re-

Manuscript of this paper was received April 25, 1918.

sistance before the short circuit current could reach a value which would cause flashing, seemed the most logical way to solve the problem, although it was appreciated that the action of the device must be much more rapid than any commercial circuit-opening device previously produced. An investigation was conducted along these lines and two distinct types of high-speed breakers developed, which will be described separately.

A flash at the commutator starts from excessive sparking. Sparking is produced by the breaking of current in the coils short-circuited by the brush as each segment of the commutator passes from under the brush. As the coil is inductive, the spark or arc tends to hold and, if the arc is of sufficient volume, the vapor produced thereby forms a low resistance path between segments and from brush to brush or to frame; through which a large current may pass. See Figs. 1 and 2.

Sparking may be prevented by providing a magnetic field of proper strength and distribution to influence the coils during reversal of their current as they pass through short circuit by the brushes. To provide the correct commutating field for all conditions of load has been the object of designers but success has been only partial. At high loads, saturation of magnetic circuits and distorting influence prevent attainment of the desired field, and for sudden changes in load the changes in field cannot be properly synchronized. It is more difficult to avoid sparking with rapidly varying loads than with gradually changing or steady load, but if a sudden load which would cause flashing is of short enough duration, the arcing at brushes may not produce enough conducting vapor to establish an arc supported by the main voltage. *The value of load that causes flashing when applied suddenly (short circuit) is a function of the time required to throw it off.* The quicker the circuit is opened the higher the value of current that will not cause arcing.

With the ordinary circuit breaker which begins to open in about 0.15 second, there is a certain maximum load which cannot be exceeded for each commutating machine without causing flashing. If feeders have sufficient resistance to limit the short-circuit current to this critical value, flashing will occur only on the rare occasion of a short circuit in a feeder itself. See Fig. 3. It has been the standard practise of nearly all manufacturers to recommend tapping the feeders, especially railway feeders, at a sufficient distance from the substation to insure enough resistance in the circuit to limit current in case of short circuit near the station.

Inductance may be added to the circuit to retard the rate of increase of current on short circuit to such an extent that the ordinary breaker will have time to trip before the current in the machine reaches a value that would cause flashing. The amount of inductance required to delay the rise in current sufficiently, however, introduces other disadvantages which make its use undesirable. When the current is interrupted, the increase in voltage from inductive "kick" is difficult for circuit breakers to handle and introduces the possibility of applying dangerous voltage stresses to the apparatus.

Reactors have been tried in a few instances with some success but it has always been a mooted question whether the resistance of the reactor did not give as much or more protection than the inductance of the coil, and if this is the case resistance only would be much cheaper to install. A coil to give the delay required is usually very large and expensive and occupies much valuable space, giving a total cost out of proportion to the cost of the machines protected or the protection obtained.

With special high-speed circuit-opening devices operating in about 0.005 second, the more sensitive machines, such as 60-cycle synchronous converters for railway voltages, may be short-circuited without flashing over, even though the maximum current is of higher value than would cause flashing with suddenly applied load and ordinary circuit-breaker protection.

The speed at which a circuit breaker must operate to prevent flashing depends on the amount of load thrown on the machine but, under worst conditions, our tests seem to confirm that it must be *quicker* than one half cycle of the machine to be protected. The time of operation of the breaker would be measured between the time that the current reaches the flashing value to the time that the current is again reduced to the same value after the breaker opens. If the arc formed between two segments is not blown out as they pass from one set of brushes to the next and all following segments have similar arcs formed between them, the arc would completely bridge between positive and negative brushes in one-half cycle, which would complete the flashover. Complete flashover might also occur from gases being blown by windage, magnetically, or by expansion, to increase or decrease the half cycle time.

The time of operation of circuit breakers as given herein is measured from the beginning of short circuit to the instant the breaker begins to reduce the current rise.

Investigation covering these several schemes of protection was made, which it is believed will be of interest and will be described briefly with oscillograms, reproductions from photographs, etc., showing behavior under different loads and short-circuit conditions.

All short-circuit tests were made by connecting positive and negative terminals with a 500,000 circular mil cable; the only equipment in the circuit being the necessary current shunt for the oscillograph, a contactor to close the circuit, and a circuit breaker for overload protection, in addition to the protective device being investigated. Power for the 300-kilowatt, 25-cycle and 500-kilowatt, 60-cycle, 600-volt synchronous converters, used in fuse, barrier, reactor, and high-speed circuit breaker tests, was supplied from a 6000-kilowatt frequency changer set only a few feet from the test, so that there was very little drop in the voltage of the generator or from resistance, and the oil switch was set so that it did not trip out.

High Speed Circuit Breaker

At the time this development was started it was felt that if a circuit breaker could be designed to operate *within* the time required for a commutator bar to pass from one brush to another; that is, within one half cycle, protection would be afforded against practically any short circuit. Designs were therefore begun on a circuit breaker which would open within 0.007 second, which would cover most commercial machines; *i.e.*, for 60 cycles and lower frequency.

High-speed breakers had been suggested and attempts made to produce such devices previous to this time but, as far as the writers know, had never been made to obtain as high speed as the discussion shows would be necessary.

Different types of construction were studied and samples of several preliminary models constructed without obtaining the speed desired. One of the most promising types of construction considered consisted of a knurled fly wheel operating continuously with a knurled cam, so designed and located that a current relay would insert a wedge between the wheel and the cam and trip a breaker attached to the cam by suitable toggle mechanism. This preliminary sample indicated that 0.035 second was the best speed that could be attained.

It was then decided to concentrate all energies on a circuit breaker using the well known principle of a latch, heavy spring

FIG. 1

Flashing at brushes on 1000-kw., 1500-volt generator forming part of 2000-kw., 3000-volt motor-generator set at five times load, showing different stages of arc formation.

FIG. 2 [LINEBAUGH AND BURNHAM]

High-speed photograph of flashing on 300-kw., 600-volt, 25-cycle synchronous converter with short circuit on 0.015 ohms additional in the external circuit and standard circuit breaker.

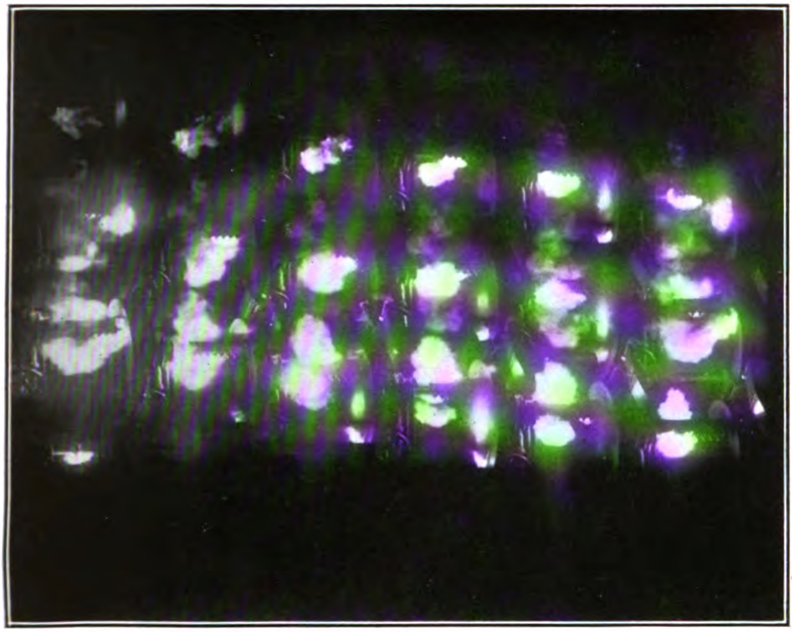


FIG. 3

High speed photograph of short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter with standard circuit breaker.

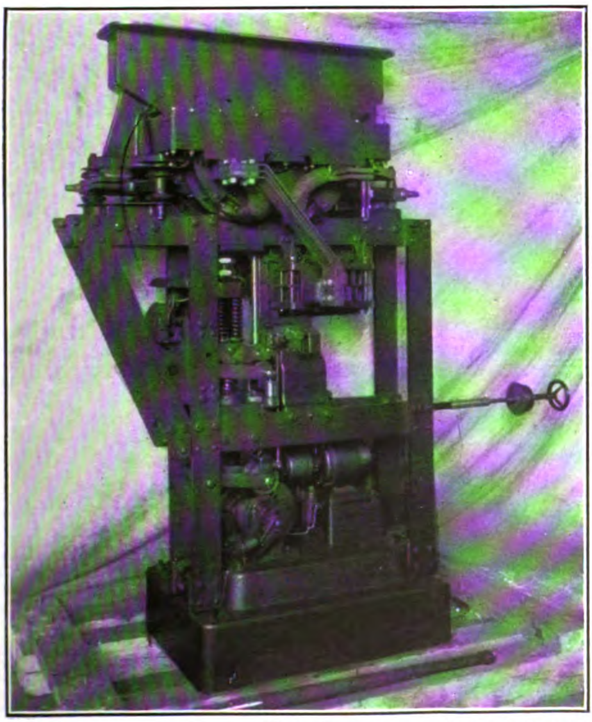


FIG. 4

[LINEBAUGH AND BURNHAM]

3000-ampere, 3600-volt, direct-current high-speed circuit breaker.

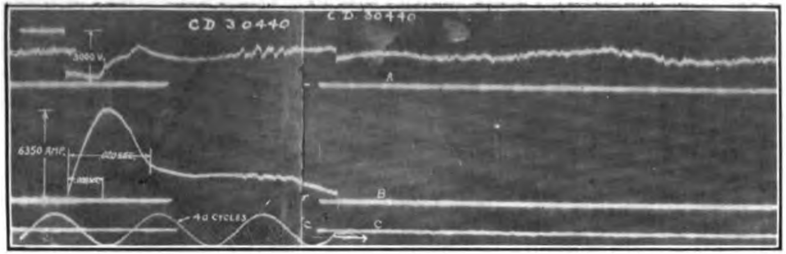


FIG. 5

Direct-current short circuit on 2000-kw., 3000-volt, motor-generator set with high-speed circuit breaker and standard 3000-volt switchboard type circuit breaker.

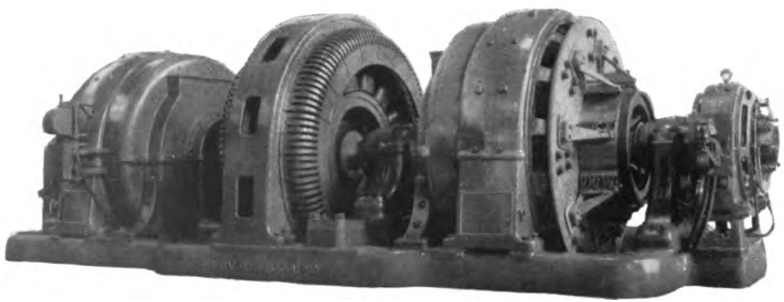
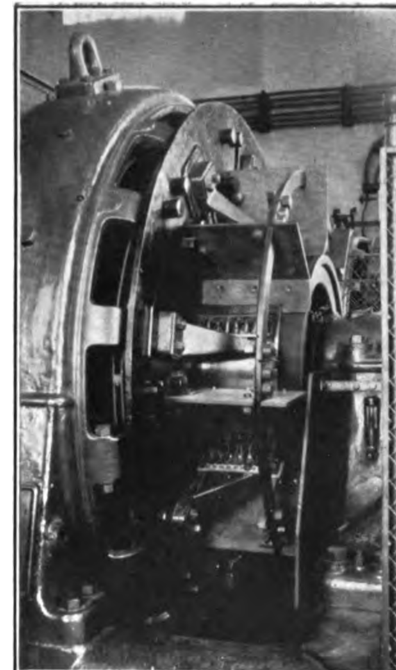


FIG. 6

2000-kw., 3000-volt, direct-current synchronous motor-generator set before assembly of flash-barriers.



[LINEBAUGH AND BURNHAM]

FIG. 7

Type of flash barrier installed on 2000-kw., 3000-volt, synchronous motor-generator set used in connection with high-speed circuit breaker.

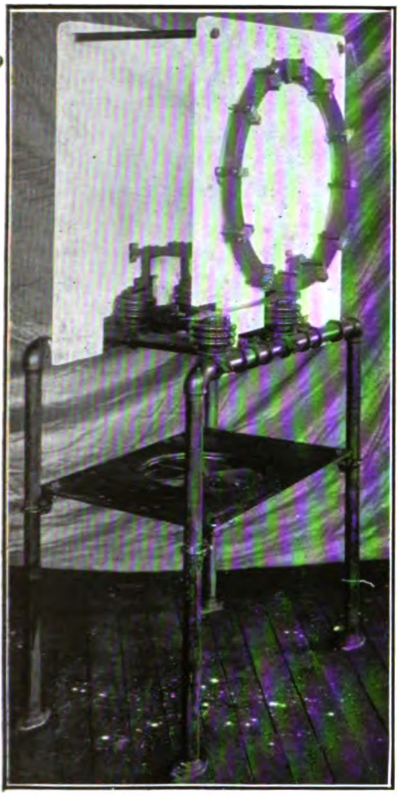


FIG. 9
High-speed air cooled fuse holder with magnetic blow-out used in test.

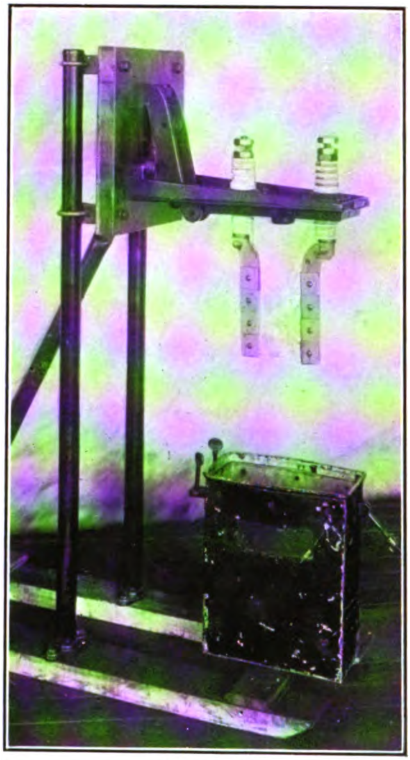


FIG. 10
High-speed oil-cooled fuse holder, used in test.

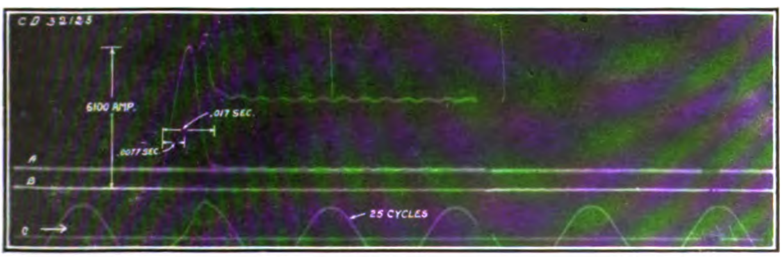


FIG. 11 [LINEBAUGH AND BURNHAM]

Short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter protected by air-cooled high-speed fuse. Curve A, voltage across fuse; Curve *B*, line current; Curve *C*, collector-ring voltage.

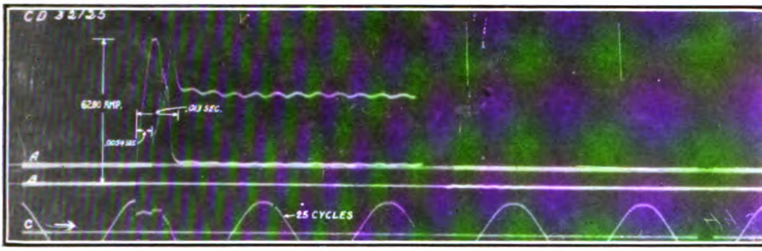


FIG. 12

Short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter protected by oil-cooled high-speed fuse. Curve A, voltage across fuse; Curve *B*, line current; Curve *C*, collector-ring voltage.

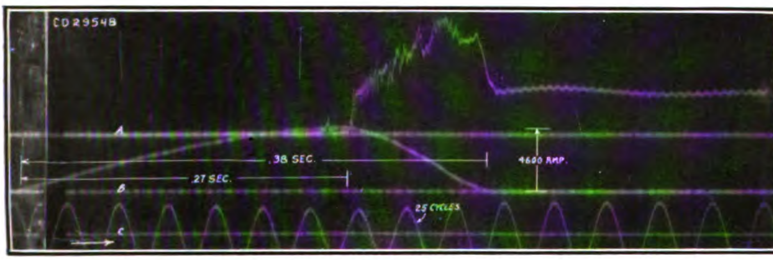


FIG. 13

Short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter protected by air-core reactor in direct-current circuit and standard circuit breaker. Curve *A*, voltage across circuit breaker; Curve *B*, line current; Curve *C*, collector-ring voltage.

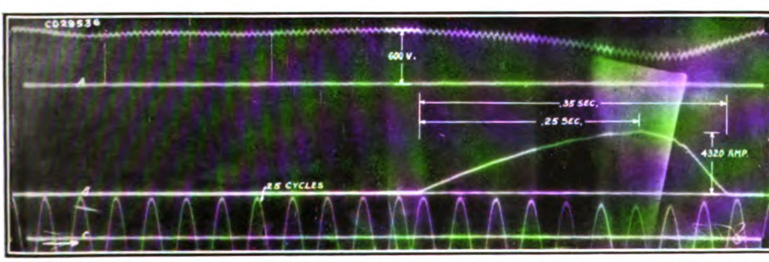


FIG. 14

Short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter protected by air-core reactor in direct-current circuit and standard circuit breaker; Curve-*A*, voltage across armature; Curve *B*, line current; Curve C, collector-ring voltage.

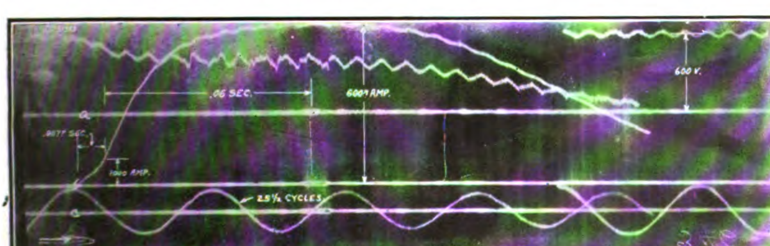


FIG. 15 [LINEBAUGH AND BURNHAM]

Short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter, protected by iron-core reactor in direct-current circuit and standard circuit breaker. Curve *A*, voltage across the armature; Curve *B*, line current; Curve *C*, collector-ring voltage.

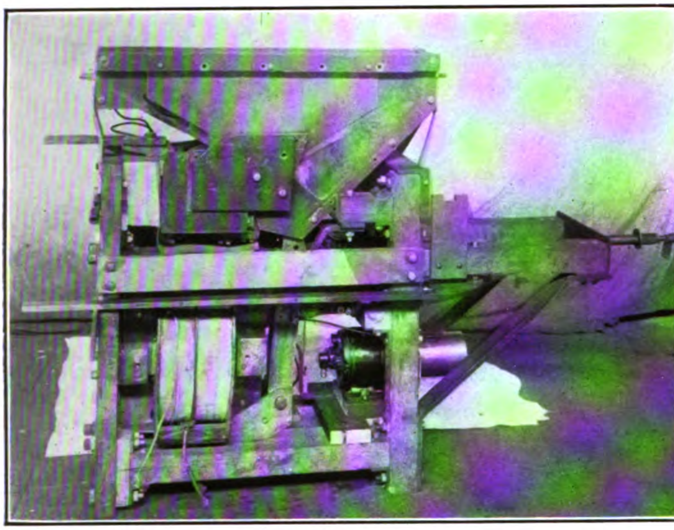


FIG. 16

Second form of high-speed circuit breaker, capacity 1500 amperes, 600 volts.

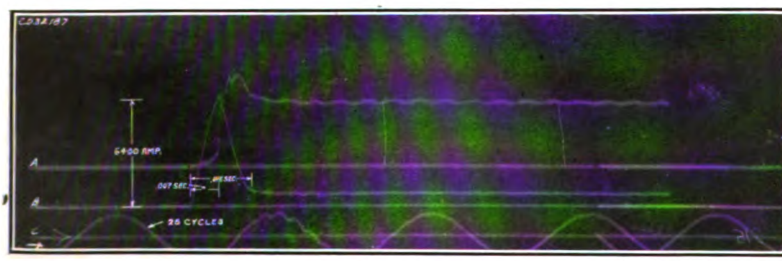


FIG. 17

Short circuit on 300-kw., 600-volt, 25-cycle synchronous converter protected by second form of high-speed circuit breaker. Curve A, voltage across circuit breaker; Curve B, line current; Curve C, collector-ring voltage.

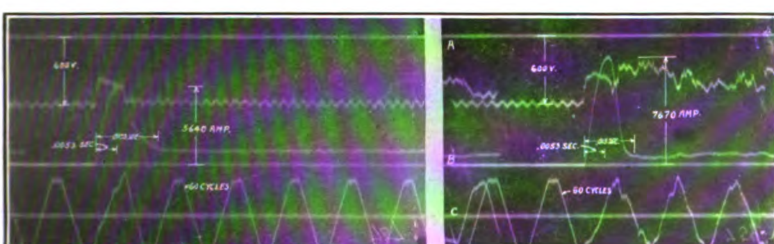


FIG. 18 [LINEBAUGH AND BURNHAM]

Short circuit on 500-kw., 600-volt, 60-cycle, synchronous converter protected by second form of high-speed circuit breaker.

Left hand curve
Load of 0.03 ohms

Right hand curve
Short circuit.

Curve A, armature volts
" B, line current
" C, collector-ring voltage.

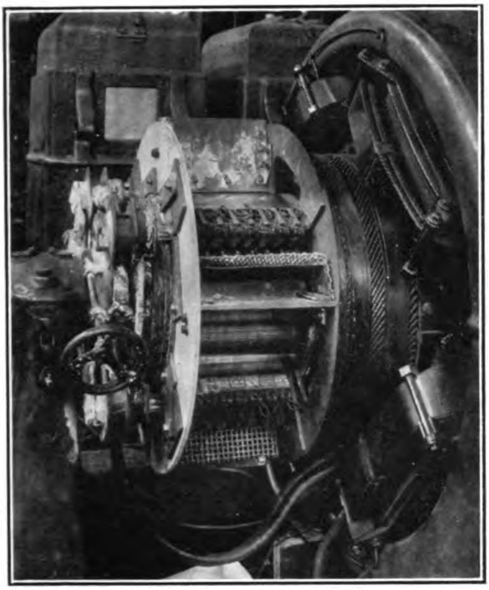


FIG. 19
Final development of flash barriers on 300-kw., 25-cycle, 600-volt synchronous converter.

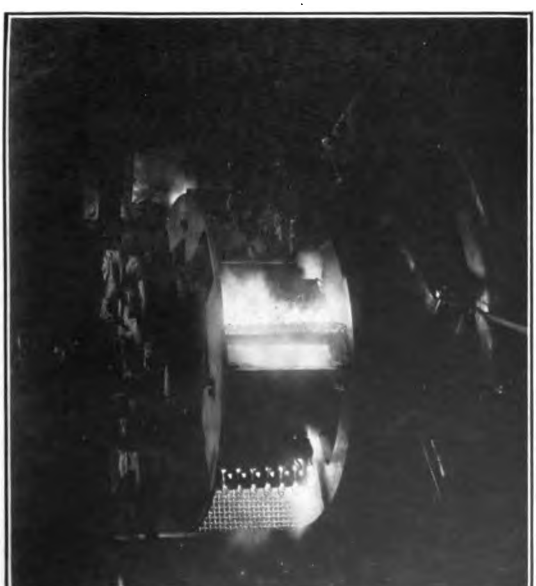


FIG. 20 [LINEBAUGH AND BURNHAM]
Short circuit on 300-kw., 25-cycle, 600-volt, synchronous converter protected by flash barriers and standard circuit breaker.

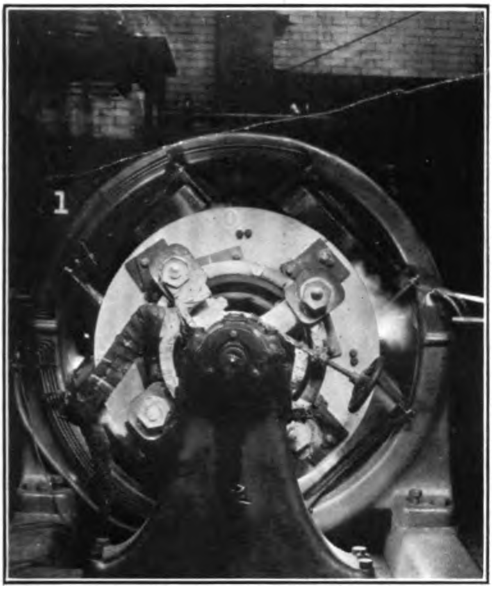


FIG. 21

Short circuit on 300-kw., 25-cycle, 600-volt synchronous converter protected by flash barriers and standard circuit breaker.

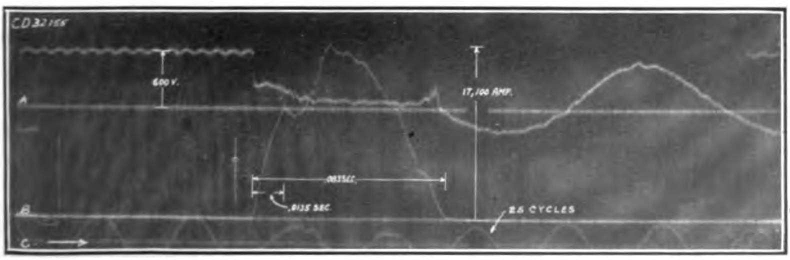


FIG. 22 [LINEBAUGH AND BURNHAM]

Short circuit on 300-kw., 600-volt, 25-cycle, synchroous converter equipped with flash barriers and standard circuit breaker. Curve A, armature volts; Curve B, line current; Curve C, collector-ring voltage.

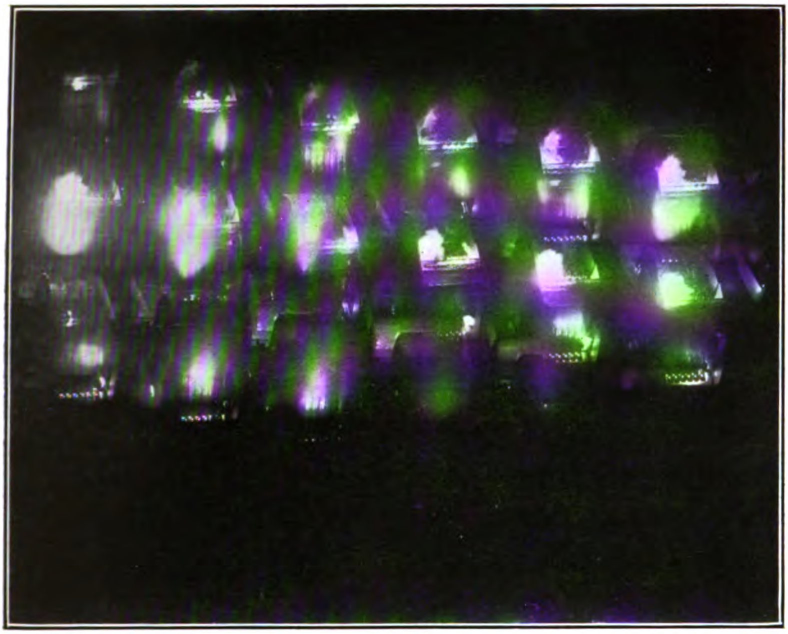


FIG. 23
High-speed photograph of short circuit on 300-kw., 600-volt 25-cycle synchronous converter protected by flash barriers and standard breaker.

FIG. 24 [LINEBAUGH AND BURNHAM]
High-speed photograph of short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter protected by flash barriers and standard circuit breakers.

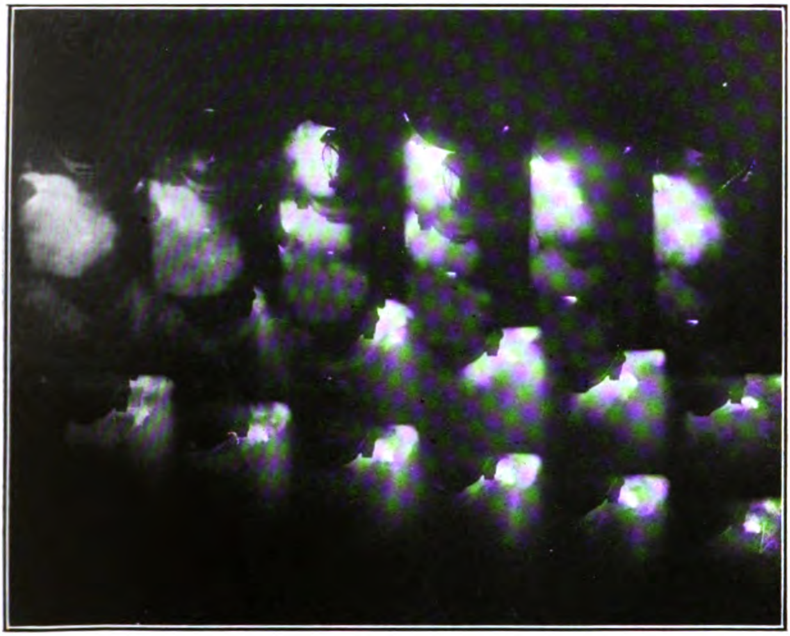


FIG. 25
High-speed photograph of short circuit on 500-kw., 600-volt, 60-cycle synchronous converter without protection.

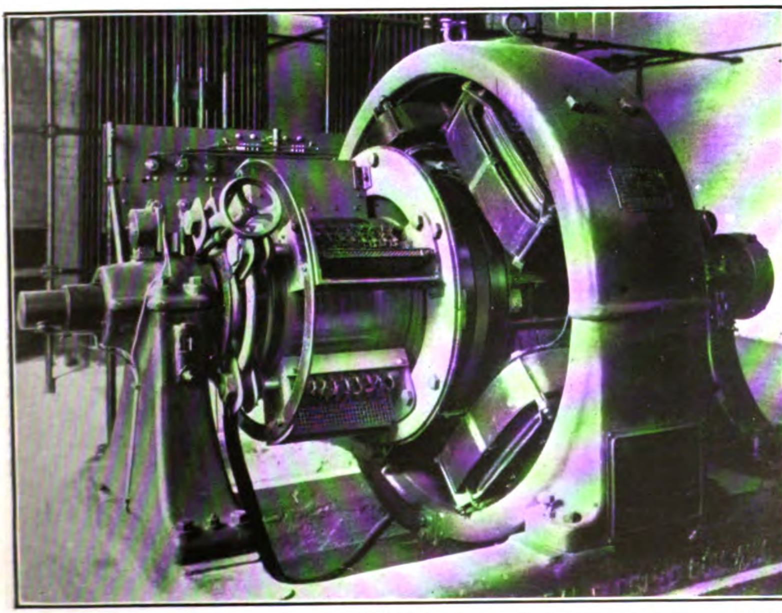


FIG. 26 [LINEBAUGH AND BURNHAM]
500-kw., 25-cycle, 600-volt, synchronous converter, installed in automatic substation, equipped with commercial form of flash barrier.

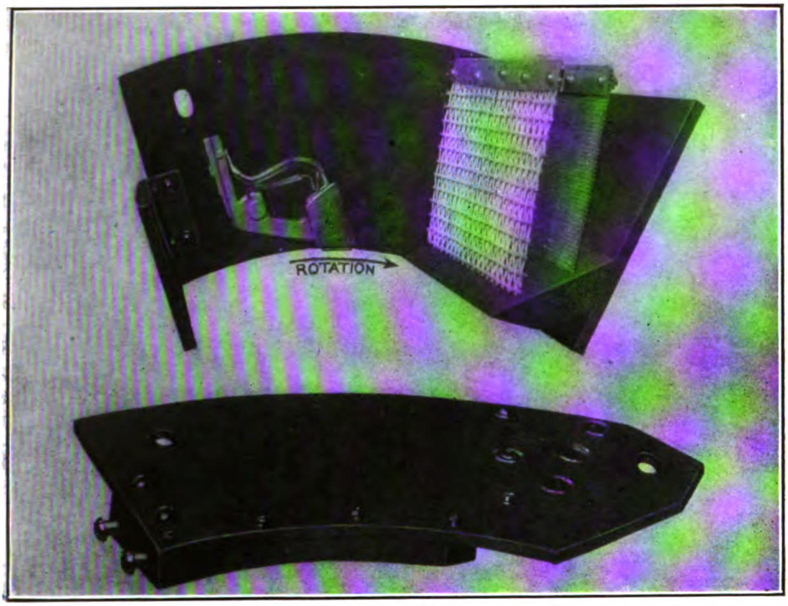


FIG. 27

Flash barrier with front removed to show location and construction of arc scoop and wire-mesh arc coolers.

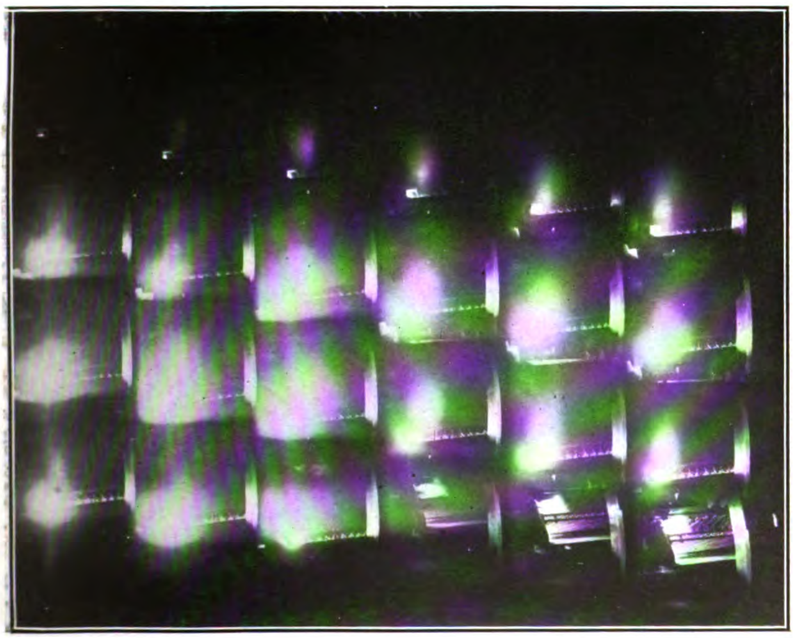


FIG. 28 [LINEBAUGH AND BURNHAM]

High-speed photograph of short circuit on 500-kw., 600-volt, 60-cycle, synchronous converter with flash barriers and standard circuit breaker with preliminary arrangement of brush rigging. Arc at outer end of commutator.

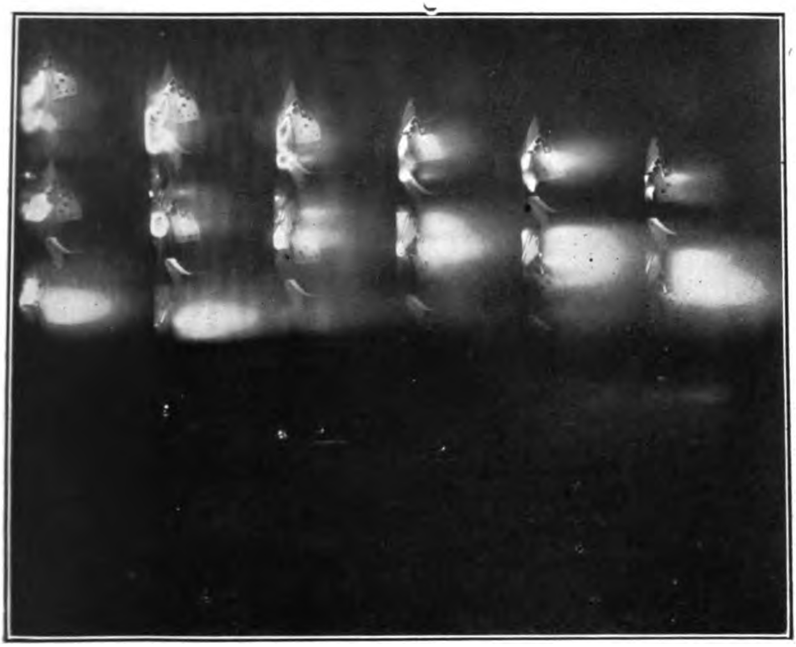


FIG. 29
High-speed photograph of short circuit on 500-kw., 600-volt, 60-cycle synchronous converter with flash barriers and standard circuit breaker with preliminary arrangement of brush rigging. Arc at outer end of brush rigging.

FIG. 30 [LINEBAUGH AND BURNHAM]
High-speed photograph of short circuit on 500-kw., 600-volt, 60-cycle synchronous converter protected by flash barriers and standard circuit breaker after arrangement of brush rigging has been changed. Uniform distribution of flashing.

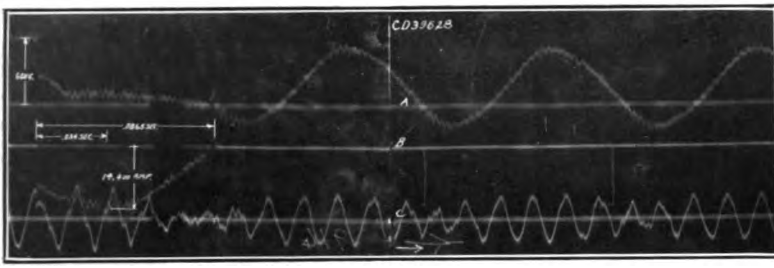


FIG. 31

Short circuit on 500-kw., 600-volt, 60-cycle, synchronous converter protected by flash barriers and standard circuit breaker after arrangement of brush rigging had been changed

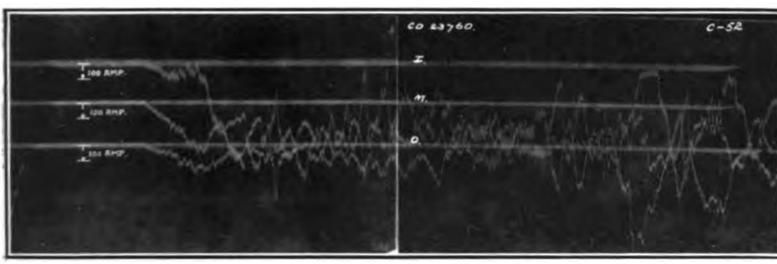


FIG. 32

Short circuit on 50-kw., 600-volt generator. Curve *O*, current in outside brush; Curve *M* current in middle brush; Curve *I*, current in inside brush.

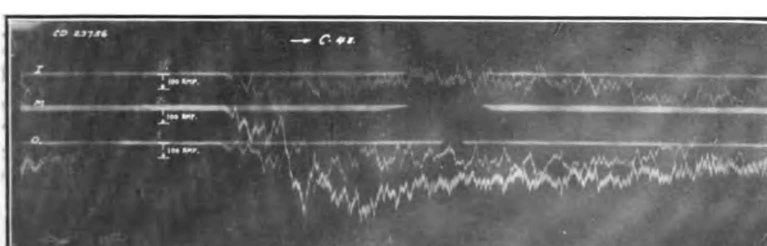


FIG. 33 [LINEBAUGH AND BURNHAM]

Short circuit on 50-kw., 600-volt generator. Curve *O*, current in outside brush; Curve *M*, current in middle brush; Curve *I*, current in inside brush.

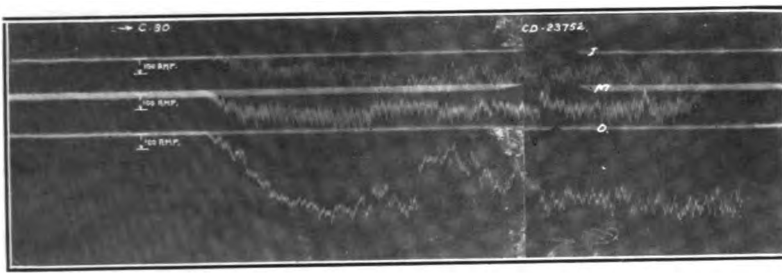


FIG. 34

Short circuit on 50-kw., 600-volt generator. Curve O, current in outside brush; Curve M, current in middle brush; Curve I, current in inside brush.

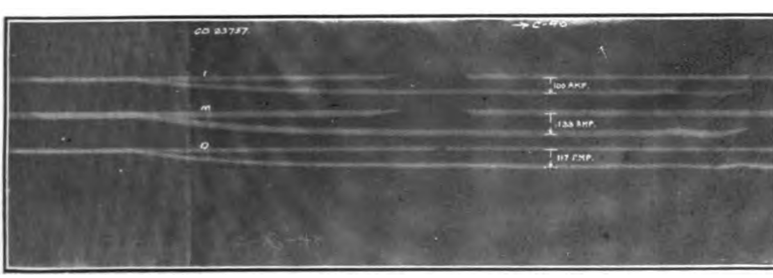


FIG. 35 [LINEBAUGH AND BURNHAM]

Current passed through 50-kw., 600-volt generator from an external source.

and series tripping coil, and the high speed breaker shown in Fig. 4 was finally built.

The problem was to obtain very quick tripping, rapid acceleration of contacts and a sufficient number of ampere turns in the magnetic blowout to insure rapid breaking of the arc. Previous ideas of design had to be abandoned when working for such high speed when a loss of 0.001 second meant a very serious increase in time of operation.

It was found that a series blowout coil had to be used, as sufficient time could not be allowed for the building up of a field after the contacts opened as is ordinarily done in circuit-breaker design, and the strength of this coil must be many times that usually used to rupture the circuit by giving the quick start and acceleration to the arc necessary for the speed desired. The breaker in question has a total of about 150,000 ampere turns at the maximum current obtained.

The moving parts must all be as light as possible, consistent with the great strength required, so that they can be started, accelerated and stopped in a very short space of time and distance. Even with this type of construction, it was found necessary to use somewhat high spring pressure; the spring being compressed to about 8000 pounds when the breaker was closed and ready for tripping.

A very special latch with very small tripping movement was designed somewhat similiar to the hair trigger on a rifle, in connection with a special high-speed tripping coil so that about 0.001 inch movement of the plunger would trip the breaker. It will assist in appreciating the speed attained when it is noted that the breaker must be arranged so that it will not trip under ordinary load condition and must be set above the tripping point of the regular substation breaker so that it will act while the current is increasing from say three and one-half times load to eight times load; current rising at the rate of about 1,000,000 amperes per second. Fig. 5 gives a very good idea of speed and limiting of current, from which it will be seen that the breaker starts to insert resistance in about 0.008 second and the load on the machine is reduced well below the flashing value in 0.020 second after the short circuit was applied.

A breaker was tested very exhaustively in connection with a 2000-kilowatt 3000-volt direct-current synchronous motor-generator set shown in Fig. 6, built for the Chicago, Milwaukee & St. Paul electrification, and found to give complete protection

from damage or burning on short circuit when equipped with barriers shown in Fig. 7.

In connection with the test, it was found that even the speed of 0.008 second obtained would not completely protect machines from flashing on the most severe short circuit, and barriers shown were designed and installed. Tests referred to with high-speed breakers were taken with these barriers, which will be described later.

It is evident that it is preferable in case of short circuit to insert resistance by a high-speed breaker to quickly limit the current to some conservative value and then open the circuit. This type of protection has been adopted as standard. All tests, investigations, etcetera, were based on this theory, although some tests were taken by opening the circuit. It was found that there was a greater tendency for machine to flash if the circuit was opened completely at one time or if too high resistance was inserted, reducing the load to too low a value. For the sake of convenience and comparison, all tests were made throwing short circuit on the machine without load.

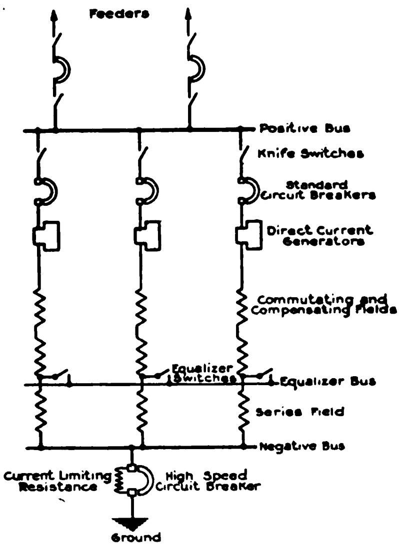


FIG. 8

Diagram of direct-current connections for substation equipped with three motor-generator sets protected by one high-speed circuit breaker connected across limiting resistance

Some of these breakers have been in service since early in 1917 in the substations of the Chicago, Milwaukee & St. Paul Railroad and have amply justified the faith of the railroad company and the designers, as they protect the apparatus from all short circuits experienced, although all feeders are tapped directly to the overhead trolley system immediately at the substation.

One of these breakers is installed in each of the substations connected between the negative bus of the station and the ground or return circuit, as this location gives maximum protection, and one breaker can be used for each machine or one for the entire substation, as shown in Fig. 8.

High Speed Fuse

It is evident that if a fuse could be developed that would melt at a very small increment of current above normal rating, it might be possible to obtain a speed which would limit the current on a short circuit along the same line as the high-speed circuit breaker just described.

A careful study of all available metals was made by Mr. P. E. Hosegood, who suggested using a silver fuse, and a number of silver fuses of different shapes were tried in the special fuse holders shown in Figs. 9 and 10. The oscillograph record, taken with air break fuse and magnetic blow-out, shown in Fig. 11, indicates that a very high speed is obtained, giving excellent protection and duplicating almost exactly the speed of the high-speed circuit breaker. It was found that a short circuit could be thrown on the 300-kilowatt, 25-cycle, 600-volt synchronous converter without flashing over and with very little sparking at the brushes. The oil-immersed fuse holder without magnetic blow-out gave practically the same result (Fig. 12), the operation being slightly better as far as speed was concerned but the mechanical difficulties of replacing the fuse, etc., being greater.

Reactors

Oscillograph records of short circuit on the 300-kilowatt, 25-cycle, 600-volt synchronous converter show an average initial current rise of about 1,300,000 amperes per second. To protect by reactance, the amount required would depend on the rate of circuit-breaker action. With coils made of 1000 feet of 500,000-circular mil cable, wound on cable reels having an inductance of approximately 0.02 henry in circuit, this particular machine could be short circuited without flashing when protected by a breaker opening in about 0.15 seconds.

An examination of records, Figs. 13 and 14, will show the severe duty on the circuit breaker and increase in voltage on the apparatus.

It was suggested that shunting the reactor by resistance might reduce duty on the circuit breaker. The coils were shunted by 14 and by 100 ohms and it was impossible to determine from either observation or oscillograph any effect due to the resistance.

The effect of an iron core in a reactor having an inductance of 0.00105 henry is shown in Fig. 15, from which it will be noted that the iron saturated at about 1000 amperes in about 0.007

seconds, after which the current rises abruptly, being limited only by the inductance of the coil as if there were no iron in its magnetic circuit. The delay of about 0.007, seconds due to the presence of iron in the coil, is far less than the time required for the usual breakers, now in use, to open. The weight of this reactor was 7 per cent of the weight of the synchronous converter and would have to be many times larger to give protection with an ordinary breaker.

Second Form of High-Speed Circuit Breaker

Mr. J. F. Tritle has more recently suggested a design for a high speed circuit breaker which is simple and substantial in construction. This device was built as shown in Fig. 16 and test indicated that the speed was even faster than the large breaker previously described, as will be seen by comparing oscillograms, Fig. 5, on the large breaker, and Fig. 17 taken with the later breaker. This device is essentially a contactor having a laminated structure with electric holding coil and series bucking coil so that it opens when the load current reaches a value sufficient to offset the ampere turns of the holding coil. Tests on the 300-kilowatt, 25-cycle synchronous converter with this device showed that a short circuit could be thrown on the machine without any tendency of the machine to flash over, and the only sparking obtained extended not over one-half inch from the brushes. Similar tests, Fig. 18, on the 60-cycle, 500-kilowatt synchronous converter showed more sparking and, although it protected the machine at times on short circuit, there were other times when the machine flashed over. When the machine was equipped with barriers, dead short circuit could be thrown on with impunity, there being no tendency to flash over and scarcely sufficient sparking to be noticeable.

This later type of high-speed breaker is a part of the more recent equipment being furnished the Chicago, Milwaukee & St. Paul Railway.

Barriers

The barriers shown in Fig. 7 in connection with the description of the high-speed circuit breaker were developed to delay time of flashover, so that the breaker would give complete protection. Such satisfactory and promising results were obtained without the breaker that it was decided to continue investigation to ascertain if it would be possible to devise barriers

that would take care of all short circuits experienced in actual service.

Under certain conditions it might be desirable to supplement rather than replace, appliances already installed or to protect from disturbances other than direct-current load which cause flashing. For instance, a synchronous converter could not be protected by a high-speed direct-current circuit breaker if flashing is caused by a.c. phase displacement. For this reason additional protection, such as barriers, to dissipate the arc when started was also needed.

Many different forms of barriers were tried on the 300-kilowatt, 25-cycle, 600-volt synchronous converter, previously mentioned. With increasing success as improvements were made to meet failures, the barriers shown in Fig. 19 were evolved. These barriers gave complete protection from flashover or damage on short circuit. Fig. 20 shows machine on short circuit giving a good idea of flashing and protection afforded, while Fig. 21 shows clearly the small amount of flash which extends beyond the barrier.

About 65 short circuits were thrown on the 300-kilowatt, 600-volt, 25-cycle machine without burning of brushes, brush connections or rigging, or damages of any kind to commutator or machine. Oscillogram, Fig. 22, shows a record of current reaching 34 times full load and gives a good idea of the protection afforded. Many of these short circuits were applied at very short intervals, even as close as one minute apart, without failure to hold and extinguish the arc when the breaker opened the circuit.

Figs. 23 and 24 are very interesting high-speed pictures of the same short circuit analyzed by means of a special high speed camera devised by Lieut. Chester Lichtenberg, and the successful high speed pictures we are able to show in this paper are mainly due to his efforts. This camera made it possible to obtain as high as 24 complete pictures of one short circuit, while the best results it was possible to obtain with a moving picture camera were two under-exposed and therefore indistinct pictures.

A little explanation is necessary to read these photographs as, due to the construction of the camera, the lower right hand picture is the first picture of the short circuit; the next picture being the one immediately to the left, and so on to the end of the plate; the first picture at the right of the the next row being

the next picture in the same order and until the end of the plate and the number of rows of pictures. These pictures show very clearly the growth of the arc, disposition on commutator and dissipation of the arc as the regular breaker opens. These permanent records eliminated the personal factors of memory and observation and showed the way for changes to give improvements in barriers. Fig. 25 illustrates very clearly what happens if the machine is short circuited without protection.

The general arrangement of a successful barrier, Fig. 26, is shown herewith.

A close fitting box of fire-proof insulating material surrounds each set of brushes and is located so as to give a small clearance between the box and the commutator.

On the side of the box towards which the commutator rotates after leaving the brush is fastened a V-shaped "scoop", Fig. 27, of fire-proof insulating material, preferably having good heat conductivity, pointing toward the brush and having small running clearance from the commutator.

Radially above the scoop, about one inch apart, are two metal screens, one coarse and one fine mesh, through which the arc is successively forced and cooled.

It was found that a moderate amount of material is required to give the necessary thermal capacity to prevent an arc from passing beyond a screen of this kind. The scoop running very close to the commutator with narrow edge and small clearance picks up the arc from the commutator and deflects it into the arc coolers which, from their construction, allow free passage of all gases generated by the arc. The cooling and condensing of the arc reduces the gas pressure so that shields at the end of the commutator, to prevent the arc being thrown from the end of the commutator and communicated to pillow block and frame, are permissible. It will be noted from the illustrations that the commutators extend beyond the end of the barrier as it was found that the arc must be prevented from being communicated to the end of the bars.

Investigation was then transferred to a 500-kilowatt, 60-cycle, 600-volt synchronous converter and barriers of similar type, but without continuous end shields, were tried.

Tests showed that these barriers did not give protection on short circuit although they prevented machine from flashing over on very high overload. The high-speed camera record in-

dicated that the arc was being thrown to the outer end of the commutator for some reason, causing such high gas pressure at the outer end of the commutator that the arc was blown under the barrier and the machine flashed over. Figs. 28 and 29.

The differences in performance was ascribed to differences of magnetic fields acting on the arc.

To demonstrate the effect of the magnetic field, various arrangements of connections of brush rigging were made, each to produce a different field where the arcing occurs. The results indicate that it is possible to arrange the brush rigging and connection to make a barrier, as described above, effective on practically all commutating machines and to prevent complete flashover. Figs. 30 and 31 show the effects of change in connection on arc distribution, giving the uniform distribution most favorable to good barrier performance.

Other tests were made to record the simultaneous short circuit current in the outer, middle and inner brushes by the oscillograph. The records in Figs. 32, 33 and 34 show typical variations of current distribution produced by different connections to brushes. The distribution of current is principally dependent on the magnetic field surrounding the brushes where the arc is formed. To show that differences of impedance have very little influence, record Fig. 35 was taken with current supplied from an exterior source with no flashing. It will be seen that current is practically the same in all brushes. With some connections the deflection of the arc can be plainly seen to follow the well-known relation, as given in Fig. 36, but with the more complicated connections the difficulty of determining resultant field from many sources makes it difficult to determine the direction of deflection of the arc except by experiment.

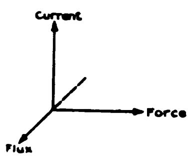


FIG. 36

Direct-current machines for use in * automatic substations are being equipped with these barriers and short-circuit tests at the substations have been taken, indicating that they will take care of any short circuit experienced in actual service. These barriers are in operation and short-circuit tests were taken on a 500-kilowatt, 600-volt, 25-cycle synchronous converter of the Des Moines Electric Railway, Des Moines, Iowa, a 500 kilowatt, 600-volt, 60-cycle synchronous converter of the Coumbus Electric Railway & Light Company, Columbus, Ohio, and a 500-kilowatt,

30-cycle, 1200-volt synchronous converter at Montieth Junction, Michigan, and other installations are now in service.

The investigations and tests indicate that if any commutating machine is equipped with barriers and the last high-speed circuit breaker described, complete protection will be given against external short circuits of all kinds so that interruption to service will not be of any greater duration than necessary for closing the circuit breaker as in ordinary overload operation.

*See paper by Taylor and Allen, A.I.E.E. TRANSACTIONS Vol. 34, 1915, page 1801.

To be presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 28, 1918.

METHOD OF SYMMETRICAL CO-ORDINATES APPLIED TO THE SOLUTION OF POLYPHASE NETWORKS

BY C. L. FORTESCUE

ABSTRACT OF PAPER

In the introduction a general discussion of unsymmetrical systems of co-planar vectors leads to the conclusion that they may be represented by symmetrical systems of the same number of vectors, the number of symmetrical systems required to define the given system being equal to its degrees of freedom. A few trigonometrical theorems which are to be used in the paper are called to mind. The paper is subdivided into three parts, an abstract of which follows. It is recommended that only that part of Part I up to formula (33) and the portion dealing with star-delta transformations be read before proceeding with Part II.

Part I deals with the resolution of unsymmetrical groups of numbers into symmetrical groups. These numbers may represent rotating vectors of systems of operators. A new operator termed the sequence operator is introduced which simplifies the manipulation. Formulas are derived for three-phase circuits. Star-delta transformations for symmetrical co-ordinates are given and expressions for power deduced. A short discussion of harmonics in three-phase systems is given.

Part II deals with the practical application of this method to symmetrical rotating machines operating on unsymmetrical circuits. General formulas are derived and such special cases, as the single-phase induction motor, synchronous motor-generator, phase converters of various types, are discussed.

INTRODUCTION

IN THE latter part of 1913 the writer had occasion to investigate mathematically the operation of induction motors under unbalanced conditions. The work was first carried out, having particularly in mind the determination of the operating characteristics of phase converters which may be considered as a particular case of unbalanced motor operation, but the scope of the subject broadened out very quickly and the writer undertook this paper in the belief that the subject would be of interest to many.

The most striking thing about the results obtained was their

Manuscript of this paper was received April 24, 1918.

symmetry; the solution always reduced to the sum of two or more symmetrical solutions. The writer was then led to inquire if there were no general principles by which the solution of unbalanced polyphase systems could be reduced to the solution of two or more balanced cases. The present paper is an endeavor to present a general method of solving polyphase network which has peculiar advantages when applied to the type of polyphase networks which include rotating machines.

In physical investigations success depends often on a happy choice of co-ordinates. An electrical network being a dynamic system should also be aided by the selection of a suitable system of co-ordinates. The co-ordinates of a system are quantities which when given, completely define the system. Thus a system of three co-planar congruent vectors are defined when their magnitude and their angular position with respect to some fixed direction are given. Such a system may be said to have six degrees of freedom, for each vector may vary in magnitude and phase position without regard to the others. If, however, we impose the condition that the vector sum of these vectors shall be zero, we find that with the direction of one vector given, the other two vectors are completely defined when their magnitude alone is given, the system has therefore lost two degrees of freedom by imposing the above condition which in dynamical theory is termed a "constraint". If we impose a further condition that the vectors be symmetrically disposed about their common origin this system will now have but two degrees of freedom.

It is evident from the above definition that a system of n coplanar congruent vectors may have $2n$ degrees of freedom and that a system of n symmetrically spaced vectors of equal magnitude has but two degrees of freedom. It should be possible then by a simple transformation to define the system of n arbitrary congruent vectors by n other systems of congruent vectors which are symmetrical and have a common point. The n symmetrical systems so obtained are the symmetrical coordinates of the given system of vectors and completely define it.

This method of representing polyphase systems has been employed in the past to a limited extent, but up to the present time there has been as far as the author is aware no systematic presentation of the method. The writer hopes by this paper to interest others in the application of the method, which will be

found to be a valuable instrument for the solution of certain classes of polyphase networks.

In dealing with alternating currents in this paper, use is made of the complex variable which in its most general form may be represented as a vector of variable length rotating about a given point at variable angular velocity or better as the resultant of a number of vectors each of constant length rotating at different angular velocities in the same direction about a given point. This vector is represented in the text by $\check{I}$, $\check{E}$, etc., and the conjugate vector which rotates at the same speed in the opposite direction is represented by $\hat{I}$, $\hat{E}$, etc. The effective value of the vector is represented by the symbol without the distinguishing mark as I, E, etc. The impedances Z_a, Z_b, Z_{ab}, etc., are generally functions of the operator, $D = \frac{d}{dt}$ and the characteristics of the circuit; these characteristics are constants only when there is no physical motion. It will therefore be necessary to carefully distinguish between $Z_a \check{I}_a$ and $\check{I}_a Z_a$ when Z_a has the form of a differential operator. In the first case a differential operation is carried out on the time variable $\check{I}_a$ in the second case the differential operator is merely multiplied by $\check{I}_a$.

The most general expression for a simple harmonic quantity e is

$$e = A \cos pt - B \sin pt$$

in exponential form this becomes

$$e = \frac{A + j B}{2} e^{j pt} + \frac{A - j B}{2} e^{-j pt}$$

$(A + j B) e^{j pt}$ represents a vector of length $\sqrt{A^2 + B^2}$ rotating in the positive direction with angular velocity p while $(A - j B) e^{-j pt}$ is the conjugate vector rotating at the same angular velocity in the opposite direction. Since $e^{j pt}$ is equal to $\cos pt + j \sin pt$, the positively rotating vector $\check{E} = (A + j B) e^{j pt}$ will be

$$\check{E} = A \cos pt - B \sin pt + j (A \sin pt + B \cos pt)$$

or the real part of $\check{E}$ which is its projection on a given axis is equal to e and therefore $\check{E}$ may be taken to represent e in phase and magnitude. It should be noted that the conjugate vector $\hat{E}$ is equally available, but it is not so convenient since the

operation $\frac{d}{dt}\, e^{-j\,pt}$ gives $-\,j\,p\,e^{-j\,pt}$ and the imaginary part of the impedance operator will have a negative sign.

The complex roots of unity will be referred to from time to time in the paper. Thus the complete solution of the equation $x^n - 1 = 0$ requires n different values of x, only one of which is real when n is an odd integer. To obtain the other roots we have the relation

$$\begin{aligned} 1 &= \cos 2\,\pi\,r + j \sin 2\,\pi\,r \\ &= e^{j\,2\,\pi\,r} \end{aligned}$$

Where r is any integer. We have therefore

$$1^{\frac{1}{n}} = e^{j\frac{2\,\pi\,r}{n}}$$

and by giving successive integral values to r from 1 to n, all the n roots of $X^n - 1 = 0$ are obtained namely,

$$\begin{aligned} a_1 &= e^{j\frac{2\,\pi}{n}} = \cos \frac{2\,\pi}{n} + j \sin \frac{2\,\pi}{n} \\ a_2 &= e^{j\frac{4\,\pi}{n}} \quad \cos \frac{4\,\pi}{n} + j \sin \frac{4\,\pi}{n} \\ a_3 &= e^{j\frac{6\,\pi}{n}} \quad \cos \frac{6\,\pi}{n} + j \sin \frac{6\,\pi}{n} \\ a_n &= e^{j\,2\,\pi} = 1 \end{aligned}$$

It will be observed that $a_2\; a_3 \ldots . a_n$ are respectively equal to $a_1^2\; a_1^3 \ldots . a_1^{(n-1)}$.

When there is relative motion between the different parts of a circuit as for example in rotating machinery, the mutual inductances enter into the equation as time variables and when the motion is angular the quantities e^{jwt} and e^{-jwt} will appear in the operators. In this case we do not reject the portion of the operator having e^{-jwt} as a factor, because the equations require that each vector shall be operated on by the operator as a whole which when it takes the form of a harmonic time function will contain terms with e^{jwt} and e^{-jwt} in conjugate relation. In some cases as a result of this, solutions will appear with indices of e which are negative time variables; in such cases the vectors with negative index should be replaced by their conjugates which rotate in the positive direction.

This paper is subdivided as follows:

Part I.—"The Method of Symmetrical Co-ordinates." Deals with the theory of the method, and its application to simple polyphase circuits.

Part II.—Application to Symmetrical Machines on Unbalanced Polyphase Circuits. Takes up Induction Motors, Generator and Synchronous Motor, Phase Balancers and Phase Convertors.

Part III. Application to Machines having Unsymmetrical Windings.

In the Appendix the mathematical representation of field forms and the derivation of the constants of different forms of networks is taken up.

The portions of Part I dealing with unsymmetrical windings are not required for the applications taken up in Part II and may be deferred in a later reading. The greater part of Part I is taken up in deriving formulas for special cases from the general formulae (**30**) and (**33**), and the reading of the text following these equations may be confined to the special cases of immediate interest.

I wish to express my appreciation of the valuable help and suggestions that have been given me in the preparation of this paper by Prof. Karapetoff who suggested that the subject be presented in a mathematical paper and by Dr. J. Slepian to whom I am indebted for the idea of sequence operators and by others who have been interested in the paper.

PART I

Method of Symmetrical Generalized Co-ordinates

Resolution of Unbalanced Systems of Vectors and Operators

The complex time function $\check{E}$ may be used instead of the harmonic time function e in any equation algebraic or differential in which it appears linearly. The reason of this is because if any linear operation is performed on $\check{E}$ the same operation performed on its conjugate $\hat{E}$ will give a result which is conjugate to that obtained from $\check{E}$, and the sum of the two results obtained is a solution of the same operation performed on $\check{E} + \hat{E}$, or $2\,e$.

It is customary to interpret $\check{E}$ and $\hat{E}$ as coplanar vectors, rotating about a common point and e as the projection of either vector on a given line, $\check{E}$ being a positively rotating vector and

$\hat{E}$ being a negatively rotating vector, and their projection on the given line being

$$e = \frac{\check{E} + \hat{E}}{2} \tag{1}$$

Obviously if this interpretation is accepted one of the two vectors becomes superfluous and the positively rotating vector $\check{E}$ may be taken to represent the variable "e" and we may define "e" by saying that "e" is the projection of the vector $\check{E}$ on a given line or else by saying that "e" is the real part of the complex variable $\check{E}$.

If $1, a, a^2 \ldots . a^{n-1}$ are the n roots of the equation $x^n - 1 = 0$ a symmetrical polyphase system of n phases may be represented by

$$\left.\begin{aligned} \check{E}_{11} &= \check{E}_{11} \\ \check{E}_{21} &= a\,\check{E}_{11} \\ \check{E}_{31} &= a^2\,\check{E}_{11} \\ &\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots \\ \check{E}_{n1} &= a^{n-1}\,\check{E}_{11} \end{aligned}\right\} \tag{2}$$

Another n phase system may be obtained by taking

$$\left.\begin{aligned} \check{E}_{12} &= \check{E}_{12} \\ \check{E}_{22} &= a^2\,\check{E}_{12} \\ \check{E}_{32} &= a^4\,\check{E}_{12} \\ &\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots \\ \check{E}_{n2} &= a^{2\,(n-1)}\check{E}_{12} \end{aligned}\right\} \tag{3}$$

and this also is symmetrical, although it is entirely different from (**2**).

Since $1 + a + a^2 + a^{n-1} = 0$, the sum of all the vectors of a symmetrical polyphase system is zero.

If $\check{E}_1\ \check{E}_2\ \check{E}_3 \ldots . \check{E}_n$ be a system of n vectors, the following identities may be proved by inspection:

$$
\left.
\begin{aligned}
\check{E}_1 \equiv & \frac{\check{E}_1 + \check{E}_2 + \check{E}_3 + \ldots . \check{E}_n}{n} \\
& + \frac{\check{E}_1 + a\,\check{E}_2 + a^2\,\check{E}_3 + \ldots . a^{n-1}\,\check{E}_n}{n} \\
& + \frac{\check{E}_1 + a^2\,\check{E}_2 + a^4\check{E}_3 + \ldots . a^{2(n-1)}\,\check{E}_n}{n} \\
& + \frac{\check{E}_1 + a^{r-1}\,\check{E}_2 + a^{2(r-1)}\,\check{E}_3 + \ldots . a^{(n-1)(r-1)}\,\check{E}_n}{n} \\
& + \ldots . \frac{\check{E}_1 + a^{-1}\,\check{E}_2 + a^{-2}\,\check{E}_3 + \ldots . a^{-(n-1)}\,\check{E}_n}{n} \\
\check{E}_2 \equiv & \frac{\check{E}_1 + \check{E}_2 + \check{E}_3 + \ldots . \check{E}_n}{n} \\
& + a^{-1}\,\frac{\check{E}_1 + a\,\check{E}_2 + a^2\,\check{E}_3 + \ldots . a^{n-1}\,\check{E}_n}{n} \\
& + a^{-2}\,\frac{\check{E}_1 + a^2\,\check{E}_2 + a^4\,\check{E}_3 + \ldots . a^{2(n-1)}\,\check{E}_n}{n} \\
& + a^{-(r-1)}\,\frac{\check{E}_1 + a^{r-1}\,\check{E}_2 + a^{2(r-1)}\,\check{E}_3 + a^{(n-1)(r-1)}\,\check{E}_n}{n} \\
& + a^{-(n-1)}\,\frac{\check{E}_1 + a^{-1}\,\check{E}_2 + a^{-2}\,\check{E}_3 + \ldots . a^{-(n-1)}\,\check{E}_n}{n} \\
& \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
& \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\check{E}_n \equiv & \frac{\check{E}_1 + \check{E}_2 + \check{E}_3 + \ldots . \check{E}_n}{n} \\
& + a^{-(n-1)}\,\frac{\check{E}_1 + a\,\check{E}_2 + a^2\,\check{E}_3 + \ldots . a^{n-1}\,\check{E}_n}{n} \\
& + a^{-2(n-1)}\,\frac{\check{E}_1 + a^2\,\check{E}_2 + a^4\,\check{E}_4 + \ldots . a^{2(n-1)}\,\check{E}_n}{n} \\
& + a^{-(n-1)(r-1)}\,\frac{\check{E}_1 + a^{r-1}\,\check{E}_2 + \ldots . a^{(n-1)(r-1)}\,\check{E}_n}{n} \\
& + a^{-1}\,\frac{\check{E}_1 + a^{-1}\,\check{E}_2 + a^{-2}\,\check{E}_3 + \ldots . a^{-(n-1)}\,\check{E}_n}{n}
\end{aligned}
\right\} \quad (4)
$$

It will be noted that in the expression for $\check{E}_1$ in the above formulae if the first term of each component is taken the result is

$n\,\dfrac{\check{E}_1}{n}$ or $\check{E}_1$. If the succeeding terms of each component involving $\check{E}_2\,\check{E}_3 \ldots \check{E}_n$ respectively, are taken separately they add up to expressions of the form $\dfrac{\check{E}_r}{n}\,(1+a+a^2+\ldots a^{n-1})$ which are all equal to zero since $(1+a+a^2+\ldots a^{n-1})$ is equal to zero. In like manner in the expression for $\check{E}_2\,\check{E}_3 \ldots \check{E}_n$ respectively, all the terms of the components involving each of the quantities $\check{E}_1\,\check{E}_2\,\check{E}_3 \ldots$ etc. excepting the terms involving that one of which the components are to be determined add up to expressions of the form $\dfrac{\check{E}_r}{n}$ $(1+a+a^2+\ldots a^{n-1})$ all of which are equal to zero, the remaining terms add up to $\check{E}_2\,\check{E}_3 \ldots E_n$ respectively. It will now be apparent that (**4**), is true whatever may be the nature of $\check{E}_1\,\check{E}_2$ etc., and therefore it is true of all numbers, real complex or imaginary, whatever they may represent and therefore similar relations may be obtained for current vectors and they may be extended to include not only vectors but also the operators.

In order to simplify the expressions which become unwieldy when applied to the general n phase system, let us consider a three phase system of vectors $\check{E}_a\;\check{E}_b\;\check{E}_c$. Then we have the following identities:

$$\left.\begin{aligned}
\check{E}_a &\equiv \frac{\check{E}_a+\check{E}_b+\check{E}_c}{3} + \frac{E_a + a\,\check{E}_b + a^2\,E_c}{3} \\
&\qquad + \frac{\check{E}_a + a^2\,\check{E}_b + a\,\check{E}_c}{3} \\
\check{E}_b &\equiv \frac{\check{E}_a+\check{E}_b+\check{E}_c}{3} + a^2\,\frac{\check{E}_a + a\,\check{E}_b + a^2\,E_c}{3} \\
&\qquad + a\,\frac{\check{E}_a + a^2\,\check{E}_b + a\,\check{E}_c}{3} \\
\check{E}_c &\equiv \frac{\check{E}_a+\check{E}_b+\check{E}_c}{3} + a\,\frac{\check{E}_a + a\,\check{E}_b + a^2\,\check{E}_c}{3} \\
&\qquad + a^2\,\frac{\check{E}_a + a^2\,\check{E}_b + a\,\check{E}_c}{3}
\end{aligned}\right\} \quad (\mathbf{5})$$

(**4**) states the law that a system of n vectors or quantities may be resolved when n is prime into n different symmetrical

groups or systems, one of which consists of n equal vectors and the remaining $(n - 1)$ systems consist of n equispaced vectors which with the first mentioned groups of equal vectors forms an equal number of symmetrical n-phase systems. When n is not prime some of the n-phase systems degenerate into repetitions of systems having numbers of phases corresponding to the factors of n.

Equation (5) states that any three vectors $\check{E}_a$ $\check{E}_b$ $\check{E}_c$ may be

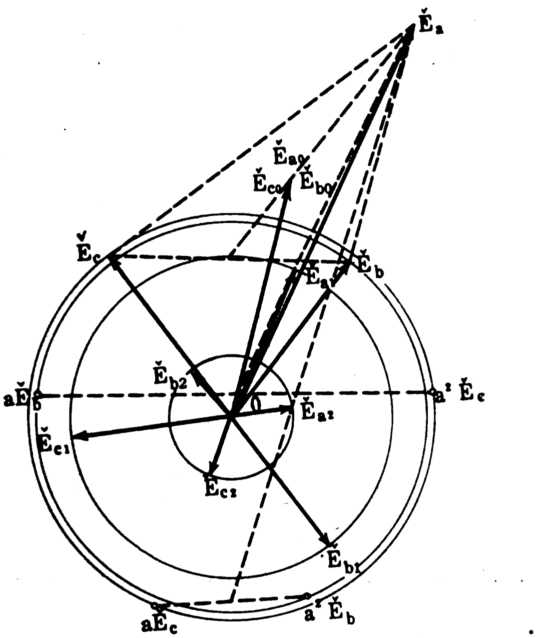


FIG. 1—GRAPHICAL REPRESENTATION OF EQUATION 5.

resolved into a system of three equal vectors $\check{E}_{a0}$ $\check{E}_{a0}$ $\check{E}_{a0}$ and two symmetrical three phase systems $\check{E}_{a1}$, $a^2\,\check{E}_{a1}$, $a\,\check{E}_{a1}$, $\check{E}_{a2}$, $a\,\check{E}_{a2}$, $a^2\,\check{E}_{a2}$, the first of which is of positive phase sequence and the second of negative phase sequence, or

$$\left.\begin{aligned}\check{E}_a &= \check{E}_{a0} + \check{E}_{a1} + \check{E}_{a2}\\ \check{E}_b &= \check{E}_{a0} + a^2\,\check{E}_{a1} + a\,\check{E}_{a2}\\ \check{E}_c &= \check{E}_{a0} + a\,\check{E}_{a1} + a^2\,\check{E}_{a2}\end{aligned}\right\} \qquad (6)$$

Similarly

$$\left.\begin{aligned} \check{I}_a &= \check{I}_{a0} + \check{I}_{a1} + \check{I}_{a2} \\ \check{I}_b &= \check{I}_{a0} + a^2\, \check{I}_{a1} + a\, \check{I}_{a2} \\ \check{I}_c &= \check{I}_{a0} + a\, \check{I}_{a1} + a^2\, \check{I}_{a2} \end{aligned}\right\} \qquad (7)$$

Figs. (1) and (2) show a graphical method of resolving three vectors into their symmetrical three-phase components corresponding to equations (5).

The system of operators $Z_{aa}\ Z_{bb}\ Z_{cc}\ Z_{ab}\ Z_{bc}\ Z_{ca}$ may be resolved in a similar manner into symmetrical groups,

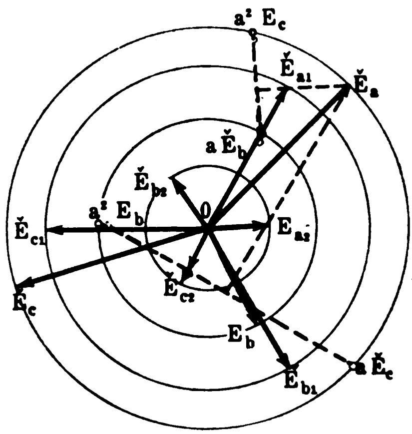


Fig. 2—Graphical Representation of Equation 5.

$$\left.\begin{aligned} Z_{aa} &= Z_{aa0} + Z_{aa1} + Z_{aa2} \\ Z_{bb} &= Z_{aa0} + a^2\, Z_{aa1} + a\, Z_{aa2} \\ Z_{cc} &= Z_{aa0} + a\, Z_{aa1} + a^2\, Z_{aa2} \end{aligned}\right\} \qquad (8)$$

$$\left.\begin{aligned} Z_{ab} &= Z_{ab0} + Z_{ab1} + Z_{ab2} \\ Z_{bc} &= Z_{ab0} + a^2\, Z_{ab1} + a\, Z_{ab2} \\ Z_{ca} &= Z_{ab0} + a\, Z_{ab1} + a^2\, Z_{ab2} \end{aligned}\right\} \qquad (9)$$

There are similar relations for *n* phase systems.

Explanation of Theory and Use of Sequence Operator

Consider the following sequences of nth roots of unity:

$$\left.\begin{array}{l} S^0 = 1, \quad 1, \quad 1 \ldots . 1 \\ S^1 = 1, \quad a^{-1}, \quad a^{-2} \ldots . a^{-(n-1)} \\ S^2 = 1, \quad a^{-2}, \quad a^{-4} \ldots . a^{-2(n-1)} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ S^r = 1, \quad a^{-r}, \quad a^{-2r} \ldots . a^{-(n-1)r} \\ S^{r+1} = 1, \quad a^{-(r+1)}, \quad a^{-2(r+1)} \ldots . a^{-(n-1)(r+1)} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ S^{n-1} = 1, \quad a^{-(n-1)}, \quad a^{-2(n-1)} \ldots . a^{-(n-1)^2} \end{array}\right\} \quad (10)$$

Consider the sequence obtained by the products of similar terms of S^r and S^1. It will be

$$S^{r+1} = 1, \quad a^{-(r+1)}, \quad a^{-2(r+1)} \ldots . a^{-(n-1)(r+1)} \quad (11)$$

Similarly

$$S^k = 1, \quad a^{-k}, \quad a^{-2k} \ldots . a^{-(n-1)k} \quad (12)$$

and the sequence obtained by products of like terms of this sequence and S^r is

$$S^{r+k} = 1, \quad a^{-(r+k)}, \quad a^{-2(r+k)} \ldots . a^{-(n-1)(r+k)} \quad (13)$$

We may therefore apply the law of indices to the products of sequences to obtain the resulting sequence.

In the case of the three-phase system we shall have the following sequences only to consider, viz.:

$$\left.\begin{array}{l} S^0 = 1, \quad 1, \quad 1 \\ S^1 = 1, \quad a^2, \quad a \\ S^2 = 1, \quad a, \quad a^2 \end{array}\right\} \quad (14)$$

The complete system of currents $\bar{I}_a \; \bar{I}_b \; \bar{I}_c$ are defined by

$$S(\bar{I}_a) = S^0 \bar{I}_{a0} + S^1 \bar{I}_{a1} + S^2 \bar{I}_{a2} \quad (15)$$

Similarly the impedances $Z_{aa} \; Z_{bb} \; Z_{cc}$ may be expressed in symmetrical form

$$S(Z_{aa}) \equiv S^0 Z_{aa0} + S^1 Z_{aa1} + S^2 Z_{aa2} \quad (16)$$

and the mutual impedances Z_{ab}, Z_{bc}, Z_{ca} are expressed by

$$S(Z_{ab}) \equiv S^0 Z_{ab0} + S^1 Z_{ab1} + S^2 Z_{ab2} \quad (17)$$

Attention is called to the importance of preserving the cyclic order of self and mutual impedances, otherwise the rule for the sequence operator will not hold. Thus, Z_{ab}, Z_{bc} and Z_{ca} are in proper sequence as also are Z_{ca}, Z_{ab}, Z_{bc}.

When it is desired to change the first term in the sequence of polyphase vectors the resulting expression will be

$$\left.\begin{aligned} S(\check{I}_b) &= S^0\,\check{I}_{a0} + S^1\,a^2\,\check{I}_{a1} + S^2\,a\,\check{I}_{a2} \\ S(\check{I}_c) &= S^0\,\check{I}_{a0} + S^1\,a\,\check{I}_{a1} + S^2\,a^2\,\check{I}_{a2} \end{aligned}\right\} \tag{18}$$

Similarly in the case of the operators $S(Z_{ab})$ we have

$$\left.\begin{aligned} S(Z_{bc}) &= S^0\,Z_{ab0} + S^1\,a^2\,Z_{ab1} + S^2\,a\,Z_{ab2} \\ S(Z_{ca}) &= S^0\,Z_{ab0} + S^1\,a\,Z_{ab1} + S^2\,a^2\,Z_{ab2} \end{aligned}\right\} \tag{19}$$

Similar rules apply to the e.m.fs. E_a E_b E_c

$$\left.\begin{aligned} S(\check{E}_a) &= S^0\,\check{E}_{a0} + S^1\,\check{E}_{a1} + S^2\,\check{E}_{a2} \\ S(\check{E}_b) &= S^0\,\check{E}_{a0} + S^1\,a^2\,\check{E}_{a1} + S^2\,a\,\check{E}_{a2} \\ S(\check{E}_c) &= S^0\,\check{E}_{a0} + S^1\,a\,E_{a1} + S^2\,a^2\,\check{E}_{a2} \end{aligned}\right\} \tag{20}$$

It should be kept in mind that any one of the several expressions $S(\check{I}_a)$ $S(\check{I}_b)$ $S(\check{I}_c)$, etc., completely specifies the system, and each of the members of the groups of equations given above is a complete statement of the system of vectors or operators and their relation.

Application to Self and Mutual Impedance Operations

We may now proceed with the current, systems $S(\check{I}_a)$, $S(\check{I}_b)$, $S(\check{I}_c)$ and the operating groups $S(Z_{aa})$ $S(Z_{bb})$ $S(Z_{cc})$ etc. and the electromotive forces in exactly the same manner as for simple a-c. circuits. Thus,

$$\begin{aligned} S(\check{E}_a) &= S(Z_{aa})\,S(\check{I}_a) + S(Z_{ab})\,S(\check{I}_b) + S(Z_{ca})\,S(\check{I}_c) \qquad \textbf{(21)} \\ &= (S^0\,Z_{aa0} + S^1\,Z_{aa1} + S^2\,Z_{aa2})\,(S^0\,\check{I}_{a0} + S^1\,\check{I}_{a1} + S^2\,I_{a2}) \\ &\quad + (S^0\,Z_{ab0} + S^1\,Z_{ab1} + S^2\,Z_{ab2}) \\ &\qquad\qquad (S^0\,\check{I}_{a0} + S^1\,a^2\,\check{I}_a + S^2\,a\,\check{I}_{a2}) \\ &\quad + (S^0\,Z_{ab0} + S^1\,a\,Z_{ab1} + S^2\,a^2\,Z_{ab2}) \\ &\qquad\qquad (S^0\,\check{I}_{a0} + S^1\,a\,\check{I}_{a1} + S^2\,a^2\,\check{I}_{a2}) \\ &= S^0\,(Z_{aa0} + 2\,Z_{ab0})\,\check{I}_{a0} + S^0\,\{Z_{aa2} + (1 + a^2)\,Z_{ab2}\}\,\check{I}_{a1} \end{aligned}$$

$$
\begin{aligned}
&+ S^0 \{Z_{aa1} + (1 + a)\, Z_{ab1}\}\, \check{I}_{a2} \\
&+ S^1 \{Z_{aa1} + (1 + a)\, Z_{ab1}\}\, \check{I}_{a0} \\
&+ S^1 \{Z_{aa0} + (a + a^2)\, Z_{ab0}\}\, \check{I}_{a1} \\
&+ S^1 \{Z_{aa2} + 2\, a\, Z_{ab2}\}\, \check{I}_{a2} \\
&+ S^2 \{Z_{aa2} + (1 + a^2)\, Z_{ab2}\}\, \check{I}_{a0} \\
&+ S^2 \{Z_{aa1} + 2\, a^2\, Z_{ab1}\}\, \check{I}_{a1} \\
&+ S^2 \{Z_{aa0} + (a + a^2)\, Z_{ab0}\}\, \check{I}_{a2} \qquad \textbf{(22)}
\end{aligned}
$$

Or since $1 + a + a^2 = 0$, $1 + a = - a^2$, $1 + a^2 = - a$ and $a + a^2 = - 1$

$$
\begin{aligned}
S(\check{E}_a) = {} & S^0 (Z_{aa0} + 2\, Z_{ab0})\, \check{I}_{a0} + S^0 (Z_{aa2} - a\, Z_{ab2})\, \check{I}_{a1} \\
&+ S^0 (Z_{aa1} - a^2\, Z_{ab1})\, \check{I}_{a2} + S^1 (Z_{aa1} - a^2\, Z_{ab1})\, \check{I}_{a0} \\
&+ S^1 (Z_{aa0} - Z_{ab0})\, \check{I}_{a1} + S^1 (Z_{aa2} + 2\, a\, Z_{ab2})\, \check{I}_{a2} \\
&+ S^2 (Z_{aa2} - a\, Z_{ab2})\, \check{I}_{a0} + S^2 (Z_{aa1} + 2\, a^2\, Z_{ab1})\, \check{I}_{a1} \\
&+ S^2 (Z_{aa0} - Z_{ab0})\, \check{I}_{a2} \qquad \textbf{(23)}
\end{aligned}
$$

Or since

$$
\begin{aligned}
S(Z_{bc}) &= S^0\, Z_{bc0} + S^1\, Z_{bc1} + S^2\, Z_{bc2} \\
&= S^0\, Z_{ab0} + S^1\, a^2\, Z_{ab1} + S^2\, a\, Z_{ab2}
\end{aligned}
$$

we may write (**23**) in the form

$$
\begin{aligned}
S(\check{E}_a) = {} & S^0 (Z_{aa0} + 2\, Z_{bc0})\, \check{I}_{a0} + S^0 (Z_{aa2} - Z_{bc2})\, \check{I}_{a1} \\
&+ S^0 (Z_{aa1} - Z_{bc1})\, \check{I}_{a2} + S^1 (Z_{aa1} - Z_{bc1})\, \check{I}_{a0} \\
&+ S^1 (Z_{aa0} - Z_{bc0})\, \check{I}_{a1} + S^1 (Z_{aa2} + 2\, Z_{bc2})\, \check{I}_{a2} \\
&+ S^2 (Z_{aa2} - Z_{bc2})\, \check{I}_{a0} + S^2 (Z_{aa1} + 2\, Z_{bc1})\, \check{I}_{a1} \\
&+ S^2 (Z_{aa0} - Z_{bc0})\, \check{I}_{a2} \qquad \textbf{(24)}
\end{aligned}
$$

which is the more symmetrical form. We have therefore from (**24**) by expressing $S(\check{E}_a)$ in terms of symmetrical co-ordinates the three symmetrical equations

$$
\left.
\begin{aligned}
S^0\, \check{E}_{a0} &= S^0 \{(Z_{aa0} + 2\, Z_{bc0})\, \check{I}_{a0} + (Z_{aa2} - Z_{bc2})\, \check{I}_{a1} \\
&\qquad + (Z_{aa1} - Z_{bc1})\, \check{I}_{a2}\} \\
S^1\, E_{a1} &= S^1 \{(Z_{aa1} - Z_{bc1})\, \check{I}_{a0} + (Z_{aa0} - Z_{bc0})\, \check{I}_{a1} \\
&\qquad + (Z_{aa2} + 2\, Z_{bc2})\, \check{I}_{a2}\} \\
S^2\, E_{a2} &= S^2 \{(Z_{aa2} - Z_{bc2})\, \check{I}_{a0} + (Z_{aa1} + 2\, Z_{bc1})\, \check{I}_{a1} \\
&\qquad + (Z_{aa0} - Z_{bc0})\, \check{I}_{a2}\}
\end{aligned}
\right\} \qquad \textbf{(25)}
$$

An important case to which we must next give consideration is that of mutual inductance between a primary polyphase circuit and a secondary polyphase circuit. The mutual impedances may be arranged in three sets. Let the currents in the secondary windings be I_u I_v and I_w, we may then express the generalized mutual impedances as follows:

$$\left.\begin{array}{r} \text{(I)}\ Z_{au}\ Z_{bv}\ Z_{cw} \\ \text{(II)}\ Z_{bw}\ Z_{cu}\ Z_{av} \\ \text{(III)}\ Z_{cv}\ Z_{aw}\ Z_{bu} \end{array}\right\} \quad (26)$$

Each set may be resolved into three symmetrical groups, so that

$$\left.\begin{array}{l} S\,(Z_{au}) = S^0\,Z_{au0} + S^1\,Z_{au1} + S^2\,Z_{au2} \\ S\,(Z_{bw}) = S^0\,Z_{bw0} + S^1\,Z_{bw1} + S^2\,Z_{bw2} \\ S\,(Z_{cv}) = S^0\,Z_{cv0} + S^1\,Z_{cv1} + S^2\,Z_{cv2} \end{array}\right\} \quad (27)$$

and we have for $S\,(\check{E}_a)$ the primary induced e.m.f. due to the secondary currents $S\,(\check{I}_u)$

$$S\,(\check{E}_a) = S\,(Z_{au})\,S\,(\check{I}_u) + S\,(Z_{av})\,S\,(\check{I}_v) + S\,(Z_{aw})\,S\,(\check{I}_w) \quad (28)$$

Substituting for $S\,(\check{I}_u)$, $S\,(\check{I}_v)$ and $S\,(\check{I}_w)$ and $S\,(Z_{au})$, $S\,(Z_{av})$, $S\,(Z_{aw})$ their symmetrical equivalents we have

$$\begin{aligned} S\,(\check{E}_a) = {} & S^0\,(Z_{au0} + Z_{bw0} + Z_{cv0})\,I_{u0} \\ & + S^0\,(Z_{au2} + a\,Z_{bw2} + a^2\,Z_{cv2})\,\check{I}_{u1} \\ & + S^0\,(Z_{au1} + a^2\,Z_{bw1} + a\,Z_{cv1})\,\check{I}_{u2} \\ & + S^1\,(Z_{au1} + a\,Z_{bw1} + a^2\,Z_{cv1})\,\check{I}_{u0} \\ & + S^1\,(Z_{au0} + a^2\,Z_{bw0} + a\,Z_{cv0})\,\check{I}_{u1} \\ & + S^1\,(Z_{au2} + Z_{bw2} + Z_{cv2})\,\check{I}_{u2} \\ & + S^2\,(Z_{au2} + a^2\,Z_{bw2} + a\,Z_{v2})\,\check{I}_{u0} \\ & + S^2\,(Z_{au1} + Z_{bw1} + Z_{cv1})\,\check{I}_{u1} \\ & + S^2\,(Z_{au0} + a\,Z_{bw0} + a^2\,Z_{cv0})\,\check{I}_{u2} \end{aligned} \quad (29)$$

On expressing $S\,(\check{E}_a)$ in symmetrical form we have the following three symmetrical equations

$$\left.\begin{aligned}
S^0\,\check{E}_{a0} &= S^0\,\{(Z_{au0} + Z_{bw0} + Z_{cv0})\,\check{I}_{u0}\\
&\qquad + (Z_{au2} + a\,Z_{bw2} + a^2\,Z_{cv2})\,\check{I}_{u1}\\
&\qquad + (Z_{au1} + a^2\,Z_{bw1} + a\,Z_{cv1})\,\check{I}_{u2}\}\\
S^1\,\check{E}_{a1} &= S^1\,\{(Z_{au1} + a\,Z_{bw1} + a^2\,Z_{cv1})\,\check{I}_{u0}\\
&\qquad + (Z_{au0} + a^2\,Z_{bw0} + a\,Z_{cv0})\,\check{I}_{u1}\\
&\qquad + (Z_{au2} + Z_{bw2} + Z_{cv2})\,\check{I}_{u2}\}\\
S^2\,E_{a2} &= S^2\,\{(Z_{au2} + a^2\,Z_{bw2} + a\,Z_{cv2})\,\check{I}_u{}^0\\
&\qquad + (Z_{au1} + Z_{bw1} + Z_{cv1})\,\check{I}_{u1}\\
&\qquad + (Z_{au0} + a\,Z_{bw0} + a^2\,Z_{cv0})\,\check{I}_{u2}\}
\end{aligned}\right\} \qquad \textbf{(30)}$$

For the e.m.f. $S\,(\check{E}_u)$ induced in the secondary by the primary currents $S\,(\check{I}_a)$ we have

$$S\,(\check{E}_u) = S\,(Z_{au})\,S\,(\check{I}_a) + S\,(Z_{bu})\,S\,(\check{I}_b) + S\,(Z_{cu})\,S\,(\check{I}_c) \qquad \textbf{(31)}$$

Since $S\,(Z_{bu})$ bears the same relation to $S\,(Z_{cv})$ as $S\,(Z_{av})$ does to $S\,(Z_{bw})$ and $S\,(Z_{au})$ bears the same relation to $S\,(Z_{bw})$ as $S\,(Z_{aw})$ does to $S\,(Z_{cv})$ to obtain $S\,(\check{E}_u)$ all that will be necessary will be tointerchange Z_{bw} and Z_{cv} in (**29**) and change $\check{I}_{u0}\ \check{I}_{u1}\ \check{I}_{u2}$ to $\check{I}_{a0}\ I_{a1}$ and $\check{I}_{a2}$ respectively, this gives

$$\begin{aligned}
S\,(\check{E}_u) &= S^0\,(Z_{au0} + Z_{bw0} + Z_{cv0})\,\check{I}_{a0}\\
&\qquad + S^0\,(Z_{au2} + a^2\,Z_{bw2} + a\,Z_{cv2})\,\check{I}_{a1}\\
&\qquad + S^0\,(Z_{au1} + a\,Z_{bw1} + a^2\,Z_{cv1})\,\check{I}_{a2}\\
&\qquad + S^1\,(Z_{au1} + a^2\,Z_{bw1} + a\,Z_{cv1})\,\check{I}_{a0}\\
&\qquad + S^1\,(Z_{au0} + a\,Z_{bw0} + a^2\,Z_{cv0})\,\check{I}_{a1}\\
&\qquad + S^1\,(Z_{au2} + Z_{bw2} + Z_{cv2})\,\check{I}_{a2}\\
&\qquad + S^2\,(Z_{au2} + a\,Z_{bw2} + a^2\,Z_{cv2})\,\check{I}_{a0}\\
&\qquad + S^2\,(Z_{au1} + Z_{bw1} + Z_{cv1})\,\check{I}_{a1}\\
&\qquad + S^2\,(Z_{au0} + Z_{bw0} + Z_{cv0})\,\check{I}_{a2}
\end{aligned} \qquad \textbf{(32)}$$

and the three symmetrical equations will be

$$\left.\begin{aligned}
S^0 \check{E}_{u0} &= S^0 \{(Z_{au0} + Z_{bw0} + Z_{cv0})\, \check{I}_{a0} \\
&\quad + (Z_{au2} + a^2 Z_{bw2} + a\, Z_{bw2} + a\, Z_{cv2})\, \check{I}_{a1} \\
&\quad + (Z_{au1} + a\, Z_{bw1} + a^2 Z_{cv1})\, \check{I}_{a2}\} \\
S^1 \check{E}_{a1} &= S^1 \{(Z_{au1} + a^2 Z_{bw1} + a\, Z_{cv1})\, \check{I}_{a0} \\
&\quad + (Z_{au0} + a\, Z_{bw0} + a^2 Z_{cv0})\, \check{I}_{a1} \\
&\quad + (Z_{au2} + Z_{bw2} + Z_{cv2})\, \check{I}_{a2}\} \\
S^2 \check{E}_{u2} &= S^2 \{(Z_{au2} + a\, Z_{bw2} + a^2 Z_{cv2})\, \check{I}_{a0} \\
&\quad + (Z_{au1} + Z_{bw1} + Z_{cv1})\, \check{I}_{a1} \\
&\quad + (Z_{au0} + a^2 Z_{bw0} + a\, Z_{cv0})\, \check{I}_{a2}\}
\end{aligned}\right\} \qquad (33)$$

The same methods may be applied to polyphase systems of any number of phase. When the number of phases is not prime the system may sometimes be dealt with as a number of polyphase systems having mutual inductance between them:—For example, a nine-phase system may be treated as three three-phase systems, a twelve phase system as three four-phase or four three-phase systems. In certain froms of dissymmetry this method is of great practical value, and its application will be taken up later.

For the present part of the paper we shall confine ourselves to the three-phase system, and dissymmetries of several different kinds.

The operators Z_{au} Z_{aa}, etc., must be interpreted in the broadest sense. They may be simple complex quantities or they may be functions of the differential operator $\frac{d}{dt}$. For if

$$i = \Sigma\, (A_n \cos n\, w\, t + B_n \sin n\, w\, t)$$

it may be expressed in the form

$$\left.\begin{aligned}
i &= \Sigma \left(\frac{A_n - j\, B_n}{2}\, \epsilon^{j n w t} + \frac{A_n + j\, B_n}{2}\, \epsilon^{-j n w t}\right) \\
&= \frac{\check{I}}{2} + \frac{\hat{I}}{2} \\
&= \text{real part of } \check{I}
\end{aligned}\right\} \qquad (34)$$

and any linear algebraic operation performed on $\check{I}/2$ will give a result which will be conjugate to that obtained by carrying out the same operation on $\hat{I}/2$ and since the true solution is the sum of these results, it may also be obtained by taking the real part of the result of performing the operation on $\check{I}$.

Modification of the General Case Met With in Practical Networks

Several symmetrical arrangements of the operator Z_{au} etc.' are frequently met with in practical networks which result in a much simpler system of equations than those obtained for the general case as in equations (**29**) to (**33**). Thus for example if all the operators in (**26**) are equal, all the operators in (**27**), except $S^0\, Z_{au0}$ $S^0\, Z_{bw0}$ and $S^0\, Z_{cv0}$ are equal to zero, and these three quantities are also equal to one another so that equation (**30**) becomes

$$\left.\begin{aligned} S^0\, \check{E}_{a0} &= S^0\, (Z_{au0} + Z_{bw0} + Z_{cv0})\, \check{I}_{u0} \\ S^1\, \check{E}_{a1} &= 0 \\ S^2\, E_{a2} &= 0 \end{aligned}\right\} \quad (35)$$

and equation (**33**)

$$\left.\begin{aligned} S^0\, \check{E}_{u0} &= S^0\, (Z_{au0} + Z_{bw0} + Z_{cv0})\, \check{I}_{a0} \\ S^1\, E_{u1} &= 0 \\ S^2\, E_{u2} &= 0 \end{aligned}\right\} \quad (36)$$

This is the statement in symmetrical co-ordinates that a symmetrically disposed polyphase transmission line will produce no electromagnetic induction in a second similar polyphase system so disposed with respect to the first that mutual inductions between all phases of the two are equal except that due to single-phase currents passing through the conductors.

If in (**26**) the quantities in each group only are equal, equations (**30**) and (**33**) become

$$\left.\begin{aligned} S^0\, \check{E}_{a0} &= S^0\, (Z_{au0} + Z_{bw0} + Z_{cv0})\, \check{I}_{u0} \\ S^1\, \check{E}_{a1} &= S^1\, (Z_{au0} + a^2\, Z_{bw0} + a\, Z_{cv0})\, \check{I}_{u1} \\ S^2\, \check{E}_{a2} &= S^2\, (Z_{au0} + a\, Z_{bw0} + a^2\, Z_{cv0})\, \check{I}_{u2} \end{aligned}\right\} \quad (37)$$

$$\left.\begin{aligned} S^0\, \check{E}_{u0} &= S^0\, (Z_{au0} + Z_{bw0} + Z_{cv0})\, \check{I}_{a0} \\ S^1\, \check{E}_{a1} &= S^1\, (Z_{au0} + a\, Z_{bw0} + a^2\, Z_{cv0})\, \check{I}_{a1} \\ S^2\, \check{E}_{a2} &= S^1\, (Z_{au0} + a^2\, Z_{bw0} + a\, Z_{cv0})\, \check{I}_{a2} \end{aligned}\right\} \quad (38)$$

Symmetrical Forms of Common Occurrence

A symmetrical form which is of importance because it is of frequent occurrence in practical polyphase networks has the terms in group (I) equation (**26**) all equal and those in group (II) $\cos \frac{2\pi}{3}$ times those in group (I) and those in group (III) $\cos \frac{4\pi}{3}$ times those in group (I).

Since $\cos \frac{2\pi}{3} = \frac{a + a^2}{2} = \cos \frac{4\pi}{3}$ we have on substituting the values of the impedances in this case,

$$\left.\begin{aligned} S^0\, \check{E}_{a0} &= S^0\, \{Z_{au0}\, (1 + a + a^2)\}\, \check{I}_{u0} = 0 \\ S^1\, \check{E}_{a1} &= S^1\, 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u1} \\ S^2\, E_{a2} &= S^2\, 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u2} \end{aligned}\right\} \quad (39)$$

$$\left.\begin{aligned} S^0\, E_{u0} &= S^0\, \{Z_{au0}\, (1 + a + a^2)\}\, \check{I}_{a0} = 0 \\ S^1\, \check{E}_{u1} &= S^1\, 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{a1} \\ S^2\, \check{E}_{u2} &= S^2\, 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{a2} \end{aligned}\right\} \quad (40)$$

The elements in group I may be unequal but groups II and III may be obtained from group I by multiplying by $\cos \frac{4\pi}{3}$ and $\cos \frac{2\pi}{3}$ respectively.

The members of the three groups will then be related as follows, the same sequence being used as before,

$$\left.\begin{aligned} &(\mathrm{I}) \quad Z_{au}, \quad Z_{bv}, \quad Z_{cw} \\ &(\mathrm{II}) \quad \frac{a + a^2}{2}\, Z_{cw}, \quad \frac{a + a^2}{2}\, Z_{au}, \quad \frac{a + a^2}{2}\, Z_{bv} \\ &(\mathrm{III}) \quad \frac{a + a^2}{2}\, Z_{bv}, \quad \frac{a + a^2}{2}\, Z_{cw}, \quad \frac{a + a^2}{2}\, Z_{au} \end{aligned}\right\} \quad (41)$$

Consequently the following relations are true:

$$\left.\begin{aligned}
S^0\, Z_{bw0} &= \frac{a + a^2}{2}\, S^0\, Z_{au0} \\
S^0\, Z_{cv0} &= \frac{a + a^2}{2}\, S^0\, Z_{au0} \\
S^1\, Z_{bw1} &= \frac{1 + a^2}{2}\, S^1\, Z_{au1} \\
S^2\, Z_{bw2} &= \frac{1 + a}{2}\, S^2\, Z_{au2} \\
S^1\, Z_{cv1} &= \frac{1 + a}{2}\, S^1\, Z_{au1} \\
S^2\, Z_{cv2} &= \frac{1 + a^2}{2}\, S^2\, Z_{au2}
\end{aligned}\right\} \qquad (42)$$

Substituting these relations in (**30**) and (**33**) we have for this system of mutual impedances

$$\left.\begin{aligned}
Z_{au0} + Z_{bw0} + Z_{cv0} &= 0 \\
Z_{au0} + a\, Z_{bw0} + a^2\, Z_{cv0} &= 1\tfrac{1}{2}\, Z_{cv0} \\
Z_{au0} + a^2\, Z_{bw0} + a\, Z_{cv0} &= 1\tfrac{1}{2}\, Z_{aw0}
\end{aligned}\right\} \qquad (43)$$

$$\left.\begin{aligned}
Z_{au1} + Z_{bw1} + Z_{cv1} &= 1\tfrac{1}{2}\, Z_{au1} \\
Z_{au1} + a\, Z_{bw1} + a^2\, Z_{cv1} &= 1\tfrac{1}{2}\, Z_{au1} \\
Z_{au1} + a^2\, Z_{bw1} + a\, Z_{cv1} &= 0
\end{aligned}\right\} \qquad (44)$$

$$\left.\begin{aligned}
Z_{au2} + Z_{bw2} + Z_{cv2} &= 1\tfrac{1}{2}\, Z_{au2} \\
Z_{au2} + a\, Z_{bw2} + a^2\, Z_{cv2} &= 0 \\
Z_{au2} + a^2\, Z_{bw2} + a\, Z_{cv2} &= 1\tfrac{1}{2}\, Z_{au2}
\end{aligned}\right\} \qquad (45)$$

which on substitution in (**30**) and (**33**) gives

$$\left.\begin{aligned}
S^0\, \check{E}_{a0} &= 0 \\
S^1\, \check{E}_{a1} &= S^1\, \{1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{u0} + 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{u2}\} \\
S^2\, \check{E}_{a2} &= S^2\, \{1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{u0} + 1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u2}\}
\end{aligned}\right\} \qquad (46)$$

$$\left.\begin{aligned}
S^0\, \check{E}_{u0} &= S^0\, \{1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{a1} + 1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{a2}\} \\
S^1\, \check{E}_{u1} &= S^1\, \{1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{a1} + 1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{a2}\} \\
S^2\, \check{E}_{u2} &= S^2\, \{1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{a1} + 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{a2}\}
\end{aligned}\right\} \qquad (47)$$

The above symmetrical forms in which the factors $\cos \frac{2\pi}{3}$ and $\cos \frac{4\pi}{3}$ occur apply particularly to electromagnetic induction between windings distributed over the surfaces of co-axial cylinders; where if the plane of symmetry of one winding be taken as the datum plane, the mutual impedance between this winding and any other is a harmonic function of the angle between its plane of symmetry and the datum plane. In other words, the mutual impedances are functions of position on the circumference of a circle and may therefore be expanded by Fourier's theorem in a series of integral harmonics of the angle made by the planes of symmetry with the datum plane. Since the same procedure applies to all the terms of the expansion it is necessary only to consider the simple harmonic case. In the partially symmetrical cases of mutual induction, such as that taken up in the preceding discussion, there will be a difference between two possible cases, viz:—Symmetrical primary, unsymmetrical secondary, which is the case just considered, and unsymmetrical primary and symmetrical secondary in which the impedances of (**26**) will have the following values

$$\left.\begin{array}{llll}
\text{(I)} & Z_{au}, & Z_{bc}, & Z_{cw} \\
\text{(II)} & \dfrac{a+a^2}{2} Z_{bc}, & \dfrac{a+a^2}{2} Z_{cw}, & \dfrac{a+a^2}{2} Z_{au} \\
\text{(III)} & \dfrac{a+a^2}{2} Z_{cw}, & \dfrac{a+a^2}{2} Z_{au}, & \dfrac{a+a^2}{2} Z_{bc}
\end{array}\right\} \quad (\mathbf{48})$$

The results may be immediately set down by symmetry from equations (**46**) and (**47**), but the difference between the two cases will be better appreciated by setting down the component symmetrical impedances, thus we have

$$\left.\begin{array}{l}
S^0 Z_{bw0} = \dfrac{a+a^2}{2} S^0 Z_{au0} \\
S^0 Z_{cv0} = \dfrac{a+a^2}{2} S^0 Z_{au0} \\
S^1 Z_{bw1} = \dfrac{1+a}{2} S^1 Z_{au1} \\
S^2 Z_{bw2} = \dfrac{1+a^2}{2} S^2 Z_{au2} \\
S^1 Z_{cv1} = \dfrac{1+a^2}{2} S^1 Z_{cu1} \\
S^2 Z_{cv2} = \dfrac{1+a}{2} S^2 Z_{au2}
\end{array}\right\} \quad (\mathbf{49})$$

Substituting these relations in the impedances used in (**30**) and (**33**) they become

$$\left.\begin{aligned} Z_{au0} + Z_{bw0} + Z_{cv0} &= 0 \\ Z_{au0} + a\, Z_{bw0} + a^2\, Z_{cv0} &= 1\tfrac{1}{2}\, Z_{au0} \\ Z_{au0} + a^2\, Z_{bw0} + a\, Z_{cv0} &= 1\tfrac{1}{2}\, Z_{au0} \end{aligned}\right\} \quad (50)$$

$$\left.\begin{aligned} Z_{au1} + Z_{bw1} + Z_{cv1} &= 1\tfrac{1}{2}\, Z_{au1} \\ Z_{au1} + a\, Z_{bw1} + a^2\, Z_{cv1} &= 0 \\ Z_{au1} + a^2\, Z_{bw1} + a\, Z_{cv1} &= 1\tfrac{1}{2}\, Z_{au1} \end{aligned}\right\} \quad (51)$$

$$\left.\begin{aligned} Z_{au2} + Z_{bw2} + Z_{cv2} &= 1\tfrac{1}{2}\, Z_{au2} \\ Z_{au2} + a\, Z_{bw2} + a^2\, Z_{cv2} &= 1\tfrac{1}{2}\, Z_{au2} \\ Z_{au2} + a^2\, Z_{bw2} + a\, Z_{cv2} &= 0 \end{aligned}\right\} \quad (52)$$

And we have from (**30**) and (**33**), or by symmetry

$$\left.\begin{aligned} S^0\, \check{E}_{a0} &= S^0\, \{1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{u2}\} \\ S^1\, \check{E}_{a1} &= S^1\, \{1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{u2}\} \\ S^2\, \check{E}_{a2} &= S^2\, \{1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u2} \end{aligned}\right\} \quad (53)$$

$$\left.\begin{aligned} S^0\, \check{E}_{u0} &= 0 \\ S^1\, \check{E}_{u1} &= S^1\, \{1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{a0} + 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{u2}\} \\ S^2\, \check{E}_{u2} &= S^2\, \{1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{a0} + 1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u2}\} \end{aligned}\right\} \quad (54)$$

If the angle between the planes of symmetry of the coils and the datum plane are subject to changes, $\cos \dfrac{2\pi}{3}$ and $\cos \dfrac{4\pi}{3}$ in the preceding discussion must be replaced by

$$\left.\begin{aligned} \cos\left(\frac{2\pi}{3} + \theta\right) &= \frac{a}{2}\, e^{j\theta} + \frac{a^2}{2}\, e^{-j\theta} \\ \cos\left(\frac{4\pi}{3} + \theta\right) &= \frac{a}{2}\, e^{-j\theta} + \frac{a^2}{2}\, e^{j\theta} \end{aligned}\right\} \quad (55)$$

where θ is measured from the datum plane

In the strictly symmetrical case of co-axial cylindrical surface windings in which the members of each group of mutual

impedances are equal, the result of substituting (**55**) in the equations for induced e.m.f. will be

$$\left.\begin{aligned} S^0\,\check{E}_{a0} &= 0 \\ S^1\,\check{E}_{a1} &= S^1\,(1\tfrac{1}{2}\,Z_{au0}\,e^{j\theta}\,\check{I}_{u1}) \\ S^2\,\check{E}_{a2} &= S^2\,(1\tfrac{1}{2}\,Z_{au0}\,e^{-j\theta}\,\check{I}_{u2}) \end{aligned}\right\} \qquad (56)$$

$$\left.\begin{aligned} S^0\,\check{E}_{u0} &= 0 \\ S^1\,\check{E}_{u1} &= S^1\,(1\tfrac{1}{2}\,Z_{au0}\,e^{-j\theta}\,\check{I}_{a1}) \\ S^2\,\check{E}_{u2} &= S^2\,(1\tfrac{1}{2}\,Z_{au0}\,e^{j\theta}\,\check{I}_{a2}) \end{aligned}\right\} \qquad (57)$$

In the case having symmetrical primary and unsymmetrical secondary in which members of each group are different, but in which there are harmonic relations between corresponding members of the different groups, the impedances are

$$\left.\begin{aligned} &(\mathrm{I})\; Z_{au},\quad Z_{bv},\quad Z_{cw} \\ &(\mathrm{II})\left(\frac{a}{2}\,e^{j\theta}+\frac{a^2}{2}\,e^{-j\theta}\right)Z_{cw} \\ &\left(\frac{a}{2}\,e^{j\theta}+\frac{a^2}{2}\,e^{-j\theta}\right)Z_{au},\ \left(\frac{a}{2}\,e^{j\theta}+\frac{a^2}{2}\,e^{-j\theta}\right)Z_{bv} \\ &(\mathrm{III})\left(\frac{a^2}{2}\,e^{j\theta}+\frac{a}{2}\,e^{-j\theta}\right)Z_{bv}, \\ &\left(\frac{a^2}{2}\,e^{j\theta}+\frac{a}{2}\,e^{-j\theta}\right)Z_{cw},\ \left(\frac{a^2}{2}\,e^{j\theta}+\frac{a}{2}\,e^{-j\theta}\right)Z_{au} \end{aligned}\right\} \qquad (58)$$

The symmetrical component mutual impedances will have the following values in terms of $Z_{au0}\ Z_{au1}\ Z_{au2}$

$$\left.\begin{aligned} S^0\,Z_{bw0} &= \left(\frac{a}{2}\,e^{j\theta}+\frac{a^2}{2}\,e^{-j\theta}\right)S^0\,Z_{au0} \\ S^0\,Z_{cv0} &= \left(\frac{a^2}{2}\,e^{j\theta}+\frac{a}{2}\,e^{-j\theta}\right)S^0\,Z_{au0} \\ S^1\,Z_{bw1} &= \left(\frac{a^2}{2}\,e^{j\theta}+\frac{e^{-j\theta}}{2}\right)S^1\,Z_{au1} \\ S^2\,Z_{bw2} &= \left(\frac{e^{j\theta}}{2}+\frac{a}{2}\,e^{-j\theta}\right)S^2\,Z_{au2} \\ S^1\,Z_{cv1} &= \left(\frac{a}{2}\,e^{j\theta}+\frac{e^{-j\theta}}{2}\right)S^1\,Z_{au1} \\ S^2\,Z_{cv2} &= \left(\frac{e^{j\theta}}{2}+\frac{a^2}{2}\,e^{-j\theta}\right)S^2\,Z_{au2} \end{aligned}\right\} \qquad (59)$$

Substituting these relations in the impedances of equations (**30**) and (**33**) they become

$$\left.\begin{aligned}
Z_{au0} + Z_{bw0} + Z_{cv0} &= 0\\
Z_{au0} + a\, Z_{bw0} + a^2\, Z_{cv0} &= 1\tfrac{1}{2}\, Z_{au0}\, e^{-j\theta}\\
Z_{au0} + a^2\, Z_{bw0} + a\, Z_{cv0} &= 1\tfrac{1}{2}\, Z_{au0}\, e^{j\theta}
\end{aligned}\right\} \tag{60}$$

$$\left.\begin{aligned}
Z_{au1} + Z_{bw1} + Z_{cv1} &= 1\tfrac{1}{2}\, Z_{au1}\, e^{-j\theta}\\
Z_{au1} + a\, Z_{bw1} + a^2\, Z_{cv1} &= 1\tfrac{1}{2}\, Z_{au1}\, e^{j\theta}\\
Z_{au1} + a^2\, Z_{bw1} + a\, Z_{cv1} &= 0
\end{aligned}\right\} \tag{61}$$

$$\left.\begin{aligned}
Z_{au2} + Z_{bw2} + Z_{cv2} &= 1\tfrac{1}{2}\, Z_{au2}\, e^{j\theta}\\
Z_{au2} + a\, Z_{bw2} + a^2\, Z_{cv2} &= 0\\
Z_{au2} + a^2\, Z_{bw2} + a\, Z_{cv2} &= 1\tfrac{1}{2}\, Z_{au}\, e^{-j\theta}
\end{aligned}\right\} \tag{62}$$

which on substitution in (**30**) and (**33**) give

$$\left.\begin{aligned}
S^0\, \check{E}_{a0} &= 0\\
S^1\, \check{E}_{a1} &= S^1\, \{1\tfrac{1}{2}\, Z_{au1}\, e^{j\theta}\ \check{I}_{u0} + 1\tfrac{1}{2}\, Z_{au0}\, e^{j\theta}\ \check{I}_{u1}\\
&\qquad + 1\tfrac{1}{2}\, Z_{au2}\, e^{j\theta}\ \check{I}_{u2}\}\\
S^2\, \check{E}_{a2} &= S^2\, \{1\tfrac{1}{2}\, Z_{au2}\, e^{-j\theta}\, \check{I}_{u0} + 1\tfrac{1}{2}\, Z_{au1}\, e^{-j\theta}\, \check{I}_{u1}\\
&\qquad + 1\tfrac{1}{2}\, Z_{au0}\, e^{-j\theta}\, \check{I}_{u2}\}
\end{aligned}\right\} \tag{63}$$

$$\left.\begin{aligned}
S^0\, \check{E}_{u0} &= S^0\, \{1\tfrac{1}{2}\, Z_{au2}\, e^{-j\theta}\, \check{I}_{a1} + 1\tfrac{1}{2}\, Z_{au1}\, e^{j\theta}\ \check{I}_{a2}\}\\
S^1\, \check{E}_{u1} &= S^1\, \{1\tfrac{1}{2}\, Z_{au0}\, e^{-j\theta}\, \check{I}_{a1} + 1\tfrac{1}{2}\, Z_{au2}\, e^{j\theta}\ \check{I}_{a2}\}\\
S^2\, \check{E}_{u2} &= S^2\, \{1\tfrac{1}{2}\, Z_{au1}\, e^{-j\theta}\, \check{I}_{a1} + 1\tfrac{1}{2}\, Z_{au0}\, e^{j\theta}\ \check{I}_{a2}\}
\end{aligned}\right\} \tag{64}$$

In the case of unsymmetrical primary and symmetrical secondary, we have for the value of the impedance in terms of Z_{au0} Z_{au1} and Z_{au2}

$$\left.\begin{aligned}
&\text{(I)}\ Z_{au},\quad Z_{bv},\quad Z_{cw}\\
&\text{(II)} \left(\frac{a}{2}\, e^{j\theta} + \frac{a^2}{2}\, e^{-j\theta}\right) Z_{bc},\\
&\left(\frac{a}{2}\, e^{j\theta} + \frac{a^2}{2}\, e^{-j\theta}\right) Z_{cw},\ \left(\frac{a}{2}\, e^{j\theta} + \frac{a^2}{2}\, e^{-j\theta}\right) Z_{au}\\
&\text{(III)} \left(\frac{a^2}{2}\, e^{j\theta} + \frac{a}{2}\, e^{-j\theta}\right) Z_{cw},\\
&\left(\frac{a^2}{2}\, e^{j\theta} + \frac{a}{2}\, e^{-j\theta}\right) Z_{au},\ \left(\frac{a^2}{2}\, e^{j\theta} + \frac{a}{2}\, e^{-j\theta}\right) Z_{bc}
\end{aligned}\right\} \tag{65}$$

The symmetrical component mutual impedances in terms of Z_{au0}, Z_{au1}, Z_{au2} are

$$\left.\begin{aligned}
S^0 Z_{bw0} &= \left(\frac{a}{2} e^{j\theta} + \frac{a^2}{2} e^{-j\theta}\right) S^0 Z_{au0} \\
S^0 Z_{cv0} &= \left(\frac{a^2}{2} e^{j\theta} + \frac{a}{2} e^{-j\theta}\right) S^0 Z_{au0} \\
S^1 Z_{bw1} &= \left(\frac{e^{j\theta}}{2} + \frac{a}{2} e^{-j\theta}\right) S^1 Z_{au1} \\
S^2 Z_{bw2} &= \left(\frac{a^2}{2} e^{j\theta} + \frac{e^{-j\theta}}{2}\right) S^2 Z_{au2} \\
S^1 Z_{cv1} &= \left(\frac{e^{j\theta}}{2} + \frac{a^2}{2} e^{-j\theta}\right) S^1 Z_{au1} \\
S^2 Z_{cv2} &= \left(\frac{a}{2} e^{j\theta} + \frac{e^{-j\theta}}{2}\right) S^2 Z_{au2}
\end{aligned}\right\} \quad (66)$$

And the impedances of equations (**30**) and (**33**) become

$$\left.\begin{aligned}
Z_{au0} + Z_{bw0} + Z_{cv0} &= 0 \\
Z_{au0} + a\, Z_{bw0} + a^2 Z_{cv0} &= 1\tfrac{1}{2} Z_{au0}\, e^{-j\theta} \\
Z_{au0} + a^2 Z_{bw0} + a\, Z_{cv0} &= 1\tfrac{1}{2} Z_{au0}\, e^{j\theta}
\end{aligned}\right\} \quad (67)$$

$$\left.\begin{aligned}
Z_{au1} + Z_{bw1} + Z_{cv1} &= 1\tfrac{1}{2} Z_{au1}\, e^{j\theta} \\
Z_{au1} + a\, Z_{bw1} + a^2 Z_{cv1} &= 0 \\
Z_{au1} + a^2 Z_{bw1} + a\, Z_{cv1} &= 1\tfrac{1}{2} Z_{au1}\, e^{-j\theta}
\end{aligned}\right\} \quad (68)$$

$$\left.\begin{aligned}
Z_{au2} + Z_{bw2} + Z_{cv2} &= 1\tfrac{1}{2} Z_{au2}\, e^{-j\theta} \\
Z_{au2} + a\, Z_{bw2} + a^2 Z_{cv2} &= 1\tfrac{1}{2} Z_{au2}\, e^{j\theta} \\
Z_{au2} + a^2 Z_{bw} + a\, Z_{cv2} &= 0
\end{aligned}\right\} \quad (69)$$

And on substitution in (**30**) and (**33**), or by symmetry from (**63**) and (**64**), we have

$$\left.\begin{aligned}
S^0 \check{E}_{a0} &= S^0 \left\{1\tfrac{1}{2} Z_{au2}\, e^{j\theta}\ \check{I}_{u1} + 1\tfrac{1}{2} Z_{au1}\, e^{-j\theta}\ \check{I}_{u2}\right\} \\
S^1 \check{E}_{a1} &= S^1 \left\{1\tfrac{1}{2} Z_{au0}\, e^{j\theta}\ \check{I}_{u1} + 1\tfrac{1}{2} Z_{au2}\, e^{-j\theta}\ \check{I}_{u2}\right\} \\
S^2 \check{E}_{a2} &= S^2 \left\{1\tfrac{1}{2} Z_{au1}\, e^{j\theta}\ \check{I}_{u1} + 1\tfrac{1}{2} Z_{au0}\, e^{-j\theta}\ \check{I}_{u2}\right.
\end{aligned}\right\} \quad (70)$$

$$\left.\begin{aligned} S^0 \check{E}_{u0} &= 0 \\ S^1 \check{E}_{u1} &= S^1 \{1\tfrac{1}{2}\, Z_{au1}\, e^{-j\theta}\, \check{I}_{a0} + 1\tfrac{1}{2}\, Z_{au0}\, e^{-j\theta}\, \check{I}_{u1} \\ &\qquad + 1\tfrac{1}{2}\, Z_{au2}\, e^{-j\theta}\, \check{I}_{a2}\} \\ S^2 \check{E}_{u2} &= S^2 \{1\tfrac{1}{2}\, Z_{au2}\, e^{j\theta}\, \check{I}_{a0} + 1\tfrac{1}{2}\, Z_{au1}\, e^{j\theta}\, \check{I}_{a1} \\ &\qquad + 1\tfrac{1}{2}\, Z_{au0}\, e^{j\theta}\, \check{I}_{a2}\} \end{aligned}\right\} \qquad (71)$$

A fuller discussion of self and mutual impedances of co-axial cylindrical windings will be found in the Appendix. It will be sufficient to note here that in the case of self inductance and mutual inductance of stationary windings symmetrically disposed if they are equal

$$\left.\begin{aligned} M_{ab} = M_{bc} = M_{ca} &= \Sigma \left(A_n \cos \frac{2\, n\, \pi}{3} \right) \\ L_{aa} = L_{bb} = L_{cc} = M_{aa} = M_{bb} = M_{cc} &= \Sigma\, A_n \end{aligned}\right\} \qquad (72)$$

If the windings are symmetrically disposed but have different number of turns

$$\left.\begin{aligned} L_{aa} &= M_{aa} = \Sigma\, A_n \\ L_{bb} &= M_{bb} = \Sigma\, B_n \\ L_{cc} &= M_{cc} = \Sigma\, C_n \end{aligned}\right\} \qquad (73)$$

$$\left.\begin{aligned} M_{ab} &= \Sigma \left(\sqrt{A_n\, B_n} \cos \frac{2\, n\, \pi}{3} \right) \\ M_{bc} &= \Sigma \left(\sqrt{B_n\, C_n} \cos \frac{2\, n\, \pi}{3} \right) \\ M_{ca} &= \Sigma \left(\sqrt{C_n\, A_n} \cos \frac{2\, n\, \pi}{3} \right) \end{aligned}\right\} \qquad (74)$$

If the coils are alike but unsymmetrically spaced $L_{aa}\ L_{bb}\ L_{cc}$ have the same values, namely $\Sigma\, A_n$ and

$$\left.\begin{aligned} M_{ab} &= \Sigma \left\{ (A_n \cos n\, \theta_1) \cos \frac{2\, n\, \pi}{3} + (A_n \sin n\, \theta_1) \sin \frac{2\, n\, \pi}{3} \right\} \\ M_{bc} &= \Sigma \left\{ (A_n \cos n\, \theta_2) \cos \frac{2\, n\, \pi}{3} + (A_n \sin n\, \theta_2) \sin \frac{2\, n\, \pi}{3} \right\} \\ M_{ca} &= \Sigma \left\{ (A_n \cos n\, \theta_3) \cos \frac{2\, n\, \pi}{3} + (A_n \sin n\, \theta_3) \sin \frac{2\, n\, \pi}{3} \right\} \end{aligned}\right\} \qquad (75)$$

If they are unequal as well as unsymmetrically disposed but are otherwise similar L_{aa} L_{bb} L_{cc} have values as in (64) and

$$\left.\begin{aligned}
M_{ab} &= \Sigma \left\{ (\sqrt{A_n B_n} \cos n\,\theta_1) \cos \frac{2\,n\,\pi}{3} \right. \\
&\qquad \left. + (\sqrt{A_n B_n} \sin n\,\theta_1) \sin \frac{2\,n\,\pi}{3} \right\} \\
M_{bc} &= \Sigma \left\{ (\sqrt{B_n C_n} \cos n\,\theta_2) \cos \frac{2\,n\,\pi}{3} \right. \\
&\qquad \left. + (\sqrt{B_n C_n} \sin n\,\theta_2) \sin \frac{2\,n\,\pi}{3} \right\} \\
M_{ca} &= \Sigma \left\{ (\sqrt{C_n A_n} \cos n\,\theta_1) \cos \frac{2\,n\,\pi}{3} \right. \\
&\qquad \left. + (\sqrt{C_n A_n} \sin n\,\theta_3) \sin \frac{2\,n\,\pi}{3} \right\}
\end{aligned}\right\} \tag{76}$$

Where the windings are dissimilar in every respect the expressions become more complicated. A short outline of this subject is given in the Appendix.

In the case of mutual inductance between two coaxial cylindrical systems, one of which A, B, C is the primary and the other U, V, W the secondary, the following conventions should be followed:

(a) All angles are measured, taking the primary planes of symmetry as data in a positive direction.

(b) The datum plane for all windings is the plane of symmetry of the primary A phase.

(c) All mechanical motions unless otherwise stated shall be considered as positive rotations of the secondary cylinder about its axis.

(d) The conventional disposition of the phases and the direction of rotation of the secondary winding are indicated in Fig. 3.

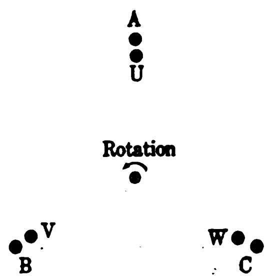


FIG. 3—CONVENTIONAL DISPOSITION OF PHASES AND DIRECTION OF ROTATION.

We shall consider five cases; Case 1 being the completely symmetrical case and the rest being symmetrical in one winding, the other winding being unsymmetrical in magnitude and phase, or both, but all windings having the same form and distribution of coils.

Case I. All Windings Symmetrical.

$$\left.\begin{aligned}
M_{au} &= M_{bv} = M_{cw} = \Sigma A_n \cos n\,\theta \\
M_{bw} &= M_{cu} = M_{av} = \Sigma A_n \cos n\left(\frac{2\,\pi}{3} + \theta\right) \\
M_{cv} &= M_{aw} = M_{bu} = \Sigma A_n \cos n\left(\frac{4\,\pi}{3} + \theta\right)
\end{aligned}\right\} \qquad (77)$$

Case II. Primary Windings equal and Symmetrical, Secondary Windings unequal but otherwise Symmetrical.

$$\left.\begin{aligned}
M_{au} &= \Sigma A_n \cos n\,\theta,\ M_{bv} = \Sigma B_n \cos n\,\theta,\ M_{cw} = \Sigma C_u \cos n\,\theta \\
M_{bw} &= \Sigma C_u \cos n\left(\frac{2\,\pi}{3} + \theta\right), \\
M_{cu} &= \Sigma A_n \cos n\left(\frac{2\,\pi}{3} + \theta\right), \\
M_{av} &= \Sigma B_n \cos n\left(\frac{2\,\pi}{3} + \theta\right) \\
M_{cv} &= \Sigma B_n \cos n\left(\frac{4\,\pi}{3} + \theta\right), \\
M_{aw} &= \Sigma C_n \cos n\left(\frac{4\,\pi}{3} + \theta\right), \\
M_{bu} &= \Sigma A_n \cos n\left(\frac{4\,\pi}{3} + \theta\right)
\end{aligned}\right\} \qquad (78)$$

Case III. Primary Winding Unequal but Otherwise Symmetrical, Secondary Winding &qual and Symmetrical.

$$\left.\begin{aligned}
M_{au} &= \Sigma A_n \cos n\,\theta,\ M_{bv} = \Sigma B_n \cos n\,\theta,\ M_{cw} = \Sigma C_n \cos n\theta \\
M_{bw} &= \Sigma B_n \cos n\left(\frac{2\,\pi}{3} + \theta\right), \\
M_{cu} &= \Sigma C_n \cos n\left(\frac{2\,\pi}{3} + \theta\right) \\
M_{av} &= \Sigma A_n \cos n\left(\frac{2\,\pi}{3} + \theta\right) \\
M_{cv} &= \Sigma C_n \cos n\left(\frac{4\,\pi}{3} + \theta\right), \\
M_{aw} &= \Sigma A_n \cos n\left(\frac{4\,\pi}{3} + \theta\right), \\
M_{bu} &= \Sigma B_n \cos n\left(\frac{4\,\pi}{3} + \theta\right)
\end{aligned}\right\} \qquad (79)$$

Case IV. Same as Case II except in addition to inequality Secondary Windings are Displaced from Symmetry by angles α_1 α_2 and α_3 whose sum is zero.

$$
\left.
\begin{aligned}
M_{au} &= \Sigma\,(A_n \cos\alpha_1 \cos n\,\theta + A_n \sin\alpha_1 \sin n\,\theta) \\
M_{bv} &= \Sigma\,(B_n \cos\alpha_2 \cos n\,\theta + B_n \sin\alpha_2 \sin n\,\theta) \\
M_{cw} &= \Sigma\,(C_n \cos\alpha_3 \cos n\,\theta + C_n \sin\alpha_3 \sin n\,\theta) \\
M_{bw} &= \Sigma \left\{ C_n \cos\alpha_3 \cos n\left(\frac{2\,\pi}{3} + \theta\right) + C_n \sin\alpha_3 \sin n\left(\frac{2\,\pi}{3} + \theta\right)\right\} \\
M_{cu} &= \Sigma \left\{ A_n \cos\alpha_1 \cos n\left(\frac{2\,\pi}{3} + \theta\right) + A_n \sin\alpha_1 \sin n\left(\frac{2\,\pi}{3} + \theta\right)\right\} \\
M_{av} &= \Sigma \left\{ B_n \cos\alpha_2 \cos n\left(\frac{2\,\pi}{3} + \theta\right) + B_n \sin\alpha_2 \sin n\left(\frac{2\,\pi}{3} + \theta\right)\right\} \\
M_{cv} &= \Sigma \left\{ B_n \cos\alpha_2 \cos n\left(\frac{4\,\pi}{3} + \theta\right) + B_n \sin\alpha_2 \sin n\left(\frac{4\,\pi}{3} + \theta\right)\right\} \\
M_{aw} &= \Sigma \left\{ C_n \cos\alpha_3 \cos n\left(\frac{4\,\pi}{3} + \theta\right) + C_n \sin\alpha_3 \sin n\left(\frac{4\,\pi}{3} + \theta\right)\right\} \\
M_{bu} &= \Sigma \left\{ A_n \cos\alpha_1 \cos n\left(\frac{4\,\pi}{3} + \theta\right) + A_n \sin\alpha_1 \sin n\left(\frac{4\,\pi}{3} + \theta\right)\right\}
\end{aligned}
\right\} \quad (80)
$$

Case V. Same as Case III except that the Primary Windings are Unsymmetrically disposed with respect to one another as well as being unequal.

$$\left.\begin{aligned}
M_{au} &= \Sigma\,(A_n \cos\alpha_1 \cos n\,\theta + A_n \sin\alpha_1 \sin n\,\theta)\\
M_{bv} &= \Sigma\,(B_n \cos\alpha_2 \cos n\,\theta + B_n \sin\alpha_2 \sin n\,\theta)\\
M_{cw} &= \Sigma\,(C_n \cos\alpha_3 \cos n\,\theta + C_n \sin\alpha_3 \sin n\,\theta)\\
M_{bw} &= \Sigma\left\{B_n \cos\alpha_2 \cos n\left(\frac{2\,\pi}{3} + \theta\right)\right.\\
&\qquad \left. + B_n \sin\alpha_2 \sin n\left(\frac{2\,\pi}{3} + \theta\right)\right\}\\
M_{cu} &= \Sigma\left\{A_n \cos\alpha_1 \cos n\left(\frac{2\,\pi}{3} + \theta\right)\right.\\
&\qquad \left. + A_n \sin\alpha_1 \sin n\left(\frac{2\,\pi}{3} + \theta\right)\right\}\\
M_{av} &= \Sigma\left\{C_n \cos\alpha_3 \cos n\left(\frac{2\,\pi}{3} + \theta\right)\right.\\
&\qquad \left. + C_u \sin\alpha_3 \sin n\left(\frac{2\,\pi}{3} + \theta\right)\right\}\\
M_{cv} &= \Sigma\left\{C_n \cos\alpha_3 \cos n\left(\frac{4\,\pi}{3} + \theta\right)\right.\\
&\qquad \left. + C_n \sin\alpha_3 \sin n\left(\frac{4\,\pi}{3} + \theta\right)\right\}\\
M_{aw} &= \Sigma\left\{A_n \cos\alpha_1 \cos n\left(\frac{4\,\pi}{3} + \theta\right)\right.\\
&\qquad \left. + A_n \sin\alpha_1 \sin n\left(\frac{4\,\pi}{3} + \theta\right)\right\}\\
M_{bu} &= \Sigma\left\{B_n \cos\alpha_2 \cos n\left(\frac{4\,\pi}{3} + \theta\right)\right.\\
&\qquad \left. + B_n \sin\alpha_2 \sin n\left(\frac{4\,\pi}{3} + \theta\right)\right\}
\end{aligned}\right\} \quad (81)$$

The expressions for dissymmetry in both windings and for unsymmetrically wound coils, etc., are more complicated and will be dealt with in the Appendix.

The impedances Z_{aa} Z_{bb}, etc., Z_{au} Z_{bv}, etc., are functions of M_{ac} M_{bb}, etc., M_{au} M_{bv}, etc., and the resistances of the system. The component of e. m. f. proportional to the current due to

mutual impedance is so small that it may generally be neglected so that Z_{au} becomes $\frac{d}{dt}$ M_{au}, $Z_{br} = \frac{d}{dt} M_{br}$ and so forth.

If the secondary winding is rotating at an angular velocity α, θ in equation (55) becomes $\alpha\, t$ and the operators Z_{aa}, etc. operate on such products as $e^{j\alpha t}\ \check{I}_{u1}\ \ e^{j\alpha t}\ \check{I}_{u2}$ where $\check{I}_{u1}$ and $\check{I}_{u2}$ are three variables.

The following relations will be found useful in the application of the method in actual examples.

If D denotes the operator $\frac{d}{d\,x}$ and $\varphi\,(Z)$ is a rational algebraic function of Z

$$\left.\begin{aligned} &\varphi\,(D)\; e^{ax} = \varphi\,(a)\; e^{ax} \\ &\varphi\,(D)\;\{e^{ax}\;\; X\} = e^{ax}\;\; \varphi\,(D + a)\; X \\ &\varphi\,(D)\; Y = e^{ax}\, \varphi\;(D + a)\;\; Y\, e^{-ax} \end{aligned}\right\} \quad (82)$$

Where X and Y may be any function of x.

Star and Delta e.m.fs. and Currents in Terms of Symmetrical Components

It has been shown in the preceding portion of this paper that the e. m. fs. $\check{E}_a$ $\check{E}_b$ and $\check{E}_c$ and the currents $\check{I}_a$ $\check{I}_b$ and $\check{I}_c$ whatever their distortion, may be represented by the sum of symmetrical systems of e. m. fs. or currents so that the two expressions

$$\left.\begin{aligned} S\;(\check{E}_a) &= S^0\; \check{E}_{a0} + S^1\; \check{E}_{a1} + S^2\; \check{E}_{a2} \\ S\;(\check{I}_a) &= S^0\; \check{I}_{a0} + S^1\; \check{I}_{a1} + S^2\; \check{I}_{a2} \end{aligned}\right\} \quad (83)$$

completely define these two systems.

If we take the delta e. m. fs. and currents corresponding to $S^0\,\check{E}_{a0}$, $S^1\,\check{E}_{a1}$ and $S^2\,\check{E}_{a2}$, $S^1\,\check{I}_{a1}$, $S^2\,\check{I}_{a2}$, we have, since $\check{E}_{bc1}$ leads $\check{E}_{a1}$ by $\frac{\pi}{2}$ and $\check{E}_{bc2}$ lags behind $\check{E}_{a2}$ by the same angle

$$\left.\begin{aligned} S^0\; \check{E}_{bc0} &= 0 \\ S^1\; \check{E}_{bc1} &= j\,\sqrt{3}\; S^1\; \check{E}_{a1} \\ S^2\; \check{E}_{bc2} &= -\,j\,\sqrt{3}\; S^2\; \check{E}_{a2} \\ S^0\; \check{I}_{bc0} &= \text{indeterminate from } S\;(\check{I}_a) \\ S^1\; \check{I}_{bc1} &= \;\; j\,\frac{1}{\sqrt{3}}\; S^1\; \check{I}_{a1} \\ S^2\; \check{I}_{bc2} &= -j\,\frac{1}{\sqrt{3}}\; S^2\; \check{I}_{a2} \end{aligned}\right\} \quad (84)$$

And therefore if we take $\check{E}_{ab}$ as the principal vector

$$\left.\begin{aligned} S^0\,\check{E}_{ab0} &= 0 \\ S^1\,E_{ab1} &= \quad j\,a\,\sqrt{3}\,\check{E}_{a1} \\ S^2\,\check{E}_{ab2} &= -j\,a^2\,\sqrt{3}\,\check{E}_{a2} \\ S\,(\check{E}_{ab}) &= S^1\,\check{E}_{ab1} + S^2\,\check{E}_{ab2} \end{aligned}\right\} \qquad (85)$$

The last equation of group (85) when expanded gives

$$\left.\begin{aligned} \check{E}_{ab} &= j\,\sqrt{3}\,(a\,\check{E}_{a1} - a^2\,\check{E}_{a2}) \\ \check{E}_{bc} &= j\,\sqrt{3}\,(\check{E}_{a1} - \check{E}_{a2}) \\ \check{E}_{ca} &= j\,\sqrt{3}\,(a^2\,\check{E}_{a1} - a\,\check{E}_{a2}) \end{aligned}\right\} \qquad (86)$$

which may also be obtained direct from (83) by means of the relations

$$\begin{aligned} \check{E}_{ab} &= \check{E}_b - \check{E}_a \\ \check{E}_{bc} &= \check{E}_c - \check{E}_b \\ \check{E}_{ca} &= \check{E}_a - \check{E}_0 \end{aligned}$$

Similarly

$$\begin{aligned} S^0\,\check{I}_{ab} &= \text{indeterminate from } S\,(\check{I}_a) \\ S^1\,\check{I}_{ab1} &= j\,a\,\frac{1}{\sqrt{3}}\,\check{I}_{a1} \\ S^2\,\check{I}_{ab2} &= j\,a^2\,\frac{1}{\sqrt{3}}\,\check{I}_{a2} \qquad\qquad (87) \\ S\,(\check{I}_{ab}) &= S^0\,\check{I}_{ab0} + S^1\,I_{ab1} + S^2\,\check{I}_{ab2} \end{aligned}$$

with similar expression for $\check{I}_{ab}$ $\check{I}_{bc}$ and $\check{I}_{ca}$ which may be verified by means of the relations

$$\begin{aligned} \check{I}_a &= \check{I}_{ca} - \check{I}_{ab} + \check{I}_{a0} \\ \check{I}_b &= \check{I}_{ab} - \check{I}_{bc} + \check{I}_{a0} \\ \check{I}_c &= \check{I}_{bc} - \check{I}_{ca} + \check{I}_{a0} \end{aligned}$$

Conversely to (84) we have the following relations

$$\left.\begin{aligned}
S^0\, \check{E}_{a0} &= \text{indeterminate from } S\,(\check{E}_{ab}) \\
S^1\, \check{E}_{a1} &= -\,j\frac{1}{\sqrt{3}}\, S^1\, \check{E}_{bc1} = -j\,\frac{a^2}{\sqrt{3}}\, S^1\, \check{E}_{ab1} \\
S^2\, \check{E}_{a2} &= \quad j\,\frac{1}{\sqrt{3}}\, S^2\, E_{bc2} = \quad j\,\frac{a}{\sqrt{3}}\, S^2\, E_{ab2} \\
S^0\, \check{I}_{a0} &= \text{indeterminate from } S\,(\check{I}_{ab}) \\
S^1\, \check{I}_{a1} &= -\,j\,\sqrt{3}\, S^1\, I_{bc1} = -\,j\,a^2\,\sqrt{3}\, S^1\, \check{I}_{ab1} \\
S^2\, \check{I}_{a2} &= \quad j\,\sqrt{3}\, S^2\, \check{I}_{bc} = \quad j\,a\,\sqrt{3}\, S^2\, \check{I}_{ab2}
\end{aligned}\right\} \qquad (88)$$

It will be sufficient in order to illustrate the application of the principle of symmetrical coordinates to simple circuits to apply it to a few simple cases of transformer connections before proceeding to its application to rotating polyphase systems to which it is particularly adapted.

Unsymmetrical Bank of Delta-Delta Transformers Operating on a Symmetrical Circuit Supplying a Balanced System

Let the transformer effective impedances be $Z_{AB}\ Z_{BC}\ Z_{CA}$ and let the secondary load currents be $\check{I}_U\ \check{I}_V$ and $\check{I}_W$ and let the star load impedance be Z. One to one ratio of transformation will be assumed, and the effect of the magnetizing current will be neglected. The symmetrical equations are

$$\left.\begin{aligned}
O &= S^0\,(Z_{AB0}\,\check{I}_{ab0} + Z_{AB2}\,\check{I}_{ab1} + Z_{AB1}\,\check{I}_{ab2}) \\
S^1\,\check{E}_{uv1} &= S^1\,\check{E}_{ab1} - S^1\,(Z_{AB1}\,\check{I}_{ab0} + Z_{AB0}\,\check{I}_{ab1} + Z_{AB2}\,\check{I}_{ab2}) \\
S^2\,\check{E}_{uv2} &= 0 - S^2\,(Z_{AB2}\,\check{I}_{ab0} + Z_{AB1}\,\check{I}_{ab1} + Z_{AB0}\,\check{I}_{ab2}) \\
S^0\,\check{I}_{u0} &= 0 \\
S^1\,Z\,\check{I}_{u1} &= \check{E}_{u1} \\
S^2\,Z\,\check{I}_{u2} &= \check{E}_{u2}
\end{aligned}\right\} \qquad (89)$$

Since the transformation ratio is unity and the effects of magnetizing currents are negligible $S^1\,\check{I}_{ab1} = S^1\,\check{I}_{UV1}$, $S^2\,\check{I}_{ab2} = S^2\,\check{I}_{UV2}$. And therefore by means of the relations (**85**), the last two equations may be expressed

$$\left.\begin{aligned} S^1\,\check{E}_{uv1} &= S^1\,3\,Z\,\check{I}_{ab1} \\ S^2\,\check{E}_{uv2} &= S^2\,3\,Z\,\check{I}_{ab2} \end{aligned}\right\} \quad (\mathbf{90})$$

in other words, the symmetrical components appear in the secondary as independent systems, 3 Z being the delta load impedance equivalent to the star impedance Z.

Substituting from (**90**) in the second and third equation and eliminating $\check{I}_{ab0}$ by means of the first equation, and we have

$$\left.\begin{aligned} S^1\,\check{E}_{ab1} &= S^1\left\{\left(3\,Z + Z_{AB0} - \frac{Z_{AB1}\,Z_{AB2}}{Z_{AB0}}\right)\check{I}_{ab1}\right. \\ &\qquad \left. + \left(Z_{AB2} - \frac{Z_{AB1}{}^2}{Z_{AB0}}\right)\check{I}_{ab2}\right\} \\ S^2\,O &= S^2\left\{\left(Z_{AB1} - \frac{Z^2{}_{AB2}}{Z_{AB0}}\right)\check{I}_{ab1}\right. \\ &\qquad \left. + \left(3\,Z + Z_{AB0} - \frac{Z_{AB1}\,Z_{AB2}}{Z_{AB0}}\right)\check{I}_{ab2}\right\} \end{aligned}\right\} \quad (\mathbf{91})$$

which, when S^1 and S^2 are removed, give two simultaneous equations in $\check{I}_{ab1}$ and $\check{I}_{ab2}$.

A modification of the problem may occur even when the load impedances are symmetrical, as they may have symmetrical but unequal impedances Z_1 and Z_2, to the two components $\check{I}_{v1}$ and $\check{I}_{v2}$ respectively, as in the case of a load consisting of a symmetrical rotating machine. The equations corresponding to (**89**), (**90**) and (**91**) then become

$$\left.\begin{aligned} O &= S^0\,(Z_{AB0}\,\check{I}_{ab0} + Z_{AB2}\,\check{I}_{ab1} + Z_{AB1}\,\check{I}_{ab2}) \\ S^1\,\check{E}_{uv1} &= S^1\,\check{E}_{ab1} - S^1\,(Z_{AB1}\,\check{I}_{ab0} + Z_{AB0}\,\check{I}_{ab1} + Z_{AB2}\,\check{I}_{ab2}) \\ S^2\,E_{uv2} &= O - S^2\,(Z_{AB2}\,\check{I}_{ab0} + Z_{AB1}\,\check{I}_{ab1} + Z_{AB0}\,\check{I}_{ab2}) \\ S^0\,\check{I}_{u0} &= O \\ S^1\,Z_1\,\check{I}_{u1} &= \check{E}_{u1} \\ S^2\,Z_2\,\check{I}_{u2} &= \check{E}_{u2} \end{aligned}\right\} \quad (\mathbf{92})$$

$$\left.\begin{aligned} S^1\,\check{E}_{uv1} &= S^1\,3\,Z_1\,\check{I}_{ab1} \\ S^2\,\check{E}_{uv2} &= S^2\,3\,Z_2\,\check{I}_{ab2} \end{aligned}\right\} \quad (\mathbf{93})$$

$$\left.\begin{aligned} S^1 \check{E}_{ab1} &= S^1 \left\{ \left(3\, Z_1 + Z_{AB0} - \frac{Z_{AB1}\, Z_{AB2}}{Z_{AB0}} \right) \check{I}_{ab1} \right. \\ &\quad \left. + \left(Z_{AB2} - \frac{Z^2_{AB1}}{Z_{AB0}} \right) \check{I}_{ab2} \right\} \\ S^2\, O &= S^2 \left\{ \left(Z_{AB1} - \frac{Z^2_{AB1}}{Z_{AB0}} \right) \check{I}_{ab1} \right. \\ &\quad \left. + \left(3\, Z_2 + Z_{AB0} - \frac{Z_{AB1}\, Z_{AB2}}{Z_{AB0}} \right) \check{I}_{ab2} \right\} \end{aligned} \right\} \quad (94)$$

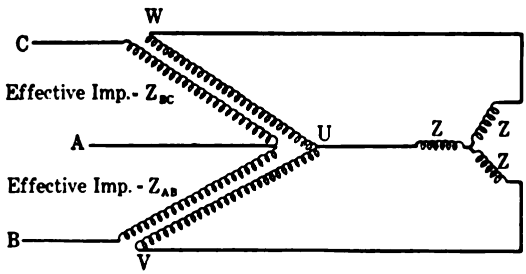


FIG. 4—OPEN DELTA OR V CONNECTION.

In an open delta system $Z_{AB1} = Z_{AB2} = Z_{AB0} - Z_{AB}$ the transformers in this case being both the same, equation (**91**) becomes in this particular case where Z_{AB0} is infinite

$$\left.\begin{aligned} S^1 \check{E}_{ab1} &= S^1 \{(3\, Z + 2\, Z_{AB})\, \check{I}_{ab} + Z_{AB}\, \check{I}_{ab2}\} \\ S^2\, O &= S^2 \{Z_{AB}\, \check{I}_{ab1} + (3\, Z + 2\, Z_{AB})\, \check{I}_{ab2}\} \end{aligned}\right\} \quad (95)$$

and we have

$$\check{I}_{ab0} = -\check{I}_{ab1} - I_{ab2} \quad (96)$$

Similarly, instead of (**94**) we have

$$\left.\begin{aligned} S^1 \check{E}_{ab1} &= S \{(3\, Z_1 + 2\, Z_{AB})\, \check{I}_{ab1} + Z_{AB}\, \check{I}_{ab2}\} \\ S^2\, O &= S^2 \{Z_{AB}\, \check{I}_{ab1} + (3\, Z_2 + 2\, Z_{AB})\, \check{I}_{ab2}\} \end{aligned}\right\} \quad (97)$$

The secondary voltages are obtained from (**90**) and (**93**) for this latter case.

The solution of (95) gives

$$
\left.
\begin{aligned}
\check{I}_{ab1} &= \frac{3\,Z_1 + 2\,Z_{AB}}{(3\,Z_1 + 3\,Z_{AB})\;(3\,Z_1 + Z_{AB})}\;\check{E}_{ab} \\
\check{I}_{ab2} &= -\;\frac{Z_{AB}}{(3\,Z_1 + 3\,Z_{AB})\;(3\,Z_1 + Z_{AB})}\;\check{E}_{ab} \\
\check{I}_{ab0} &= -\;\frac{1}{3\,Z_1 + 3\,Z_{AB}}\;\check{E}_{ab}
\end{aligned}
\right\} \qquad (98)
$$

And we have

$$
\left.
\begin{aligned}
S^1\,\check{I}_{a1} &= S^1\,\frac{3\,Z_1 + 2\,Z_{AB}}{3\,(Z_1 + Z_{AB})\left(Z_1 + \dfrac{Z_{AB}}{3}\right)}\;\check{E}_a \\
S^2\,\check{I}_{a2} &= S^2\,\frac{Z_{AB}}{3\,(Z_1 + Z_{AB})\left(Z_1 + \dfrac{Z_{AB}}{3}\right)}\;\check{E}_b
\end{aligned}
\right\} \qquad (99)
$$

And therefore

$$
\left.
\begin{aligned}
\check{I}_a &= \frac{\check{E}_a}{Z_1 + \dfrac{Z_{AB}}{3}} + \frac{\frac{1}{3}\,Z_{ab}}{(Z_1 + Z_{AB})\left(Z_1 + \dfrac{Z_{AB}}{3}\right)}\;\check{E}_{ab} \\
\check{I}_b &= \frac{\check{E}_b}{Z_1 + \dfrac{Z_{AB}}{3}} - \frac{\frac{1}{3}\,Z_{AB}}{(Z_1 + Z_{AB})\left(Z_1 + \dfrac{Z_{AB}}{3}\right)}\;\check{E}_{ab} \\
\check{I}_c &= \frac{\check{E}_c}{Z_1 + \dfrac{Z_{AB}}{3}}
\end{aligned}
\right\} \qquad (100)
$$

Three Phase System with Symmetrical Waves Having Harmonics

We may express $\check{E}_a$ in the following form:

$$
\left.
\begin{aligned}
\check{E}_a &= E_1\,e^{jwt} + E_2\,e^{j2wt} + E_3\,e^{j3wt} + \;.\;.\; \\
&= \Sigma\,E_n\,e^{jnwt}
\end{aligned}
\right\} \qquad (101)
$$

where E_n is in general a complex number.

If the system is symmetrical three-phase $\check{E}_b$ is obtained by displacing the complete wave by the angle $-\dfrac{2\,\pi}{3}$ or

$$\check{E}_b = e^{-j\frac{2\pi}{3}} E_1 e^{jw\tau} + e^{-j\frac{4\pi}{3}} E_2 e^{j2w\tau} + e^{-j\frac{6\pi}{3}} E_3 e^{j3w\tau} + \ldots$$

$$E_c = e^{j\frac{2\pi}{3}} E_1 e^{jw\tau} + e^{j\frac{4\pi}{3}} E_2 e^{j2w\tau} e^{j\frac{6\pi}{3}} E_3 e^{j3w\tau} + \ldots$$

or since $e^{-j\frac{2\pi}{3}} = a^2$, $e^{j\frac{2\pi}{3}} = a$ etc.

$$\left.\begin{aligned} \check{E}_a &= E_1 e^{jwt} + E_2 e^{j2wt} + E_3 e^{j3wt} + \ldots \\ \check{E}_b &= a^2 E_1 e^{jwt} + a E_2 e^{j2wt} + E_3 e^{j3wt} + \ldots \\ \check{E}_c &= a E_1 e^{jwt} + a^2 E_2 e^{j\tau wt} + E_3 e^{j3wt} + \ldots \end{aligned}\right\} \qquad (102)$$

or

$$\left.\begin{aligned} S(\check{E}_a) = {} & S^0 \{E_3 e^{j3wt} + E_6 e^{j6wt} + E_9 e^{j9wt} + \ldots\} \\ & + S^1 \{E_1 e^{jwt} + E_4 e^{j4wt} + E_7 e^{j7wt} + \ldots\} \\ & + S^2 \{E_2 e^{j2wt} + E_5 e^{j5wt} + E_8 e^{j8wt} + \ldots\} \end{aligned}\right\} \qquad (103)$$

$$S(\check{E}_a) = S^0 \Sigma (E_{3n} e^{j3nwt}) + S^1 \Sigma (E_{3n-2} e^{j(3n-2)w\tau}) + S^2 \Sigma (E_{3n-1} e^{j(3n-1)w\tau}) \qquad (104)$$

This shows that a symmetrical three-phase system having harmonics is made up of positive and negative phase sequence harmonic systems and others of zero phase sequence, that is to say of the same phase in all windings, which comprise the group of third harmonics. These facts are not generally appreciated though they are factors that may have an appreciable influence in the performance of commercial machines. It should be particularly noted that in three phase generators provided with dampers the fifth, eleventh, seventeenth, and twenty-third harmonics produce currents in the damper windings.

In dealing with the complex variable it will be convenient to use for the amplitude the root mean square value for each harmonic. When instantaneous values are required, the real part of the complex variable should be multiplied by $\sqrt{2}$. In the remainder of this paper this convention will be adopted.

Power Presentation in Symmetrical Co-ordinates

Since the power in an alternating current system is also a harmonically varying scalar quantity, it may therefore be represented in the same manner as the current or electromotive force,

that is to say by a complex variable which we shall denote by $(P + j\,Q) + P_H + j\,Q_H)$ $P + j\,Q$ being the mean value, is the term of the complex variable of zero frequency, P representing the real power and Q the wattless power $\sqrt{P^2 + Q^2}$ will be the volt-amperes.

The value of the complex variable $(P + j\,Q) + (P_H + j\,Q_H)$ may be taken as

$$(P + j\,Q) + (P_H + j\,Q_H) = \check{E}\,\hat{I} + \check{E}\,\check{I} \qquad \textbf{(105)}$$

with the provision that for all terms having negative indices the conjugate terms must be substituted, these terms being present in the product $\hat{E}\,\check{I} + \hat{E}\,\hat{I}$, which is the conjugate of the product (**105**). A similar rule holds good for the symmetrical vector system

$$\left.\begin{aligned} S(\check{E}_a) &= S^0\,\check{E}_{a0} + S^1\,\check{E}_{a1} + \quad . \quad . \quad . \qquad S^{n-1}\,E_{a(n-1)} \\ S(\check{I}_a) &= S^0\,\check{I}_{a0} + S^1\,\check{I}_{a1} + \quad . \quad . \quad . \qquad S^{n-1}\,I_{a(n-1)} \end{aligned}\right\} \qquad \textbf{(106)}$$

The conjugate of $S\,\check{I}_a$ is

$$S(\hat{I}_a) = S^0\,\hat{I}_{a0} + S^{(n-1)}\,\hat{I}_{a1} + \quad . \quad . \quad . \qquad S^1\,\hat{I}_{a(n-1)} \qquad \textbf{(107)}$$

and the Power is represented by

$$(P + P_b) + j\,(Q + Q_b) = \Sigma\,\{S(\check{E}_a)\,S(\hat{I}_a) + S(\check{E}_a)\,S(\check{I}_a)\} \qquad \textbf{(108)}$$

with the same provision for terms having negative indices the sign Σ signifies that all the products in each sequence are added together.

$$\left.\begin{aligned} \Sigma\,\{S(\hat{I}_a)\,S(\check{E}_a)\} = {} & \Sigma\,S^0\,\{\hat{I}_{a0}\,\check{E}_{a0} + \hat{I}_{a1}\,\check{E}_{a1} + \quad . \quad . \\ & \qquad \hat{I}_{a(n-1)}\,\check{E}_{a(n-1)}\} \\ & + \Sigma\,S^1\,\{\hat{I}_{a0}\,\check{E}_{a1} + \hat{I}_{a1}\,\check{E}_{a2} + \hat{I}_{a2}\,\check{E}_{a3} + \quad . \quad . \quad . \\ & \qquad \hat{I}_{a(n-1)}\,\check{E}_{a0}\} \\ & + \Sigma\,S^2\,\{\hat{I}_{a0}\,\check{E}_{a2} + \hat{I}_{a1}\,\check{E}_{a3} + \hat{I}_{a2}\,\check{E}_4 + \quad . \quad . \quad . \\ & \qquad \check{I}_{a(n-1)}\,\check{E}_{a1}\} \\ & . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ & + \Sigma\,S^{(n-1)}\,\{\hat{I}_{a0}\,\check{E}_{a(n-1)} + \hat{I}_{a1}\,\check{E}_{a0} + \quad . \quad . \quad . \\ & \qquad + \hat{I}_{a(n-1)}\,\check{E}_{a(n-2)}\} \end{aligned}\right\} \qquad \textbf{(109)}$$

The terms prefixed by S^1, S^2, S^3 . . . $S^{(n-1)}$ all become zero and since S^0 becomes n

$$\Sigma\, S(\hat{I}_a)\, S(\check{E}_a) = n\,\{\hat{I}_{a0}\check{E}_{a0} + \hat{I}_{a1}\check{E}_{a1} + \;\dots\; \hat{I}_{a(n-1)}\check{E}_{a(n-1)}\} \qquad \textbf{(110)}$$

In a similar manner it may be shown that

$$\Sigma\, S(\check{I}_a)\, S(\check{E}_a) = n\,\{\check{I}_{a0}\check{E}_{a0} + \check{I}_{a1}\check{E}_{a(n-1)} + \check{I}_{a2}\check{E}_{a(n-2)} + \;\dots\; \check{I}_{a(n-1)}\check{E}_{a1}\} \qquad \textbf{(111)}$$

and therefore

$$\begin{aligned}(P + jQ) + (P_H + jQ_H) = n\,\{\hat{I}_{a0}\check{E}_{a0} + \hat{I}_{a1}\check{E}_{a1} + \;\dots\; \hat{I}_{a(n-1)}\check{E}_{a(n-1)}\} \\ + n\,\{\check{I}_{a0}\check{E}_{a0} + \check{I}_{a1}\check{E}_{a(n-1)} + \;\dots\; \check{I}_{a(n-1)}\check{E}_{a1}\}\end{aligned} \qquad \textbf{(112)}$$

For a three-phase system the expression reduces to

$$\begin{aligned}(P + jQ) + (P_H + jQ_b) = 3\,(\hat{I}_{a0}\check{E}_{a0} + \hat{I}_{a1}\check{E}_{a1} + \hat{I}_{a2}\check{E}_{a2}) \\ + 3\,(\check{I}_{a0}\check{E}_{a0} + \check{I}_{a1}\check{E}_{a2} + \check{I}_{a2}\check{E}_{a1})\end{aligned} \qquad \textbf{(113)}$$

In the above expression $P + P_H$ is the value of the instantaneous power on the system, P being the mean value and P_H the harmonic portion. When the currents are simple sine waves, Q may be interpreted to be the mean wattless power of the circuit or the sum of the wattless voltamperes of each circuit. In rotating machinery since the coefficients of mutual induction may be complex harmonic functions of the angular velocity, this is not strictly true for all cases; but if the effective impedances to the various frequencies of the component currents be used, it will be found to be equal to the mean wattless voltamperes of the system with each harmonic considered independent.

In a balanced polyphase system P_H and Q_H both become zero.

The instantaneous power is a quantity of great importance in polyphase systems because the instantaneous torque is proportional to it and this quantity enters into the problem of vibrations which is at times a matter of great importance, especially when caused by unbalanced e. m. fs. A system of currents and e. m. fs. may be transformed to balanced polyphase by means of transformers alone, provided that the value of P_H is zero, while on the other hand polyphase power cannot be supplied from a pulsating power system without means for

supplying the necessary storage to make a continuous flow of energy.

PART II

Application of the Method to Rotating Polyphase Networks

The methods of determining the constants Z_a Z_u, M, etc., of co-axial cylindrical networks is taken up in Appendix I of this paper. It will be assumed that the reader has familiarized himself with these quantities and understands their significance. We shall first consider the case of symmetrically wound machines taking up the simple cases first and proceeding to more complex ones.

Symmetrically Wound Induction Motor Operating on Unsymmetrical Polyphase Circuit

Denoting the pole pitch angle by π let the synchronous angular velocity be ω_0 and let the angular slip velocity be ω_1. And let $S^1 E_{a1}$ $S^2 E_{a2}$ be the symmetrical components of impressed polyphase e. m. f. Let R_a be the primary resistance and R_u the secondary resistance. The primary self-inductance being M_{aa}, that of the secondary being M_{uu} and corresponding symbols being used to denote the mutual inductances between the different pairs of windings. Then by means of **(39)**, **(40)**, **(56)** and **(57)**

$$\left.\begin{aligned}
S^1 \breve{E}_a{}^1 &= S^1 \left\{ R_a \breve{I}_{a1} + 1\tfrac{1}{2}\, M_{aa} \frac{d}{d\,t}\, \breve{I}_{a1} \right. \\
&\qquad \left. + 1\tfrac{1}{2}\, M_{au} \frac{d}{d\,t}\, e^{j(w_0 - w_1)t}\, \breve{I}_{u1} \right\} \\
S^2 \breve{E}_{a2} &= S^2 \left\{ R_a \breve{I}_{a2} + 1\tfrac{1}{2}\, M_{aa} \frac{d}{d\,t}\, \breve{I}_{a2} \right. \\
&\qquad \left. + 1\tfrac{1}{2}\, M_{au} \frac{d}{d\,t}\, e^{-j(w_0 - w_1)t}\, \breve{I}_{u2} \right\} \\
S^1 \breve{E}_{u1} = O &= S^1 \left\{ R_u \breve{I}_{u1} + 1\tfrac{1}{2}\, M_{uu} \frac{d}{d\,t}\, \breve{I}_{u1} \right. \\
&\qquad \left. + 1\tfrac{1}{2}\, M_{au} \frac{d}{d\,t}\, e^{-j(w_0 - w_1)t}\, \breve{I}_{a1} \right\} \\
S^2 \breve{E}_{u2} = O &= S^2 \left\{ R_u \breve{I}_{u2} + 1\tfrac{1}{2}\, M_{uu} \frac{d}{d\,t}\, \breve{I}_{u2} \right. \\
&\qquad \left. + 1\tfrac{1}{2}\, M_{au} \frac{d}{d\,t}\, e^{j(w_0 - w_1)t}\, \breve{I}_{a2} \right\}
\end{aligned}\right\} \quad (114)$$

denote $1\frac{1}{2}\ M_{aa}$ by L_a and $1\frac{1}{2}\ M_{uu}$ by L_u, $1\frac{1}{2}\ M_{au}$ by M, the equations (**1**) become

$$\left.\begin{aligned}
S^1\,\check{E}_{a1} &= S^1\left\{\left(R_a + L_a\frac{d}{d\,t}\right)\check{I}_{a1} + M\frac{d}{d\,t}\,e^{j\,(w_0 - w_1)\,t}\,\check{I}_{u1}\right\}\\
S^2\,\check{E}_{a2} &= S^2\left\{\left(R_a + L_a\frac{d}{d\,t}\right)\check{I}_{a2} + M\frac{d}{d\,t}\,e^{-j\,(w_0 - w_1)\,t}\,\check{I}_{u2}\right\}\\
S^1\,\check{E}_{u1} = O &= S^1\left\{\left(R_u + L_u\frac{d}{d\,t}\right)\check{I}_{u1} + M\frac{d}{d\,t}\,e^{-j\,(w\ - w_1)\,t}\,\check{I}_{a1}\right\}\\
S^2\,\check{E}_{u2} = O &= S^2\left\{\left(R_u + L_u\frac{d}{d\,t}\right)\check{I}_{u2} + M\frac{d}{d\,t}\,e^{j\,(w_0 - w_1)\,t}\,\check{I}_{a2}\right\}
\end{aligned}\right\} \qquad (115)$$

From the last two equations we have

$$\check{I}_{u1} = -\frac{M\,\dfrac{d}{d\,t}}{R_u + L_u\,\dfrac{d}{d\,t}}\,e^{-j\,(w_0 - w_1)}\cdot\,\check{I}_{a1} \qquad (116)$$

$$\check{I}_{u2} = -\frac{M\,\dfrac{d}{d\,t}}{R_u + L_u\,\dfrac{d}{d\,t}}\,e^{j\,(w_0 - w_1)\,t}\,\check{I}_{a2} \qquad (117)$$

Substituting these in the first two equations of (**115**) we obtain

$$S'\,\check{E}_{a1} = S^1\left[\left(R_a + L_a\frac{d}{d\,t}\right) - \frac{M^2\,\dfrac{d}{d\,t}\left\{\dfrac{d}{d\,t} - j\,(w_0 - w_1)\right\}}{R_u + L_u\left\{\dfrac{d}{d\,t} - j\,(w_0 - w_1)\right\}}\right]\check{I}_{a1} \qquad (118)$$

$$S^2 \check{E}_{a2} = S^2 \left[\left(R_a + L_a \frac{d}{d\,t}\right) - \frac{M^2 \frac{d}{d\,t} \left\{ \frac{d}{d\,t} + (j\,w_0 - w_1) \right\}}{R_u + L_u \left\{ \frac{d}{d\,t} - j\,(w_0 - w_1) \right\}} \right] \check{I}_{a2} \qquad (119)$$

If $\check{E}_{a1} = E_{a1}\, e^{jwt}$ and $\check{E}_{a2} = E_{a2}\, e^{jwt}$ the solution for $\check{I}_{a1}$ and $\check{I}_{a2}$ will be

$$\check{I}_{a1} = \frac{\check{E}_{a1}}{Z_1} \qquad (120)$$

$$\check{I}_{a2} = \frac{\check{E}_{a2}}{Z_2} \qquad (121)$$

Where

$$Z_1 = R_a + j\,w_0\,L_a + \frac{w_0\,w_1\,M^2}{R_u^2 + w_1^2\,L_u^2}\,(R_u - j\,w_1\,L_u) \qquad (122)$$

$$Z_2 = R_a + j\,w_0\,L_a + \frac{w_0\,(2\,w_0 - w_1)\,M^2}{R_u^2 + (2\,w_0 - w_1)^2\,L_u^2}\,\{R_u - j\,(2\,w_0 - w_2)\,L_u\} \qquad (123)$$

The impedances Z_1 and Z_2 will be found more convenient to use in the form

$$Z_1 = (R_a + K_1^2\,R_u) + j\,w_0\,(L_a - K_1^2\,L_u) + \frac{w_0 - w_1}{w_1}\,K_1^2\,R_u \qquad (124)$$

$$Z_2 = (R_a + K_2^2\,R_u) + j\,w_0\,(L_a - K_2^2\,L_u) - \frac{w_0 - w_1}{Z\,w_0 - w_1}\,K_2^2\,R_u \qquad (125)$$

Where, as we will see later, K_1^2 and K_2^2 are the squares of the transformation ratios between primary and secondary currents of positive and negative phase sequence.

The last real term in each expression is the virtual resistance due to mechanical rotation and when combined with the mean square current represents mechanical work performed, the positive sign representing work performed and the negative sign work required.

Thus, for example, to enable the currents $S^2\,\check{I}_{a2}$ to flow, the mechanical work $3\,I_{a2}^2\,\frac{w_0 - w_1}{2\,w_0 - w_1}\,K_1^2\,R_u$ must be applied to the shaft of the motor.

The phase angles of the symmetrical systems $S^1 \check{I}_{a1}$ $S^2 \check{I}_{a2}$ with respect to their impressed e. m. f., $S^1 \check{E}_{a1}$ and $S^2 \check{E}_{a2}$ are given by these impedances so that the complete solution of the primary circuit is thus obtained.

The secondary currents are given by equations (**116**) and (**117**) and are

$$\check{I}_{u1} = -\frac{j\, w_1\, M}{R_u + j\, w_1\, L_u}\, I_{a1}\, e^{j w_1 \tau} = \check{K}_1\, I_{a1} e^{j w_1 \tau} \qquad (\mathbf{126})$$

$$\check{I}_{u2} = -\frac{j\,(2\, w_0 - w_1)\, M}{R_u + j\,(2\, w_0 - w_1)\, L_u}\, I_{a2}\, e^{j(2w_0 - w_1)\tau} = \check{K}_2\, I_{a2}\, e^{j(2w_0 - w_1)\tau} \qquad (\mathbf{127})$$

In the results just given, M is not the maximum value of mutual inductance between a pair of primary and secondary windings but is equal to the total mutual inductance due to a current passing through the two coils W and V through the coil

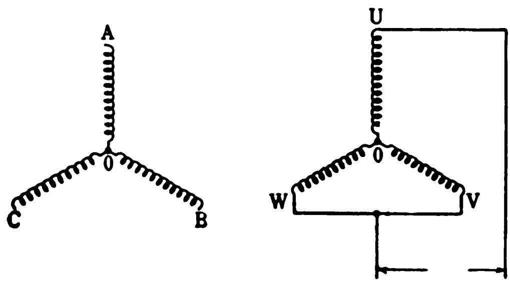


FIG. 5

U as shown in the sketch Fig 5 and the winding "A" when A and U have their planes of symmetry coincident.

Where the windings are symmetrical the induced e. m. f. is independent of the division of current between W and V, but this quantity must not be used in unsymmetrical windings, or with star windings having a neutral point connection so that $\check{I}_{a0}$ is not zero.

The appearance of M in this equation follows from the equation

$$\check{I}_u + \check{I}_v + \check{I}_w = 0$$

so that

$$\check{I}_u = -(\check{I}_v + \check{I}_w)$$

The power delivered by the motor is

$$P_o = 3\left\{\frac{w_0 - w_1}{w_1}\, K_1^2\, I_{a1}^2\, R_u - \frac{w_0 - w_1}{2\, w_0 - w_1}\, K_2^2\, I_{a2}^2\, R_u\right\} \qquad (\mathbf{128})$$

The copper losses are given by

$$P_L = 3\,\{I_{a1}^2\,(R_p + K_1^2\,R_u) + I_{a2}^2\,(R_p + K_2^2\,R_u)\} \qquad \textbf{(129)}$$

The iron loss is independent of the copper loss and power output. The iron loss and windage may be taken as

$$P_r = \text{Iron loss and windage} \qquad \textbf{(130)}$$

The power input as

$$P_1 = P_0 + P_L + P_r \qquad \textbf{(131)}$$

The mechanical power output is P_0 less friction and windage losses.

$$\text{Torque} = 3\left\{\frac{1}{w_1}\,K_1^2\,I_{a1}^2\,R_u - \frac{1}{2\,w_0 - w_1}\,K_2^2\,I_{a2}^2\,R_u\right\} \times 10^7 \text{ dyne-cm.} \qquad \textbf{(132)}$$

The kv-a. at the terminals is

$$\sqrt{P_1^2 + Q_1^2} = \text{The effective value of } 3\,(E_{a1}\,I_{a1} + E_{a2}\,I_{a2}) \qquad \textbf{(133)}$$

This last result may be arrived at in the following way

$$\left.\begin{aligned} S\,(\check{E}_a) &= S^1\,\check{E}_{a1} + S^2\,\check{E}_{a2}) \\ S\,(\hat{I}_a) &= S^2\,\hat{I}_{a1} + S^1\,\hat{I}_{a2} \end{aligned}\right\} \qquad \textbf{(134)}$$

Since $S^2\hat{I}_{a1}$ is conjugate to $S^1\,\check{I}_{a1}$, etc.

The product of $\check{E}_{s\cdot a}$ and $\hat{I}_{sa}$ is the power product of the two vectors, $S\,(\check{E}_a)$ and $S\,(\check{I}_a)$ and omits the harmonic variation as a double frequency quantity, the average wattless appears as an imaginary non-harmonic quantity.

$$P_1 + j\,Q_1\,\Sigma\,(S^0\,\check{E}_{a1}\,\hat{I}_{a1} + S^0\,\check{E}_{a2}\,\hat{I}_{a2} + S^1\,\check{E}_{a2}\,\hat{I}_{a1} + S^2\,\check{E}_{a1}\,\hat{I}_{a2}) \qquad \textbf{(135)}$$

The S^1 and S^2 products have zero values, since the sum of the terms of each sequence is zero, hence—

$$P_1 + j\,Q_1 = 3\,(\check{E}_{a1}\,\hat{I}_{a1} + \check{E}_{a2}\,\hat{I}_{a2}) \qquad \textbf{(136)}$$

$$\sqrt{P_1^2 + Q_1^2} = \text{The effective value of } 3\,(\check{E}_{a1}\,\hat{I}_{a1} + \check{E}_{a2}\,\hat{I}_{a2}) \qquad \textbf{(137)}$$

The solution for the general case of symmetrical motor operating on an unsymmetrical circuit is not of as much interest as

certain special cases depending thereon. Some of the most important of these will be taken up in the following paragraphs.

Case I. Single-Phase e. m. f. Impressed across one phase of three-phase motor.

Assuming the single-phase voltage to be $\check{E}_{bc}$ impressed across the terminals $B\ C$. The known data or constraints are

$$\left.\begin{aligned} \check{E}_{bc} &= j\sqrt{3}\,(\check{E}_{a1} - \check{E}_{a2}) \\ \check{I}_a &= O,\ \check{I}_b = -\check{I}_c \end{aligned}\right\} \qquad (138)$$

and therefore

$$\check{I}_{a1} = -\check{I}_{a2} \qquad (139)$$

$$\frac{\check{E}_{a1}}{Z_1} = -\frac{\check{E}_{a2}}{Z_2}$$

$$\check{E}_{a2} = -\frac{Z_2}{Z_1}\check{E}_{a1} \qquad (140)$$

Substituting in (**138**)

$$\left.\begin{aligned} \check{E}_{a1} &= -j\frac{\check{E}_{bc}}{\sqrt{3}} \cdot \frac{Z_1}{Z_1 + Z_2} \\ \check{E}_{a2} &= \ \ j\frac{\check{E}_{bc}}{\sqrt{3}} \cdot \frac{Z_2}{Z_1 + Z_2} \end{aligned}\right\} \qquad (141)$$

and therefore

$$\left.\begin{aligned} \check{I}_{a1} &= -j\frac{\check{E}_{bc}}{\sqrt{3}} \cdot \frac{1}{Z_1 + Z_2} \\ \check{I}_{a2} &= \ \ j\frac{\check{E}_{bc}}{\sqrt{3}} \cdot \frac{1}{Z_1 + Z_2} \end{aligned}\right\} \qquad (142)$$

Since $\check{I}_b = \check{I}_{b1} + \check{I}_{b2} = a^2\,\check{I}_{a1} + a\,\check{I}_{a2}$

$$\check{I}_b = -I_c = -\frac{\check{E}_{bc}}{Z_1 + Z_2} \qquad (143)$$

$$P_0 = \left(\frac{w_0 - w_1}{w_1}\,K_2^2\,R_u - \frac{w_0 - w_1}{2\,w_0 - w_1}\,K_2^2\,R_u\right)I_0^2 \qquad (144)$$

$$P_1 + j\,Q_1 = I_b^2\,(Z_1 + Z_2) + P_F \qquad (145)$$

The power factor is obtained from (**145**) by the formula

$$\cos\alpha = \frac{P_1}{\sqrt{P_1^2 + Q_1^2}} \qquad (146)$$

Substituting from (**142**) in equation (**126**) and (**127**) of the general case we obtain for the secondary currents

$$\left.\begin{aligned} \check{I}_{u1} &= -\, j\, \check{K}_1 \frac{E_{bc}}{Z_1 + Z_2}\, \epsilon^{j w_1 t} \\ \check{I}_{u2} &= \quad j\, \check{K}_2 \frac{E_{bc}}{Z_1 + Z_2}\, \epsilon^{j (2 w_0 - w_1) t} \end{aligned}\right\} \qquad (\mathbf{147})$$

Many unsymmetrical cases may be expressed in terms of the operation of coupled symmetrical motors operating on symmetrical systems. This is invariably the case with symmetrical polyphase motors operating on single phase circuits. Since the physical interpretations are useful in impressing the facts on ones memory they will be given whenever they appear to be useful.

Equations (**141**) and (**142**) show that single-phase operation is exactly equivalent to operating two duplicate motors in series with a symmetrical polyphase e. m. f. $S^1\, E_{ab}$ impressed across one motor, the other being connected in series with the first but with phase sequence reversed, the two motors being directly coupled.

Case II. B and C connected together e. m. f. impressed across A B.

The data given by the conditions of constraint are

$$\left.\begin{aligned} \check{E}_{ab} &= -\, \check{E}_{ca} \\ \check{E}_{bc} &= O = j\, \sqrt{3}\, (\check{E}_{a1} - \check{E}_{a2}) \end{aligned}\right\} \qquad (\mathbf{148})$$

We therefore have

$$\check{E}_{a1} = \check{E}_{a2} = -\, \frac{\check{E}_{ab}}{3} \qquad (\mathbf{149})$$

and

$$\left.\begin{aligned} \check{I}_{a1} &= -\, \frac{\check{E}_{ab}}{3\, Z_1} \\ \check{I}_{a2} &= -\, \frac{\check{E}_{ab}}{3\, Z_2} \end{aligned}\right\} \qquad (\mathbf{150})$$

The remainder follows from the general solution and need not be repeated here.

(**150**) shows that a motor operated in this manner is the exact equivalent in all respects to two duplicate mechanically coupled polyphase motors, one of which has sequence reversed, operating in parallel on a balanced three-phase circuit of e. m. f. $S^1\, \dfrac{E_{ab}}{\sqrt{3}}$.

The secondary currents follow from substitution of (**150**) in equations (**126**) and (**127**) of the general case.

Case III. B and C connected together by the terminals of a balance coil, the impressed e. m. f. E_{AD} applied between A and the middle point of the balance coil. Resistance and reactance of balance coil negligible.

The data furnished by the connection in this case is

$$\check{I}_b = \check{I}_c = -\frac{\check{I}_a}{2} \qquad (151)$$

and therefore

$$\check{I}_{a1} = \frac{\check{I}_a - a\dfrac{\check{I}_a}{2} - a^2\dfrac{\check{I}_a}{2}}{3} = \frac{\check{I}_a}{2}$$

$$\check{I}_{a2} = \check{I}_{a1} = \frac{\check{I}_a}{2}$$

We therefore have

$$\left.\begin{aligned} \check{E}_{a1} &= \frac{Z_1\check{I}_a}{2} \\ \check{E}_{a2} &= \frac{Z_2\check{I}_a}{2} \end{aligned}\right\} \qquad (152)$$

we have

$$\check{E}_{ab} = j\sqrt{3}\,(a\,\check{E}_{a1} - a^2\,\check{E}_{a2})$$

$$= j\sqrt{3}\,\frac{\check{I}_a}{2}\,(a\,Z_1 - a^2\,Z_2)$$

$$\check{E}_{bc} = j\sqrt{3}\,\frac{\check{I}_a}{2}\,(Z_1 - Z_2)$$

$$\left.\begin{aligned} \check{E}_{ad} &= \left(\check{E}_{ab} + \frac{\check{E}_{bc}}{2}\right) \\ &= j\sqrt{3}\,\frac{\check{I}_a}{2}\,\{(a + \tfrac{1}{2})\,Z_1 - (a^2 + \tfrac{1}{2})\,Z_2\} \\ &= -\tfrac{3}{4}\,\check{I}_a\,(Z_1 + Z_2) \end{aligned}\right\} \qquad (153)$$

and therefore,

$$\check{I}_a = -1\tfrac{1}{3}\,\frac{\check{E}_{ad}}{Z_1 + Z_2} \qquad (154)$$

$$P_0 = \tfrac{3}{4} \left\{ \frac{w_0 - w_1}{w_1} K_1^2 - \frac{w_0 - w_1}{2\, w_0 - w_1} K_2^2 \right\} I_a^2 R_s \qquad (155)$$

$$P_1 + j\, Q_1 = \tfrac{3}{4} I_a^2 (Z_1 + Z_2) + P_r \qquad (156)$$

$$\cos \alpha = \frac{P_1}{\sqrt{P_1^2 + Q_1^2}} \qquad (157)$$

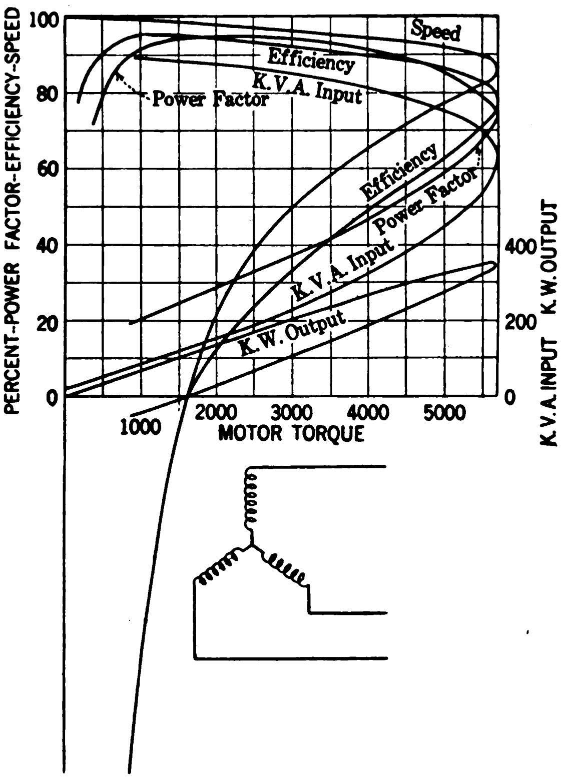


Fig. 6—Characteristics of Three-Phase Induction Motor—Balanced Three-Phase

Evidently (155), (156) and (157) are identical to (144), (145) and (146) if I_a is equal to $I_b \div \frac{\sqrt{3}}{2}$. This will be the case if the value of $E_{ad} = \frac{\sqrt{3}}{2}$ times that of E_{bc}. The total heating of

the motors will be the same in each case but the heating in one phase for Case III will be one-third greater than for Case I.

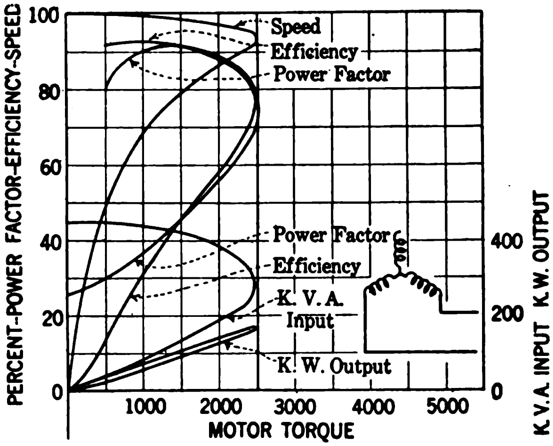


FIG. 7—CHARACTERISTICS OF THREE-PHASE INDUCTION MOTOR—SINGLE-PHASE OPERATION —ONE LEAD OPEN

This method of operation is therefore, as far as total losses, etc. are concerned, the exact counterpart of two polyphase

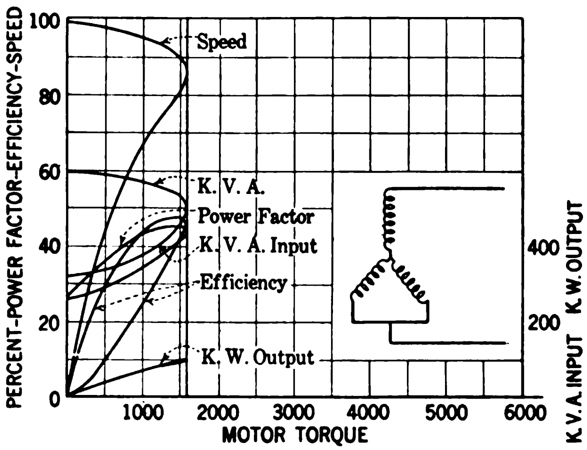


FIG. 8—CHARACTERISTICS OF THREE-PHASE INDUCTION MOTOR—SINGLE-PHASE OPERATION

motors connected in series with shafts mechanically connected, one of which has its phase sequence reversed.

Figs. 6, 7 and 8 show characteristic curves of a three-phase

induction motor operating respectively on a symmetrical circuit, according to Case I and according to Case II.

Synchronous Machinery

The Symmetrical Three-Phase Generator Operating on Unsymmetrically Loaded Circuit

The polyphase salient pole generator is not strictly a symmetrical machine, the exciting winding is not a symmetrical polyphase winding and it therefore sets up unsymmetrical trains of harmonics in exactly the same way as they are set up in an induction motor with unsymmetrical secondary winding. These cases will therefore be taken up later on. A three-phase generator may however be wound with a distributed polyphase winding to serve both as exciting and damper winding and if properly connected will be perfectly symmetrical. Such a machine will differ from an induction motor only in respect to the fact that it operates in synchronism and has internally generated symmetrical e. m. fs. which we will denote by $S^1 \check{E}_{a1}$, $S^2 \check{E}_{a2}$ the negative phase sequence component being zero; an e. m. f. $S^0 \check{E}_{a0}$ may exist but since in all the connections that will be considered there will be no neutral connection its value may be ignored. If the load impedances be Z_a', Z_b' and Z_c' they may be expressed by

$$Z_{sa}{}^1 = S^0 Z_{a0}{}^1 + S^1 Z_{a1}' + S^2 Z_{a2}'$$

and the equations of the generator will be

$$\left.\begin{aligned}
S^1 \check{E}_{a1} &= S^1 \left\{ \left(R_a + L_a \frac{d}{d\,t}\right) \check{I}_{a1}' + Z_{a0}' I_{a1}' \right. \\
&\qquad \left. + Z_{a2}' I_{a2}' + M \frac{d}{d\,t} e^{j u v t} \check{I}_{u1}' \right\} \\
O &= S^2 \left\{ \left(R_a + L_a \frac{d}{d\,t}\right) \check{I}_{a2} + Z_{a0} I_{a2} \right. \\
&\qquad \left. + Z_{a1}{}^1 \check{I}_{a1}' + M \frac{d}{d\,t} e^{-j u v t} \check{I}_{u2}' \right\} \\
O &= \left(R_u + L_u \frac{d}{d\,t}\right) \check{I}_{u1}' + M \frac{d}{d\,t} e^{-j u v t} \check{I}_{a1}' \\
O &= \left(R_u + L_u \frac{d}{d\,t}\right) \check{I}_{u2}' + M \frac{d}{d\,t} e^{j u v t} \check{I}_{a2}'
\end{aligned}\right\} \quad (158)$$

The last two equations give

$$\left.\begin{aligned} I_{u1}' &= -\frac{M\dfrac{d}{d\,t}}{R_u + L_u\dfrac{d}{d\,t}}\,e^{-jw_0 t}\,\check{I}_{a1}' \\ I_{u2}' &= -\frac{M\dfrac{d}{d\,t}}{R_u + L_u\dfrac{d}{d\,t}}\,e^{jw_0 t}\,I_{a2}' \end{aligned}\right\} \qquad (159)$$

which on substitution in the first two equations of (158) give the equations

$$\left.\begin{aligned} &\left\{R_a + L_a\frac{d}{d\,t} - \frac{M^2\dfrac{d}{d\,t}\left(\dfrac{d}{d\,t} - j\,w_0\right)}{R_u + L_u\left(\dfrac{d}{d\,t} - j\,w_0\right)}\right\}\check{I}_{a1}' \\ &\qquad\qquad + Z_{a0}^1\,\check{I}_{a1}' + Z_{a2}'\,\check{I}_{a2}' = \check{E}_{a1} \\ &Z_{a1}^1\,\check{I}_{a1}' + \left\{R_a + L_a\frac{d}{d\,t}\right. \\ &\left. - \frac{M^2\dfrac{d}{d\,t}\left(\dfrac{d}{d\,t} + j\,w_0\right)}{R_u + L_u\left(\dfrac{d}{d\,t} + j\,w_0\right)}\right\}\check{I}_{a2}' + Z_{a0}^1\,\check{I}_{a2}' = 0 \end{aligned}\right\} \qquad (160)$$

or if

$$\check{E}_{a1} = E_{a1}\,e^{jw_0 t} \qquad (161)$$

the impedances Z_{a0}, Z_{a1}, Z_{a2} become ordinary impedance for an electrical angular velocity w_0 and equations (160) become

$$\left.\begin{aligned} &(R_a + j\,w\,L_a + Z_{a0}^1)\,\check{I}_{a1}' + Z_{a2}'\,\check{I}_{a\,2}' = \check{E}_{a1} \\ &Z_{a1}'\,\check{I}_{a1}' + \{Z_{a0}' + (R_a + K_2^2\,R_u) + j\,2\,w_0\,(L_a - K_2^2\,L_u) \\ &\qquad\qquad - \tfrac{1}{2}\,K_2^2\,R_u\}\,\check{I}_{a2}' = O \end{aligned}\right\} \qquad (162)$$

It is apparent that in the generator the impedances

$$R_a + j\,w_0\,L_a = Z_1'$$

and $\{(R_a + K_2^2\,R_u) + j\,2\,w_0\,(L_a - K_2^2\,L_u) - \frac{1}{2}\,K^2 R_u\} = Z_2'$

take the place of Z_1 and Z_2 in the symmetrical induction motor operating on an unsymmetrical circuit, and we may express equation (**162**)

$$\left.\begin{aligned} (Z_{a0}' + Z_1')\, \check{I}_{a1}' + Z_{a2}'\, \check{I}_{a2}' &= \check{E}_{a1} \\ Z_a'\, \check{I}_{a1}' + (Z_{a0}' + Z_2')\, I_{a2}' &= O \end{aligned}\right\} \quad \textbf{(163)}$$

which gives

$$\check{I}_{a2}' = - \frac{Z_{a1}'}{Z_{a0} + Z_2'}\, \check{I}_{a1}'$$

$$\check{I}_{a1}' = \frac{\check{E}_{a1}}{(Z_{a0}' + Z_1') - \dfrac{Z_{a1}'\, Z_{a2}'}{Z_{a0}' + Z_2'}}$$

Or in more symmetrical form

$$\left.\begin{aligned} \check{I}_{a1}' &= \frac{Z_{a0}' + Z_2'}{(Z_{a0}' + Z_1')\,(Z_{a0}' + Z_2') - Z_{a1}'\, Z_{a2}'}\, \check{E}_{a1} \\ \check{I}_{a2}' &= - \frac{Z_{a1}^1}{(Z_{a0}' + Z_1')\,(Z_{a0}' + Z_2') - Z_{a1}'\, Z_{a2}'}\, \check{E}_{a1} \end{aligned}\right\} \quad \textbf{(164)}$$

From (**159**) we have for the damper currents

$$\left.\begin{aligned} \check{I}_{u1}' &= O \text{ if } R_u > O \\ \check{I}_{u2}' &= - \check{K}_2\, I_{a2}\, e^{j\, 2 u \omega t} \\ &\text{where } \check{K}_2 = j\, \frac{2\, w_0\, M}{R_u + j\, 2\, w_0\, L_u} \end{aligned}\right\} \quad \textbf{(165)}$$

A particular case of interest is when the load is a Synchronous Motor or Induction Motor with unsymmetrical line impedances in series—Equation (**163**) becomes

$$\left.\begin{aligned} (Z_{a0}' + Z_1' + Z_1)\, \check{I}_{a1}' + Z_{a2}'\, I_{a2}' &= \check{E}_{a2} \\ Z_{a1}\, \check{I}_{a1}' + (Z_{a0}' + Z_2' + Z_2)\, I_{a2}' &= O \\ I_{a1}^1 &= \frac{Z_{a0}' + Z_2' + Z_2}{(Z_{a0}'+Z_1'+Z_1)\ (Z_{a0}'+Z_2'+Z_2) - Z_{a1}\, Z_{a2}}\, \check{E}_{a1} \\ I_{a2}^1 &= \frac{Z_{a1}}{(Z_{a0}'+Z_1'+Z_1)\ (Z_{a0}'+Z_2'\ Z_2) - Z_{a1}\, Z_{a2}}\, \check{E}_{a1} \end{aligned}\right\} \quad \textbf{(166)}$$

An important case is that of a generator feeding into a symmetrical motor and an unsymmetrical load. Let the motor currents be

$\check{I}_a$, $\check{I}_b$, $\check{I}_c$, those of the load I_a', I_b', I_c' and the load impedances Z_a', Z_b', Z_c'. The equations of this system will be

$$\left.\begin{aligned}
S^1\,\check{E}_{a1} &= S^1\,\{Z_1'(\check{I}_{a1} + \check{I}_{a1}') + Z_{a0}'\,\check{I}_{a1}' + Z_{a2}'\,\check{I}_{a2}'\}\\
S^1\,\check{E}_{a1} &= S^1\,\{Z_1'(\check{I}_{a1} + \check{I}_{a1}') + Z_1\,\check{I}_{a1}\}\\
S^2\,O &= S^2\,\{Z_2'(\check{I}_{a2} + \check{I}_{a2}') + Z_{a0}'\,\check{I}_{a2} + Z_{a1}'\,\check{I}_{a1}'\}\\
S^2\,O &= S^2\,\{Z_2'(\check{I}_{a2} + \check{I}_{a2}') + Z_2\,\check{I}_{a2}\}
\end{aligned}\right\} \quad (167)$$

Or, omitting the sequence symbols and re-arranging—

$$\left.\begin{aligned}
\check{E}_{a1} &= Z_1'\,\check{I}_{a1} + (Z_1' + Z_{a0}')\,\check{I}_{a1}' + Z_{a2}'\,\check{I}_{a2}^1\\
\check{E}_{a1} &= (Z_1' + Z_1)\,\check{I}_{a1} + Z_1'\,\check{I}_{a1}^1\\
O &= Z_2'\,\check{I}_{a2} + Z_{a1}'\,\check{I}_{a1}^1 + (Z_2' + Z_{a0}')\,\check{I}_{a2}'\\
O &= (Z_2' + Z_2)\,\check{I}_{a2} + Z_2'\,\check{I}_{a2}'
\end{aligned}\right\} \quad (168)$$

These equations can be further simplified as follows:

$$\left.\begin{aligned}
O &= (Z_2' + Z_2)\,\check{I}_{a2} + Z_2'\,\check{I}_{a2}'\\
O &= -\,Z_2\,\check{I}_{a2} + Z_{a1}'\,\check{I}_{a1}' + Z_{a0}'\,\check{I}_{a2}'\\
O &= -\,Z_1\,\check{I}_{a1} + Z_{a0}'\,\check{I}_{a0}' + Z_{a2}'\,I_{a2}'\\
\check{E}_{a1} &= (Z_1' + Z_1)\,\check{I}_{a1} + Z_1'\,\check{I}_{a1}'
\end{aligned}\right\} \quad (169)$$

A set of simultaneous equations which may be easily solved.

The Single-Phase Generator is an Important Case of the Three-Phase Generator Operated on an Unbalanced Load

Let the impedance of the single-phase load be Z and let us suppose it to be made up of three star connected impedances

$$Z_a' = 3\,Z_x + \frac{Z}{2}$$

$$Z_b' = \frac{Z}{2}$$

$$Z_c' = \frac{Z}{2}$$

the value of Z_x in the limit being infinity. Then we have

$$\left.\begin{aligned} Z_{a0}' &= Z_x + \frac{Z}{2} \\ Z_{a1}' &= Z_x \\ Z_{a2}' &= Z_x \end{aligned}\right\} \qquad (170)$$

Equation (**164**) in the limit when Z_x becomes infinite reduces to

$$\begin{aligned} \check{I}_{a1}' &= \frac{\check{E}_{a1}}{Z + Z_1^1 + Z_2^1} \\ \check{I}_{a2} &= -\frac{\check{E}_{a1}}{Z + Z_1' + Z_2'} \end{aligned} \qquad (171)$$

The single-phase load being across the phase $B\,C$, the single-phase current I will therefore be equal to $\check{I}_c$ or

$$\left.\begin{aligned} \check{I} &= \frac{j\sqrt{3}\,\check{E}_{a1}}{Z + Z_1' + Z_2'} \\ &= \frac{\check{E}_{bc}}{Z + Z_1' + Z_2'} \end{aligned}\right\} \qquad (172)$$

$$\left.\begin{aligned} \check{I}_{u1} &= O \text{ if } R_u > O \\ \check{I}_{u2} &= -j\,\frac{1}{\sqrt{3}}\,\check{K}_2\,\check{I}\,e^{j u_0 t} \\ \check{I}_{u2} &= -\frac{j\,\check{K}_2}{\sqrt{3}}\,I\,e^{j w_0 t} \end{aligned}\right\} \qquad (173)$$

$\check{I}_{u2}$ is double normal frequency

$$\left.\begin{aligned} P_1 + j\,Q_1 &= 3\,I^2\,Z \\ P_L + j\,Q_L &= 3\,I^2\,(Z_1' + Z_2') \\ (P + j\,Q) + (P_H + j\,Q_H) &= 3\,\check{E}_{bc}\,(\hat{I} + \check{I}) \end{aligned}\right\} \qquad (174)$$

In the case of the generally unbalanced three-phase load

$$\left.\begin{aligned} P_1 + j\,Q_1 &= 3\,\{(I_{a1}^2 + I_{a2}^2)\,Z_{a0}' \\ &\qquad + \check{I}_{a1}\,\hat{I}_{a2}\,Z_{a2}' + \hat{I}_{a1}\,\check{I}_{a2}\,Z_{a1}'\} \\ P_L + j\,Q_L &= 3\,\{I_{a1}^2\,Z_1' + I_{a2}^2\,Z_2'\} \\ (P + j\,Q)\,j\,(P_H + j\,Q_H) &= 3\,\check{E}_{a1}\,(\check{I}_{a2} + \hat{I}_{a2}) \end{aligned}\right| \qquad (175)$$

When the generator has harmonics in its wave form equations (**162**) must be written

$$\left.\begin{aligned}&(R_a + j\,w\,L_a + Z_{a0}')\,\check{I}_{a1}' + Z_{a2}'\,I_{a2}' = \check{E}_{a1}\\&\quad Z_{a1}'\,I_{a1}' + \{Z_{a0}' + (R_a + K_2^2\,R_u)\\&\qquad\qquad + j\,2\,w\,(L_a - K_2^2\,L_u) - \tfrac{1}{2}\,a^2\,R_u\}\,I_{a2}' = \check{E}_{a2}\end{aligned}\right\}\qquad \textbf{(176)}$$

Where $\check{E}_{a1}$ is finite, $\check{E}_{a2}$ is zero and vice versa, the frequencies being different in each case, we have therefore a solution for each frequency depending on the phase and amplitude and phase sequence of the e. m. f. of this frequency generated. Of course the values of Z_1' and Z_2' change with each frequency on account of the change in the reactance with frequency, and a value must be taken for w conforming with the frequency of the harmonic under consideration.

Symmetrical Synchronous Motor, Synchronous Condenser, Etc.

As in the case of the generator, the synchronous motor has two impedances, one to the positive phase sequence current of a given frequency and the other to the negative phase sequence current of the same frequency. But, since there is no quantity in the positive phase sequence impedance corresponding to the virtual resistance which indicates mechanical work in an induction motor, its equivalent is furnished by the excitation of the field. Let us denote the e. m. f. due to the field excitation by $S^1\,\check{E}_{a1}'$ assuming it to be for the present a simple harmonic three-phase system. Let P_0 be the output of the motor which will include the windage and iron losses assumed to be constant. Then for the synchronous motor on a balanced circuit of e. m. f. $S^1\,\check{E}_{a1}$ we have

$$S^1\,\check{E}_{a1} = S^1\,\{\check{I}_{a1}\,(R_a' + j\,w\,L_a') + \check{E}_{a1}'\} \qquad \textbf{(177)}$$

$$S^0\,\check{E}_{a1}\,\hat{I}_{a1} = S^0\left\{I_{a1}^2\,(R_a' + j\,w\,L_a') + \frac{P_0}{3} - j\,\frac{Q_0}{3}\right\} \qquad \textbf{(178)}$$

Where Q_0 is the imaginary part of the product, $\check{E}_{a1}'\,\hat{I}_{a1}$. (**178**) reduces to

$$E_{a1}\,I_{a1}\cos\alpha = I_{a1}^2\,R_a^1 + \frac{P_0}{3} \qquad \textbf{(179)}$$

Where $\cos\alpha$ is the required operating power factor. Solving for I_{a1}

$$I_{a1} = \frac{E_{a1} \cos \alpha}{2 R_a^1} \left\{ 1 \pm \sqrt{1 - \frac{4 R_a' P_0}{3 E_{a1}^2 \cos^2 \alpha}} \right\} \quad (180)$$

$$\check{I}_{a1} = \check{E}_{a1} \frac{\cos \alpha}{2 R_{a1}} \left\{ 1 \pm \sqrt{1 - \frac{4 R_{a1}' P_0}{3 E_{a1}^2 \cos^2 \alpha}} \right\} (\cos \alpha - j \sin \alpha) \quad (181)$$

The apparent impedance of the motor is

$$\frac{2 R_1 \sec \alpha}{1 \pm \sqrt{1 - \frac{4 P_0}{3 E_a^2 \cos^2 \alpha}}} (\cos \alpha + j \sin \alpha) \quad (182)$$

and

$$\check{E}_{a1}^1 = \check{E}_{a1} \left[1 - \frac{\cos \alpha}{2 R_a^1} \left\{ 1 \pm \sqrt{1 - \frac{4 R_a' P_0}{3 E_{a1}^2 \cos^2 \alpha}} \right\} (\cos \alpha - j \sin \alpha) (R_a' + j w L_a') \right] \quad (183)$$

The same equations apply to the case of the synchronous condenser with the difference that the mechanical work is that required to overcome the iron and windage losses only.

If we take

$$\left. \begin{aligned} \check{E}_{a1} &= E_{a1} (\cos \alpha + j \sin \alpha) e^{jw_0 t} = (A_1 + j B_1) e^{jw_0 t} \\ \check{E}_{a1}' &= (A_1' + j B_1') e^{jw_0 \tau} \end{aligned} \right\} \quad (184)$$

we have

$$\check{I}_{a1} = \frac{A_1}{2 R_a^1} \left(1 \pm \sqrt{1 - \frac{4 R_a' P_0}{3 A_1^2}} \right) e^{jw_0 t} \quad (185)$$

$$A_1' = \frac{A_1}{2} \left(1 \pm \sqrt{1 - \frac{4 R_a' P_0}{3 A_1^2}} \right) e^{jw_0 t} \quad (186)$$

$$B_1' = \left\{ R_1 - \frac{j w L_a' A_1}{2 R_a'} \left(1 \pm \sqrt{1 - \frac{4 R_a' P_0}{3 A_1^2}} \right) \right\} e^{jw_0 t} \quad (187)$$

Since α may be a positive or negative angle, the sine may be positive or negative for a positive cosine, and therefore the power factor will be leading or lagging accordingly as B_1 is negative or positive respectively. The double signs throughout are due to the fact that for any given load and power factor there are always two theoretically possible running conditions. However, since

we are concerned only with that one which will give the max. operating efficiency, that is the condition that gives I_{a1} the lesser value, for a given value of P_0 the equations may be written

$$\left.\begin{aligned} \check{I}_{a1} &= \frac{A_1}{2\,R_a^1}\left(1 - \sqrt{1 - \frac{4\,R_a'\,P_0}{3\,A_1^2}}\right) e^{j\,u\,w\,t} \\ A_1' &= \frac{A_1}{2}\left(1 + \sqrt{1 - \frac{4\,R_a'\,P_0}{3\,A_1^2}}\right) e^{j\,u\,w\,t} \\ B_1' &= \left\{ B_1 - \frac{j\,w_0\,L_a'\,A_1}{2\,R_a'}\left(1 - \sqrt{1 - \frac{4\,R_a'\,P_0}{3\,A_1^2}}\right)\right\} \end{aligned}\right\} \quad (188)$$

And corresponding values for (**180**), (**181**), (**182**) and (**183**) may be obtained by omitting the positive sign in these equations.

Another condition of operation is obtained by inspection of (**180**), due to the fact that I_{a1} must be a real quantity

$$\frac{4\ R_a'\ P_0}{3\ E_{a1}^2 \cos^2 \alpha} \text{ must be } > 1 \quad (189)$$

this is the condition of stability. In terms of (**184**) it becomes

$$\frac{4\,R_a'\,P_0}{3\,A_1^2} \text{ must be } > 1 \quad (190)$$

The same conditions apply to the synchronous condenser, the total mechanical load in this case being the iron loss and windage and friction losses.

Proceeding now to operation with unbalanced circuits having sine waves the motor also having a sine wave. In addition to equation (**177**) we shall have

$$S^2\,\check{E}_{a2} = S^2\,Z_2'\,\check{I}_{a2} \quad (191)$$

The mechanical power delivered through the operation of this negative phase sequence e. m. f. is given by P_N where

$$P_N = -\,3\ I_{a2}^2\,\frac{R_a'}{2} \quad (192)$$

this quantity must therefore be subtracted from the value of P_0 in all the equations in which P_0 appears when unbalanced circuits are used in connection with equations (**177**) to (**190**) inclusive. These equations, however, give the conditions for maintaining a given mechanical load and a given power factor in the positive phase sequence component, but in practice what is re-

quired is the combined power factor of the whole system, or the conditions to give a certain combined factor while delivering a given mechanical load; this may be obtained as follows:

The negative phase sequence component is a perfectly definite impedance and is independent of the load, and therefore the zero frequency part of the product $E_{a2}\, I_{a2}$ may be set down as

$$\check{E}_{a2}\, \hat{I}_{a2} = \frac{P_2}{3} + j\,\frac{Q_2}{3} \qquad \textbf{(193)}$$

we have also for the positive phase sequence power delivered

$$(A_1 + j\, B_1)\; I_{a1} = I_{a1}^2\, R_a^1 + \frac{P_0}{3} - \frac{P_N}{3}$$
$$+\, j\,(w\, I_{a1}\, L_a^1 + B_1^1)\; I_{a1} \qquad \textbf{(194)}$$

And the power factor is given by

$$\tan\alpha = \frac{I_{a1}\, B_1 + \dfrac{Q_2}{3}}{I_{a1}\, A_1 + \dfrac{P_2}{3}} \qquad \textbf{(195)}$$

From (**194**) we have

$$A_1\, I_{a1} = I_{a1}^2\, R_a^1 + \frac{P_0}{3} - \frac{P_N}{3} \qquad \textbf{(196)}$$

$$B_1 = w\, I_{a1}\, L_a^1 + B_1^1 \qquad \textbf{(197)}$$

$$A_1^2 + B_1^2 = E_{a1}^2 \qquad \textbf{(198)}$$

The simplest method of solving these equations is by means of curves. Taking arbitrary values of I_{a1}, B_1 and A_1 are chosen consistent with (**198**) so as to satisfy (**195**), $\dfrac{P_0}{3}$ A_1' and B_1' are then obtained from (**196**) and (**197**). If there are harmonics in the impressed e. m. f. but there are none in the wave form of the machine, the machine will have a definite impedance to the positive and negative phase sequence components of each harmonic, so that there will be a definite amount of mechanical work contributed by each harmonic which must be subtracted from the total work to be done to give the amount of work contributed by the positive phase sequence fundamental component, the equations will be identical to (**193**), (**194**), (**195**), (**196**),

(**197**) and (**198**), if we take P_N to mean the total mechanical work done by the harmonics both positive and negative phase sequence and P_2 and Q_2 to represent the products

$$\Sigma \ ({}_n\check{E}_{a1} \quad {}_n\hat{I}_{a1} + {}_n\check{E}_{a2} \quad {}_n\hat{I}_{a2})$$

the zero frequency part only being taken into account.

When harmonics are present both in the impressed wave and in the generated wave, the problem becomes too complicated to treat generally, but specific cases can be worked out without much difficulty.

Phase Converters and Balancers

The phase converter is a machine to transform energy from single-phase or pulsating form to polyphase or non-pulsating form or vice versa to transform energy from polyphase to single-phase. The transformation may not be complete, that is to say, the polyphase system may not be perfectly balanced when supplied from a single-phase source through the medium of a phase converter. Phase converters may be roughly divided into two classes, namely—shunt type and series type.

Induction Motor or Synchronous Condenser Operating as a Phase Converter of the Shunt Type to Supply a Symmetrical Induction Motor or Synchronous Motor

Let Z_1 and Z_2 be the positive and negative phase sequence impedances of the motor, Z_1', Z_2' those of the phase converter. Let $S^1\ \check{E}_{a1}$ and $S^2\ \check{E}_{a2}$ be the positive and negative phase sequence components of the star e. m. f. impressed on the motor as a result of the operation. The single-phase supply will be one side of the delta e. m. f. $S\ \check{E}_{bc}$ which has positive and negative phase sequence components $S^1\ E_{bc1}$ and $S^2\ E_{bc2}$ the single-phase supply being $\check{E}_{bc} = \check{E}_{bc1} + \check{E}_{bc2}$.

The value of Z_2' may be considered fixed for all practical purposes and since in the induction motor phase converter the speed is practically no-load speed, Z' is practically the no-load impedance plus a real part obtained by increasing the real part of the no-load impedance by the ratio of the normal no-load losses to these same losses plus ½ the secondary losses due to the phase converter currents. The latter may be calculated roughly as even a large error in its value will have an inappreciable effect on the actual results. We have therefore

$$\left.\begin{aligned} S^1 \check{E}_{a1} &= -\,S^1 j \frac{\check{E}_{bc}}{\sqrt{3}} \\ S^2 \check{E}_{a2} &= \quad S^2 j \frac{\check{E}_{bc2}}{\sqrt{3}} \end{aligned}\right\} \tag{199}$$

$$\left.\begin{aligned} S_1^1 \check{I}_{a1}' &= -\,S^1 j \frac{\check{E}_{bc1}}{\sqrt{3}\, Z_1'} \\ S^1 \check{I}_{a1} &= -\,S^1 j \frac{\check{E}_{bc1}}{\sqrt{3}\, Z_1} \end{aligned}\right\} \tag{200}$$

$$\left.\begin{aligned} S^2 \check{I}_{a2}' &= \quad S^2 j \frac{\check{E}_{bc2}}{\sqrt{3}\, Z_2'} \\ S^2 \check{I}_{a2} &= \quad S^2 j \frac{\check{E}_{bc2}}{\sqrt{3}\, Z_2} \end{aligned}\right\} \tag{201}$$

In the common lead of motor and converter we have

$$\check{I}_{a1}' + \check{I}_{a2}' + \check{I}_{a1} + \check{I}_{a2} = O \tag{202}$$

or, substituting from (**200**) and (**201**)

$$\check{E}_{bc2}\left(\frac{1}{Z_2'} + \frac{1}{Z_2}\right) = \check{E}_{bc1}\left(\frac{1}{Z_1'} + \frac{1}{Z_1}\right) \tag{203}$$

$$\frac{\check{E}_{bc1}}{\check{E}_{bc2}} = \frac{\dfrac{1}{Z_2'} + \dfrac{1}{Z_2}}{\dfrac{1}{Z_1'} + \dfrac{1}{Z_1}} \tag{204}$$

$$\check{E}_{bc1} = \frac{\dfrac{1}{Z_2} + \dfrac{1}{Z_2'}}{\left(\dfrac{1}{Z_1} + \dfrac{1}{Z_1'}\right) + \left(\dfrac{1}{Z_2} + \dfrac{1}{Z_2'}\right)}\, \check{E}_{br} \tag{205}$$

$$\check{E}_{bc2} = \frac{\dfrac{1}{Z_1} + \dfrac{1}{Z_1'}}{\left(\dfrac{1}{Z_1} + \dfrac{1}{Z_1'}\right) + \left(\dfrac{1}{Z_2} + \dfrac{1}{Z_2'}\right)}\, \check{E}_{bc} \tag{206}$$

which give the complete solution for all the quantities required with the aid of equations (**200**) and (**201**). For the supply current $\check{I}$

$$\left.\begin{aligned} \check{I} &= \check{I}_{bc1} + \check{I}_{bc2} + I_{bc1}' + I_{bc2}' \\ S\, \check{I}_{bc} &= S^1\, \check{I}_{bc1} + S^2\, \check{I}_{bc2} \\ S\, \check{E}_{bc} &= S^1\, \check{E}_{bc1} + S^2\, E_{bc2} \end{aligned}\right\} \quad (\mathbf{207})$$

$$P_1 + j\, Q_1 = \check{E}_{bc}\, \hat{I} \quad (\mathbf{208})$$

In order to obtain a perfect balance we may consider the addition of an e. m. f. $S^2\, j\, \dfrac{E_{x2}}{\sqrt{3}}$ in series with the phase converter whose value must be a function of the load and the phase converter impedances, and therefore equation (**201**) will be replaced by

$$S^2\, \check{I}_{a2}' = S^2 \left(j\, \frac{\check{E}_{bc2}}{\sqrt{3}\, Z_2'} + j\, \frac{\check{E}_{x2}}{\sqrt{3}\, Z_2'}\right) \quad (\mathbf{209})$$

$$S^2\, \check{I}_{a2} = S^2\, j\, \frac{\check{E}_{bc}}{\sqrt{3}\, Z_2}$$

and since the balance is perfect $\check{E}_{bc2}$ is zero, and therefore

$$S^2\, j\, \frac{\check{E}_{x2}}{\sqrt{3}} = S^2\, Z_2'\, \check{I}_{a2}' \quad (\mathbf{210})$$

An e. m. f. equal and of opposite phase to the negative phase sequence drop through the phase converter is required to produce a perfect balance.

Carrying out the solution in the same manner as in the imperfect converter, we obtain

$$\check{E}_{bc2} = \frac{\dfrac{1}{Z_1} + \dfrac{1}{Z_1'}}{\dfrac{1}{Z_2} + \dfrac{1}{Z_2'}}\, E_{bc} - \frac{\dfrac{1}{Z_2'}}{\dfrac{1}{Z_2} + \dfrac{1}{Z_2'}}\, \check{E}_{x2} \quad (\mathbf{211})$$

and since $\check{E}_{bc2}$ is zero and $\check{E}_{bc1} = \check{E}_{bc}$ the single-phase impressed e. m. f., we obtain

$$\check{E}_{x2} = Z_2' \left(\frac{1}{Z_1} + \frac{1}{Z_1^1}\right) \check{E}_{bc} \quad (\mathbf{212})$$

and therefore from (**210**)

$$S^2\, \check{I}_{a2}' = S^2\, j \left(\frac{1}{Z_1} + \frac{1}{Z_1'}\right) \frac{\check{E}_{bc}}{\sqrt{3}} \quad (\mathbf{213})$$

$$S^1 \check{I}_{a1}{}^1 = - S^1 j \frac{\check{E}_{bc}}{\sqrt{3} Z_1'} \tag{214}$$

$$S^2 \check{I}_{a2} = O \tag{215}$$

$$S^1 \check{I}_{a1} = - S^1 j \frac{\check{E}_{bc}}{\sqrt{3}\ Z_1} \tag{216}$$

Figs. 9, 10, 11 and 12 are vector diagrams of some of the principal compensated shunt type phase converters. There will be no

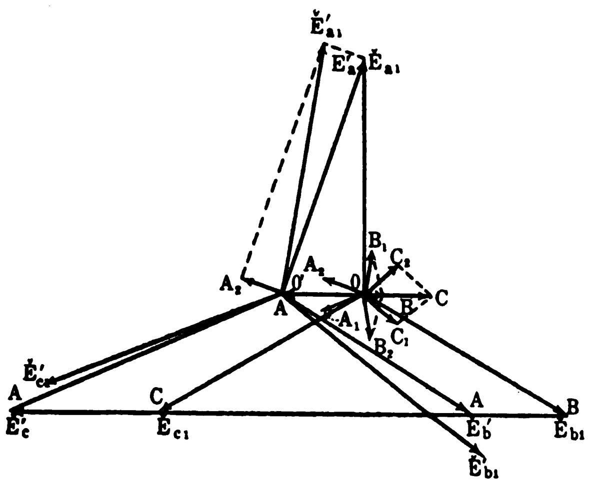


FIG. 9—VECTOR DIAGRAM OF SHUNT-TYPE PHASE CONVERTER OPERATED FROM TRANSFORMER SO AS TO DELIVER BALANCED CURRENTS

Terminal voltages of phase converter $S\check{E}^1{}_a$

Terminal voltages of motor $S^1\check{E}_{a1}$

Nega ive phase sequence e.m.fs. in phase converter $S^2\ (OA_2)$

difficulty in following out these diagrams if the principles of this paper have been grasped.

The Phase Balancer is a device to maintain symmetry of e. m. fs. at a given point in a polyphase system. It may consist of an induction motor or synchronous condenser with an auxiliary machine connected in series to supply an e. m. f. always proportional to the product of the negative phase sequence current passing through the machine and the negative phase sequence impedance of the balancer. It therefore has the effect of annulling the impedance of the machine to the flow of negative phase sequence current. Thus, in a symmetrical polyphase

network, where we have an unbalanced system of currents due to certain conditions

$$S \check{I}_a = S^1 \check{I}_{a1} + S^2 \check{I}_{a2} \tag{217}$$

If a balancer be placed at the proper point the component $S^2 \check{I}_{a2}$ will circulate between the loads and the phase balancer, the other component $S^1 I_{a1}$ being furnished from the power house. On the other hand, if there be a dissymmetry in the impedance of the system up to the phase balancer, the latter will draw a negative phase sequence current sufficient to counteract the unbalance

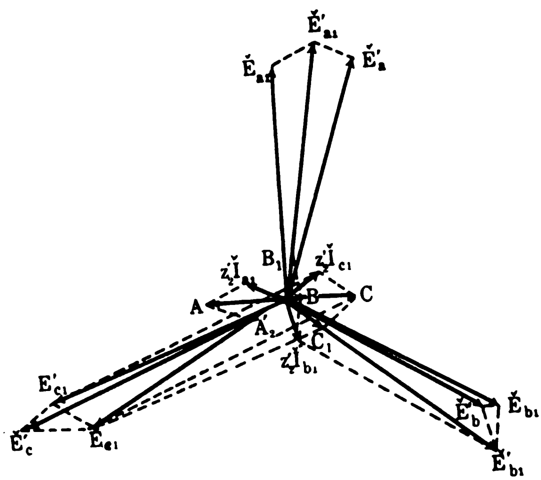


FIG. 10—VECTOR DIAGRAM SHOWING RELATIONS BETWEEN MOTOR TERMINAL E.M.F'S., CONVERTER TERMINAL E.M.FS., AND SYMMETRICAL GENERATED E.M.F'S., SAME CONNECTION AS FOR FIG. 9.

Negative phase sequence drops in phase converter $S^2 Z_2{}^1 \check{I}_{a1}$
Conjugate positive phase sequence e.m.fs. $S^1(ABC)$

due to any symmetrical load by causing the proper amount of negative phase sequence current to flow to produce a balance.

The balancer may be made inherently self-balancing by inserting in series with it a machine which is self-exciting and is able to furnish an e. m. f. equal to the negative phase sequence impedance drop. The combination thus has zero impedance to negative phase sequence currents. If in the neighborhood of a phase balancer the loads have impedances

$$S Z_a = S^0 Z_{a0} + S^1 Z_{a1} + S^2 Z_{a2}$$

The equations of the system are

$$\left.\begin{aligned} S^1 \check{E}_{a1} &= S^1 Z_{a0} \check{I}_{a1} + S^1 Z_{a2} \check{I}_{a2} \\ S^2 E_{a2} &= O = S^2 Z_{a0} \check{I}_{a2} + S^2 Z_{a1} \check{I}_{a1} \end{aligned}\right\} \quad (\mathbf{218})$$

The currents in the phase converter are

$$- S^2 \check{I}_{a2} \text{ and } S^1 \frac{\check{E}_{a1}}{Z_1{}^1}$$

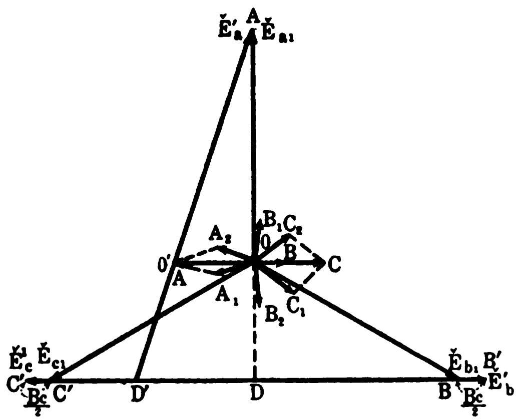


Fig. 11—Vector Diagram of Shunt Type Phase Converter Scott Connected with Compensation by Transformer Taps
Terminal voltages of converter O^1A and B^1C^1
Terminal voltages of motor $S^1\check{E}_{a1}$

The solution of (**218**) gives $S^2\ \check{I}_{a2}$ and $S^1\ \check{I}_{a1}$, the former of which are the phase balancer currents. The solution is

$$\left.\begin{aligned} \check{I}_{a1} &= \frac{Z_{a0}}{Z_{a0}{}^2 - Z_{a1} Z_{a2}} \check{E}_{a1} \\ \check{I}_{a2} &= - \frac{Z_{a1}}{Z_{a0}{}^2 - Z_{a1} Z_{a2}} \check{E}_{a1} \end{aligned}\right\} \quad (\mathbf{219})$$

The phase balancer is a voltage balancer and will maintain balanced e. m. f. for any condition of impedance, and if the impedance of the mains is unsymmetrical it will draw a sufficient amount of wattless negative phase sequence current through these mains to produce an e. m. f. balance at its terminals. Hence the complete solution requires consideration of all the

connections in the network between the supply point and the balancer. Two equations for each mesh and connection are required, one of the positive phase sequence e. m. fs. and the other of the negative phase sequence e. m. f., and these equations may be solved in the usual way.

Series Phase Converter. In discussing the various reaction in rotating machines we have made use of the terms "positive phase sequence impedance" and "negative phase sequence impedance." These terms are definite enough when dealing with relations between machines whose generated e. m. fs. all have the

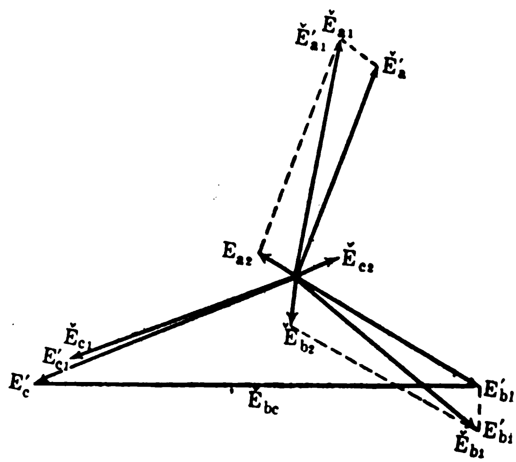


FIG. 12—VECTOR DIAGRAM OF SHUNT-TYPE PHASE CONVERTER WITH AUXILIARY ROTATING COMPENSATOR TO EFFECT A PERFECT BALANCE

Terminal voltages of phase converter $S\ \check{E}_a{}^1$
Terminal voltages of motor $S^1\check{E}_{a1}$
Terminal voltages of compensator $S^2\check{E}_{a2}$

same phase sequence, but require further definition when we are dealing with relations between machines whose e. m. fs. have different phase sequence. We shall retain the symbols Z_1 and Z_2 for the values of the positive and negative phase sequence impedances, depending upon the sequence symbol S to define whether these impedances apply to a negative or positive phase sequence current. Thus, the phase sequence of the currents and e. m. f. will be defined by the apparatus supplying and receiving power and the impedances of the transmitting apparatus will be defined in relation to these currents. As an example a motor

series connected in counter phase sequence relation in a circuit and driven in a positive direction will have impedances

$$\begin{aligned} &\text{positive phase sequence } Z_2 \\ &\text{negative phase sequence } Z_1 \end{aligned} \qquad (220)$$

Where an auxiliary machine is defined as being of negative phase sequence relation to other machines, it will have impedances as given above to the positive and negative phase sequence currents passing through the other machines.

A single-phase transformer winding tapped at the middle point

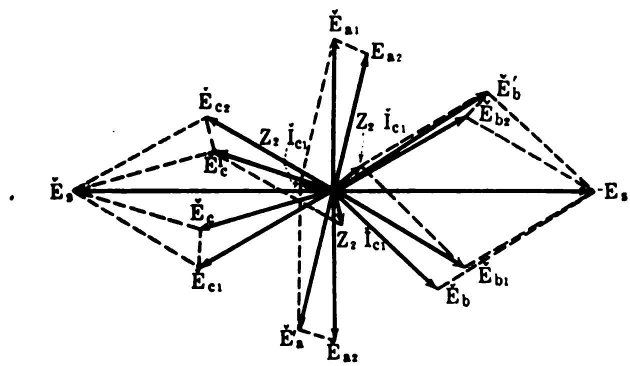


Fig. 13—Vector Diagram of Series-Type Converter.
No Load e.m.f's. Across Motor Terminals $S_1\check{E}_{a1}$
No Load e.m.f's. Across Converter Terminals $S^2\check{E}_{a2}$
Single-Phase e.m.f's. $2\check{E}_s$
e.m.f. Across Terminal of Motor Under Load $\check{E}_a\check{E}_a\check{E}_c$
e.m.f. Across Terminal of Converter Under Load $\check{E}_a\check{E}_b\check{E}_c$

may be regarded as an unbalanced three-phase system where

$$\check{E}_a = O \quad \check{E}_b = +\check{E}_s \quad \check{E}_c = -\check{E}_s$$

$2\check{E}_s$ being the single-phase e. m. f The system may be represented by the equation

$$S\check{E}_a = S^1\check{E}_{a1} + S^2\check{E}_{a2}$$

$$\left.\begin{aligned} \text{where } \check{E}_{a1} &= \quad j\frac{\check{E}_s}{\sqrt{3}} \\ \check{E}_{a2} &= -j\frac{\check{E}_s}{\sqrt{3}} \end{aligned}\right\} \qquad (221)$$

If, therefore between the single-phase source of power and the load we interpose a polyphase machine with e. m. f. $- S^2 \check{E}_{a2}$, we shall have at the load terminals the e. m. f. $S^1 \check{E}_{a1}$. If we use an induction type phase converter it will have impedances to motor currents as follows

$$\left.\begin{array}{l} \text{To positive phase sequence } Z_2' \\ \text{To negative phase sequence } Z_1' \end{array}\right\} \qquad (222)$$

we therefore have the relations

$$S^1 \check{E}_{a1} = S^1 \check{I}_{a1} (Z_1 + Z_2') \qquad (223)$$

$$S^2 \check{E}_{a2} = S^2 \check{I}_{a2} (Z_2 + Z_1') \qquad (224)$$

If the converter is doing no mechanical work, Z_1' is large compared with Z_2' or Z_2, and therefore the component of negative phase sequence is small in the motor. The value of Z_1' depends upon the slip of the phase converter which will depend on the mechanical load it carries as well as on the load carried by the motors. Approximately the load currents due to the motors produce the equivalent at the phase converter of a mechanical load equal to one-half the rotor loss of the phase converter due to these load currents. Substituting the values given in (**221**) for $S^1 \check{E}_{a1}$ and $S^2 \check{E}_{a2}$, we obtain

$$\left.\begin{array}{rl} S^1 j \dfrac{\check{E}_s}{\sqrt{3}} & = S^1 \check{I}_{a1} (Z_1 + Z_2') \\ - S^2 j \dfrac{E_s}{\sqrt{3}} & = S^2 \check{I}_{a2} (Z_2 + Z_1') \end{array}\right\} \qquad (225)$$

$$\begin{array}{rl} S^1 \check{I}_{a1} = & S^1 j \dfrac{\check{E}_s}{\sqrt{3}\,(Z_1 + Z_2')} \\ S^2 \check{I}_{a2} = & - S^2 j \dfrac{\check{E}_s}{\sqrt{3}\,(Z_2 + Z_1')} \end{array} \qquad (226)$$

If instead of an induction type phase converter a synchronous phase converter is used an e. m. f. of negative phase sequence $S^2 \check{E}_{a2}$ *the generated e. m. f. of the phase converter must be introduced in equations* (**224**) *and* (**225**) *and the value and phase of these e. m. fs. will depend upon the load on the phase converter shaft as well as the load carried by the motors.* The equations will be

$$S^1\,\check{E}_{a1} = S^1\,\check{I}_{a1}\,(Z_1 + Z_2') \tag{227}$$

$$S^2\,\check{E}_{a2} = S^2\,\check{I}_{a2}\,(Z_2 + Z_1') + S^2\,\check{E}_{a2}' \tag{228}$$

or

$$\left.\begin{aligned} S^1\,j\,\frac{\check{E}_s}{\sqrt{3}} &= S^1\,\check{I}_{a1}\,(Z_1 + Z_2') \\ -\,S^2\,j\,\frac{\check{E}_s}{\sqrt{3}} &= S^2\,\check{I}_{a2}\,(Z_2 + Z_1') + S^2\,\check{E}_{a2}' \end{aligned}\right\} \tag{229}$$

The last member of equations (**229**) is the equation of a synchronous condenser. Assuming its windage, iron loss and increased losses due to secondary reactions to be P_0, we have by equation (**160**) of the Section on Synchronous Motors

$$\frac{E_s}{\sqrt{3}}\,I_{a2}\cos\alpha = I_{a2}^2\,(R_2 + R_1') + \frac{P_0}{3} \tag{230}$$

Let

$$\check{I}_{a2} = a_2 + j\,b_2 \tag{231}$$

then (**230**) becomes

$$\frac{E_s}{\sqrt{3}}\,a_2 = (a_2^2 + b_2^2)\,(R_2 + R_1') + \frac{P_0}{3} \tag{232}$$

Of the two quantities a_2 and b_2, b_2 alone is arbitrary and depends upon the excitation, a_2 will depend upon the value of b_2 and also upon the losses. Solving therefore for a_2 in terms of b_2, we have

$$a_2 = \frac{E_s}{2\sqrt{3}\,(R_2 + R_1')}\left\{1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,\{3\,b_2^2\,(R_2 + R_1') + P_0\}}{E_s^2}}\right\} \tag{233}$$

Since b_2 is arbitrary we may now determine $\cos\alpha_2 = \dfrac{a_2}{\sqrt{a_2^2 + b_2^2}}$ and the value of $\check{I}_{a2}$ in terms of the impressed e. m. f. will be by (**181**) of Section on Synchronous Motors

$$S^2\,\check{I}_{a2} = -\,S^2\left[j\,\frac{\check{E}_s}{\sqrt{3}}\;\frac{\cos\alpha_2}{2\,(R_2 + R_1')}\left\{1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2\,\cos^2\alpha_2}}\right\}\epsilon^{j\alpha}\right] \tag{234}$$

The effective value of $\check{I}_{a2}$ in terms of the effective value of $\check{E}_s$ will then be

$$I_{a2} = \frac{E_s}{\sqrt{3}} \frac{\cos \alpha_2}{2\,(R_2 + R_1')} \left\{ 1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}} \right\} \tag{235}$$

and since the component of the e.m.f. generated in phase with the current is determined only by the magnitude of $\check{I}_{a2}$ and the motor losses, if we define its value by A_2' the quadrature component being B_2' we shall have

$$A_2' = \frac{E_s}{\sqrt{3}} \frac{\cos \alpha_2}{2} \left(1 + \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}} \right) \tag{236}$$

and

$$B_2' = - \frac{E_s}{\sqrt{3}} \sin \alpha_2 - \frac{w\,(L_2 + L_2')}{A_2^1} \tag{237}$$

$$= - \frac{E_s}{\sqrt{3}} \left\{ \sin \alpha_2 + \frac{3\,w\,(L_2 + L_1')}{P_0} \cdot \frac{\cos \alpha_2}{2\,(R_2 + R_1')} \left(1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}} \right) \right\} \tag{238}$$

and therefore we have

$$\check{E}_2' = - j \frac{E_s}{\sqrt{3}} \left[\frac{\cos \alpha_2}{2} \left(1 + \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}} \right) - j \left\{ \sin \alpha_2 + \frac{3\,w\,(L_2 + L_1') \cos \alpha_2}{2\,P_0\,(R_2 + R_1')} \left(1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}} \right) \right\} \right] \tag{239}$$

The impedance of the phase converter to the flow of negative phase sequence current is

$$\frac{2\,(R_2 + R_1') \sec \alpha}{1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha}}} \tag{240}$$

The balance will be at its best when $\check{I}_{a2}$ is a minimum with $\cos \alpha_2$ as the independent variable. This will be the case when $\cos \alpha_2$ is unity; that is to say when b_2 is zero.

Recapitulating the results given above, we have for the general case taking the single-phase e. m. f. $\check{E}_s$ as reference

$$S^1 \check{I}_{a1} = S^1 j \frac{\check{E}_s}{\sqrt{3}\,(Z_1 + Z_2')} \tag{241}$$

$$S^2 \check{I}_{a2} = -j\,(a_2 + j\,b_2) \tag{242}$$

where b_2 is arbitrary and

$$a_2 = \frac{E_s}{2\sqrt{3}\,(R_2 + R_1')} \left\{ 1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,\{3\,b_2^2\,(R_2 + R_1') + P_0\}}{E_s^2}} \right. \tag{243}$$

Since b_2 is arbitrary $\cos \alpha_2$ is determined by

$$\cos \alpha_2 = \frac{a_2}{\sqrt{a_2^2 + b_2^2}} \tag{244}$$

we may express $\check{I}_{a2}$ in terms of $\check{E}_s$ by

$$S^2 I_{a2} = -S^2 j \frac{\check{E}_s}{\sqrt{3}} \frac{\cos \alpha_2}{2\,(R_2 + R_1')} \left\{ 1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}} \right\} e^{j\alpha_2} \tag{245}$$

The effective value of $\check{I}_{a2}$ will be

$$I_{a2} = \sqrt{a_2^2 + b_2^2} = \frac{E_s}{\sqrt{3}} \frac{\cos \alpha_2}{2\,(R_2 + Q_1')} \left\{ 1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}} \right\} \tag{246}$$

If A_2' and B_2' are components of $\check{E}_{a2}'$ these being the generated e. m. f. in phase and in quadrature with the current $\check{I}_{a2}$ we shall have

$$\check{E}_{a2}' = -j\,(A_2' + j\,B_2') \tag{247}$$

and A_2' and B_2' will have the following values

$$A_2' = \frac{E_s}{\sqrt{3}} \; \frac{\cos \alpha_2}{2} \left(1 + \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}}\right) \tag{248}$$

$$B_2^1 = -\frac{E_s}{\sqrt{3}} \left\{ \sin \alpha_2 + \frac{3\,w\,(L_2 + L_1')}{P_0} \cdot \frac{\cos \alpha_2}{2\,(R_2 + R_1')} \left(1 - \sqrt{1 - \frac{4\,(R_2 + Q_1')\,P_0}{E_s^2 \cos \alpha^2}}\right)\right\} \tag{249}$$

and $\check{E}_{a2}'$ expressed in terms of $\check{E}_s$ becomes

$$\check{E}_2' = -j\,\frac{\check{E}_s}{\sqrt{3}} \left[\frac{\cos \alpha_2}{2} \left(1 + \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}}\right) - j \left\{ \sin \alpha_2 + \frac{3\,w\,(L_2 + L')\cos \alpha_2}{2\,P_0\,(R_2 + R_1')} \left(1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}}\right)\right\}\right] \tag{250}$$

The effective impedance of the phase converter to the flow of negative phase sequence currents is

$$\frac{2\,(R_2 + R_1')\sec \alpha_2}{1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha}}} \,(\cos \alpha_2 - j \sin \alpha_2) \tag{251}$$

or

$$\frac{E_s^2}{P_0} \cdot \frac{\cos \alpha_2}{2} \left(1 + \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}}\right) e^{-j\alpha_2} \tag{252}$$

In the above equations $\cos \alpha_2$ is arbitrary or b_2 may be considered arbitrary and $\cos \alpha_2$ will then be determined.

Minimum Unbalance is obtained when $\cos \alpha_2$ is made unity or when b_2 is made zero in equations (**241**) and (**252**).

Perfect Balance is obtained by driving the phase converter mechanically so as to supply the mechanical power P_0 from a separate or symmetrical source. Under this condition a_2 and b_2 both become zero when $\cos \alpha_2$ is unity. The only equation of the system is then (**241**).

Currents and Power Factor in the Single-Phase Supply Circuit. The e. m. f. is $2\,\check{E}_s$ and the current supplied is

$$\check{I}_s = \frac{\check{I}_b - \check{I}_c}{2}$$

$$= \frac{\check{I}_{b1} - \check{I}_{c1}}{2} + \frac{\check{I}_{b2} - \check{I}_{c2}}{2} \tag{253}$$

If we take

$$S^1\,\check{I}_{a1} = S^1 j\,(a_1 - j\,b_1) \tag{254}$$

$$\frac{\check{I}_{b1} - \check{I}_{c1}}{2} = \frac{\sqrt{3}}{2}\,(a_1 - j\,b_1) \tag{255}$$

Similarly, since under the same conditions

$$S^2\,\check{I}_{a2} = -\,S^2 j\,(a_2 + j\,b_2) \tag{256}$$

$$\frac{\check{I}_{b2} - \check{I}_{c2}}{2} = \frac{\sqrt{3}}{2}\,(a_2 + j\,b_2) \tag{257}$$

and therefore

$$\check{I}_s = \frac{\sqrt{3}}{2}\,\{(a_1 + a_2) - j\,(b_1 - b_2)\} \tag{258}$$

where a_1, b_1, a_2, b_2 are to be obtained by means of equations (**243**) to (**254**). The single-phase power factor is given by

$$\tan\theta = \frac{b_1 - b_2}{a_1 + a_2} \tag{259}$$

of these quantities a_2 is usually the smallest and its value may be obtained approximately by assigning to b_2 a value which will make the ratio $\frac{b_1 - b_2}{a_1}$ equal to $\tan\theta$, and obtaining the corresponding value of a_2 by (**242**), the value of b_2 may then be recalculated from (**259**) by substituting the tentative value obtained for a_2. This procedure may be repeated until sufficient accuracy has been obtained.

Single Phase Power Factor in Shunt Type Phase Converter

The simplest procedure is to obtain a curve of admittances for varying excitation of the converter and plot the power factor obtained by varying the admittance with a fixed load. The true

and wattless power is obtained easily by means of (**208**) whether the system is balanced or unbalanced.

Figs. 14, 15, 16 and 17 are vector diagrams of several methods of using phase converters to supply a balanced 3-phase e. m. f. to a symmetrical load such as an induction motor. The diagram are all based on a main machine having the same negative phase sequence impedance and the system in each case is

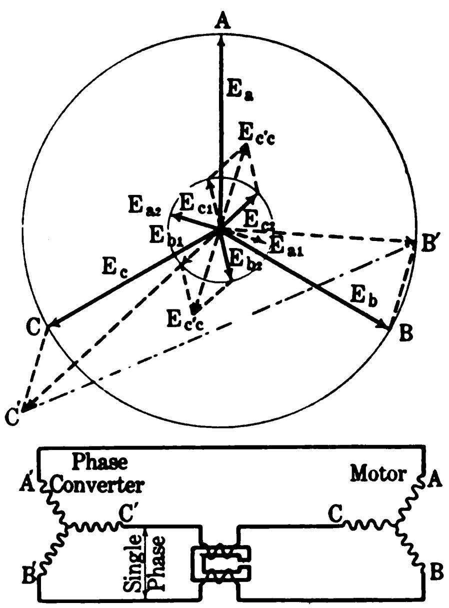


FIG. 14
SINGLE -PHASE IMPRESSED E.M.F. = $B'C'$
MOTOR E.M.F. = BC
NEGATIVE PHASE SEQUENCE E.M.FS. $\check{E}_{a2}\check{E}_{b2}\check{E}_{c2}$
CONJUGATE POSITIVE PHASE SEQUENCE E.M.FS. $\check{E}_{a1}\check{E}_{b1}\check{E}_{c1}$
PHASE CONVERTER TERMINAL E.M.F. $AB'C'$

delivering the same amount of power at the same voltage and 3-phase power factor without supplying any wattless power. It will be noted that the scheme Fig. 14 has the lowest single phase power factor, Fig. 16 the highest and the rest arcing alike. It may be remarked, however, that with the shunt type schemes adjustments can be made for power factor correction which will result also in better regulation.

APPENDIX I

Cylindrical Fields in Fourier Harmonics

When we have a diametrical coil around a cylinder concentric with another cylinder which forms the return magnetic path, and the length of the gap is uniform and the coil dimension very small, the field across the gap takes the form of a square topped

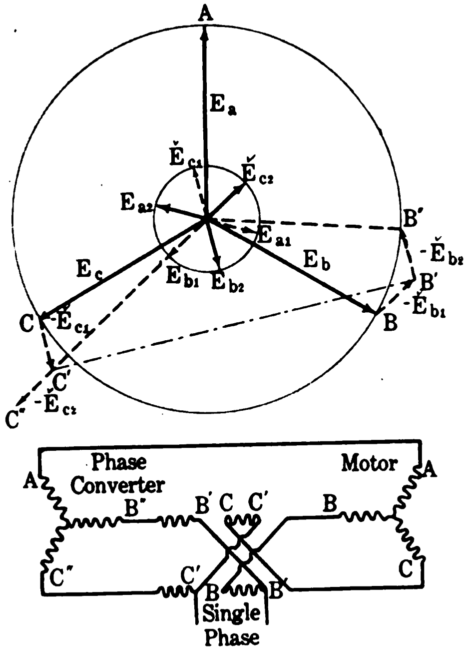


FIG. 15

SINGLE PHASE IMPRESSED E.M.F. = $B'C'$
MOTOR E.M.F. = BC
PHASE CONVERTER E.M.F. = $B''C''$
NEGATIVE PHASE SEQUENCE E.M.F $\check{E}_{a2}\check{E}_{b2}\check{E}_{c2}$
CONJUGATE POSITIVE PHASE SEQUENCE E.M.F. $\check{E}_a{}^1 E_{b1} E_{c1}$
PHASE CONVERTER TERMINAL E.M.F. $AB''C''$

wave, which may be expressed in the form of a Fourier series with the plane of symmetry of the coil as reference plane, and its Fourier expansion is

$$\mathfrak{B} = \frac{4\,B}{\pi}\left(\cos\theta - \tfrac{1}{3}\cos 3\,\theta + \frac{1}{S}\cos S\,\theta - \ldots + \ldots\right) \qquad (1)$$

where B is the average induction in the air gap.

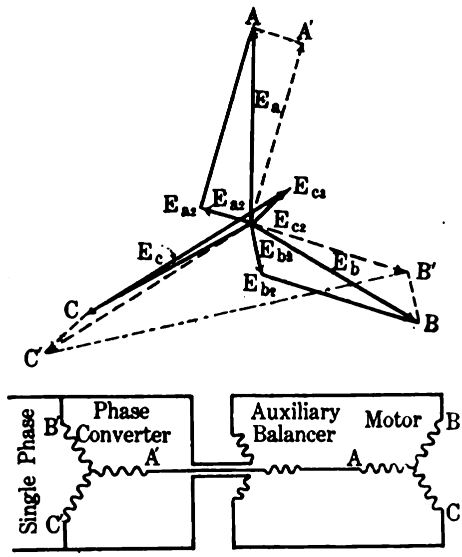


FIG. 16—PHASE CONVERTER WITH AUXILIARY BALANCER.

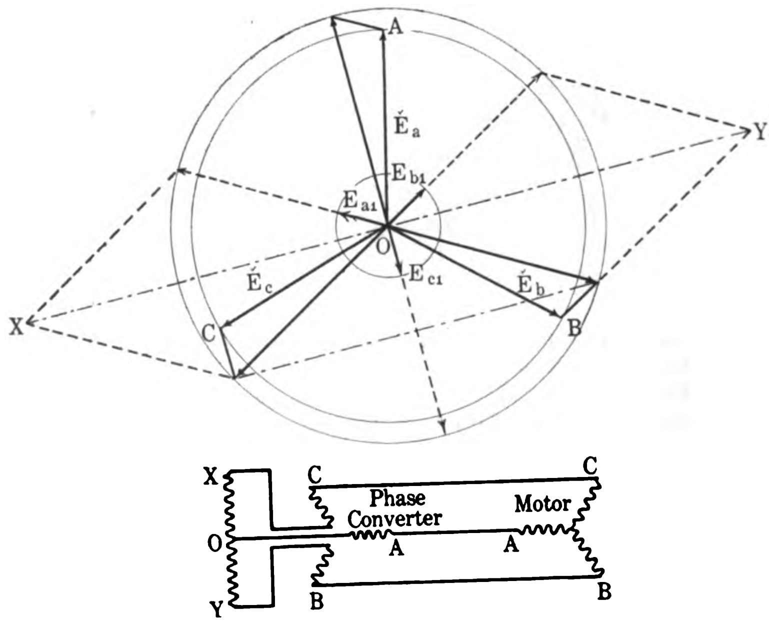


FIG. 17

SINGLE PHASE IMPRESSED E.M.F. = XY

MOTOR E.M.F. = BC

THERE IS A 2 TO 1 TRANSFORMATION OF E.M.F. FROM SINGLE-PHASE TO THREE-PHASE IN THIS CONNECTION

With pitch less than π the curve will have a different form, the amplitude being greater on one side of the plane of the coils than on the other, the areas of each wave will remain the same and second harmonic terms will appear. Let $2\, m_0\, \pi$ be the new pitch then the average amplitude of the induction will be the same as before, namely B, and the value on one side of the coil will be $2\,(1 - m_0)B$ and on the other side $2\, m_0\, B$ so that the total flux will be the same on either side. To obtain the values of the coefficients we have

$$2\,(1 - m_0 B \int_0^{m_0 \pi} \cos n\,\theta\, d\,\theta + 2\, m_0\, B \int_{m_0 \pi}^{2\pi} \cos n\,\theta\, d\,\theta = \frac{\pi}{2} A_n$$

$$2\,(1 - m_0)\, B \left[\frac{1}{n} \sin n\,\theta \right]_0^{m_0 \pi} - 2\, m_0\, B \left[\frac{1}{n} \sin n\,\theta \right]_{m_0 \pi}^{2\pi} = \frac{\pi}{2} A_n$$

$$A_n = \frac{4\,B}{\pi} \left\{ \frac{(1\ \ m_0) + m_0}{n} \sin n\, m_0\, \pi \right\}$$

$$A_n = \frac{4\,B}{\pi} \left(\frac{1}{n} \sin n\, m_0\, \pi_1 \right) \tag{2}$$

Let $2\, m_0\, \pi = \frac{2}{3}\pi$, then $(1 - m_0)\,\pi = \frac{2}{3}\pi$ and

$$\mathfrak{B} = \frac{2\sqrt{3}\,B}{\pi} \left(\cos\theta + \frac{1}{2}\cos 2\,\theta - \frac{1}{4}\cos 4\,\theta - \frac{1}{5}\cos 5\,\theta \right.$$
$$\left. + \frac{1}{7}\cos 7\,\theta + \frac{1}{8}\cos 8\,\theta - \frac{1}{10}\cos 10\,\theta\ .\ \ .\ \ . \right) \tag{3}$$

A general expression for $\mathfrak{B}$ where B is the average of the positive and negative, maximum value for any pitch coil would be

$$\mathfrak{B} = \frac{4\,B}{\pi} \Sigma \left(\frac{1}{n} \sin n\, m_0\, \pi \cos n\,\theta \right) \tag{4}$$

and includes all possible coil pitches. If the number of teeth in a pole pitch be n_T; in addition to the average induction as indicated by (4), there will also be a tooth ripple of flux, the maximum value of which will depend upon the average value of the induction at each point. The value of m_0 must be a fraction having n_T as denominator and an integral numerator. The

value of the integral numerator is therefore always $m_0\, n_T$. The correct value for the max. induction will therefore be

$$\mathfrak{B}_m = \left\{ \frac{4\,B}{\pi}\, \Sigma \left(\frac{1}{n} \sin n\, m_0\, \pi \cos n\, \theta \right) \right\} (1 - (-1)^{m_0 n_T} K_T \cos n_T\, \theta) \qquad (5)$$

where K_T is the ratio of the average to the min. air gap. "m_0" must always be chosen so that $m_0\, n_T$ is an integer.

If the length of the average effective air gap in centimeters be d the value of B is given by

$$B = \frac{4\,\pi}{10}\ \frac{I\,N}{2\,d} \text{ gauss}$$

where I is the maximum value of the current in the coil and N is the number of turns. If d is given in inches we may write

$$B = \frac{4\,\pi}{10}\ \frac{I\,N}{2\,d} \times 2.54 \quad \text{maxwells per square inch.}$$

If we integrate (5) between the limits $(\theta - m_0\,\pi)$ and $(\theta + m_0\,\pi)$ we shall have the total flux φ through the coil

$$\varphi = \frac{4\,B\,r\,l}{\pi} \int_{\theta - m_0\,\pi}^{\theta + m_0\,\pi} \Sigma \left(\frac{1}{n} \sin n\, m_0\, \pi \cos n\, \theta \right) d\,\theta$$

$$- \frac{4\,B\,r\,l}{\pi} (-1)^{m_0\, n_T} \int_{\theta - m_0\,u}^{\theta + m_0\,\pi} \Sigma \left(\frac{1}{n} \sin n\, m_0\, (\cos n\, \theta \right) K_T \cos n_T\, \theta\, d\, 0$$

$$= \frac{4\,B\,re}{\pi} \left[\frac{1}{n^2} \sin n\, m_0\, \pi \sin n\, \theta \right]_{\theta - m_0\,\pi}^{\theta + m_0\,\pi}$$

$$- \frac{4\,B\,r\,l}{\pi} (-1)^{m_0\, n_T} K_T\, \Sigma \frac{1}{n}\, \theta\, n\, m_0\, n\, \pi \left[\frac{\sin (n - n_T)\, \theta}{2\,(n - n_T)} + \frac{\sin (n + n_T\, \theta)}{2\,(n + n_T)} \right]_{\theta - m_0\,\pi}^{\theta + m_0\,\pi} \qquad (6)$$

The second expression is zero for all values of θ which are integral multiples of the tooth pitch angle, so long as $m_0\, n$ is also an integer and therefore it is zero for all mutual inductive relations of similar coils on a symmetrical toothed core we therefore have:

The induction through a coil displaced an angle θ from the axis of a similar coil carrying a current giving a mean induction B both coils being wound on the same symmetrical toothed core is

$$\varphi = \frac{8 B r l}{\pi} \Sigma \left(\frac{1}{n^2} \sin^2 n\, m_0\, \pi \cos n\, \theta \right) \tag{7}$$

The second term in equation (6) also becomes zero when n_T becomes infinite independent of the value of θ. We may therefore safely make use of an imaginary uniformly distributed winding when considering self and mutual impedances. It will also be shown later on that with certain groupings of windings the second term may be reduced to zero for every value of θ.

If N_1 be the total number of complete loops in one complete pole pitch, we may take $\dfrac{N_1}{2\pi}$ as the density of winding per unit angle of the complete pole pitch. The mutual induction per turn in a coil angularly displaced an angle θ from another coil of winding density $\dfrac{N_1}{2\pi}$ with an effective total air gap $2\, d$ and with windings subtending an angle $2\, m_1\, \pi$ is given by

$$M_1 = \frac{8\, N_1\, r\, l}{10^9\, \pi\, d} \int_{-m_1\pi}^{+m_1\pi} \Sigma \left\{ \frac{1}{n^2} \sin^2 n\, m_0\, \pi \cos n(\theta + \theta_1) \right\} d\, \theta' \text{ henrys} \tag{8}$$

$$= \frac{8\, N_1\, r\, l}{10^9\, \pi\, d} \Sigma \frac{1}{n^3} \sin^2 n\, m_0\, \pi \left[\sin n\, (\theta + \theta_1)\right]_{\theta' = -m_1\pi}^{\theta' = m_1\pi} \text{ henrys}$$

$$M_1 = \frac{16\, N_1\, r\, l}{10^9\, \pi\, d} \Sigma \left(\frac{1}{n^3} \sin^2 n\, m_0\, \pi \sin n\, m_1\, \pi \cos n\, \theta \right) \text{henrys} \tag{9}$$

Next, if the loop of which M_1 is the mutual inductance is part of a winding having distribution density of winding $\dfrac{N_2}{2\pi}$ and subtending an angle $2\, m_2\, \pi$ its mutual inductance with the other winding will be

$$M_{12} = \frac{8\, N_1\, N_2\, r\, l}{10^9\, \pi^2\, d} \int_{-m_2\pi}^{m_2\pi} \Sigma \frac{1}{n^3} \sin^2 n\, m_0\, \pi \sin n\, m_1\, \pi \cos n\, (\theta + \theta^1)\, d\, \theta' \text{ henry} \tag{10}$$

$$= \frac{8\, N_1\, N_2\, r\, l}{10^9\, \pi^2\, d} \Sigma \frac{1}{n^4} \sin^2 n\, m_0\, \pi \sin n\, m_1\, \pi \left[\sin n\, (\theta + \theta_1)\right]_{\theta' = -m_2\pi}^{\theta' = m_2\pi} \text{ henrys}$$

$$M_{12} = \frac{16\, N_1\, N_2\, r\, l}{10^9\, \pi^2\, d}\, \Sigma \left(\frac{1}{n^4} \sin^2 n\, m_0\, \pi \sin n\, m_1\, \pi \sin n\, m_2\, \pi \cos n\, \theta \right) \text{ henrys} \qquad (11)$$

This is the general expression for the mutual inductance between two groups of connected coils of like form on the same cylindrical core. It should be noted how much the harmonics have been reduced due to grouping.

When the coils are not of like design as in the case of a rotor and stator and the pitch of the coils is different in one from the other, $\sin n\, m_0\, \pi$ will not appear twice in the equation but one of its values must be replaced by $\sin n\, m_x\, \pi$ where $2\, m_x\, \pi$ is the pitch of the new coil. Equation (**11**) then becomes

$$M_{1a} = \frac{16\, N_1\, N_a\, r\, e}{10^9\, \pi^2\, d}\, \Sigma \left(\frac{1}{n^4} \sin n\, m_0\, \pi \sin n\, m_x\, \pi \sin n\, m_1\, \pi \sin n\, m_2\, \pi \cos n\, \theta \right) \text{ henrys} \qquad (12)$$

This formula is strictly correct when m_x is an integer and when θ is an integral multiple of the tooth pitch. It is true for all values of θ if either m_0 or m_x or both are unity.

By considering the axes of two similar groups of coils as coincident we obtain the value of $\Delta_1\, L_1$ which is part of the self inductance of the group, thus

$$\Delta_1\, L_1 = \frac{16\, N_1^2\, r\, e}{10^9\, \pi^2\, d}\, \Sigma \left(\frac{1}{n^4} \sin^2 n\, m_0\, \pi \sin^2 n\, m_1\, \pi \right) \qquad (13)$$

The other factor that enters into the self inductance is the slot leakage inductance which depends upon the number of turns in a coil, the number of coils in a group and the width and depth of the slot and the length of the air gap. Since with the value of $\Delta_1\, L_1$ all the field which links the secondary winding has been included, only the portion of the slot leakage which does not link all the turns in the opposed secondary coil should be considered. No hard and fast rule can be made for determining this quantity since it depends upon the shape of the slots, there should be little trouble in making the calculation when the data is given. Denoting this quantity by $\Delta_2\, L_1$ we have

$$L_1 = \Delta_1\, L_1 + \Delta_2\, L_1 \qquad (14)$$

Symmetrically Grouped Windings. The above formulae give the mutual impedance between groups of coils, each group of which may be unsymmetrical. Generally machines are designed so that, although the individual groups of coils due to fractional pitch may be unsymmetrical, the complete winding is symmetrical. When two coils are together in a slot this may be done by connecting one group of coils opposite the north pole in series with the corresponding group opposite the south pole; that is to say, the group displaced electrically by the angle π. If therefore we take equation (**11**) and consider the mutual induction as due to a group having axis at θ = zero and another having its axis at $\theta = \pi$ with a similarly arranged group of coils having its axis at θ, we find that (**11**) becomes

$$M_{12} = \frac{16\, N_1\, N_2\, r\, l}{10^9\, \pi^2\, d}\, \Sigma \left\{ \frac{1}{n^4} \sin^2 n\, m_0\, \pi \sin n\, m_1\, \pi \right.$$

$$\left. \sin n\, m_2\, \pi\, (1 - \cos n\, \pi)^2 \cos n\, \theta \right\} \text{henrys} \qquad (15)$$

Similarly

$$M_{1a} = \frac{16\, N_1\, N_a\, r\, l}{10^9\, \pi^2\, d}\, \Sigma \left\{ \frac{1}{n^4} \sin n\, m_1\, \pi \sin n\, m_x \right.$$

$$\left. \pi \sin n\, m_1\, \pi \sin n\, m_a\, \pi\, (1 - \cos n\, \pi)^2 \cos n\, \theta \right\} \text{henrys} \qquad (16)$$

Since $1 - \cos n\pi$ is zero for all even values of n it is evident that (**15**) and (**16**) contain no even harmonics, moreover the above formulae give the mutual induction between two similarly connected groups of windings, but if $(1 - \cos n\, \pi)$ is used only with the first power these formulas give the mutual impedance between one pair of such symmetrically grouped windings and another single group with axis inclined at an angle θ.

The value of self induction is

$$\Delta_1 L_1 = \frac{16\, N_1^2\, r\, e}{10^9\, \pi^2\, d}\, \Sigma \left\{ \frac{1}{n^4} \sin^2 n\, m_0\, \pi \sin^2 n\, m_1\, \pi \right.$$

$$\left. (1 - \cos n\, \pi)^2 \right\} \qquad (17)$$

$\Delta_2 L_1$ is found in the same manner as before

$$L_1 = \Delta_1 L_1 + \Delta_2 L_1 \qquad (18)$$

It is obvious from (**15**) and (**16**) that the effect of dissymmetry is to introduce more or less double frequency into the wave form of generated e. m. f.

It will be seen from an examination of (**15**) and (**17**) that, for example, a winding of pitch $\frac{2\pi}{3}$ and subtending an angle $\frac{\pi}{3}$ when connected in a symmetrical group of two has the same field form and characteristics as a full pitch winding of the same number of turns subtending an angle $\frac{2\pi}{3}$.

There are many symmetrical forms of winding but all will be found to be covered by the formulas (**15**) and (**16**).

Unsymmetrical Windings. These may take many forms which may be classified:

(1) Dissymmetry of flux form due to even harmonics.

(2) Dissymmetry in axial position of polyphase groups.

(3) Dissymmetry in windings due to incorrect grouping of coils.

(4) Dissymmetry due to unsymmetrical magnetic characteristics of the iron.

Of these various forms of dissymmetry the most common is a combination of (**1**), (**2**) and (**3**). These forms of unsymmetrical windings may all be calculated by the formulas (**11**) to (**16**).

It is to be noted that the mutual inductance between a symmetrical and an unsymmetrical winding is harmonically symmetrical. Hence, if the field of a machine is harmonically symmetrical, the e. m. f. generated will be also harmonically symmetrical whatever may be the form of the windings.

The reciprocal nature of M is fully established by its form, for it is immaterial in obtaining (**16**) whether we start out with the winding whose pitch is m_x or with that whose pitch is m_0, the result will be the same. The effect of saturation will be to tend to alter the values of the coefficients of M but the general form will not vary appreciably. We shall now consider some standard windings of Generators and Motors.

Three-Phase Symmetrical Full Pitch. Here m_0, m_1 and m_2 are 0.5, 0.1666 and 0.1666 respectively. Using formula (**15**) all the even harmonics disappear and $(1 - \cos n\,\pi)^2$ is equal to 4 or zero.

$$M_{12} = \frac{16\,N_1\,N_2\,r\,l}{10^9\,\pi^2\,d}\left(\cos\theta + \frac{4}{81}\cos 3\,\theta + \frac{1}{625}\cos 5\,\theta + \frac{1}{2401}\cos 7\,\theta + \frac{4}{6561}\cos 9\,\theta + \ldots\right) \qquad (\mathbf{19})$$

Theoretical Symmetrical Three-Phase Winding. Here $m_0 = 0.5$, $m_1 = m_2 = 0.333$. Using formula (**11**)

$$M_{12} = \frac{3}{4}\,\frac{16\,N_1\,N_2\,r\,l}{10^9\,\pi^2\,d}\left(\cos\theta + \frac{1}{625}\cos 5\,\theta\right.$$

$$\left. + \frac{1}{2401}\cos 7\,\theta + \frac{1}{14641}\cos 11\,\theta + \ldots\right) \qquad (\mathbf{20})$$

Here the third group of harmonics is entirely eliminated.

Three-Phase Symmetrical $\frac{2\,\pi}{3}$ *Pitch Winding.* Here $m_0 = 0.333$, $m_1 = m_2 = 0.166$. Using formula (**15**)

$$M = \frac{3}{4}\,\frac{16\,N_1\,N_2\,r\,l}{10^9\,\pi^2\,a}\left(\cos\theta + \frac{1}{625}\cos 5\,\theta\right.$$

$$\left. + \frac{1}{2461}\cos 7\,\theta + \frac{1}{14641}\cos 11\,\theta + \ldots\right) \qquad (\mathbf{21})$$

which gives the same result as (**20**).

Formulas for Salient Pole Machines

The formulas given in the preceding discussion are appropriate for distributed winding and non-salient poles. Where salient poles are used the field form due to the poles with a given winding will be arbitrary so that with the polar axis as reference we shall have

$$\mathfrak{B} = \frac{2\,\pi\,N_a\,I_a}{d}\,\Sigma\,(A_n \cos n\,\theta) \qquad (\mathbf{22})$$

Where $\mathfrak{B}$ is the induction through the armature or stator. When the poles are symmetrical $A_n \cos n\,\theta$ might be chosen at once for this condition and in this case we do not require coefficients of mutual induction between pole windings, since the value of $\mathfrak{B}$ is obtained by considering the mutual reaction between pole windings to be such as will produce symmetry. We may however assume $\mathfrak{B}$ to be perfectly general in form in which case the flux through a coil of pitch $2\,m_0\,\pi$ is

$$\varphi = \frac{4\,\pi\,N_a\,I_a\,r\,l}{10\,d}\,\Sigma\left(\frac{A_n}{n}\sin n\,m_0\,\pi\cos n\,\theta\right) \qquad (\mathbf{23})$$

We have therefore for the mutual induction between one pole and a group of coils at an angle θ and subtending an angle $2\,m_1\,\pi$

$$M_{a1} = \frac{4\,N_a\,N_1\,r\,l}{10^9\,d}\,\Sigma\left(\frac{A_n}{n^2}\sin n\,m_0\,\pi\sin n\,m_1\,\pi\cos n\,\theta\right) \qquad \mathbf{(24)}$$

and where there is symmetry due to grouping of windings, we have

$$M_{a1} = \frac{4\,N_a\,N_1\,r\,l}{10^9\,d}\,\Sigma\left\{\frac{A_n}{n^2}\sin n\,m_0\,\pi\sin n\,m_1\,\pi\right.$$

$$\left.(1-\cos n\,\pi)^2\cos n\,\theta\right\} \qquad \mathbf{(25)}$$

where N_a is the number of turns for one pole and (**25**) applies to one pair of poles and the corresponding group of coils. When there are more than one pair of poles in series and the corresponding groups of winding are also in series, if it is desired to consider the mutual inductance of the complete winding, the result given above must be multiplied by the number of pairs of. poles.

If in equation (**16**) we take

$$\left.\begin{aligned} \frac{N_a}{2\,\pi}\,\frac{1}{n}\sin n\,m_a\,\pi &= N_a \\ \text{and} \qquad \frac{1}{\pi\,n} &= B_n \end{aligned}\right\} \qquad \mathbf{(26)}$$

it becomes

$$M_{1a} = \frac{32\,N_1\,N_a\,r\,l}{10^9\,d}\,\Sigma\left\{\frac{B_n}{n^2}\sin n\,m_x\,\pi\sin n\,m_0\,\pi\right.$$

$$\left.\sin n\,m_1\,\pi\,(1-\cos n\,\pi)^2\cos n\,\theta\right\} \qquad \mathbf{(27)}$$

which is the expression corresponding to (**25**) starting with the winding flux form. (**25**) and (**27**) must therefore be identical and we have

$$\frac{32\,N_1\,N_a\,r\,e}{10^9\,d}\,B_n\sin n\,m_x\,\pi - \frac{4\,N_a\,N_1\,r\,e}{10^9\,d}\,A_n$$

or

$$\mathcal{B}_n = \frac{A_u}{8\sin n\,m_x\,\pi} \qquad \mathbf{(28)}$$

and

$$\mathcal{B}_1 = \frac{2\,\pi\,I_1}{10\,d}\,\Sigma\,(B_n\sin n\,m_0\,\pi\cos n\,\theta) \qquad \mathbf{(29)}$$

and is the induction wave form for a single turn of the winding.

The expression for the mutual inductance between windings of the same core for salient poles is obtained in terms of the pole flux wave form by substituting in the formulas $\frac{A_n}{8 \sin n\, m_x\, \pi}$ for $\frac{1}{n\,\pi}$. We have therefore the following formulas for salient poles.

General expression considering only one pole and one group of coils.

$$\mathcal{B}_a = \frac{2\,\pi\,N_a\,I_a}{10\,d}\,\Sigma\,(A_n \cos n\,\theta) \tag{a}$$

$$\mathcal{B}_1 = \frac{\pi\,I_1}{20\,d}\,\Sigma\left(A_n\,\frac{\sin n\,m_0\,\pi}{\sin n\,m_a\,\pi}\cos n\,\theta\right) \tag{b}$$

$$M_{a1} = \frac{4\,N_a\,N_1\,r\,l}{10^9\,d}\,\Sigma\left(\frac{A_n}{n^2}\sin n\,m_0\,\pi\,\sin n\,m_1\,\pi\,\cos n\,\theta\right) \tag{c}$$

$$M_{12} = \frac{2\,N_1\,N_2\,r\,l}{10^9\,\pi\,d}\,\Sigma\left(\frac{A_n}{n^3}\,\frac{\sin n\,m_0\,\pi}{\sin n\,m_x\,\pi}\sin n\,m_0\,\pi\,\sin n\,m_1\,\pi\right.$$
$$\left.\sin n\,m_2\,\pi\,\cos n\,\theta\right) \tag{d}$$

$$\Delta_1 L_a = \frac{4\,\pi\,N_a^2\,r\,l}{10^9\,d}\,\Sigma\left(\frac{A_n}{n}\sin n\,m_x\,\pi\right) \tag{e}$$

$$\Delta_1 L_1 = \frac{2\,N_1^2\,r\,l}{10^9\,\pi\,d}\,\Sigma\left(\frac{A_n}{n}\,\frac{\sin^2 n\,m_0\,\pi\,\sin^2 n\,m_1\,\pi}{\sin n\,m_x\,\pi}\right) \tag{f}$$

General expressions considering only poles to be symmetrical. Considered on the basis of two poles, N_a being turns on one pole.

$$\mathcal{B}_a = \frac{2\,\pi\,N_a\,I_a}{10\,d}\,\Sigma\,\{A_n\,(1-\cos n\,\pi)\cos n\,\theta\} \tag{a$'$}$$

$$\mathcal{B}_1 = \frac{\pi\,I_1}{20\,d}\,\Sigma\left\{A_n\,\frac{\sin n\,m_0\,\pi}{\sin n\,m_x\,\pi}\,(1-\cos n\,\pi)\cos n\,\theta\right\} \tag{b$'$}$$

$$M_{a1} = \frac{4\,N_a\,N_1\,r\,l}{10^9\,d}\,\Sigma\left\{\frac{A_n}{n^2}\sin n\,m_0\,\pi\,\sin n\,m_1\,\pi\right.$$
$$\left.(1-\cos n\,\pi)\cos n\,\theta\right\} \tag{c$'$}$$

$$M_{12} = \frac{2 N_1 N_2 r l}{10^9 \pi d} \Sigma \left\{ \frac{A_n}{n^3} \frac{\sin n m_0 \pi}{\sin n m_x \pi} \sin n m_0 \pi \sin n m_1 \pi \right.$$

$$\left. \sin n m_2 \pi \cos n \theta \right\} \text{henrys} \quad (\mathbf{d}')$$

$$\Delta_1 L_a = \frac{4 \pi N_a^2 r l}{10^9 d} \Sigma \left\{ \frac{A_n}{n} \sin n m_x \pi (1 - \cos n \pi) \right\} \quad (\mathbf{e}')$$

$$\Delta_1 L_1 = \frac{2 N_1^2 r l}{10^9 \pi d} \Sigma \left\{ \frac{A_n}{n} \frac{\sin^2 n m_0 \pi \sin^2 n m_1 \pi}{\sin n m_x \pi} \right\} \quad (\mathbf{f}')$$

General expression with both polar and winding symmetry.

$$\mathfrak{B}_a = \frac{2 \pi N_a I_a}{10 d} \Sigma \{A_n (1 - \cos n \pi) \cos n \theta) \quad (\mathbf{a}'')$$

$$\mathfrak{B}_1 = \frac{\pi I_1}{20 d} \Sigma \left\{ A_n \frac{\sin n m_0 \pi}{\sin n m_x \pi} (1 - \cos n \pi) \cos n \theta \right\} \quad (\mathbf{b}'')$$

$$M_{a1} = \frac{4 N_a N_1 r l}{10^9 d} \Sigma \left\{ \frac{A_n}{n^2} \sin n m_0 \pi \sin n m_1 \pi (1 - \cos n \pi)^2 \right.$$

$$\left. \cos n \theta \right\} \quad (\mathbf{c}'')$$

$$M_{12} = \frac{2 N_1 N_2 r l}{10^9 \pi d} \Sigma \left\{ \frac{A_n}{n^3} \frac{\sin n m_0 \pi}{\sin n m_x \pi} \sin n m_0 \pi \sin n m_1 \pi \right.$$

$$\left. \sin n m_2 \pi \ (1 - \cos n \pi)^2 \cos n \theta \right\} \quad (\mathbf{d}'')$$

$$\Delta_1 L_a = \frac{4 \pi N_a^2 r l}{10^9 d} \Sigma \left\{ \frac{A_n}{n} \sin n m_x \pi (1 - \cos n \pi)^2 \right\} \quad (\mathbf{e}'')$$

$$\Delta_1 L_1 = \frac{2 N_1^2 r l}{10^9 \pi d} \Sigma \left\{ \frac{A_n}{n} \frac{\sin^2 n m_0 \pi \sin^2 n m_1 \pi}{\sin n m_x \pi} \right.$$

$$\left. (1 - \cos n \pi)^2 \right\} \quad (\mathbf{f}'')$$

In using any of the formulas given above for machines having more than two poles, it must be divided by the number of pairs of poles and likewise the expression for M or $\Delta_1 L$ must be multiplied by the number of pairs of poles, which leaves the formula for these quantities unchanged.

Let us next consider the actual induction in the air gap with a distributed winding operating with three-phase currents. Let i_{m1} be the magnetizing current of the first phase i_{m2} and i_{m3} those of the other phases. The induction due to one group of coils of phase 1 is

$$\mathfrak{B}_1 = \frac{8\, N_1\, i_{m1}}{10\,\pi\, d}\,\Sigma\left\{\frac{1}{n^2}\sin n\, m_0\,\pi\sin n\, m_1\,\pi\cos n\,\theta\right\} \tag{30}$$

and if the phase displacement of 2 and 3 from 1 be φ_{12} and φ_{13}

$$\mathfrak{B}_2 = \frac{8\, N_2\, i_{m2}}{10\,\pi\, d}\,\Sigma\left\{\frac{1}{n^2}\sin n\, m_0\,\pi\sin n\, m_2\,\pi\cos(n\theta - \varphi_{12})\right\} \tag{31}$$

$$\mathfrak{B}_3 = \frac{8\, N_3\, i_{m3}}{10\,\pi\, d}\,\Sigma\left\{\frac{1}{n^2}\sin n\, m_0\,\pi\sin n\, m_2\,\pi\cos(n\,\theta - \varphi_{13})\right\} \tag{32}$$

For symmetrically grouped coils the formulas become

$$\mathfrak{B}_1 = \frac{8\, N_1\, i_{m1}}{10\,\pi\, d}\,\Sigma\left\{\frac{1}{n^2}\sin n\, m_0\,\pi\sin n\, m_1\,\pi\,(1-\cos n\,\pi)\cos n\,\theta\right\} \tag{33}$$

$$\mathfrak{B}_2 = \frac{8\, N_2\, i_{m2}}{10\,\pi\, d}\,\Sigma\left\{\frac{1}{n^2}\sin n\, m_0\,\pi . \sin n\, m_2\,(1-\cos n\,\pi)\cos m\,(\theta - \phi_{12})\right\} \tag{34}$$

$$\mathfrak{B}_3 = \frac{8\, N_3\, i_{m3}}{10\,\pi\, d}\,\Sigma\left\{\frac{1}{n^2}\sin n\, m_0\,\pi\sin n\, m_3\,\pi\,(1-\cos n\,\pi)\cos m\,(\theta - \varphi_{13})\right\} \tag{35}$$

For a symmetrical three-phase motor with full pitch coils $m_0 = 0.5$, $m_1 = m_2 = m_3 = 0.166$ (**33**), (**39**) and (**35**) become of the four

$$\mathfrak{B}_1 = \frac{8\, N_1\, i_{m1}}{10\,\pi\, d}\left\{\cos\theta - \frac{2}{9}\cos 3\,\theta + \frac{1}{25}\cos 5\,\theta + \frac{1}{49}\cos 7\,\theta - \frac{2}{81}\cos 9\,\theta + \frac{1}{121}\cos 11\,\theta + \frac{1}{169}\cos 13\,\theta + \right\} \tag{36}$$

which is the field due to one group of coils alone. The wave is flattened by the third group of harmonics but all the other harmonics are peaking values. There is therefore a decided gain in such a wave form of flux since it permits of high fundamental flux density.

The maximum value of flux is approximately

$$B_{max} = 0.823 \cdot \frac{8\, N_1\, i_m}{10\, \pi\, d} \text{ gaus} \qquad \textbf{(37)}$$

where d is given in centimeters.

$$B_{max} = \frac{1.67\, N_1\, i_m}{\pi\, d} \quad \text{maxwells per square inch,}$$

with d given in inches.

For the total winding the resultant induction will be the sum of B_1, B_2, and B. If we take the symmetrical winding with angles between planes of symmetry

$\varphi_{12} = \dfrac{2\,\pi}{3}$ and $\varphi_{13} = \dfrac{4\,\pi}{3}$, we have

$$\left.\begin{aligned} \cos n\,\theta &= \frac{e^{jn\theta}}{2} + \frac{e^{-jn\theta}}{2} \\ \cos n\left(\theta - \frac{2\,\pi}{3}\right) &= a^{-n}\,\frac{e^{jn\theta}}{2} + a^{n}\,\frac{e^{-jn\theta}}{2} \\ \cos n\left(\theta - \frac{4\,\pi}{3}\right) &= a^{n}\,\frac{e^{jn\theta}}{2} + a^{-n}\,\frac{e^{-jn\theta}}{2} \end{aligned}\right\} \qquad \textbf{(38)}$$

If we multiply these three quantities successively by $\bar{I}_{m1}$, $a^2\bar{I}_{m1}$, $a\bar{I}_{m1}$ and add, we have

$$\bar{I}_{m1}\left\{\frac{e^{jn\theta}}{2}\,(1 + a^{-(n-2)} + a^{(n+1)})\left(+ \frac{e^{-jn\theta}}{2}\right.\right.$$

$$\left.\left. \times\,(1 + a^{n+2} + a^{-(n-1)})\right)\right\} \qquad \textbf{(39)}$$

and giving n successive odd values from 1 up, we find for **(39)** the following values

$n = 1$ (**39**) becomes $\dfrac{3}{2}\,\bar{I}_{m1}\,e^{-j\theta}$

$n = 3$ " " 0

$n = 5$ " " $\frac{3}{2} \bar{I}_{m1} e^{j5\theta}$

$n = 7$ " " $\frac{3}{2} \bar{I}_{m1} e^{-j7\theta}$

$n = 7$ " " 0

$n = 11$ " " $\frac{3}{2} \bar{I}_{m1} e^{j11\theta}$

$n = n$ " " $2 \bar{I}_{m1} \sin^2 \frac{2 n \pi}{3} e^{-j \frac{2}{\sqrt{3}} \sin \frac{2 n \pi}{3} n \theta}$

We may therefore express $\mathfrak{B}$ by

$\mathfrak{B}$ = real part of

$$\frac{16 N_1 \bar{I}_{m1}}{10 \pi d} \Sigma \left\{ \frac{1}{n^2} \sin n m_0 \pi \sin n m_1 \pi (1 - \cos n \pi) \right.$$

$$\left. \times \sin^2 \frac{2 n \pi}{3} e^{-j \frac{2}{\sqrt{3}} \sin \frac{2 n \pi}{3} n \theta} \right\} \tag{40}$$

It will be obvious that if we proceed around the cylinder in the negative direction of rotation at an angular speed w and $\bar{I}_{m1}$ is equal to $I_{m1} e^{jwt}$, for $n = 1$ the value of B_1 will remain constant and real, hence B_1 must be a constant field rotating at angular velocity w in the negative direction. The value of B may be expressed in harmonic form, but in this form it does not illustrate the rotating field theory so aptly. The harmonic form is given below and is simpler in appearance than (**40**).

$$\mathfrak{B} = \frac{16 N_1 i_{m1}}{10 \pi d} \Sigma \left(\frac{1}{n^2} \sin n m_0 \pi \sin n m_1 \pi (1 - \cos n \pi) \right.$$

$$\left. \sin^2 \frac{2 n \pi}{3} \cos n \theta \right) \tag{41}$$

For a symmetrical three-phase motor with full pitch coil ($m_0 = 0.5\ m_1 = 0.166$) $\mathfrak{B}$ becomes

$$\mathfrak{B} = \frac{12 N_1 i_{m1}}{10 \pi d} \Sigma \left\{ \cos \theta + \frac{1}{25} \cos 5 \theta + \frac{1}{49} \cos 7 \theta \right.$$

$$\left. + \frac{1}{121} \cos 11 \theta + \frac{1}{169} \cos 13 \theta + \ldots \right\} \tag{42}$$

This gives for the maximum induction approximately

$$\mathfrak{B}_{max} = \frac{1.075 \times 12\, N_1\, i_{m1}}{10\, \pi\, d} = \frac{1.29\, N_1\, i_m}{\pi\, d} \text{ gauss} \qquad (43)$$

where d is measured in centimeters.

$$\mathfrak{B}_{max} = \frac{3.28 \times N_1\, i_{m1}}{\pi\, d} \text{ maxwell per square inch} \qquad (44)$$

where d is measured in inches and N is the total number of turns per pair of poles.

REPORT OF THE BOARD OF DIRECTORS

FOR FISCAL YEAR ENDING APRIL 30, 1918

The Board of Directors of the American Institute of Electrical Engineers presents herewith to the membership its Thirty-fourth Annual Report, for the fiscal year ending April 30, 1918. A General Balance Sheet showing the condition of the Institute's finances on April 30, 1918, together with other detailed financial statements, is included herein.

Directors' Meetings.—The Board of Directors held ten regular meetings during the year and one adjourned meeting. Eight meetings were held in New York, one in Philadelphia in October, and one in Cleveland in March. The adjourned meeting was held in New York in March.

The Executive Committee held two meetings in New York, on January 5 and January 22, 1918. Both of these meetings were held for the purpose of considering matters relating to the Engineering Council.

In accordance with the practise established years ago, the Board has endeavored to keep the membership informed of its proceedings by the monthly publication of a résumé of the business transacted at each meeting. These notices have appeared in each issue of the PROCEEDINGS, but they are not, however, a complete report of the work done by the Board at any one meeting, for the reason that many important matters are referred to committees for further consideration, and publicity in such cases must generally be deferred pending their final disposition. Information relating to such matters is usually published in subsequent issues.

Annual Meeting.—The Annual Business Meeting was held at Institute headquarters, New York, on May 18, 1917. The Annual Report of the Board of Directors for the fiscal year ending May 1, 1917, was presented as published in full in the June 1917 issue of the PROCEEDINGS. The Tellers Committee presented its report upon the election of officers for the year beginning August 1, 1917.

The principal feature of the meeting was the presentation of the Edison Medal to Nikola Tesla; this ceremony following the business meeting.

Annual Convention.—The Thirty-fourth Annual Convention was originally scheduled to be held at Hot Springs, Va., June 26-29, 1917. In view of the national crisis which developed in April it was decided by the Board of Directors at the May meeting to cancel the Convention and to hold instead a special meeting in New York City, for the presentation of

the papers originally scheduled for presentation at the Convention. This special meeting was held on June 27-28.

Pacific Coast Convention.—The Pacific Coast Convention for 1917 was to have been held under the auspices of the Los Angeles Section in September 1917. At the May meeting of the Board of Directors, upon the recommendation of the Los Angeles Section and the Meetings and Papers Committee, the Convention was cancelled.

Philadelphia Meeting.—The first Institute meeting of the administrative year of 1917-18 was held in Philadelphia on October 8, 1917. There were two technical sessions, at which three papers were presented.

Three-City Meeting.—A Three-City Meeting was held in Boston, January 8, New York, January 11, and Chicago January 14, 1918, the same paper being presented at all three meetings. This innovation was tried for the purpose of providing an opportunity for the local members in all three cities to participate in the discussion, all of which will be published in the TRANSACTIONS.

Midwinter Convention, New York.—The Sixth Midwinter Convention was held in New York on February 15 and 16, 1918. Four technical sessions were held. The attendance was about 400. On the evening of February 15 an informal dinner was held which was attended by 225 members and guests.

Cleveland Meeting.—An inter-section meeting in which the Toledo, Toronto, Detroit, Pittsburgh and Cleveland Sections participated, was held in Cleveland, Ohio, on March 8, 1918. Four technical papers were presented at two technical sessions. An informal dinner was held between the afternoon and evening sessions. The latter session being held jointly with the Association of Iron and Steel Electrical Engineers, one of the four papers was presented on behalf of the A. I and S. E. E. The meeting was attended by Presidents E. W. Rice, Jr., of the A. I. E. E., and C. A. Mink, of the A. I. and S. E. E., and members were present from all the sections mentioned above. The attendance at this meeting was about 150.

Pittsburgh-New York Meeting.—The Institute meeting for April was held in Pittsburgh on April 9 and in New York on April 12, under an arrangement similar to the meetings held in Boston, New York and Chicago in January 1918. The same two papers were presented at both the New York and Pittsburgh meetings.

New York Meetings.—In addition to the meetings referred to above, Institute meetings were also held in New York in November and April. The attendance at the November meeting was 175 and one paper was presented; the attendance at the April meeting was 250 and two papers were presented. The April meeting was preceded by an informal buffet supper under the auspices of the New York Membership Acquaintance committee.

National Defense.—In the annual reports for 1916 and 1917 the published references to the Institute's connection with matters pertaining to National Defense were limited to the various actions taken during the year covered by each report. It has been thought, in view of the entry of the United States into the war, that the membership will gain a better perspective of the services actually rendered by the Institute if in preparing the present report there be included not only a record of the developments of the past year, but a summary of all of the defense work done during the three years from the time when the co-operation of the engineering societies was first suggested in connection with national preparedness. It is felt that such a statement will also be useful as a record for future reference, and the following has been prepared with these objects in view.

Engineer Officers' Reserve Corps.—Early in 1915, when the subject of an Officers' Reserve Corps in connection with the Army reorganization plan was under consideration in Congress, the suggestion was made that the National Engineering Societies tender their assistance to the War Department in the formation of an Engineer Reserve, there being no adequate provision at the time for any large body of engineers in the Army. Acting on this suggestion the Institute and other national engineering societies formed a joint committee for the purpose of taking such steps as seemed desirable to assist in the organization of such a corps as a part of the regular Army. Conferences were held with the Secretary of War, officers of the General Staff and the War College, and the chairmen of the House and Senate Committees on Military Affairs. Largely as a result of the work of this joint committee, the National Defense Act of 1916, which became effective on July 1st of that year, embodied a provision for an Engineer Reserve Corps as part of the Officers' Reserve Corps. A circular containing an abstract of the engineer section of this Act was mailed to all members of the Institute on July 5, 1916.

Upon completion by the War Department of the details of the requirements and qualifications for commissions in the Officers' Reserve Corps as provided in the new law, a second circular was issued by the Institute to the membership under date of September 12, 1916, giving complete information regarding the qualifications for commissions in the Reserve, thus enabling interested members to apply for commissions promptly. Many applications were filed at that time by members of the Institute who are now in active service.

Naval Consulting Board.—In July 1915 the Institute was invited by the Honorable Josephus Daniels, Secretary of the Navy, to select two members for appointment by him upon a proposed Naval Advisory Board, to be presided over by Mr. Thomas A. Edison, and to be composed of men recognized throughout the country for their inventive genius and engineering achievements, to assist the Navy Department, instructively and critically, in the development of such new ideas for naval advance as might be found worthy of consideration. The underlying idea was to make available to the Navy Department the latent inventive and engineering genius of the country to improve the Navy and to bring the officers of the service into more intimate contact with the industrial resources of the

country. Similar invitations were extended to ten other scientific and engineering organizations.

Industrial Iuventory.—The good work of the Institute's representatives upon the Naval Consulting Board is attested to in the following letter and invitation received by the Institute from the President of the United States, under date of January 13, 1916:

"The White House
Washington. January 13, 1916.

My dear Sir:

The work which the American Institute of Electrical Engineers has done through its members on the Naval Consulting Board is a patriotic service which is deeply appreciated. It has been so valuable that I am tempted to ask that you will request the Institute to enlarge its usefulness to the Government still further by nominating for the approval of the Secretary of the Navy a representative from its membership for each state in the Union to act in conjunction with representatives from the American Society of Mechanical Engineers, the American Society of Civil Engineers, the American Chemical Society, and the American Institute of Mining Engineers, for the purpose of assisting the Naval Consulting Board in the work of collecting data for use in organizing the manufacturing resources of the country for the public service in case of emergency. I am sure that I may count upon your cordial cooperation.

With sincere regard,

Cordially yours,
(Signed) Woodrow Wilson.

Mr. John J. Carty, President,
American Institute of Electrical Engineers,
New York City."

This invitation was accepted by the Board of Directors of the Institute at a special meeting called on January 21, 1916, and the nominees selected by the Institute were subsequently appointed by the Secretary of the Navy. The resulting organization became officially known as the *State Directors of the Organization for Industrial Preparedness and Associate Members of the Naval Consulting Board of the United States.* These State Directors made a canvass of all the industrial establishments in their respective states by means of a confidential industrial inventory form giving in great detail data regarding their manufacturing and producing resources. The statistics obtained by the Government as the result of this work fully justified the confidence shown by the President of the United States in entrusting to engineers a task of such magnitude and detail.

General Co-operation with Government.—On February 5, 1917, during the diplomatic exchanges between the American and German Governments, resulting from the announced policy of the latter to sink merchant shipping in certain prescribed zones without warning, the following joint telegram, signed by the Presidents of the Civil, Mining, Mechanical and Electrical Engineers, was sent to the President of the United States:

"To the President,
Executive Mansion,
Washington, D. C.

We, the presidents of the National Societies of Civil, Mining, Mechanical and Electrical Engineers, and of the United Engineering Society, with a membership of thirty thousand,

cordially unite in supporting Congress and the Administration in the stand for freedom and safety on the seas, and we are confident that we represent the membership of the four societies in offering to assist toward the organization of engineers for service to our country in case of war."

This was followed by a special meeting of the Executive Committee on February 6, at which it was decided to issue immediately, on behalf of the Board of Directors, to all Institute members in the United States, a circular letter calling attention to this telegram and to the opportunities existing for patriotic service to the nation in case of emergency. This circular letter was issued under date of February 8, 1917, and was accompanied by a simple classification sheet which members were requested to fill out and return to Institute headquarters, indicating whether or not they would be available for military or naval service if required. Over two thousand of these classification sheets were filled out and returned to Institute headquarters and subsequently were placed at the disposal of the War Department. A considerable number of the members who sent in data sheets was communicated with by the branches most urgently in need of their services.

On April 2, 1917, in co-operation with several other engineering societies, the Institute issued to the membership two pamphlets containing instructions and methods of procedure for engineers who desired to offer their services in the Army or Navy of the United States. The pamphlets also contained all information available regarding all of the branches of the Army and Navy in which the services of electrical engineers could be utilized to good advantage. The circulation of these pamphlets aroused great interest and resulted in the filing of a great many more applications by Institute members for commissions in the Officers' Reserve Corps and in the Naval Reserve, many of which were favorably acted upon.

During the past year the Institute has maintained its connection with the Government through a number of agencies, among which may be mentioned: the Naval Consulting Board, the Subcommittee on Wires and Cables of the Standards Committee, the National Research Council, the Engineering Council, and the General Engineering Committee of the Advisory Commission of the Council of National Defense.

Personnel.—In the work of obtaining technically trained men and men of special qualifications for Government service, the Institute has been able to render considerable assistance to the Government. It has been called upon repeatedly by various branches of the Army, and the Navy for technical men for special services, also by industrial corporations and various Government bureaus, to recommend individuals or small groups of men for special work.

On September 20, 1917, Rear-Admiral L. C. Palmer, Chief of the Bureau of Navigation, Navy Department, in a letter addressed to the President of the Institute, informed him that the Secretary of the Navy had authorized the commissioning of one hundred graduate electrical engineers as Lieutenants, junior grade, in the U. S. Naval Reserve Force, that three organizations, namely, the Naval Consulting Board, the National Research Council and the American Institute of Electrical Engineers, were each requested to nominate eighty-five men possessing certain specified

qualifications, and that from the total of two hundred and fifty-five candidates thus nominated, a Board of Naval Examiners would select the one hundred men to be commissioned. A circular containing this information was mailed to the entire membership of the Institute in the United States, so that every member might have an opportunity to apply for one of these commissions or to recommend desirable candidates.

At the Directors meeting held on October 8, 1918, a special committee was appointed to examine the applications received by the Secretary and to select therefrom the Institute's eighty-five nominees. This committee, consisting of five members of the Board of Directors, met at Institute headquarters on October 16 and 17, and from the applications received up to that date, selected, solely upon their merits, eighty-five nominees whose names were transmitted to Rear-Admiral Palmer on October 18. The Naval Consulting Board and the National Research Council transmitted their nominations upon the same date.

Of the eighty-five men nominated by the Institute, thirty-eight were included in the first hundred commissioned. It was learned later from reports received from individual members of the Institute that in addition to the one hundred men who were originally commissioned a considerable number of other applicants received commissions.

Under date of March 22, 1918, the Institute was requested by Rear-Admiral Palmer to nominate, in groups of twenty-five each, specially qualified electrical engineers, for training as submarine officers in the Naval Reserve Force. The successful candidates are to be commissioned as Ensigns, and after satisfactorily completing a special technical course of instructions they will become part of the active Submarine Officer Complement of the Navy. The first group of nominations was submitted to the Navy Department, early in April.

This request was supplemented several weeks later with a call for an additional 50 nominees for the rank of Ensign in general service in the Naval Reserve Force. Work on the selection of these nominees is now progressing.

Other requests received by the Institute for co-operation in obtaining the services of technical men include the following:

Navy Department: Bureau of Navigation, candidates for aviation inspection duty for training as Ensigns in the Naval Reserve Flying Corps; Bureau of Yards and Docks, candidates for appointment in the Civil Engineer Corps of the Naval Reserve Force, for the ranks of Ensign, Lieutenant (junior grade) and Lieutenant.

Bureau of Chemistry: electrical engineers for special investigations in grain-dust explosions in mills throughout the country.

War Department: Signal Corps, electrical men for radio division, radio operators and mechanicians for Air Service, and graduate electrical engineers for the aviation section; Coast Artillery Corps, electrical graduates for training school at Ft. Monroe, Va. with opportunity for commissions; Ordnance Department, Trench Warfare Section, technical men for commissions in Ordnance Officers Reserve Corps; Corps of Engineers, enlisted men for engineering and railway units and replacement regiments of engineers.

Eight hundred and twenty-four members of the Institute are now serving with the uniformed forces in the Army or Navy of the United States, and in addition, a large number are serving the Government in various civilian capacities. A great many members are also giving their services as members of various committees engaged in war activities.

General Engineering Committee.—Shortly after the entry of the United States into the war and at the suggestion of President Wilson, each of the four National Engineering Societies designated two representatives upon a committee of the Advisory Commission of the Council of National Defense under Dr. Hollis Godfrey, Commissioner of Engineering and Education, as chairman, to serve as the medium through which the engineering societies might serve the Government. In September last this committee was reorganized as the "General Engineering Committee," with Prof. C. A. Adams as chairman.

Besides acting on minor questions referred to it by the Government, this committee was active on four important tasks: 1. the preparation of a scheme of organization for war purchases, which has since partly been put into operation; 2. electric welding in shipbuilding; 3. development of cast steel anchor chain; 4. specifications for shipboard cable for the Navy Department. (For details regarding the three latter items see statement herein relating to the Standards Committee.)

The General Engineering Committee dissolved when the Council of National Defense discontinued all of its advisory committees. The work was then reorganized and continued under other agencies.

National Research Council.—The National Research Council was formed at the request of the President of the United States by the President of the National Academy of Sciences for the purpose of furthering scientific research in its broadest aspects. Its field includes educational institutions, technical, scientific and medical organizations, governmental departments and the industries. Upon invitation the Institute designated representatives upon the Engineering Committee of the Council. This committee was organized early in 1917 with offices in Washington and New York. It is in contact with the work of the other committees of the Council and since its organization it has been engaged on a wide range of engineering problems brought to its notice by the several branches of the Government. Weekly meetings have been held and weekly reports of its activities have been sent to each member of the committee. The confidential nature of these reports and of the problems which have been under consideration has rendered it inexpedient to publish detailed statements regarding the work which has been carried on.

Membership Service Classification.—Early in February 1917 the Institute offered its services to the Government in case of emergency. It soon became evident that in order to extend the fullest possible cooperation and to respond promptly and effectively to the numerous demands which were being made upon the Institute by the different governmental departments and industrial establishments engaged in Government work, for technical men of special qualifications, it would be necessary to have available for immediate reference at Institute headquarters de-

tailed information regarding the many members who were willing and anxious to serve the Government. After conferences with the other National Engineering Societies, the matter was brought to the attention of the Board of Directors of the Institute at its meeting of October 8, 1917, at which meeting a proof of a suggested form of questionnaire which had been prepared was submitted with a request for an appropriation sufficient to distribute the form to the membership and to compile an index of the replies. · Recognizing the importance of such a classification, the Board granted an appropriation of $1,000 for this purpose and authorized the Secretary to proceed with the work after opportunity had been given for final revision of the form. A special committee was appointed to have supervision of the classification and indexing.

Although primarily intended as an aid in selecting technical men for service in the present emergency, it is proposed to maintain a permanent file of this data at Institute headquarters to be revised from year to year for use after peace has again been restored. Such a classification has been contemplated for several years in connection with the regular work of the Institute and its committees.

Employment.—It is over three years since the Institute first initiated the plan for assisting its members in obtaining employment and employers in obtaining desirable employees. It has not attempted to conduct an employment department in the generally accepted sense of the term, but has simply acted as a medium for placing men in touch with opportunities, through the publication of announcements in the montly Institute PROCEEDINGS, under the heading of "Engineering Service Bulletin."

While the service might have been developed to a greater extent had it been possible to appropriate funds for carrying on the work, the results attained since the plan was inaugurated are, nevertheless, very gratifying. There is no doubt that it offers a wide sphere of usefulness, and that the service is highly appreciated; not only by the individual member who is seeking a position, but also by the many corporations and men in posts of responsibility who have been assisted by the Institute in obtaining the services of high-class technical men. The co-operation of the membership in bringing vacancies to the attention of the Secretary will be very helpful in the future successful conduct of this work.

United Engineering Society.—In the Directors' Annual Report for 1917 reference was made to the addition of three stories to the Engineering Societies Building as a result of an agreement whereby the American Society of Civil Engineers was to be admitted into the fraternity of founder societies and take up its headquarters in the building.

Full details regarding this agreement were published in the PROCEEDINGS for September 1916.

This work was completed in the Fall of 1917 and on December 7, 1917, a joint meeting was held under the auspices of the Institute, the American Society of Mechanical Engineers, and the American Institute of Mining Engineers, at which the American Society of Civil Engineers was formerly welcomed into the fraternity of the founder societies and the occupancy of its quarters in the enlarged Engineering Societies Building. These

four great National Engineering Societies are thus brought together under one roof with all of the obvious advantages for closer cooperative action, realizing more fully the purpose of the donor of the building, Mr. Andrew Carnegie, that it should be the home and headquarters of the engineering profession of America.

Engineering Council.—The Engineering Council represents the result of an organized effort inaugurated in the latter part of 1916 by four of the leading national engineering societies—the American Society of Civil Engineers, American Society of Mechanical Engineers, American Institute of Mining Engineers and the American Institute of Electrical Engineers, to establish a central body to deal with matters of common interest to engineers and to serve as a connecting medium between the engineer on the one hand and the public welfare on the other so far as such matters relate to the engineering profession, in order that united action may be possible.

The first meeting of the Council was held on June 27, 1917. The Annual Meeting of the Council was held on February 21, 1918. Officers were elected and the following committees were appointed: Executive, Finance, Rules, Public Affairs, American Engineering Service, War, Fuel Conservation and Patents.

For details regarding future plans of the Council, its field, and aims, members are referred to the abstracts from a statement issued by the Council in February 1918, published in the Institute PROCEEDINGS for April 1918.

Sections Committee.—The encroachment of the many demands of these unprecedented times has been reflected in the activities of the Sections and Branches. Many of the leading men have been called away on war duties, while others have assumed additional burdens at home. The number of meetings held during the past year has for these reasons been reduced, yet it is gratifying to observe that the total attendance in the Section meetings has actually increased.

This is undoubtedly due to the fact that these meetings have been arranged to present topics of current interest. It has been evident to any one who has seen the notices of these various meetings over the country that the thought of service and knowledge of present day problems were uppermost in the mind of the engineer.

The list of Sections has been increased by the addition of Erie, Pa., which qualified on January 11, 1918, under conditions which inspire confidence in its future.

The Branches have, however, been severely affected by the influences of war. There our young engineers have forfeited their present educational advantages and have gone into the ranks of the service in large numbers. Several Branches have suspended activity, temporarily, while two, the Queens University Branch, Kingston, Ontario, and the Michigan Agricultural College Branch, East Lansing, Michigan, were added, respectively, on January 11 and March 15, 1918. Two other Branches, the Iowa State College Branch, and the Rhode Island State College

Branch, were terminated by the Board of Directors at their request. The tabulation following shows the reduction in Branch activity.

	For Fiscal Year Ending					
	May 1 1913	May 1 1914	May 1 1915	May 1 1916	May 1 1917	May 1 1918
Sections						
Number of Sections.........	29	30	31	32	32	34
Number of Section meetings held.....................	244	233	246	251	265	245
Total Attendance...........	22,825	22,626	23,507	28,553	31,299	34,614
Branches						
Number of Branches........	47	47	52	54	59	59
Number of Branch meetings held.....................	357	306	328	360	368	268
Attendance................	11,808	11,617	12,712	15,166	16,107	10,683

On the whole the situation in the Sections and Branches is one for encouragement rather than otherwise. It augurs well for the participation of the engineer in the country's large affairs and insures the continued maintenance of the activities of the Institute.

Standards Committee.—The Standards Committee has held monthly meetings throughout the year except in January and the summer months.

Owing to the numerous demands made upon many members of the committee by war work, all non-essential subcommittees were allowed to mark time thus reducing the number of active subcommittees from over 40 to about 20.

Subcommittee on Wires and Cables.—The Subcommittee on Wires and Cables of the Standards Committee was appointed by the Board of Directors in response to the request of Rear-Admiral Griffith, Chief of the Bureau of Steam Engineering, to assist the Navy Department in the solution of problems relating to wires and cables with special reference to the high-tension cables to be used on the new electrically-driven warships.

This committee, after conducting a considerable number of investigations and much experimental research, has sent reports to the Navy Department on several subjects. The committee is still co-operating with the Navy Department and expects to extend its activities to other departments of the Government.

Electric Welding in Ship Construction. Another important piece of war work which originated in the Standards Committee is the application of electric welding to shipbuilding. This was started in August 1917 as the Electric Welding Subcommittee of the Standards Committee, and was adopted by the General Engineering Committee of the Council of National Defense in September. Finally, after the Council of National

Defense had dropped all of its advisory committees, this subcommittee was appointed in February by the U. S. Shipping Board, Emergency Fleet Corporation, as its Electric Welding Committee, with C. A. Adams as chairman and with ample financial support. An enormous amount of valuable work has been and is being done by this committee, which includes representatives of: The Emergency Fleet Corporation, the Classification Societies, (Lloyds Register of Shipping and the American Bureau of Shipping), U. S. Navy, Bureau of Standards, electric welding manufacturers and users, electrical engineers and metallurgists.

The Research Subcommittee of the Electric Welding Committee is also attached to the Engineering Division of the National Research Council.

The principal object of the Electric Welding Committee is to save time, labor, material and expense by the extension of the application of electric welding to shipbuilding. This is being done not only by the extension of the application to minor parts of ships, but also by demonstrating its suitability and economy on the capital parts of ships.

British Conference. As a result of an invitation from the Engineering Standards Committee of Great Britain, Mr. H. M. Hobart was sent to London in September 1917 as a delegate from the Standards Committee to a joint conference on standards for electrical machinery. Mr. Hobart brought back numerous suggestions which have been acted upon by the Standards Committee.

Mr. Hobart also investigated the electric welding situation in England and rendered a report which has proved very satisfactory in the work of the Electric Welding Committee.

Another result of this visit was that the Emergency Fleet Corporation, engaged Captain James Caldwell, in charge of the electric welding work for the British Admiralty, to spend a couple of months here and to give the Electric Welding Committee the benefit of the experience of Great Britain. This visit has proved most helpful all around.

Standardization Rules. The result of the year's work of the Standards Committee as far as the Standardization Rules are concerned, does not involve many radical changes, although some of the additions are distinctly valuable. The changes and additions will be published in a revised supplement. A complete revision of form and arrangement is under way and will be completed for a new edition in 1919.

American Engineering Standards Committee.—In January 1917 a joint committee was appointed with three representatives from each of the Four National Engineering Societies, and the American Society for Testing Materials, to prepare a plan of organization for an American Engineering Standards Committee. This committee completed its work on June 19, 1917, and rendered its report to the five societies interested.

The governing boards of the four National Engineering Societies, A. S. C. E., A. S. M. E., A. I. M. E., and A. I. E. E., approved the plan of organization and appointed three representatives each upon the proposed American Engineering Standards Committee. The executive committee of the A. S. T. M., presented certain suggestions and additions. The organization committee was therefore continued and proceeded to consider the suggestions of the A. S. T. M.

A revision of the constitution and rules of procedure governing the proposed American Engineering Standards Committee, satisfactory to the representatives of all the societies, has finally been completed and a report has been submitted to the governing boards of the respective societies for approval.

Meetings and Papers Committee.—The Meetings and Papers Committee has held six monthly meetings during the past year, at which the programs of the Institute meetings and conventions during the season were arranged. One meeting was held in Philadelphia in October 1917, four in New York during the winter, and one in Cleveland in March 1918.

The policy of the Institute of holding a number of its regular monthly meetings outside of New York City was thoroughly discussed at the opening meeting and while there was considerable sentiment in favor of continuing this policy as in the previous year, the difficulty of railroad travel and the fact that the present year was an extremely busy one for a large proportion of the Institute membership led the committee to limit the meetings outside of New York to two; one of which was held in Philadelphia in October, and the other in Cleveland in March.

About 80 papers have been considered by the committee during the year and owing to the abnormally high cost of printing and publishing at the present time the selection of papers has been made with unusual care. Every paper which has been offered during the past year has been reported upon by one or two members of the technical committee to which it belonged before final action has been taken by the Meetings and Papers Committee.

In order to bring the papers presented at regular meetings before as large a number of the Institute members as possible, a new arrangement for meetings has been devised which has been called Inter-section Meetings, in which one paper is presented as nearly simultaneously as possible before several different Sections. Two of these meetings have been held this year with considerable success; one in January 1918 in Boston, New York and Chicago, and another in April in Pittsburgh and New York. The meeting at Cleveland was held under the joint auspices of the Cleveland, Pittsburgh, Toledo, Toronto and Detroit Sections, and a joint technical session was held with the Association of Iron and Steel Electrical Engineers. This participation by the different Sections in regular Institute meetings is believed to have added considerable stimulus to the activities of the Sections participating and has been generally beneficial to the Institute.

Editing Committee.—The Editing Committee, which now has entire supervision of the TRANSACTIONS, announced last year that future issues of TRANSACTIONS would be published semi-annually. Owing to war considerations, however, which resulted in a considerable reduction in the amount of material published, it was found that the papers for the entire year would make only one volume of the usual size and it was subsequently decided to publish the 1917 TRANSACTIONS in one volume. This plan was followed to avoid the publication of two small books instead of one of the usual size, and it also results in a very considerable reduction in the expense of binding and distribution.

The volume is now completed and is being distributed approximately five months earlier than in previous years.

The system of handling papers and discussions for the TRANSACTIONS has been gradually evolved from experience with different methods. It has now been practically standardized for several years and appears to meet with general satisfaction.

All authors and discussors have the opportunity of revising their contributions both in the manuscript of the stenographer and in the proof. Each author has the privilege of reading the proof of all discussion before the final revision of his closure.

All papers published in the PROCEEDINGS are not necessarily published in the bound TRANSACTIONS. The responsibility for a decision in this matter now rests with the Editing Committee.

The committee suggests that papers of the character that are accepted for publication without presentation be printed only in the Annual TRANSACTIONS.

Public Policy Committee.—The Public Policy Committee held one formal meeting during the year at which a communication regarding the bill to incorporate the American Academy of Engineers, referred to the committee by a vote of the Board of Directors, was considered. A report upon this matter was subsequently made to the Board of Directors by the committee.

A majority of the members of the Public Policy Committee, being members also of the Engineering Council, official action by the committee on matters other than that above referred to has not seemed desirable, as such matters now come within the scope of the Engineering Council, thus resulting in a large measure in cooperation between the four National Engineering Societies.

Code Committee.—The Code Committee has continued to represent the Institute on the Electrical Committee of the National Fire Protection Association and representatives of the committee attended the biennial meeting of the National Fire Protection Association. The name of this committee was recently changed by an amendment to the by-laws of the Institute from "Code Committee" to "Committee on Safety Codes."

U. S. National Committee, International Electrotechnical Commission.—Two meetings were held by this committee during the year, both at Institute headquarters; the first on December 14, 1917, the second on March 13, 1918, the latter meeting being held jointly with the Institute's Standards Committee.

The International Electrotechnical Commission has been prevented by the War from holding any international gathering, but it has been the expressed desire of the U. S. National Committee that an attempt should be made to hold such a meeting as soon as conditions will permit, as the lack of intercommunication between the various national committees of the allied countries since 1913 has naturally served to retard generally electrotechnical development.

At the March meeting attention was given to certain inquiries made by the French National Committee concerning such changes as may have

taken place in the A. I. E. E. Standardization Rules since the last convention of the International Electrotechnical Commission in 1913. The necessity for such explanations would not have arisen if the work of the Commission had not been held in abeyance by the world-war. Some progress, however, has been made independently among the various individual national committees.

Committee on Code of Professional Conduct.—The only matter which this committee acted upon during the year was the formulation of definitions of an "Electrical Engineer." A report embodying several definitions has been prepared and filed at Institute headquarters.

Board of Examiners.—The Board of Examiners held 18 meetings during the year, averaging about three hours each. It considered and referred to the Board of Directors with its recommendations a total of 1889 applications of all kinds. In addition to these the Board reviewed or reconsidered 33 applications for a second and third time.

The demand upon the time of the Board has been greater during the past year than in any previous year, owing to the large number of applications filed for admission or transfer to the higher grades. Such applications are considered in great detail and all of the evidence submitted by the applicants, including the record and communications from references and others, is read by the Board.

The result of the Board's work for the year is given in the following tabulated statement:

APPLICATIONS FOR ADMISSION.

Recommended for grade of Associate	1036		
Not recommended for grade of Associate	10	1046	
Recommended for grade of Member	74		
Not recommended for the grade of Member	50	124	
Recommended for grade of Fellow	11		
Not recommended for Fellow	5	16	
Recommended for enrolment as students	576	576	

APPLICATIONS FOR TRANSFER.

Recommended for grade of Member	66		
Not recommended for grade of Member	41	107	
Recommended for grade of Fellow	11		
Not recommended for grade of Fellow	9		20
Total number of applications considered		1889	
Applications reconsidered		33	
Total		1922	

Membership Committee.—The work of the committee has resulted in the filing of 1235 new applications. Much credit for the success of the committee's work is due to the loyal support of the Sections and more particularly the chairmen of the local membership committees.

The following tabulated statement shows the number of members in each grade, the total membership of the Institute on April 30, and the additions and deductions which have been made during the year. It is not intended, however, to show the number of applications received, as a considerable number is still in the preliminary stages and cannot, therefore, be embodied in a list of the members of the Institute.

	Honorary Member	Fellow	Member	Associate	Total
Membership, April 30, 1917.	4	455	1223	7028	8710
Additions:					
Transferred	....	14	76	....	....
New Members Qualified	2	3	66	937	....
Reinstated	....	2	8	54	....
Deductions:					
Died	....	4	7	26	....
Resigned	....	1	3	111	....
Transferred	....	....	4	84	....
Dropped	....	5	27	318	....
Membership, April 30, 1918..	6	464	1332	7480	9282

Net increase in membership during the year 572

Deaths.—The following deaths have occurred during the year:

Fellows.—Albert F. Ganz, F. B. H. Paine, John K. Robinson, Karl Von Krogh.

Members.—Harry Bottomley, Eugene F. Roeber, Henry R. Ford, J. G. Lorrain, Osborn P. Loomis, E. W. Stevenson, E. P. Warner.

Associates.—L. R. Pomeroy, S. H. Harvey, Stuart A. Nims, W. K. Kretsinger, P. H. Goodwin, W. S. Horry, Arthur Gunn, William G. Bee, T. Ohta, John Sachs, W. H. Peberdy, George Scharfe, Percy L. Cobb, John Gilmartin, R. C. Carter, Charles O. Smith, William Duddell, John Hesketh, John H. Goehst, Robert L. Stevenson, O. Zell Howard, Bernard W. Capen, Cyril F. Mickler, St. John Chilton, James A. Barkley, Henry N. Brooks.

Total deaths, 37.

Edison Medal.—The Edison Medal for 1916, which had been awarded to Nikola Tesla by the Edison Medal Committee in December of that year "for his early original work in polyphase and high frequency electric currents" was presented to Mr. Tesla with appropriate ceremonies at the Annual Meeting of the Institute held in New York on May 18, 1917.

The Edison Medal for 1917 has been awarded to Col. John J. Carty "for his work in the science and art of telephone engineering," and the

presentation will take place at the Annual Meeting of the Institute which will be held in New York on May 17, 1918.

John Fritz Medal.—The John Fritz Medal for 1917 was awarded to Dr. Henry Marion Howe "for his investigations in metallurgy, especially in the metallography of iron and steel." The presentation was made in the Engineering Societies Building in New York on May 10, 1917.

The John Fritz Medal for 1918 was presented to Mr. J. Waldo Smith in the Engineering Societies Building, New York, at a joint meeting held on April 17, 1918, for "achievement as engineer in providing the City of New York with a supply of water."

Finance Committee.—During the year the committee has held monthly meetings, has passed upon the expenditures of the Institute for various purposes, and otherwise performed the duties prescribed for it in the Constitution and By-laws.

Haskins and Sells, certified public accountants, have audited the books, and their report is included herein. It will be noted that in the report a readjustment of the Institute's equity in the property held in trust by the United Engineering Society has been made, due to the admission of a fourth founder society to the United Engineering Society.

In company with the Secretary, the Treasurer, and a representative of Haskins and Sells, the chairman of the committee examined the securities held by the Institute and found them to be as stated in the accountants' report.

NEW YORK CHICAGO DETROIT ST. LOUIS CLEVELAND BALTIMORE PITTSBURGH	**HASKINS & SELLS** CERTIFIED PUBLIC ACCOUNTANTS CABLE ADDRESS "HASKSELLS" 30 BROAD STREET NEW YORK	SAN FRANCISCO LOS ANGELES SEATTLE DENVER ATLANTA WATERTOWN LONDON

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

CERTIFICATE

We have audited the books and accounts of the American Institute of Electrical Engineers for the year ended April 30, 1918, and

WE HEREBY CERTIFY that the accompanying General Balance Sheet properly sets forth the financial condition of the Institute on April 30, 1918, that the Statement of Income and Profit & Loss for the year ended that date is correct, and that the books of the Institute are in agreement therewith.

HASKINS & SELLS,

Certified Public Accountants.

NEW YORK,

May 14, 1918.

AMERICAN INSTITUTE OF

Exhibit A. **General Balance Sheet**

Assets.

Real Estate:			
One-fourth Interest in United Engineering Society's Real Estate, No. 25 to 33 West 39th Street:			
Land and Building		$472,500.00	
Real Estate Equipment, etc.		14,292.79	
Total Real Estate			$486,792.79
Equipment:			
Library—Volumes and Fixtures		$ 40,031.55	
Works of Art, Paintings, etc.		3,001.35	
Office Furniture and Fixtures		12,274.65	
Total		$ 55,307.55	
Less Reserve for Depreciation		9,419.77	
Remainder—Equipment			$ 45,887.78
Investments:			
Bonds—City of Wilmington, Delaware, 4½%, 1934, Par Value $15,000.00		$ 15,834.19	
United States Liberty Loan, 4¼% Bonds		10,000.00	
Total Investments			25,834.19
Working Assets:			
Publications Entitled "Transactions," etc.		$ 14,049.00	
Paper and Cover Paper		1,046.28	
Badges		676.70	
Total Working Assets			15,771.98
Current Assets:			
Cash		$ 12,270.07	
Accounts Receivable:			
Members for Past Dues		9,559.82	
Advertisers		769.70	
Miscellaneous		470.76	
Accrued Interest on Investments		56.25	
Accrued Interest on Bank Balances		358.66	
Total Current Assets			23,485.26
Funds:			
Life Membership Fund:			
Cash	$ 438.67		
Chicago, Burlington & Quincy Railroad Company Bonds, 4%, 1958, Par Value $5,000.00	4,868.75		
Accrued Interest	33.33	$ 5,340.75	
International Electrical Congress of St. Louis—Library Fund:			
Cash	$ 943.99		
New York City Bonds, 4½%, 1957, Par Value $2,000.00	2,248.71		
Accrued Interest	45.00	3,237.70	
Mailloux Fund:			
Cash	$ 167.35		
New York Telephone Company Bond, 4½%, 1939	1,000.00		
Accrued Interest	22.50		
		1,189.85	
Midwinter Convention Fund—Cash		163.58	
International Electrical Congress of San Francisco—Cash		40.50	
Total Funds			$ 9,972.38
Dues Paid in Advance—International Electrotechnical Commission, London, England			500.00
Total			$608,244.38

ELECTRICAL ENGINEERS.

APRIL 30, 1918.

LIABILITIES

CURRENT LIABILITIES:		
Accounts Payable—Subject to Approval by the Finance Committee	$ 6,304.01	
Due United Engineering Society Account Building Addition	10,000.00	
Dues Received in Advance	2,382.87	
Entrance Fees and Dues Advanced by Applicants for Membership	179.50	
Total Current Liabilities		$ 18,866.38
FUND RESERVES:		
Life Membership Fund	$ 5,340.75	
International Electrical Congress of St. Louis—Library Fund	3,237.70	
Mailloux Fund	1,189.85	
Midwinter Convention Fund	163.58	
International Electrical Congress of San Francisco	40.50	
Total Fund Reserves		$ 9,972.38
SURPLUS: Per Exhibit "B"		$579,405.62
Total		$608,244.38

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

Statement of Income and Profit and Loss.

For the Year Ended April 30, 1918.

Exhibit B.

Revenue:			
Entrance Fees		$ 5,545.00	
Dues		96,897.93	
Student's Dues		3,771.00	
Transfer Fees		940.00	
Advertising		8,189.02	
Subscriptions		3,286.20	
Sales of "Transactions," etc		2,574.24	
Badges Sold	$2,253.75		
Less Cost	1,992.12		
		261.63	
Interest on Investments		675.00	
Interest on Bank Balances		1,027.43	
Exchange		30.74	
Total			$123,198.19
Expenses:			
Meetings and Paper Committee:			
Salaries		$ 5,560.00	
Binding and Mailing Proceedings		4,384.97	
Printing Proceedings		6,052.07	
Engraving Proceedings		1,523.22	
Paper and Cover Paper		4,581.05	
Envelopes		723.20	
Stationery and Miscellaneous Printing		177.96	
General Expenses		250.35	
Meetings		3,193.14	
Editing Committee:			
Volume No. 34		136.22	
Volume No. 35		12,370.15	
Volume No. 36		5,372.29	
Total		$ 44,324.62	
Deduct Increase in Inventory of Publications:			
April 30, 1917	$12,884.25		
April 30, 1918	14,049.00	1,164.75	$43,159.87
Executive Department:			
Salaries		$ 17,698.50	
General Expenses		2,737.33	
United Engineering Society—Assessments		4,800.00	
Express		311.13	
Postage		1,980.65	
Advertising		2,623.74	
Stationery and Miscellaneous Printing		3,027.68	
Year Book and Catalgue		3,487.90	
Office Furniture and Fixtures—Discarded		51.20	
Total			36,718.13
Sections Committee:			
Section Meetings		$ 6,162.15	
Branch Meetings		208.85	
Salaries, New York Office		2,340.00	
Stationery and Printing, New York Office		698.70	
Express on Advance Copies		8.09	
Total			$ 9,417.79
Forward			$ 89,295.79
Revenue—(Forward)			$123,198.19

REVENUE—(Forward)			$123,198.19
EXPENSES—(Forward)			$ 89,295.79
General:			
Library Committee		$ 4,000.01	
Membership Committee		1,079.38	
Finance Committee		150.00	
Standards Committee		1,221.72	
Code Committee		30.00	
Annual Dues, International Electrotechnical Commission		250.00	
National Defense—Miscellaneous Expenses		1,488.42	
John Fritz Medal Award		94.00	
Honorary Secretary		4,000.00	
United Engineering Society, Engineering Council		1,600.00	
American Engineering Standards Committee		359.37	
Membership Classification Service		776.65	15,049.55
Total			$104,345.34
Deduct:			
Decrease in Accounts Payable—Subject to Approval by the Finance Committee, Expenses Undistributed at:			
May 1, 1917—As Adjusted		$ 7,869.81	
April 30, 1918		6,304.01	1,565.80
Total Expenses			$102,779.54
NET REVENUE			$ 20,418.65
PROFIT AND LOSS CREDITS:			
Accessions to Library Volumes and Fixtures		$ 151.75	
Refund of Unexpended Balance of Contribution of American Institute of Electrical Engineers to proposed International Electrical Congress 1915		321.99	
Total			473.74
GROSS SURPLUS FOR THE YEAR			$ 20,892.39
PROFIT AND LOSS CHARGES:			
Uncollectible Dues Written Off		$ 4,003.75	
Provision for Depreciation of Furniture and Fixtures		1,393.49	
Amortization of Premium on City of Wilmington, Delaware—4½% Bonds of 1934		52.14	
Total			5,449.38
NET SURPLUS FOR THE YEAR			$ 15,443.01
SURPLUS, May 1, 1917		$623,016.43	
ADD—REAL ESTATE EQUIPMENT AND PRELIMINARY EXPENSES CREDITED TO THE INSTITUTE BY THE UNITED ENGINEERING SOCIETY BUT NOT HERETOFORE SHOWN ON THE BOOKS		15,710.44	
Total		$638,726.87	
DEDUCT:			
Value of Equity Adjusted to One-fourth Interest:			
Real Estate Equipment, etc	$ 4,764.26		
Land and Building	132,500.00		
Total	$137,264.26		
Less Increase in Equity in Building by Addition of Three Stories	62,500.00		
Net Adjustment of Equity Value		74,764.26	
Surplus as Adjusted			563,962.61
SURPLUS, APRIL 30, 1918			$579,405.62

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

Statement of Cash Receipts and Donations for Designated Purposes. Also Disbursements, for the Year Ended April 30, 1918.

Exhibit C.

Receipts:	
Life Membership Fund	$217.68
International Electrical Congress of St. Louis Library Fund—Interest and Royalties	93.40
Mailloux Fund—Interest	45.00
Midwinter Convention Fund—Contributions and Interest	74.53
Total	$430.61
Disbursements:	
Life Membership Fund	217.68
Midwinter Convention Fund	6.75
Total	$224.43

RECEIPTS AND DISBURSEMENTS PER YEAR PER MEMBER.

During each fiscal year for the past eight years.

Year ending April 30	1911	1912	1913	1914	1915	1916	1917	1918
Membership, April 30, each year	7117	7459	7654	7876	8054	8212	8710	9282
Receipts per Member	$13.37	$13.19	$13.45	$14.08	$14.06	$13.62	$13.30	$13.17
Disbursements per Member	11.03	12.44	15.57	12.86	13.54	13.74	12.75	11.99
Credit Balance per Member	$2.34	$ .75	*$2.12	$1.22	$ .52	*$ .12	$ .55	$1.18

*Deficit.

Respectfully submitted for the Board of Directors,

F. L. HUTCHINSON, *Secretary.*

New York, May 17, 1918.

To be presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 27, 1918.

THE DESIGN OF TRANSPOSITIONS FOR PARALLEL POWER AND TELEPHONE CIRCUITS

BY HAROLD S. OSBORNE

ABSTRACT OF PAPER

This paper presents the results obtained in a recent design of transposition systems for telephone circuits exposed to induction from circuits of other kinds, particularly from three-phase power circuits, and the coordinate arrangements of transpositions in the power circuits. The new arrangements are the result of a systematic investigation of the degree of flexibility which could be obtained in the coordination of telephone transpositions and transpositions in outside disturbing circuits, particularly three-phase power circuits.

In presenting the results obtained, a discussion is first given of the requirements which must be met by systems of transpositions for telephone circuits in general, and by the new "exposed line" system in particular. An outline is given of the methods used in the design work and the theory upon which it is based. The diagrams in the paper show the arrangements of transpositions for all circuits on eight crossarms of telephone line. The results to be obtained from the use of these diagrams are outlined, and the suitable locations of coordinate transpositions in parallel power circuits are discussed.

The application of these and other arrangements of coordinated transpositions to specific cases must take into account the variations in separation and other changes in the power and telephone circuits. It is considered to be beyond the scope of the present paper to go into a discussion of the problems involved in these specific applications.

INTRODUCTION

IN a well-engineered telephone system the power transmitted into a long-distance telephone circuit is almost entirely absorbed in line losses. The circuit terminates in that marvelous little electric motor, the telephone receiver, which requires only a few millionths of a watt for its operation. As the power delivered to the line by the telephone transmitter may be measured in hundredths of a watt, only a fraction of one per cent of the transmitted energy need be received at the far end of the line. This is fortunate, for as the telephone currents are of relatively high frequency, the energy losses suffered by transmission over a given circuit are very much greater than would be the losses for 60-cycle current. Moreover, the telephone currents must be

Manuscript of this paper was received April 25, 1918.

carried over very great distances. It has therefore been necessary to do an enormous amount of work in developing means for increasing the efficiency of long telephone circuits in order that country-wide service may be given with a network of conductors which should not be prohibitively expensive.

The large ratio between the energies at the transmitting end and at the receiving end of a long distance telephone line imposes very severe limitations on the amount of inductive effect which can be permitted between different telephone circuits. If the transfer of energy between two circuits is more than a millionth of the energy transmitted there is danger that one can overhear on one circuit what is said on the adjacent end of the other. This must of course be avoided, and an even stricter limitation of the transferred energy is desirable.

The method which is used to prevent excessive transfer of energy between telephone circuits on the same pole line is the installation of a systematic arrangement of transpositions of the conductors of each circuit.

Present standard telephone practise for open-wire circuits provides ten wires spaced about one foot (0.3 m.) apart on a crossarm and any number of crossarms spaced two feet (0.6 m.) apart on a pole. The currents carried by these circuits are of very high frequency compared with those used in power practise and tend to make much of small mutual impedances or admittances between the circuits. It is small wonder then that when in 1886 the first long distance telephone line was built between New York and Philadelphia it was found that, listening at the distant end, an observer could not tell which circuit was used for transmission but could hear equally well on all of them. The systematic study of telephone transpositions began at that moment and has continued until the present day.

One factor which has much complicated the study of telephone transpositions is the common use of phantom circuits. These are more accurately described by the French terms *derived* or *superposed* circuits. A phantom circuit is formed by superposing a third circuit on two 2-wire telephone circuits called the side circuits of the phantom. The phantom and its side circuits thus use the same copper conductors, and it is evident that great care in balancing the inductive relations is necessary to prevent the transfer of energy between circuits so intimately related.

Long distance telephone lines have not grown up without

neighbors, and in the design of telephone transposition systems it has been necessary to take them into account. Telegraph systems and power transmission systems have been contemporaries with the telephone system in the process of development. These systems have not in general been obliged, for any reason of their own, to restrict their electric and magnetic fields as has been necessary with telephone circuits. From the first, however, the limitation of the effect of these outside fields has been a factor in the design of telephone transposition systems.

It is evident that when a telephone circuit is paralleled by an electric power circuit carrying hundreds or even thousands of kilowatts, the mutual inductive connection between the two circuits must be extremely minute if we are to avoid induction in the telephone circuit of power of the same magnitude as that of the voice currents or even much more. In fact, this difficult problem would probably be quite impractical of solution if we had to rely alone upon balance produced by transpositions. Fortunately, another very weighty factor works for the solution of the problem. The fundamental frequency of commercial power circuits is out of the easily audible range and quite out of the range of frequencies which the telephone line must transmit in order to reproduce intelligible speech. As a matter of fact the noise produced by power circuits in telephone circuits is practically not at all due to the fundamental but due to the harmonics or high-frequency components of the power circuit which lie within the range of telephone frequencies. The energy represented by these harmonics is represented by kilowatts or even by watts rather than by thousands of kilowatts. In spite of this fact the degree of balance required in cases of close parallel is very precise indeed.

Of all the means used to reduce the inductive effect between power circuits and telephone circuits the coordinate transposition of both classes of circuit is probably the most generally important. The application of this means to parallels between telephone and power circuits is in practical cases attended with difficulties because instead of two circuits running accurately parallel for long distances the separation usually varies at frequent intervals and very irregularly.

The usual case, in which the power circuit is three-phase, presents another complication which is fundamental to this type of circuit. The telephone circuit is a single-phase circuit and its conductors have only two relative arrangements on the pins.

The three-phase circuit has three possible arrangements of its conductors on the pins, and three sections of line, each with a different arrangement, are necesssary for what is called a "barrel" of the power circuit. The combinations of relative pin positions of the two circuits when both are transposed are, therefore, six in number. The difficulties introduced by this relation have been discussed in an earlier paper* before the Institute.

Although the importance of neutralizing as far as practicable the effect of outside circuits has always been a factor in the design of transposition systems, the possibilities of coordinating the telephone transpositions with the transpositions in three-phase power circuits were not at first fully developed. With the great growth in both telephone circuits and three-phase power circuits this matter has been growing rapidly more important in telephone transmission.

In connection with the very extensive investigations of the Joint Committee on Inductive Interference in California it was therefore considered advisable to undertake the development of a new system of telephone transpositions which would embody the maximum possibilities of coordination of transpositions in the two classes of circuit. There has resulted from this work a system of transpositions known as the "exposed-line" transposition system.

Outline of Methods of Design

The design of a transposition system requires the mastery of a very difficult and complicated technique. It is attempted here to give only a general outline of the methods employed and some of the difficulties encountered and overcome. Some of the mathematical work is given in the appendices.

Conductive and Inductive Connections. The mutual connection between two circuits may be of any of the following characters

Resistance
Leakage
Inductance
Capacitance

Two circuits may be said to be unbalanced with respect to each other by a mutual connection of any of the four above characters when the mutual connection enters into the circuit in such a way that current in one circuit tends to cause a current in the other circuit.

**Inductive Interference as a Practical Problem.* Messrs. Griswold & Mastick, TRANSACTIONS, 1916, p. 1051.

By resistance and leakage we denote conductive connections respectively series and shunt. Although two metallic circuits which have no resistance in common cannot have a mutual resistance unbalance, care is necessary to avoid a mutual resistance unbalance between a phantom and its side circuits. This is however a matter apart from transpositions. The leakage from telephone circuits is almost all directly to ground. This is normally kept very low because of the large effect which high leakage would have upon the efficiency of the circuits. For these reasons leakage is not an appreciable factor in the interference between telephone circuits, except under abnormal conditions, such as those produced by baling wire thrown over a telephone line.

Two circuits strung side by side have both mutual inductance and mutual capacity unbalances. The two terms represent the coefficients by which are measured the unbalanced action of the mutual electromagnetic and electrostatic fields respectively, and both represent inductive rather than conductive effects. It is the purpose of transpositions to correct for these as largely as may be.

Inductive Effects Between Telephone Circuits. The general equations for the mutual inductive effects between long parallel circuits are hopelessly complex. For the benefit of those interested a discussion is given in Appendix A of the mathematics by which these inductive effects may be computed in some simple special cases. In Appendix B are given equations for the most simple general case, that is, the case of inductive effects between two long symmetrical parallel wires with ground return. Examination of equations (**7**) to (**10**) in this appendix shows that the current or voltage in either conductor is represented by the sum of two complex exponential terms. Little consideration is required to convince one that similar general equations for forty parallel wires would be impracticable either to establish or to use.

However, in studying the inductive effects between telephone circuits (commonly called cross-talk), use can fortunately be made of a beautiful simplification. Although the circuits are very closely associated on the same pole line, the requirement for good operation after the circuits are transposed is that the transfer of energy from one circuit to another shall be exceedingly small. This being the case, the reaction of the induced current on the inducing circuit is evidently negligible. The prob-

lem can therefore be studied by computing the current and voltage of the inducing circuit as though no currents whatever were induced in other circuits and then computing the inductive effect of these currents and voltages on each adjacent circuit under the assumption that no current flows in any of the other circuits.

In order that this computation may be carried out, it is necessary that the coefficients of induction between the circuits be determined. The mutual inductance unbalance between all pairs of circuits on a 40-wire pole lead can be readily computed. It was estimated, however, that to compute the direct capacities between circuits would require the time of one man for at least ten years. It was therefore found much cheaper and more practical to make measurements of the capacities between the wires in a short section of line under construction. The results obtained in this way are given in Table I.

Resultant Unbalance between Long Circuits. With the data and by the use of the approximations outlined above, it is possible to compute the inductive effect between any two short lengths of telephone circuit on the pole lead. In carrying out the computation of the effects for long circuits, however, a complication arises due to the fact that the current and voltage in the disturbing circuit are different both in magnitude and phase angle at different points of the line. The change in magnitude, because of attenuation effect, varies from 0.05 per cent to over 0.5 per cent per kilometer on open wire telephone circuits of different construction, and the change in phase angle varies from a small value up to nearly 10 degrees per kilometer. Also, in the propagation of induced current along the disturbed circuit to its terminal, this current undergoes a similar change in magnitude and in phase angle. Therefore, the current induced between two circuits in one kilometer length of line cannot be perfectly balanced against that induced in another kilometer length, and no matter how often or under what arrangement the transpositions are placed, they cannot serve to perfectly balance out the induced effects. There is always a resultant effect which is known as the "type unbalance". The transposition system must be so designed that this type unbalance is kept below the desired limits for all combinations of the circuits on the lead.

A brief mathematical discussion of type unbalance is given in Appendix C.

The type unbalance does not measure the entire inductive effect between two telephone circuits, for this is contributed to also by another factor which is sometimes very important. It is impracticable to locate the telephone transpositions at exactly the theoretically correct points. In laying out a new line it is sometimes possible to set poles exactly at the theoretical transposition point, and when this cannot be done the nearest pole is generally chosen as the transposition pole. However, the transposition itself occupies two pole spans. Moreover, small changes in separation of wires cannot be avoided, even going round a curve introduces a certain amount of irregularity in the separation of wires. These and other irregular factors increase the inductive effects above those due to the type unbalances and must be taken into account in designing the transposition system.

Phantom Circuits. One factor which introduces a large complication into the design of transposition systems is the general

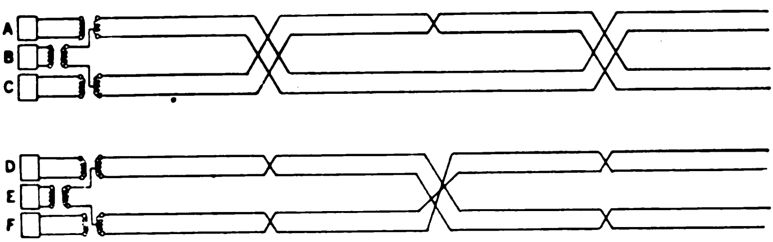


FIG. 1—PHANTOM TELEPHONE CIRCUITS

use of phantomed circuits in long-distance telephone lines. This is standard practise for long distance telephone circuits, and makes it possible to obtain three circuits from each two pairs of wires instead of two. The principle involved in phantoming is generally understood. The arrangement is indicated diagrammatically in Fig. 1.

By the transposition of a phantom circuit the pin positions of its side circuits are interchanged, and the coefficients of inductive effect between the side circuit and other circuits on the lead are changed. In order that the phantoming may be possible without interference, it is therefore necessary not only to very carefully balance the terminal transformers and other apparatus connected to the telephone lines, but also to use a transposition system that provides for a very high degree of balance of the inductive effects between the phantom and its side circuits and also between these circuits and the other circuits on the pole line.

It is evident that in designing the transpositions in the side circuits it is necessary to take into account the transpositions in the phantom. For example, in the figure, side circuit *C* would be considered balanced to side circuit *F* if the transpositions in these two circuits only were to be considered. However, the transpositions in the phantom circuits change the coefficients of induction between circuits *C* and *F* in different parts of the section shown and therefore prevent the transpositions in the side circuits from balancing this section.

Equations showing the method of computing the equivalent

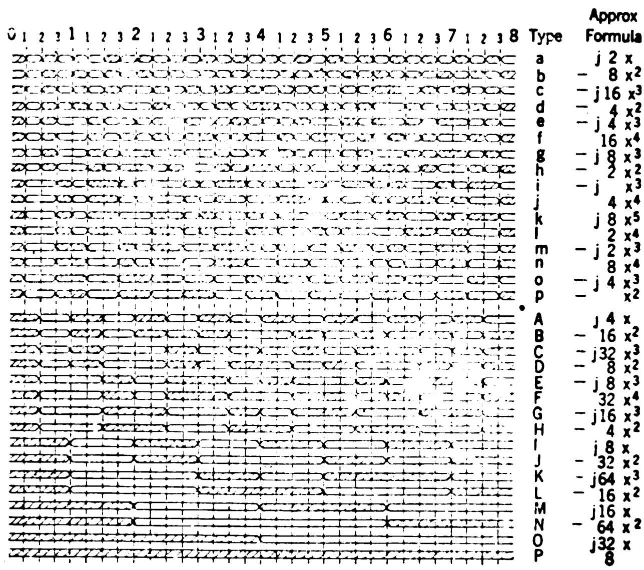


Fig. 2—Typical Arrangements of Transpositions

exposure and the type unbalance between circuits which are phantomed are given in Appendix *D*.

In order to fit in with standard telephone practises a transposition system must provide for the phantoming of any two horizontally adjacent pairs of circuits and any two vertically adjacent pole pairs. The system must also provide that any of these combinations of circuits may be phantomed and any others non-phantomed.

Typical Arrangements of Transpositions. In carrying out the design of a transposition system it is convenient and economical to arrange the transpositions for each circuit in accordance with

one or another of a list of selected typical arrangements. In Fig. 2 are shown the types, 32 in number, suitable for use in a transposition section made up of 32 parts of equal length. Reading from bottom to top, the types are arranged in the drawing so that each type has one more transposition per section than the preceding type.

The type unbalance between two telephone circuits is determined by the relative transposition of the two circuits, that is, the points at which one circuit is transposed but not both. The typical arrangements of transpositions have the characteristic that the exposure between any two circuits so transposed can be represented by some other type; that is, the exposure between any two types is the same as the exposure between some other type and a circuit without transpositions. For example, exposure f to $k =$ exposure F to $K = A$, exposure J to $m = k$. For transposition sections divided into 64 equal parts, 32 more typical arrangements can be added to the 32 shown in the figure by adding a transposition at the midpoint of each elemental length of each type.

Typical Arrangements for Exposed Lines. The purpose of the exposed-line transposition system is particularly to provide increased facility in balancing the induction from outside circuits in so far as this can be done by transpositions. The effect of outside circuits in inducing voltages between the telephone wires and ground which may be productive of very serious interference cannot be remedied by transpositions in the telephone circuits, but can frequently be greatly reduced by transpositions in the outside circuits. The telephone circuit transpositions must be designed to equalize the voltages induced in the two sides of each telephone circuit with the greatest possible flexibility in the location of the transpositions in the outside circuits. This imposes great limitations on the typical arrangements of transpositions which can be used.

The outside disturbing circuits can be classified as follows:

1. Non-transposed circuits such as telegraph circuits. This applies also to the residual components of current and voltage in power circuits.

2. Two-wire transposed circuits such as single-phase power circuits.

3. Three-phase power circuits.

In order that the disturbing effect of circuits of the first class may be reduced as far as possible, no typical arrangement of

telephone transpositions can be used whose type unbalance factor to non-transposed circuits exceeds a certain value. This eliminates a few of the types having the least numbers of transpositions.

Two-wire outside circuits can best be transposed in accordance with some of the typical arrangements for telephone circuit transpositions. A balance to these circuits requires then that certain types be set aside for their use and that the exposures between the transposed telephone circuits and the outside circuits transposed in accordance with these types be kept below the limiting values set for type unbalance for these cases.

The inherent complication in balancing the inductive effects

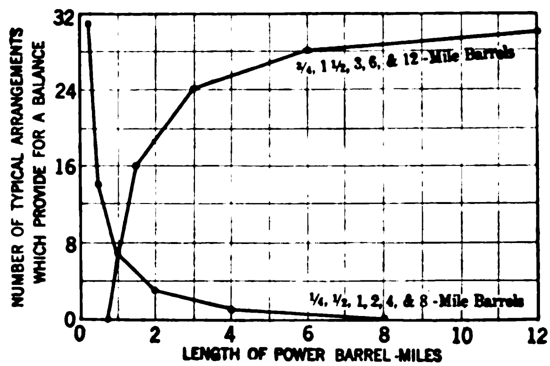


FIG. 3—NUMBER OF TYPICAL ARRANGEMENTS OF TELEPHONE TRANSPOSITIONS WHICH COORDINATE WITH THREE-PHASE POWER LINES TRANSPOSED WITH DIFFERENT LENGTHS OF BARREL

between telephone circuits and three-phase power circuits arises, as has been noted, from the fact that a three-phase circuit has three different arrangements of conductors on the pin positions and a two wire telephone circuit has but two. There are, therefore, six combinations of relative pin positions of the two circuits, all to be used for equal lengths of line.

In considering the severe restrictions placed on the selection of telephone transposition types by the necessity of balancing to transposed three-phase power circuits, the following conditions may be noted:

1. If the barrels of the power line are very long, they can be made to coordinate with almost all of the typical arrangements by having each non-transposed section of power circuit, that is, each third of a barrel, equal in length to one-half or one-fourth of the telephone transposition

section, for most of the typical arrangements balance within this length. This is shown in Fig. 3, assuming 12.8 km. (8 miles) to be the length of the telephone transposition section. Thus with a 19.2-km. (12-mile) power barrel the telephone circuits may be balanced for each 6.4-km. (4-mile) non-transposed length of the power circuit. However, as the length of the power barrel is progressively decreased to 9.6 km. (6 miles), 4.8 km. (3 miles) 2.4 km. (1 ½ miles) etc., the number of typical arrangements of telephone transposition which can be used very rapidly decreases.

2. If the barrels of the power line are very short, they may be made to coordinate with all the typical arrangements by having a complete barrel between telephone transpositions. In the case shown in Fig. 3 this requires a 0.4-km. (¼ mile) barrel. As the length of barrel is increased, however, to 0.8 km. (½ mile) 1.6 km. or 3.2 km. (1 to 2 miles) etc., the number of typical arrangements of telephone transposition which can be used, very rapidly decreases.

3. Barrels in the power line of relative lengths other than those mentioned do not in general coordinate with telephone transposition arrangements.

As a chief requirement of the exposed-line system was that it should provide a maximum possible flexibility in the location of transpositions in parallel three-phase power circuits, this was made a matter of most careful study. The results obtained are outlined in the description of the system given in the next section of this paper.

Determination of Limiting Permissible Exposures. Before the attempt can be made to select, from among the typical arrangements of transpositions which can be used, a type for each telephone circuit on the lead, a table is made up of the limiting type unbalances which can be permitted between pairs of circuits, assuming that a given limit of the induced current between circuits cannot be exceeded. The procedure is as follows:

1. A computation is made of the total amount of unbalance which could be permitted between each combination of circuits on the lead, taken two at a time. This unbalance is expressed in terms of the length of untransposed exposure between the circuits which would give the limiting value of induced current, no current being induced in the rest of the circuit.
2. The total allowable unbalance for each pair of circuits is reduced by a figure representing the unbalance which must be expected to arise from irregularities with the type of construction to be employed.
3. The remaining unbalance represents the limiting permissible effect of type unbalance in a long length of line.
4. From data regarding the attenuation of currents along the telephone circuits, and from the total limiting effect of type unbalance, a computation is made of the limiting type unbalance per transposition section.

This sets a direct limit on the relative exposure type between each pair of circuits.

The Design of a Transposition System. With the information regarding the requirements of the proposed transposition system prepared as indicated above, the creation of a design which shall meet the requirements of maximum economy and efficiency is a problem requiring a great deal of skillful and conscientious application on the part of a trained expert. The problem is of course one of selecting from the typical arrangements which can be used the best type for each circuit such that the type unbalance between no two circuits will exceed the established limit.

In order that a design may be complete it must provide for any number of crossarms on the telephone lead. However, it is generally desirable to take into consideration only four crossarms at the beginning of the design. Considering all the circuits, phantom and two-wire, which may be placed on the lead, taken in combinations of two there are about 2500 such combinations on a standard 40-wire pole line.

In carrying out the design of the exposed-line transposition system it was desired to make the requirements both of efficiency and economy as exacting as could possibly be met. These requirements were set, therefore, at the beginning of the work, so high that no design could meet them, and then gradually reduced until it was possible to complete the design. As the design depends on a number of interdependent requirements, this process not only requires a very large amount of work but demands the continual exercise of trained judgment on the part of the designer to bring about the mutual adjustment of requirements so as to produce the best final result.

The process of determining whether or not a design can be established to meet a given set of requirements is very technical and will not be described here in detail. It involves a systematic consideration of all the possible arrangements. These possibilities are very great in number—about seven million in the case of the E section design—and are, of course, not considered individually, but in groups, and successive groups of possibilities are rejected when it has been proved that they could not meet the requirements.

The systematic grouping of the possibilities, and the means adopted for the examination and elimination of the groups depends a great deal upon the experience and skill of the designer. The general method which has been successfully employed can

be roughly described as a consideration of the design in steps. First the attempt is made to carry through a preliminary design in which pairs of circuits which cannot be exposed to each other untransposed for the entire length of the transposition section (*P* exposures) are provided with different typical arrangements of transpositions, but in which no other requirements are considered. If this design is possible, the other balance limitations of the types selected are added, one by one.

When a design is finally reached it is usually found that any one of a considerable number of arrangements will meet the balance requirements almost equally well. The most economical of these is chosen for use. In determining upon the relative economy of different designs, account is taken of the extent to which circuits on different pin positions will exist in the telephone plant and the extent to which it might be possible, in retransposing existing lines, to use the present transpositions as a part of the transpositions called for in accordance with the new system.

Exposed-Line Transposition System

Special Electrical Requirements. The exposed-line transposition system was designed to meet the following special electrical requirements:

1. The greatest practicable flexibility for the arrangement of transpositions in power circuits paralleling the telephone circuits, the power circuit transpositions co-ordinating with those in the telephone circuits so as to produce a balance of the inductive effects. This requirement to be met for transpositions both in three-phase circuits and in single-phase circuits.

2. A suitable balance against the inductive effects of telegraph circuits or other ground return circuits paralleling the telephone circuits.

3. A degree of freedom from interference between the telephone circuits (crosstalk) suitable for modern high grade telephone circuits used in connection with loading and with telephone repeaters.

General Requirements. In addition to these special requirements, it must meet the following requirements for any satisfactory transposition system:

1. *Neutral Points.* The system must be arranged so that neutral points, that is, points at which a maximum degree of balance is obtained between all circuits on the telephone line are

provided where there are discontinuities in line construction or in the parallel. In the case of open wire toll lines of the Bell telephone system, there must be neutral points at regular intervals of about 12.8 km. (actually 7.88 miles). In addition there must be neutral points wherever there is an important junction between telephone lines and at other points of discontinuity. It is, therefore, desirable that the system include a design intended for a maximum length of 12.8 km. and another design or designs for shorter maximum distances.

2. *Cost.* The completed section must be designed for as low a cost as practicable without sacrifice of the electrical requirements. This means that as far as other conditions of

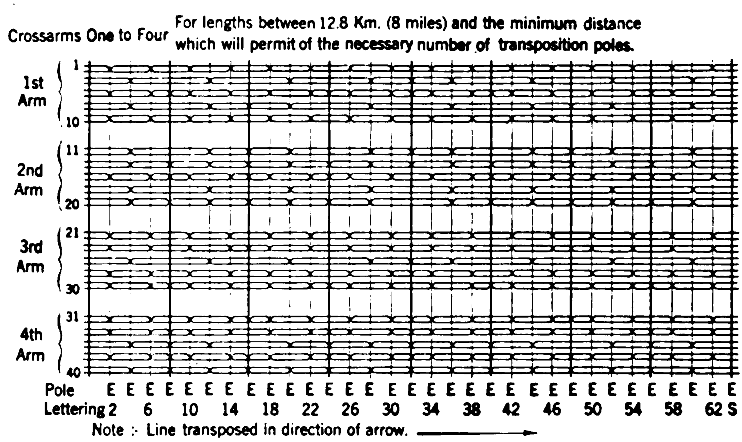


FIG. 4—*E* SECTION—TRANSPOSITION SYSTEM FOR NON-PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES

design permit, the typical arrangements having the least numbers of transpositions should be placed on the circuits most often used. In general, it is desirable that phantom circuits have fewer transpositions than two-wire circuits as phantom circuit transpositions are the more expensive.

3. *Transposition Poles.* It is desirable that the transposition poles be as few in number as practicable not only on account of expense but because the use of large numbers of transposition poles limits the minimum length of a transposition section. The use of many poles may also tend to increase the induction between telephone circuits due to irregularities.

E Section and L Section. As a result of the study, two designs have been completed which are called respectively the *E*

section and the L section. Diagrams showing transpositions according to the E section for crossarms 1 to 8 of a pole line are shown in Figs. 4 to 9, inclusive, and according to the L section in Figs. 10 to 15, inclusive. It will be noted by examination of the figures that the designs provide for the phantoming of any two horizontally adjacent pairs or of any two vertically adjacent pole pairs on the lead. In carrying out the designs provision has been made for extension if necessary to any number of crossarms.

The E section is designed for a maximum length of approximately 12.8 km. It was found impossible to meet the electrical

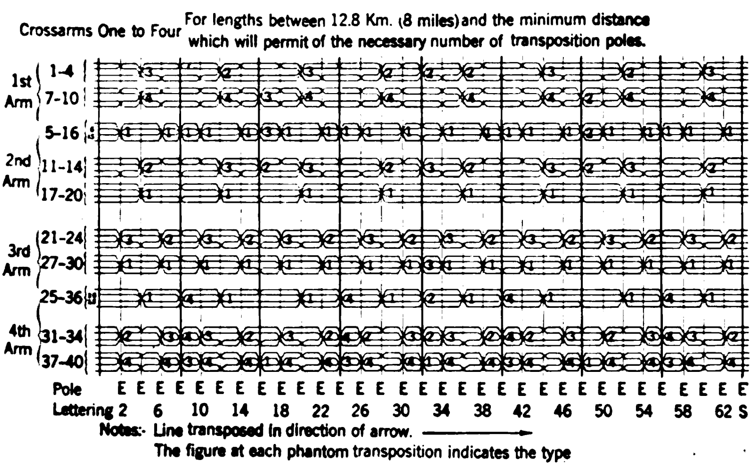


FIG. 5—E SECTION—TRANSPOSITION SYSTEM FOR PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR PHANTOMING ALL CIRCUITS

requirements for this case with the use of 32 transposition poles per section. It was possible, however, to design the E section so that in the first four crossarms the non-phantom circuits and the phantom circuits which are most commonly used require only 32 transposition poles, so that in some toll lines not exceeding four crossarms, the larger number of transposition poles will not be necessary. For circuits on crossarms below the fourth and for some infrequently used phantomed circuits on the upper crossarms, 64 transposition poles are required.

The E section may be used for as short a length of line as supplies the required number of transposition poles. For distances less than 6.4 km., however, it is economical to use the

L section. It has just half the number of transposition poles for the corresponding circuits.

When two or more *L* sections are used consecutively in the same loading section, junction transpositions should be installed at the *S* poles between them in order to reduce the induction between the telephone circuits. These junction transpositions are indicated in the drawings for as many as four consecutive *L* sections.

Use of E and L Sections on Exposed Lines. The results obtained in meeting the special electrical requirements regarding balance to the inductive effects of outside circuits are indicated in the following table:

	E Section	*L Section*
1. Element of section in which each circuit is transposed at least once, *i. e.* balances to a parallel non-transposed disturbing circuit.	8th Section	Quarter-Section
2. Neutral points suitable for the location of transpositions in paralleling three-phase power circuits, separation of approximate centers of disturbing and disturbed groups of wires *more* than 6 meters.	8th Section points; *i. e.*, barrels having nominal length of 4.8 km. (3 miles) or 9.6 km. (6 miles) can be used. Five points dividing distance between any two 8th-section points into six equal parts, all five transpositions to be rotated in the same direction; *i. e.*, nominal 0.8 km. (0.5 mile) barrels can be used also.	Quarter-section points; *i.e.* nominal 4.8 km. (3 mile) barrels can be used.
3. Neutral points suitable for the location of transpositions in paralleling metallic single-phase power circuits—separation as defined above *more* than 6 meters.	8th-section points. 3 points dividing distance between any two 8th-section points into four equal parts.	Quarter-section points.
4. Neutral points suitable for the occurrence of discontinuities—separation as defined above *more* than six meters.	8th-section points	Quarter-section points
5. Neutral Points suitable for the location of transpositions in disturbing power circuits or the occurrence of discontinuities in disturbing power or telegraph circuits—separation as defined above *less* than 6 meters.	Mid-section point	Mid-section point
6. Neutral points suitable for the occurrence of discontinuities in telephone circuits.	S-poles	S-poles

The results given in the table are further illustrated in Figs. 16 and 17 which show premissible relative locations of the tele-

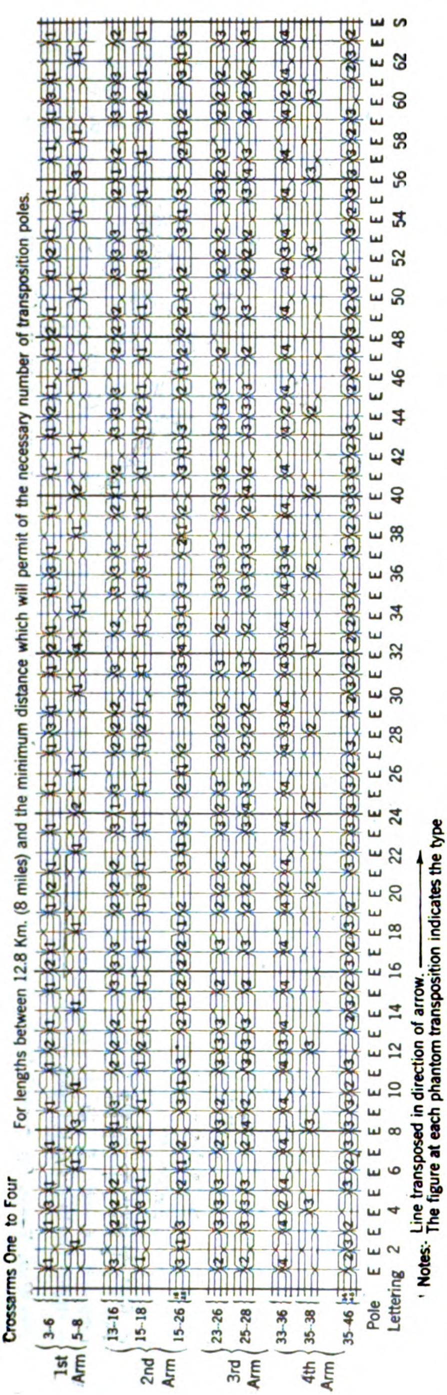


FIG. 6—*E* SECTION—TRANSPOSITION SYSTEM FOR PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR HORIZONTAL POLE PAIR PHANTOMS AND ODD VERTICAL PHANTOMS

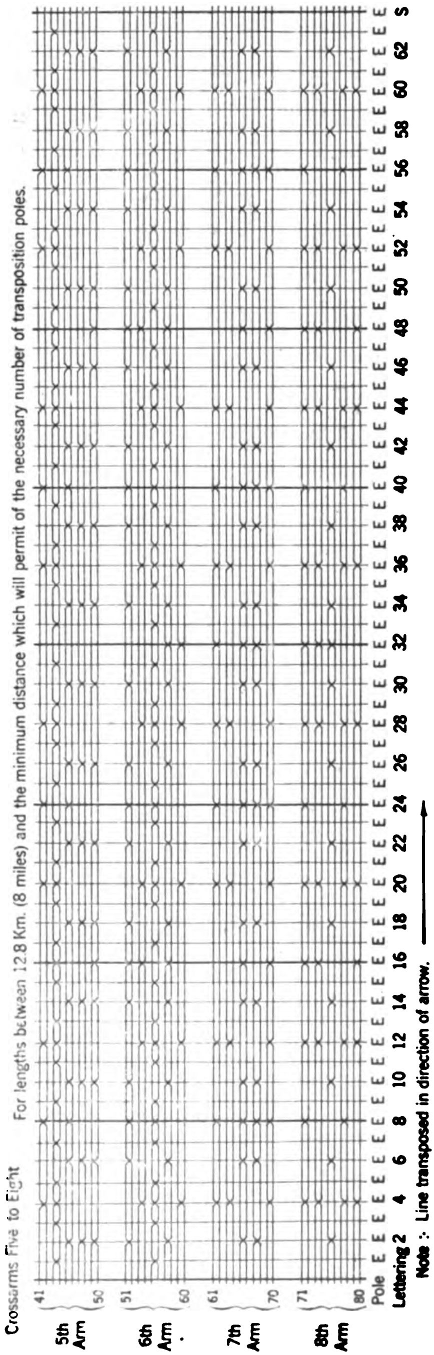


FIG. 7—*E* SECTION—TRANSPOSITION SYSTEM FOR NON-PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES

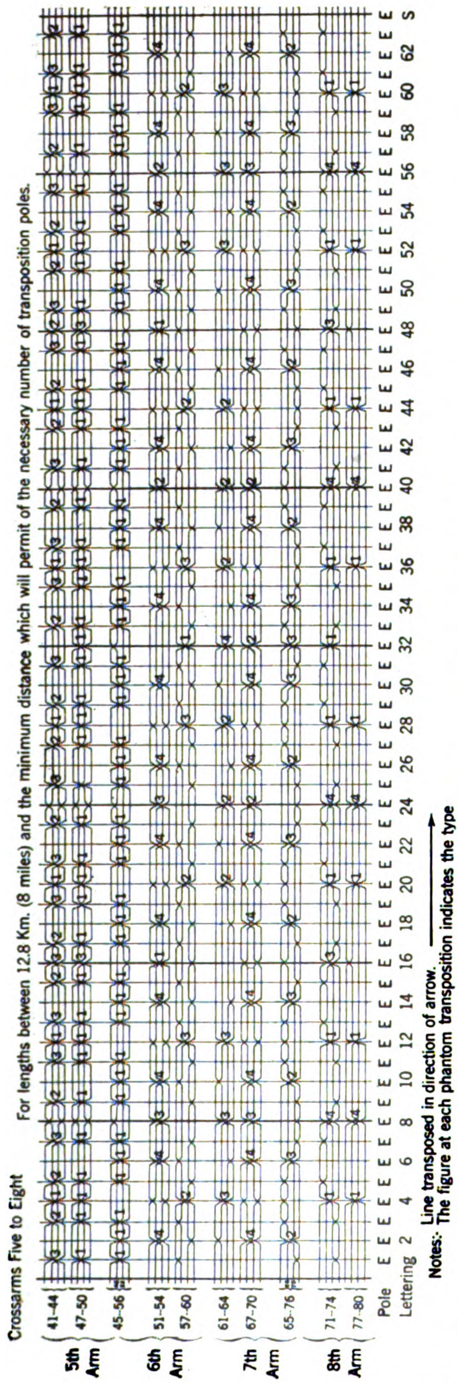
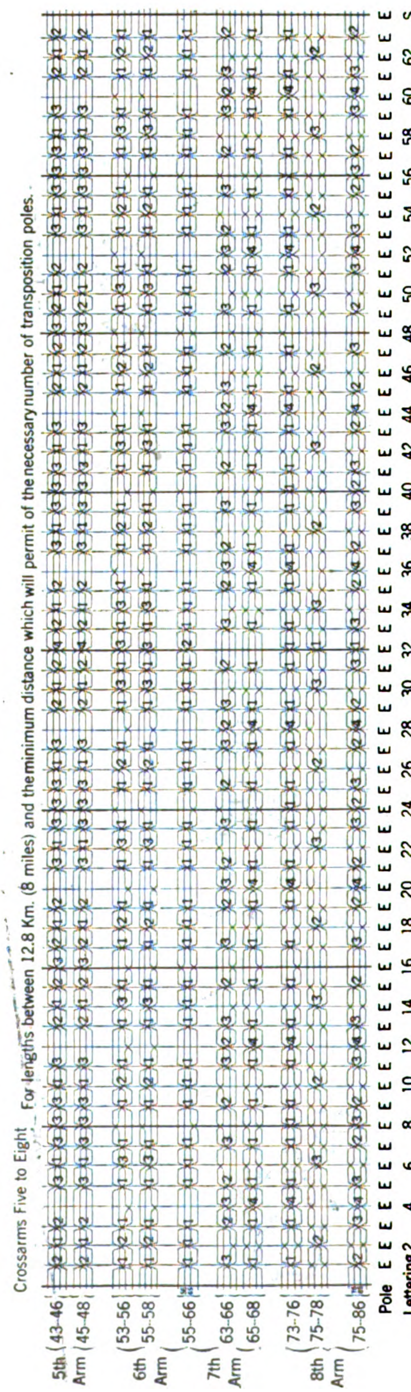


FIG. 8—*E* SECTION—TRANSPOSITION SYSTEM FOR PHANTOM CIRCUITS ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR PHANTOMING ALL CIRCUITS



Line transposed in direction of arrow ⟶

Notes: The figure at each phantom transposition indicates the type.

FIG. 9—*E* SECTION—TRANSPOSITION SYSTEM FOR PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR HORIZONTAL POLE PAIR PHANTOMS AND ODD VERTICAL PHANTOMS

phone transposition poles and of transpositions and discontinuities in the outside parallel circuits or changes in separation between the two classes of circuit.

It will be noted that the E section can be used to coordinate with transpositions in three-phase power circuits which create for the maximum length of E section 0.8 km. (½ mile), 4.8 km. (3 miles) or 9.6 km. (6 miles) in the power circuit. As it was considered very important to obtain the maximum practicable degree of flexibility in this respect, this matter was given very thorough study and a large amount of work was required to arrive at this result. It should be noted, moreover, that in the use of a 0.8-km. barrel, two barrels occupying a 1.6-km. section

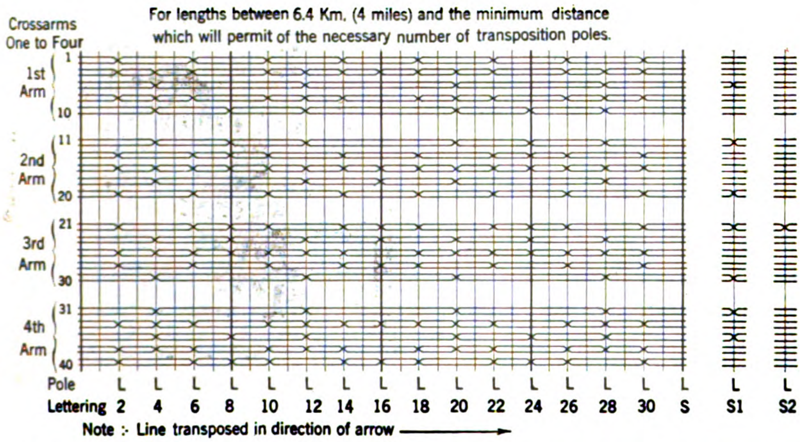


FIG. 10—L SECTION—TRANSPOSITION SYSTEM FOR NON-PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES

of the transposition system are required to obtain a balanced condition.

In the L section the requirements for the avoidance of excessive interference between the telephone circuits are more exacting than in the E section because the use of a smaller number of transposition poles makes available a correspondingly smaller number of typical arrangements of transpositions. It was found possible, however, to design this section so that it balances to a three-phase power line transposed at the quarter section points.

The exposed-line transposition system has been designed primarily for use on toll lines and not for use on joint lines with

power distribution circuits. It is, however, somewhat better for this case than the other transposition systems in use.

When large inductive effects are experienced from circuits less than 20 ft. (6 m.) away from a side circuit, the variation in separation between the telephone circuit and the exposing circuits as the side circuits shifts from one pair of pin positions to another must be taken into consideration. The *E* section is not designed to give a high degree of balance under this condition for all circuits in each eighth-section, but does give balance to uniform outside induction effects for most circuits in each eighth-section and with one exception, for all circuits in each

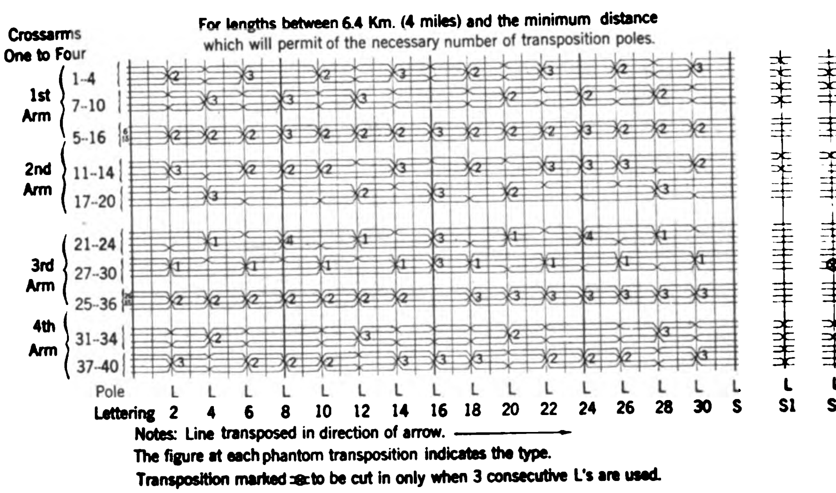


FIG. 11—*L* SECTION—TRANSPOSITION SYSTEM FOR PHANTOMED CIRCUIT ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR PHANTOMING ALL CIRCUITS

half-section. Under similar conditions the *L* section gives a balance for most circuits in each quarter section and for all circuits in each half section.

It should be clearly pointed out that in the use of the telephone transposition sections to coordinate with transpositions in power circuits, it is important that the transpositions in the two lines be coordinated both as regards relative length and as regards relative location of telephone transposition section and transpositions in the power circuits. The locations of the transpositions in either line must, therefore, be made with reference to the requirements of the other. It is frequently the case that

the locations of the transpositions in the power circuit are to a large degree determined by the changes in separation and other characteristics of the parallel.

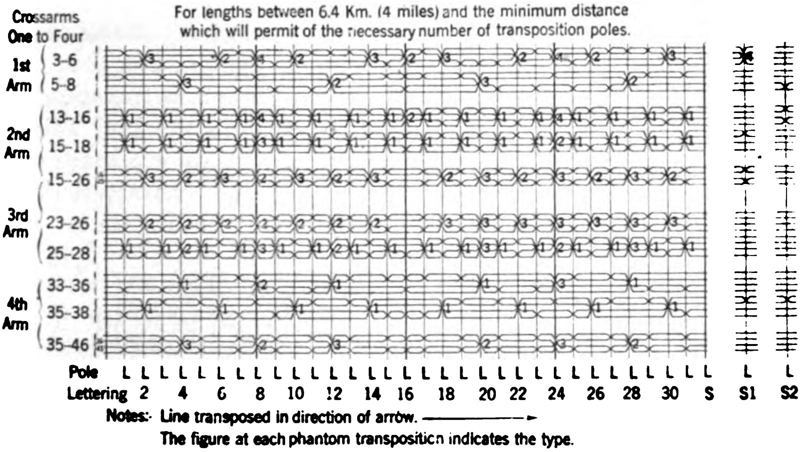


FIG. 12—*L* SECTION—TRANSPOSITION SYSTEM FOR PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR HORIZONTAL POLE PAIR PHANTOMS AND ODD VERTICAL PHANTOMS

The *E* and the *L* section are both designed to give a high degree of balance of the inductive effects between the telephone

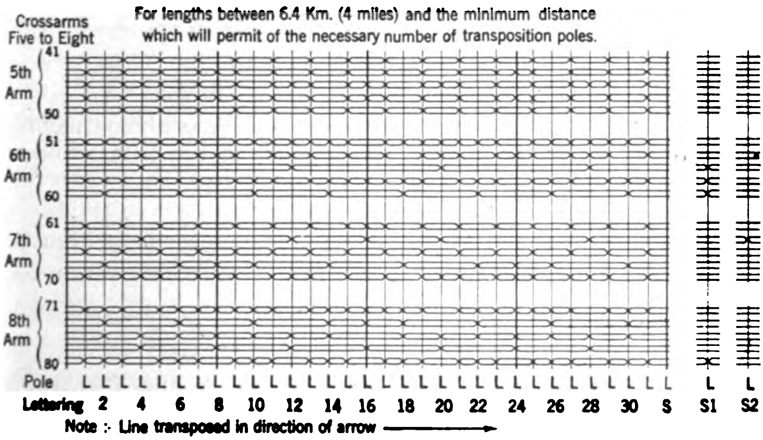


FIG. 13—*L* SECTION—TRANSPOSITION SYSTEM FOR NON-PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES

circuits. In using them, therefore, it is important that the transposition poles be spaced as accurately as practicable in

order to avoid diminishing the effectiveness of the system in this respect because of irregularities.

The design work led to a large number of possible arrangements of approximately equal electrical efficiency. In choosing from these the most economical, account was taken both of the cost of transposing when stringing new circuits and of the cost of retransposing to the new system telephone lines now transposed according to other arrangements standard with the Bell Telephone Companies. Although a most careful study was made of economy it was necessary because of the special characteristics required in this system to allow in the final sections a cost of

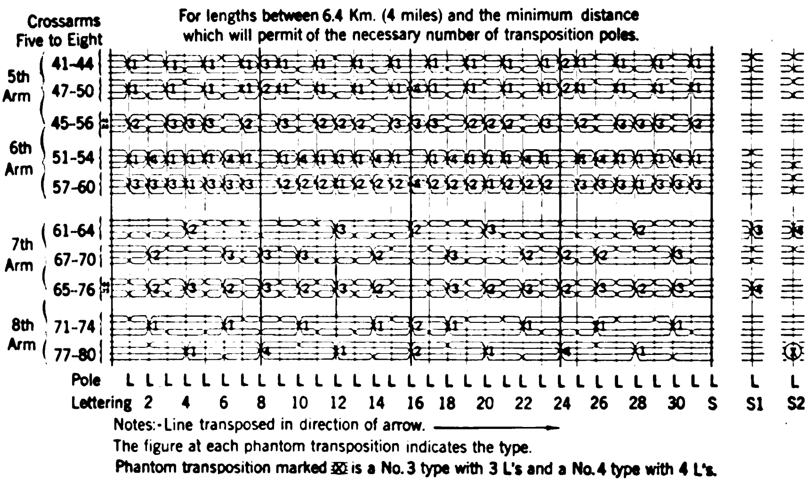


FIG. 14—*L* SECTION—TRANSPOSITION SYSTEM FOR PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR PHANTOMING ALL CIRCUITS

telephone transpositions 25 to 50 per cent greater than that of the section which is standard in the Bell Telephone System for use on non-exposed lines.

CONCLUSION

The results obtained in the design of the *E* and *L* sections were possible only through a systematic study of the problem. The work directed particularly to the design of these two sections occupied about two man-years' time of the specially trained experts who carried out the work. The investigation has been so thorough that the conclusion seems warranted that the *E* section and the *L* section are substantially the best arrangement

of transpositions which could be devised to meet the requirements established for them.

The work of many members of the engineering department of the American Telephone and Telegraph Company has contributed to the results. The theory of induction between telephone circuits and the special analytical methods of design have been developed in the department during a considerable number of years. Special mention should be made of the work of Mr. G. A. Campbell who, years ago, laid the foundations for the theory of induction between telephone circuits, and of

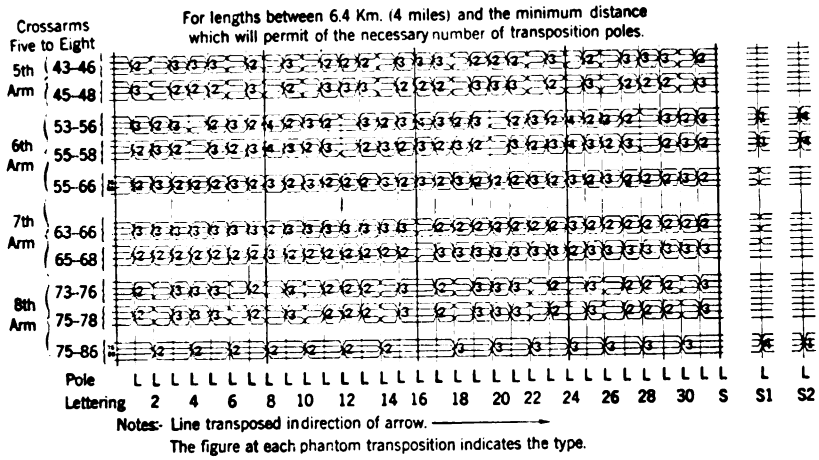


FIG. 15—*L* SECTION—TRANSPOSITION SYSTEM FOR PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR HORIZONTAL POLE PAIR PHANTOMS AND ODD VERTICAL PHANTOMS

Messrs. A. G. Chapman and R. E. Leonard, who perfected the methods of systematic search and carried out the design of the *E* and *L* sections.

APPENDIX A. INDUCTION BETWEEN PARALLEL WIRES

1. *Two Wires with Ground Return—Short Length of Exposure.* If the current and voltage in a short element of circuit No. 1 (Fig. 18) are represented by I and E, the inductive effect which this element of circuit No. 1 exercises on parallel circuit No. 2 can be represented as follows:

The voltage induced in circuit No. 2 because of current I is

$$V_2 = -j\,\omega\, I\, m\, l \qquad (1)$$

where ω equals 2π times the frequency, l the length of element of circuit considered and M the mutual impedance between the circuits per unit length. In the case of a perfectly conducting earth surface, M is made up entirely of the mutual inductance caused by the linkage of flux from circuit No. 1 with circuit No. 2. With a perfectly conducting earth surface this mutual inductance can be readily computed by the method of images. Under actual conditions the mutual inductance is affected by the resistance of the earth, which increases the linkages between the two grounded circuits because it causes the return current from circuit No. 1 to distribute more or less widely throughout the

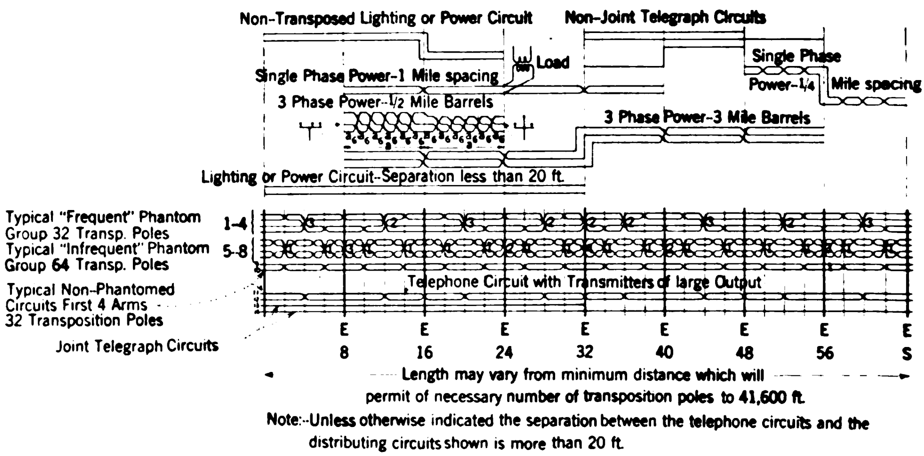


FIG. 16—CHARACTERISTICS OF EXPOSED LINE E SECTION —DIAGRAM ILLUSTRATES LENGTH, NUMBER OF TRANSMISSION POLES AND PERMISSIBLE LOCATIONS OF TRANSPOSITIONS AND DISCONTINUITIES IN EXTRANEOUS DISTURBING CIRCUITS

earth rather than to flow in a current sheet on the earth surface. In many cases, also, the fact that the two circuits both use the earth return having a finite resistance introduces an appreciable resistance term into the mutual impedance. Sometimes this term is very important.

The voltage induced in a short section of wire 2 because of the voltage between wire 1 and ground is

$$e_2 = E \frac{C_M}{C_2 + C_M} \qquad (2)$$

In this equation C_M and C_2 are the direct capacities from wire 2 to wire 1 and to the ground, respectively; that is, they are the

capacities measured by the charges on wire 1 and on the earth when wire 1 is connected to the earth and unit potential is applied between wire 2 and the combination of wire 1 and earth.

If the short section of wire 2 is connected to the earth, the charging current flowing to ground is

$$i_2 = j\ \omega\ l\ E\ C_M \quad . \qquad (3)$$

That is the induced voltage acts like a generator having a voltage e_2 connected between wire 2 and ground through a condenser whose capacity is equal to the total grounded capacity of the exposed section of wire 2. This is indicated in Fig. 19.

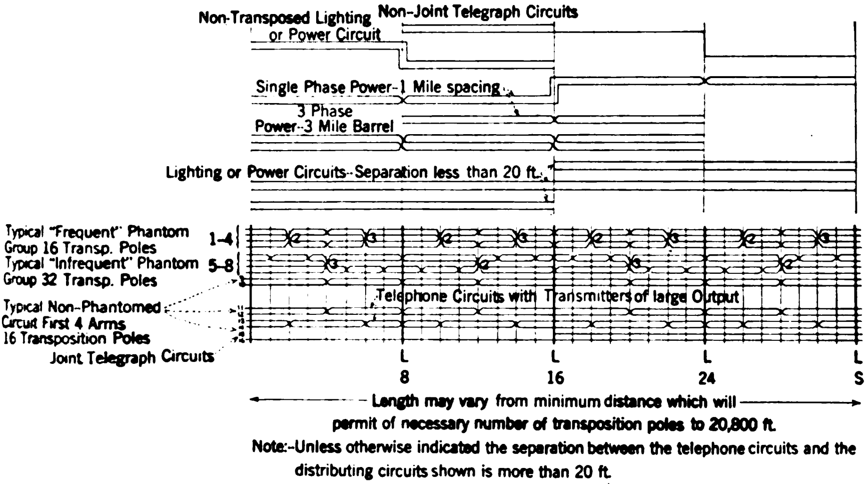


FIG. 17—CHARACTERISTICS OF EXPOSED LINE·L SECTION—DIAGRAM ILLUSTRATES LENGTH, NUMBER OF TRANSPOSITION POLES, AND PERMISSIBLE LOCATIONS OF TRANSPOSITIONS AND DISCONTINUITIES IN EXTRANEOUS DISTURBING CIRCUITS

2. *Small Induced Effects.* In cases where the induced currents and voltages are small compared with the inducing currents and voltages so that their reaction on the inducing circuit can be neglected, the magnitude of the induced effects can be obtained by solving for the values of current and voltage in the disturbing circuit, neglecting the presence of the other circuit, and then applying equations similar to those given above for each element of the parallel between the two circuits.

3. *A number of Parallel Wires.* The computation of induced currents is much complicated when there are several disturbing wires or several disturbed wires or both.

The computation of the electromagnetic induction may be complicated by the reaction between the induced currents in different disturbed circuits. In cases where the induced effect is small, however, the electromagnetic induction can be computed for this case as for a single circuit.

In determining electrostatic induction with a large number of parallel circuits, it is evident that it becomes exceedingly complicated to compute the direct capacities which are used. It is possible to greatly simplify this work in cases in which the disturbed and disturbing circuits are separated so that each group has little effect on the direct capacities of the other group. Under these conditions the direct capacities within the disturbing circuits can be computed and from [this computation a determination can be made of the charge of electricity on each of the disturbing wires taking account of the voltages on all wires. From these charges it is possible to compute the potentials between the disturbed wires and ground if isolated in space.

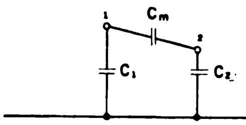


FIG. 18

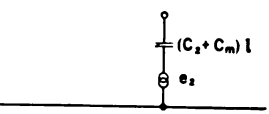


FIG. 19

These potentials may then be assumed to regulate through the direct capacities of the disturbed circuits as indicated in the above discussion of induction between two parallel wires. In computing the flow of current with many disturbed wires the effect on one wire of the other wires must, of course, be considered.

This outlines briefly the method which is in use for computing the induction between three-phase power circuits and telephone circuits. The method sometimes described in the text books is incorrect in that it does not take account of the mutual reactions between the three conductors of the three-phase circuit which materially modify their resultant electrostatic field.

4. *Long Parallel Circuits.* In the case of two long circuits which closely parallel each other without means for neutralizing inductive effects, the reaction of the induced currents on the disturbing circuit cannot be neglected. If the circuits are very long there tends to be the same distribution of current and potential at the receiving ends of the disturbing and disturbed

circuits as there would be if equal voltages were applied to the two circuits at the transmitting end. The simplest case of this sort is when the two circuits are symmetrical. The equations for this case are given in Appendix B. It may be seen that even in this case the equations are so complicated that their use in practise would be burdensome. Where there are many long parallel wires the problem becomes one of utmost complexity.

5. *Parallel Telephone Circuits.* In the case of telephone circuits many conductors are run very closely parallel for long distances. It would seem, then, that the general solution of the inductive effect between telephone conductors would be so complicated as to be hopeless for practical use.

In attacking this problem, however, we fortunately are not interested in the general case. If telephone circuits are suitable for commercial service they must be so arranged that the inductive effects between any two circuits are exceedingly small.

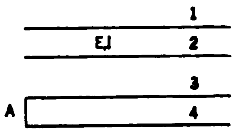


FIG. 20

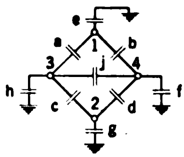


FIG. 21

This is, therefore, a case in which the reaction of the induced currents on the disturbing circuit can be neglected.

If one considers then a short section of close parallel between a disturbing circuit 1-2 carrying a voltage E and current I and a disturbed circuit 3-4 as indicated in Fig. 20 the voltage due to electromagnetic induction can be expressed as in equation (4)

$$e = j\,\omega\,I\,M\,l \tag{4}$$

In computing the current flowing at A due to electrostatic induction one may represent the direct capacities in the form of a Wheatstone bridge as in Fig. 21. In this figure the direct capacities a, b, c and d represent the Maxwell capacity coefficients. Capacities e, f, g and h represent the sum of the direct capacities between the wires, 1, 4, 2 and 3 respectively and all other wires and ground. This grouping is permissible because the induced effects all being small, the other circuits on the pole line are practically at ground potential, and have the same effect on the mutual capacity unbalance between the circuits 1, 2 and 3, 4 as they would if actually connected to earth.

The induced current is represented approximately by equation (5)

$$i = j\,\omega\,E\,\frac{a'\,d' - b'\,c'}{a' + b' + c' + d'} \tag{5}$$

$a' = a + \dfrac{h\,e}{s}$, $b' = b + \dfrac{e f}{s}$, etc., s being equal to $e + f + g + h$.

When the circuits are transposed so that the electrostatically induced current is small it results that in a balanced transposition section a', b', c' and d' are all very nearly equal. Under these conditions the magnitude of the electrostatically induced current in one small section of line is represented with a sufficient degree of accuracy by equation (6).

$$i = j\,\omega\,E\;\frac{a' - b' - c' + d'}{4} \tag{6}$$

The equations given above indicate the coefficients of induction which must be determined in order to make possible computations of the induced currents between telephone circuits. The mutual inductance unbalance between any two circuits can be computed from the geometrical positions of the conductors. The mutual capacity unbalance can be computed readily from the table of measured values of direct capacity for all wires on a 40 wire pole lead. (Table I.) The capacities of course vary with the number and to a small extent with the size of conductors on the lead. Forty wires is a typical condition and has been used as representative. When desirable, correction has been made for the use of wires of larger size.

The above equations therefore present a thoroughly practical method of computing the current induced between short sections of two telephone circuits which are well transposed against each other. The correctness of the results obtained in this way has been checked by experiment.

APPENDIX B—THE INDUCTION BETWEEN TWO LONG PARALLEL SYMMETRICAL CIRCUITS

Assume two long symmetrical wires, A and B, as shown in Fig. 22, operating with ground return. Assume the linear self impedances of the circuits to be z_1 and z_2 respectively, the linear mutual impedance to be M, and the linear direct admittances to

ground and linear mutual admittance to be y_1', y_2', and y_m respectively.

The differential equations for inductive effect between the two circuits are as follows:

$$\frac{-d\,v_1}{d\,x} = i_1\,z_1 + i_2\,M \qquad \frac{-d\,v_2}{d\,x} = i_2\,z_2 + i_1\,M$$

$$\frac{-d\,i_1}{d\,x} = v_1\,y_1 - v_2\,y_m \qquad \frac{-d\,i_2}{d\,x} = v_2\,y_2 - v_1\,y_m$$

In the above formulas, $y = y_1' + y_m$, $y_2 = y_2' + y_m$, and M and y_m are both written as positive quantities.

For the case of two symmetrical wires let

$$z_1 = z_2 = z$$

$$y_1 = y_2 = y$$

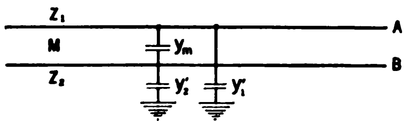


FIG. 22—INDUCTION BETWEEN TWO SYMMETRICAL GROUNDED CIRCUITS

Use is made of the following auxiliary equations:

$$A = 2\sqrt{\frac{z - M}{y + y_m}} \qquad B = \frac{1}{2}\sqrt{\frac{z + M}{y - y_m}}$$

$$\gamma_s = \sqrt{(z - M)\,(y + y_m)} \qquad \gamma_p = \sqrt{(z + M)\,(y - y_m)}$$

Equations are given below for a number of conditions which are of interest. In these equations, v_1 and i_1 represent the voltage and current on the disturbing circuit A at any distance x from the transmitting end, and v_2 and i_2 represent the voltage and current on the disturbed circuit B.

Case I. Voltage impressed on Circuit A at $x = O$ is E_1
Circuit B connected to earth at $x = O$ through impedance W
Both circuits infinite in length.

$$v_1 = E_1\,\frac{2\,B\,(A + 2\,W)\,\epsilon^{-\gamma_p x} + A\,(2\,B + W)\,\epsilon^{-\gamma_s x}}{W\,(A + 4\,B) + 4\,A\,B} \tag{7}$$

$$i_1 = E_1\,\frac{(A + 2\,W)\,\epsilon^{-\gamma_p x} + (4\,B + 2\,W)\,\epsilon^{-\gamma_s x}}{W\,(A + 4\,B) + 4\,A\,B} \tag{8}$$

$$v_2 = E_1 \frac{2\,B\,(A + 2\,W)\,\epsilon^{-\gamma_p x} - A\,(2\,B + W)\,\epsilon^{-\gamma_s x}}{W\,(A + 4\,B) + 4\,A\,B} \tag{9}$$

$$i_2 = E_1 \frac{(A + 2\,W)\,\epsilon^{-\gamma_p x} - (4\,B + 2\,W)\,\epsilon^{-\gamma_s x}}{W\,(A + 4\,B) + 4\,A\,B} \tag{10}$$

Case II. If the resistance, leakance, and internal reactance are negligible and the equivalent height of the wires above ground is the same for static and magnetic induction, $\gamma_s = \gamma_p$ and

$$\frac{A}{4\,B} = \frac{z - M}{z + M} = \frac{y - y_m}{y + y_m} \tag{11}$$

Then, if $W = \infty$, it follows that

$$v_2 = v_1 \frac{y_m}{y} \tag{12}$$

Under these conditions, ($\gamma_s = \gamma_p$ and $W = \infty$) the sending end impedance of the disturbing circuit is

$$Z_0 = \sqrt{\frac{Z\,y}{y^2 - y_m^2}} \tag{13}$$

This equation shows that under these conditions there is no reaction from circuit B at the terminal of circuit A.

Case III. If $z\,y_m + y\,M = O$ **(14)**

Then $B = A/4$ and it follows that

$$v_2 = \frac{E_1}{2}\,(\epsilon^{-\gamma_p x} - \epsilon^{-\gamma_s x})$$

$$i_2 = \frac{E_1}{A}\,(\epsilon^{-\gamma_p x} - \epsilon^{-\gamma_s x}) \tag{16}$$

that is, at $x = O$, the induced voltage and current are O.

If y_m is negligible, equation (**14**) requires that $M = O$. This may be approximated by the use of compensating transformers. The above relation suggests, however, that by properly adjusting the value of M an approximate neutralization of induced voltage might be obtained in some special cases in which the electrostatic induction is considerable.

Case IV. When $W = \infty$, then at $x = O$,

$$v_2 = E_1 \frac{y(D-1) + y_m(D+1)}{y(D+1) + y_m(D-1)} \tag{17}$$

where $\frac{\gamma_p}{\gamma_s} = D$.

The ratio of this voltage to that computed by equation (**12**) gives an idea of the error involved in the assumption which leads to equation (**12**).

Case V. If at $x = O$,

$$W = \frac{v_1}{i_1},$$

then

$$W = \sqrt{A\ B}$$

and

$$v_2 = E_1 \frac{2\sqrt{B} - \sqrt{A}}{2\sqrt{B} + \sqrt{A}} \tag{19}$$

This impedance, $\sqrt{A\ B}$, is

$$\sqrt{A\ B} = Z_0 = z_0 \sqrt[4]{\left[1 - \left(\frac{M}{z}\right)^2\right]\left[1 - \left(\frac{y_m}{y}\right)^2\right]} \tag{20}$$

Where $z_0 = \sqrt{\frac{z\ y}{y^2 - y_m^2}}$ = the infinite line impedance for a (**13**) single wire.

Case VI. At a long distance from the origin,

$$v_1 = v_2 = \frac{E_1 e^{-\gamma_p x}}{2} \text{ if } W = O. \tag{21}$$

$$\text{Also } v_1 = v_2 = E_1 \frac{4\ B}{A + 4\ B} e^{-\gamma_p x} \text{ if } W = \infty. \tag{22}$$

It will be noted that the voltages on the disturbed and disturbing wires tend to become equal at a considerable distance from the origin, and that the two expressions for v_1 and v_2 with maximum and with minimum shielding are nearly the same.

APPENDIX C—TYPE UNBALANCE BETWEEN TRANSPOSED TELEPHONE CIRCUITS

The induction between long transposed telephone circuits on the same lead is the vector sum of the induction between all

short elements of the circuits, taking into account the attenuation and phase change of the inducing currents and voltages from the transmitting point, and the attenuation and phase change of the induced currents in their propagation to the end of the circuit from the point at which the inductive effect occurs.

A simple typical arrangement of transpositions between two telephone circuits in a short balanced section is indicated in Fig. 23 in which similar telephone circuits P and Q are divided into four sections of lengths A, B, C and D by the transpositions in the two circuits. The resulting induction between the two circuits in this section is the sum of two terms:

a. Unbalance Due to Irregularity in Length. As this is a short section of line, one may consider as a first approximation that the current and voltage in the inducing circuit P are the same throughout this section. Currents induced in sections B and D,

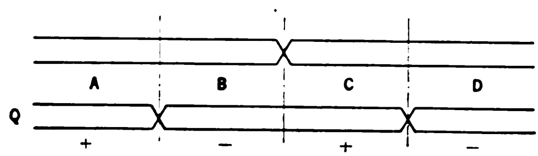


FIG. 23

therefore, tend to neutralize currents induced in sections A and C, provided the length $A + C$ is equal to the length $B + D$. When this is not the case there is a resultant induced current in circuit Q due to irregularities approximately proportional to the resultant unbalanced length $A - B + C - D$.

b. Type Unbalance. If lengths A, B, C and D are all exactly equal there will nevertheless be a resultant induced current in circuit Q because of the fact that the currents and voltages in either circuit P or Q change in magnitude and phase as they are propagated along the circuit.

In this appendix are given equations illustrating the method of computing type unbalance.

If P is a section of a long telephone circuit the current and voltage in it can be represented by equations (**23**) and (**24**).

$$I = I_0 \, e^{-\alpha l} \sin\,(\beta \, l - w\, t) \qquad \textbf{(23)}$$

$$E = E_0 \, e^{-\alpha l} \sin\,(\beta \, l - w\, t) \qquad \textbf{(24)}$$

In these equations α and β are the attenuation constant and the phase-change constant of the telephone circuit and of course vary with the frequency of the current.

The induced current in a short element of circuit a distance L from the end of the circuit, can be represented as equation (**25**).

$$d\,X = K\,d\,l\,e^{-\alpha l} \sin(\beta\,1 - w\,t) \qquad \textbf{(25)}$$

where $K\,d\,l$ is the induction between short elements at the end of the circuits and is computed with the help of equations **4** and **6** for the electromagnetic and electrostatic components. (Appendix A.) This induced current will be propagated on circuit Q back to the beginning of the circuit and under the condition for transmission without reflection, that is, terminal impedance equal to line impedance, will have when it reaches the beginning of the circuit, the value given in equation (**26**)

$$d\,X_0 = K\,d\,l\,e^{-2\alpha l} \sin(2\,\beta\,1 - w\,t) \qquad \textbf{(26)}$$

A similar equation can be written for each element of the circuit. The induction due to length A is obtained by integrating expression (**26**) for the length between zero and A as shown in equation (**27**)

$$X_A = \int_0^A K\,d\,l\,e^{-2\alpha l} \sin(2\,\beta\,1 - w\,t) \qquad \textbf{(27)}$$

The general expression for this integral is rather complicated. In practical telephone circuits, however, the effect of the attenuation in the short section of circuit between successive transpositions is very much less than the effect of the change in phase angle. Under these conditions the attenuation can be neglected for short lengths and the expression (**27**) reduces to that given in equation (**28**)

$$X_A = K\,A\,\frac{\sin\,\beta\,A}{\beta\,A}\,/\underline{\beta\,A} \qquad \textbf{(28)}$$

Similarly, the crosstalk at the end of the circuit due to section B of the exposure between the lines can if B is equal in length to A be represented as in equation (**29**).

$$X_B = -\,K\,A\,\frac{\sin\,\beta\,A}{\beta\,A}\,e^{-2\alpha A}\,/\underline{3\,\beta\,A} \qquad \textbf{(29)}$$

In practical cases the ratio $\frac{\sin\,\beta\,A}{\beta\,A}$ is practically unity

and the exponential expression $e^{-2\alpha A}$ is also practically unity. Under these conditions the induced current at the end of the circuit caused by sections A and B of the parallel can be represented approximately as in equation (**30**).

$$X_{AB} = 2\,K\,A \sin \beta\,A \;\underline{/2\,\beta\,A-\pi/2} \tag{30}$$

Similarly, the resultant induction in circuit Q from sections A, B, C and D can be represented as in equation (**31**)

$$X_{AD} = 4\,K\,A \sin \beta\,A \cos 2\,\beta\,A \;\underline{/4\,\beta\,A-\pi/2} \tag{31}$$

This simple example corresponds to one of the typical arrangements of transpositions, namely, that denoted by M in Fig. 2. A similar equation can be worked out for each typical arrangement, and numerical values worked out for any given values of β and A.

For much work a sufficiently good approximation can be made in the formulas for type unbalance by assuming that

$$\sin n\,\beta\,A = n\,\beta\,A$$

$$\cos n\,\beta\,A = 1$$

The resulting simple formulae give an easy means of judging of the approximate relative effectiveness of different arrangements. They are given for different typical arrangements under the heading *Approximate Formulas* in Fig. 2. When X represents the phase change per mile these formulas represent numerically the type unbalance for a 12.8 km. (8 mile) section. The angles of the type unbalance for different types of the same length differ only by 90 deg. or multiples of 90 deg., and these differences only are indicated in the approximate formulas.

The type unbalance varies, of course, with the frequency. In the design of telephone transpositions type unbalances are computed for a frequency found to represent the average crosstalk effect.

It remains to point out the relation between resultant unbalance due to type for a long length of line and type unbalance for one transposition section. Considering current of a single frequency, the ratio between these two for N sections is

$$\frac{1 - e^{-2N\gamma L_0}}{1 - e^{-2\gamma L_0}} \tag{32}$$

where L_0 is the length of the section.

Equation (**32**) assumes that the wires have the same relative pin positions at the beginning of each section If the pin positions are the same in alternate sections and the opposite in successive sections formula (**32**) becomes

$$2 \sin \beta L_0 \frac{1 - e^{2N\gamma L_0}}{1 - e^{-4\gamma L_0}} \tag{33}$$

For a large number of sections, and for the attenuations prevailing on good telephone toll lines equations (**32**) and (**33**) become approximately

$$\frac{1}{2\,\gamma\,L_0} \tag{34}$$

and

$$\frac{\sin\,\beta\,L_0}{2\,\gamma\,L_0} \tag{35}$$

Actually, results obtained by these simple formulas must be corrected for the fact that the telephone current is a compound of currents of different frequencies, for which the values of β and γ are different. In the case of loaded lines a correction can be made rather simply. The change in phase angle from one loading section to another is large and varies widely for different components of the telephone current. Good results are therefore obtained by assuming that the effects of successive sections add together by a root-sum-square law. The resulting ratio between long line unbalance and type unbalance for a single section is

$$\sqrt{\frac{1 - e^{-4N\alpha L_0}}{1 - e^{-4\alpha L_0}}} \tag{36}$$

Since N is large and $\alpha\,L_0$ small, this becomes approximately

$$\frac{1}{2\sqrt{\alpha\,L_0}} \tag{37}$$

APPENDIX D, FORMULAS FOR TYPE UNBALANCE IN PHANTOMED CIRCUITS

Under the heading, *Phantom Circuits*, in the paper it is pointed out that the use of phantoms in telephone line construction complicates the computation of type unbalance because each transposition of a phantom circuit interchanges the pin positions

of its two side circuits and, therefore, changes the coefficients of induction between these and other circuits.

The equations for computing the resultant induction per section due to type unbalance are given below. These equations are given for the case in which a typical arrangement of transpositions is associated with each side circuit of the phantom independent of its interchange of pin positions with the other side circuit of the phantom. This is the arrangement which has been found most suitable for use in the exposed line transposition system. Transposition systems can be designed with a typical arrangement associated with the wires on a given pair of pin positions. With this arrangement, each side circuit of the phantom is transposed partly in accordance with one typical arrangement and partly in accordance with another as it changes from one pair of pin positions to the other with transpositions of the phantom circuit. Under some conditions, this arrangement has advantages in the economy of retransposing non-phantomed circuits to form phantoms. It is used in the transposition system which is standard with the Bell Telephone Companies for non-exposed lines.

The formulas for the resultant induction per section due to type unbalance are as follows:

Suppose two circuits, 1 and 2, to be combined in a phantom R and two circuits, 3 and 4, to be combined in a circuit Q.

Denote by T_1, T_2, T_R, etc., the type unbalances corresponding to the typical arrangements of transpositions on circuits 1, 2, R, etc. Further, let K_{1R}, K_{2R}, etc. represent the induction per unit untransposed length between circuits 1 and R, between circuits 2 and R, etc.

For Side Circuit to Phantom of Which it is a Part.

$$X_{1R} = S\,T_{1R} + D\,T_1 \qquad (38)$$

$$X_{2R} = S\,T_{2R} - D\,T_2 \qquad (39)$$

$$\text{Where } S = \frac{K_{1R} + K_{2R}}{2}.$$

$$D = \frac{K_{1R} - K_{2R}}{2}$$

T_{1R} represents the type unbalance for the typical arrangement corresponding to the relative exposure between type 1 and type R.

T_{R} is similarly defined.

For Side Circuit to a Non-phantomed Circuit or a Phantom Other Than its Own.

$$X_{1Q} = S\,T_{1Q} + D\,T_{1RQ} \qquad (40)$$

$$X_{2Q} = S\,T_{2Q} - D\,T_{2RQ} \qquad (41)$$

When
$$S = \frac{K_{1Q} + K_{2Q}}{2}$$

$$D = \frac{K_{1Q} - K_{2Q}}{2}$$

The nomenclature is the same as before.

T_{1RQ} represents the type unbalance corresponding to the relative exposure between the arrangement giving unbalance T_{1R} and typical arrangement Q.

For example, if circuit 1 is transposed to Type A

" R	"	" F
" Q	"	" L

$$T_{1RQ} = T_{(AF)L} = T_{KL} = T_O$$

That is, T_{1RQ} is in this case the type factor for arrangement O, which is obtained by combining successively types A, F, and L.

For the Two Side Circuits of a Phantom

$$X_{12} = K_{12}\,T_{12} \qquad (42)$$

For Side Circuits in Two Different Phantoms

$$X_{13} = S\,T_{13} + D_a\,T_{13R} + D_b\,T_{13Q} + D_c\,T_{13RQ} \qquad (43)$$

$$X_{14} = S\,T_{14} + D_a\,T_{14R} - D_b\,T_{14Q} - D_c\,T_{14RQ} \qquad (44)$$

$$X_{24} = S\,T_{24} - D_a\,T_{24R} - D_b\,T_{24Q} + D_c\,T_{24RQ} \qquad (45)$$

$$X_{23} = S\,T_{23} - D_a\,T_{23R} + D_b\,T_{23Q} - D_c\,T_{23RQ} \qquad (46)$$

Where

$$S = \frac{K_{13} + K_{14} + K_{24} + K_{23}}{4}$$

$$D_a = \frac{K_{13} + K_{14} - K_{24} - K_{23}}{4}$$

$$D_b = \frac{K_{13} - K_{14} - K_{24} + K_{23}}{4}$$

$$D_c = \frac{K_{13} - K_{14} + K_{24} - K_{23}}{4}$$

In the above equations the nomenclature is as before.

T_{13RQ} denotes the type unbalance corresponding to the relative exposure between the types giving type unbalances of T_{13} and T_{RQ}.

The D terms in the above equations are generally not negligible compared with the S terms. This fact requires that in designing, for example, transpositions between side circuits in two different phantoms, it is necessary to take account not only of the type unbalance of the relative exposure between the two circuits, but also of type unbalances formed by building up combinations of the typical arrangements of transpositions on the two side circuits and on the phantoms of which they form a part.

To be presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 27, 1918.

ELECTRIC POWER FOR NITROGEN FIXATION

BY E. KILBURN SCOTT

Abstract of Paper

Reference is made to propaganda against processes for making nitrates from air by those interested in keeping the Allies dependant on supplies of nitrate from Chili.

A tabular comparison is made of the operations involved in the *indirect* method and the *direct* method of fixing nitrogen. The indirect method involves the manufacture of carbide of calcium and its combination with nitrogen to form calcium cyanimid, from which ammonia and in turn nitric acid are obtained. The direct method merely consists in combining nitrogen and oxygen of the air in the electric arc.

In the direct method electric energy is the only factor, whereas by the indirect much plant of a very diverse and complicated character is required. Also there are difficulties in connection with the platinum catalyst necessary to convert ammonia into nitric acid. It is claimed that the direct method is better because of the simplicity of plant and of operation, and the possibility of working with off peak power. The suggestion is made that a number of plants for making nitrates by the direct arc process should be erected at existing power houses. Keeping the generating plant more fully employed would improve the power factor and reduce costs.

By making nitrate in a number of centers the transportation of same to the explosive factories would be reduced and the risk of interruption of supplies in case of accident or sabotage would be less than in having a few very large factories.

A diagram is given showing the layout of a battery of by-product coke ovens with an electric power house worked by the surplus gas and a nitrate from air plant to use the electricity. Figures are given showing that the nitric acid made by such a plant is about the right amount to combine with the ammonia to form ammonium nitrate, a compound in great demand at the present time for explosives.

ONE of the most powerful combinations in the world is that connected with the exploitation of Chile Nitrates, and to extend the uses of that material and regulate prices, etc., there is a Chile Nitrate Committee supported by the various interests concerned.

It was created for propaganda work amongst farmers and others, to facilitate the use of nitrate as a fertilizer but since the advent of air nitrates some attention has been given to discrediting the methods of fixing nitrogen from air. This has been

Manuscript of this paper was received April 25, 1918.

done partly by paragraphs in the press throwing doubt on the financial and technical success of such methods, etc. German influence, working through political clubs and the press, also assisted the Chile nitrate propaganda while at the same time German scientists were being assisted in every possible way to develop air nitrate processes in their own country.

Years before the war, some of us saw that the question of supplies of Chile nitrate for the manufacture of explosives would be an important factor, and in 1911, at the Portsmouth meeting of the British Association, and later at a meeting of the Society of Arts in London, I sounded a note of warning.

To show how differently the Germans tackle these matters, I may say that when the war started, the German government appointed an electrical engineer, head of the Allgemeine Elektricitäts Gesellschaft, to expedite the manufacture of explosives.

On the other hand when the British government started its explosives department a lawyer politician was put in charge, and even afterwards, when a Minister of Munitions was appointed he also was a lawyer politician and had as second in command, a doctor of medicine.

The appointment of politico-legal persons to positions concerned with scientific and engineering matters, has been favorable to Chile nitrate and to German propaganda, in that it retarded developments that would have assisted to make the Allies independent of Chilean supplies.

Even after three and a half years of war, the Allies still remain practically dependent for explosives on supplies which have to be brought thousands of miles. This requires much shipping that might be used for other purposes and also occupies the attention of part of the Navy, in order to keep open the sea routes.

It is to their credit that certain scientists and engineers of this country not only saw the danger, but insisted on the authorities taking action by providing money to establish plants for the manufacture of nitrates.

At the same time, in this country as well as in England, there has been time lost, owing to certain parties maneuvering to obtain the adoption of their own process to the exclusion of others. In so large a field as nitrogen fixation there must necessarily arise numerous improvements in the various processes so that it is not possible today for anyone to gage or forecast their future relative economic values.

This is particularly the case with processes in which electrical energy plays a leading part for it is a sort of ingrained habit of the electrical engineer to simplify and revolutionize existing methods that they eventually become essentially electrical. The whole history of electrical progress, and especially of electrochemistry and metallurgy establishes that fact.

I consider that boards or committees dealing with nitrate problems should be largely made up of engineers who have expert first hand knowledge of electric power conditions and of apparatus, etc. Chemists, pure and simple are, useful but they should not have power to pass upon processes in a field which electrical engineering is capturing so completely as the production of nitrates.

In certain quarters there has been too great a readiness to listen to the tittle tattle of propaganda such as hinted of above.

I feel that the merits of the arc flame process for making nitric acid have not been adequately and sympathetically considered, and this paper is written with the special object of stating them. I wish also to remove the misconception that the arc flame process is dependent on water power and that it can only be installed economically on a very large scale. The matter is one of special interest to electrical engineers because the process is essentially an electrical one.

Method of Making Nitric Acid

One method of producing nitric acid from air which I call the *indirect* method is first to make carbide of calcium, then treat it with nitrogen to form calcium cyanamid, from which ammonia and in turn nitric acid are obtained.

Another method is the *direct* which, consists in combining nitrogen and oxygen of the air directly in the electric arc to form nitric acid.

Those interested in the indirect method have drawn comparisons between it and the direct electric arc process, with the object of showing that the indirect is the better. A tabular comparison of the operations involved in the two processes should thus prove of interest, since it is the only way in which a fair comparison can be made.

It is frequently stated that the amount of electric energy required for a given quantity of nitric acid produced by the indirect process, is less than that required by the direct, and this is put forward as a strong argument in favor of the indirect method. The

only way to compare two operations is to take into account *all* the factors which go to make up the total cost, and appraise each one at its proper value.

	Indirect Method Employing Calcium Cyanamid to Make Ammonia and Oxidising the Ammonia to Acid by a Catalyst	Direct Method Employing the Arc Flame Furnace Only
Factories	1. To make calcium carbide. 2. To make cyanamid. 3. To make nitric acid.	1. To make nitric acid.
Operations	1. Burning limestone. 2. Grinding lime. 3. Grinding coke or anthracite. 4. Mixing lime and carbon in correct proportions. 5. Making calcium carbide in electric furnaces. 6. Grinding carbide to fine powder in neutral atmospheres. 7. Making liquid air to produce nitrogen. 8. Packing calcium carbide into retorts. 9. Making calcium cyanamid by adding nitrogen and by use of electric resistors. 10. Emptying cyanamid from retorts. 11. Grinding cyanamid to a fine powder. 12. Hydrating cyanamid to rid it of carbide. 13. Making superheated steam. 14. Treatment of cyanamid with steam in autoclaves to produce ammonia. 15. Oxidation of ammonia to produce weak nitrous gases by means of a catalyst. 16. Absorption of gases in towers to produce acid.	1. Blowing air through electric arc flame to produce nitrous gases. 2. Absorption of gases in towers to produce acid.

	Indirect Method	Direct Method
Raw Materials	1. Lime.	1. Air.
	2. Coke.	2. Metal electrodes.
	3. Carbon electrodes in carbide furnaces.	3. Water.
	4. Carbon resistors in cyanamid retorts.	
	5. Pure nitrogen.	
	6. Superheated steam.	
	7. Air.	
	8. Water.	
Electric Energy for	1. Carbide furnaces.	1. Arc flame furnaces.
	2. Grinding carbide.	
	3. Cyanamid retorts.	
	4. Grinding cyanamid.	
	5. Heating catalyst.	
	6. Motors for power, etc., including several cranes.	
Skilled Labor for	1. Carbide furnaces	1. Arc flame furnace.
	2. Cyanamid retorts.	2. Absorption plant.
	3. Packing cyanamid.	
	4. Grinding machinery.	
	5. Making pure nitrogen.	
	6. Making ammonia.	
	7. Catalytic process.	
	8. Absorption plant.	

If two processes are to be compared as regards one factor only, then it may with equal justice be claimed that the electric energy represented by a few motors and lights required for a plant making acid from sodium nitrate, is less than the electric energy required by all other processes for making acid. Such a statement does not prove anything, and yet it is similar to the one put forward by the advocates of the *indirect* process.

Obviously the cost of plant using the indirect method, will be very much greater than that in the case of the direct, for if we assume that the cost of a carbide furnace and its accessories is about the same as that of an air nitrate furnace with its accessories, then, the indirect process embraces in addition:

1. A complete plant for making cyanamid.
2. A liquid air plant for making pure nitrogen.
3. Powerful machinery for grinding the carbide and the cyanamid.
4. Steam boilers and autoclaves for making ammonia.
5. A complete catalytic plant for oxidizing the ammonia to nitric acid.

In the indirect method it is essential to have all the materials, gases, etc., absolutely pure, for example at the cyanamid works at Odda in Norway it was necessary to carry a pipe up the mountain side so as to ensure a supply of pure air to the liquid air plant.

When carbide is converted into cyanamid some of the formre remains unchanged and in order to obviate danger of explosion a special treatment of the mixture is necessary to ensure a total decomposition of the remaining carbide.

Platinum is usually employed and in order to "reactivate" it, it is necessary to subject it to an acid treatment and eventually to remelt, in which process it is impossible to avoid loss of this expensive metal.

Russia is almost the sole source of platinum, and whilst our Ally, could be depended upon. Today with Germany practically controlling that country, the position is serious. The various allied Governments have had to commandeer platinum as it is essential for several war purposes. With utmost deliberation and foresight the Germans are working to control the worlds storehouse of platinum in the Ural Mountains and any processes which depend upon this rare metal are going to be very seriously handicapped. I consider that those who have had a hand in starting new processes dependent on platinum are very blameworthy. Politicians cannot be expected to know these things but those who do know should inform them.

By the direct method the cost of air is nil, and the cost of water is practically that of pumping. On the other hand, the materials required in the indirect method are very expensive and especially difficult to obtain at the present time. Over three fourths of the cost of working the indirect process is represented in materials liable to price fluctuation. These are now much higher than before the war, and will remain at the higher level after the war.

In the direct method less than one-fifth of the total cost is represented in materials dependent on market rates, and the principal item of cost, namely electric power, will, if anything, tend to come down in price.

The direct method is very simple to operate, whilst the indirect requires much skilled and unskilled labor, and some of the operations are dangerous to health. Therefore, the more labor demands increase, the more will the indirect method be handicapped in this respect. There are many separate links involving exact operating, to make the whole run smoothly and the slightest hitch in connection with any one link necessarily holds up the whole system.

The manufacture of cyanamid has to be carried out in retorts of relatively small size involving much labor to set up, etc. This is to enable the nitrogen gas to penetrate to all parts of the con-

tained carbide. The times of the reaction and the cooling down, etc., are definitely fixed and it is quite impossible to work the process with off-peak power; also should there be an accident or failure of current for a time there is every chance of ruining both the cyanamid retorts and the carbide furnaces.

All nitrate processes have a military bearing as regards preparedness, in which is involved the question of transportation. The heavy and bulky raw materials necessary for the indirect process places it at a serious disadvantage from this point of view. Especially at the present time when the railways are so congested; with the direct process there is no carriage of raw materials.

The indirect process has been strongly advocated in that after the war, cyanamid will be much used as a fertilizer. On the other hand a large number of objections have been voiced against such use, as witness the following extract from a book by Dr. Brion: "Cyanamid cannot be used with a large number of soils such as very sandy or moor soils, or with such soils as tend to become acid. Further it cannot be used for growing tobacco nor for some kinds of fodder. It is useless as a top dressing and can be applied only in dry weather when it must be plowed in at once. Cyanamid attacks the eyes of men handling it."

Whilst some of these objections may have been overcome by making the cyanamid granular and probably also some of them are over-emphasized, it still remains true that cyanamid is by no means as good a fertilizer as nitrate.

The effect of the calcium in calcium cyanamid in the presence of moisture is to cause the reversion of phosphoric acid and therefore it can only be used in limited quantities in a combined fertilizer.

In a legal action in the State of Maine between the Armour Fertilizer works and Ellis Logan, in April, 1916, there was sworn testimony that calcium cyanamid destroyed a crop of potatoes and that not more than 60 to 70 lb. of it should be used per ton of fertilizer. I believe that is only one specific instance of the general situation.

Electric Power

As a basic load for a power house the direct arc process presents the advantage that it can be established anywhere, because the raw materials being only air and water, considerations of transportation do not enter into the situation.

It is particularly suitable for off-peak or off-season loads, for there is no fused material to solidify, and little to deteriorate in case of stoppage. Some of the furnaces can be switched on and off like an arc lamp, without detriment to brickwork or structural details, or to the process of manufacture.

As there seems to be some doubt as to the possibility of running arc furnaces intermittently on a commercial scale, I would mention that about seven years ago a nitric acid factory was built at Legnano, Italy to utilize 10,000 horse power, especially during the night (see Utilization of Atmospheric Nitrogen by T. H. Norton p. 68). Of course this plant has been considerably extended especially since the war. I am also credibly informed that in Germany there is a very large arc process plant working with off-peak power. At any rate there is no difficulty in doing it, whereas it is impossible to work intermittently with any other method of fixing atmospheric introgen.

In some ways, it is an advantage to run a plant for 8000 or less hours per year, instead of the full number, because the spare time can be conveniently used for renewals and repairs, Less spare plant is thus required and the plant can be operated by two shifts of men.

Because the plants in Norway are very large and only use hydroelectric power, a mythology has grown up, that the arc flame process can only be worked commercially on a very large scale, and with water power. As a matter of fact it is well worth while to build plants of 10,000 kw.

As a matter of fact hydroelectric power may be a disadvantage because of its distance from industrial centers, for either the factory has to be placed in an out of the way position, or else the power has to be transmitted over a long transmission line. I am of the opinion that electrochemical factories should be placed near the power supply, and the ideal position is alongside the power house especially if off-peak power is used.

In a national emergency it is surely better to bring into immediate use all the surplus equipment that already exists, than to start building new power houses, whether hydraulic or steam, and seeing that the direct-arc flame process is suitable for working with off-peak power, I suggest that a number of nitrate plants be forthwith erected at existing power houses.

By erecting say, ten or more nitrate plants of say 10,000 kw. each at power houses in places near where nitrates are required there would be considerable saving in transportation; early de-

liveries of nitrate could be made. Further there would be less risk of temporary interruption of supplies in case of accident or sabotage.

As a matter of fact there are power houses which could easily spare more than 10,000 kw. for over 20 hours a day and through the week end. Also there are power houses fully equipped with steam plant which are now standing idle. In the present crisis they might just as well be brought into use even if the cost of generation is high.

In some power houses the load factor might be doubled and this would have the immediate effect of reducing costs but there has been far too much shilly shallying consideration given to questions of cost. With U-boats on the high seas trying to stop supplies of Chile nitrate, the railways congested with traffic and electrical engineering works making ammunitions, what is the use of discussing power costs. The thing to do is to jump in and make full use of plants already installed.

Recently much has been heard of the suitability of Muscles Shoals, Alabama as a site for the manufacture of nitrates because of the water power which is being developed there, but it will take at least four years to complete these hydraulic works. In the meantime a large steam power house is being built in order that the cynamid process may be put in to early operation This includes a 60,000-kw. turbo generator and should anything happen to it the nitrate plant would be stopped as the various steps of the indirect cyanamid process are so interlocked.

Viewed from this standpoint it would seem to be better in every way to have the manufacture of indispensable materials for explosives manufactured in a number of smaller plants, in widespread centers and by other processes than the indirect.

Coke Oven and Nitrate Plants

At the present time ammonium nitrate is required in very large quantities for burster charges for shells, torpedoes, mines grenades, etc. This is made from two components, viz., nitric acid and ammonia, both of which are difficult to transport, the first because it is a corrosive acid and the second because in every ton of aqua ammonia there are about 2½ tons of water. An industrial process capable of furnishing electric energy as well as a supply of ammonia would be ideal, and it so happens that this is the case with a regenerative coke oven plant. Half

the total gas made is available and this can be easily turned into electric energy whilst at the same time the nitrogen contained in the coal provides about the right amount of ammonia necessary to combine with the nitric-acid made from the electric energy by the arc flame process.

In order to show how ideal such a system is for making ammonia nitrate, I have prepared the diagram Fig. 1.

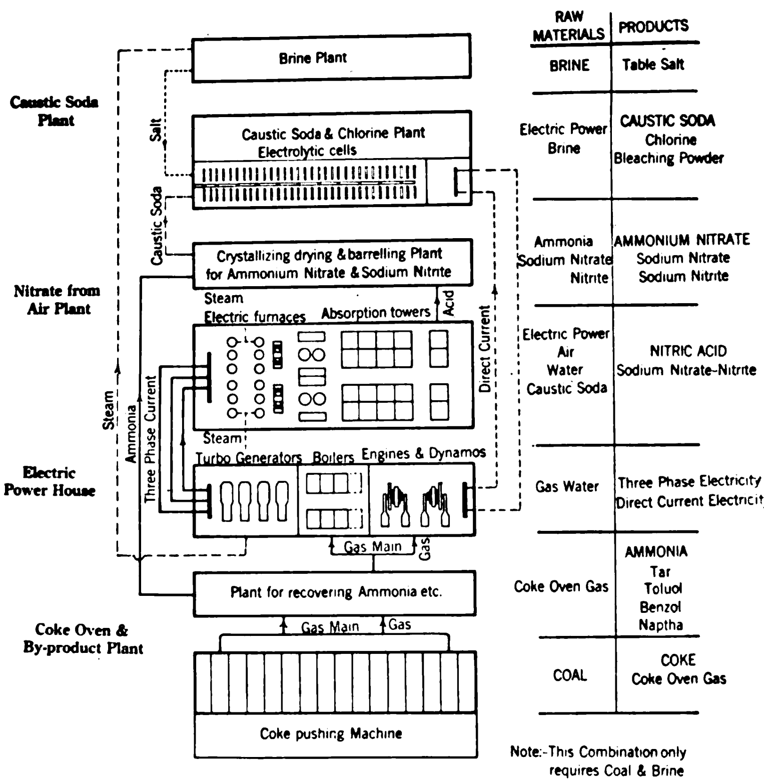


DIAGRAM LAYOUT OF NITRATE FROM AIR PLANT WITH ELECTRIC POWER HOUSE USING COKE OVEN GAS

The scheme provides for a combination of a battery of coke ovens with an ammonia recovery plant together with an electric power house in order to utilize the surplus gas. Alongside the power house, there is an electrochemical plant for the manufacture of nitric acid from air by utilizing the three-phase high-tension current. A nitrate house is provided for the purpose

of combining the ammonia from the coke ovens with the nitric acid from the elctrochemical plant.

It happens that the by-product of the acid factory is sodium nitrate-nitrite, which is made by combining the gases remaining from the acid towers with caustic soda or soda ash. Electrolytic cells may be laid down as shown in the diagram for purpose of making caustic alkali from brine.

It will thus be seen that the complete project requires only two raw materials viz., coal and brine, and on the other hand, the products which can be made are coke and ammonium nitrate together with toluol, benzol, naptha, tar and sodium nitrate-nitrite.

If electrolytic cells are used there are also the products chlorine and bleaching powder. The chlorine can be combined with the benzol to form chloro-benzol which is an important intermediate in the manufacture of dye-stuffs as well as in the manufacture of picric acid.

From the point of view of efficient management, and of elimination of transportation charges, the combination is unique, for the ammonia has only to be piped a few yards to the nitrate house and there is no carriage of acid.

As a cheap supply of coal is indispensable for the project, it would be well to locate the plants at industrial centers where this raw material is readily available and which in all probability would be locations where transportation charges are low.

In order to show what can be done with a coke oven plant the following particulars will be of interest. I take a Koppers type of oven as being the best known.

Quality of Coal	Tons per charge	Hours coking time
Low volatile coal	13½	18
Mixture containing 80 per cent high volatile 20 per cent low volatile	12½	16½
High volatile coal	11½	15

A battery of ovens varies in size but we may as well take a round number of 100 for which the average yields are as follows:

Number of ovens	100
Tons of coal per oven	12½
Hours coking time	16
Total yield of coke	72 per cent
Yield small coal and breeze	5 per cent

Net yield good coke	67 per cent
Ammonium sulphate per ton of coal	25 lb.
Reckoned as ammonia per ton of coal	6½ lb.
Tar per ton of coal	9 gal.
Light oil per ton of coal	3 gal.
Total gas per ton, of coal	11,000 cu. ft.
British thermal units	550 per cu. ft.
Surplus gas	55 per cent
Surplus gas per ton of coal	6,000 cu. ft.

Such a battery of ovens, each of which distils 12½ tons of coal in 16 hours, will deal with

$$\frac{100 \times 12.5 \times 24}{16} = 1{,}900 \text{ tons per day}$$

Assuming 6000 cu. ft. of surplus gas per ton of coal and 550 B.t.u. per cu. ft. the total heat value per hour will be

$$\frac{1900 \times 6000 \times 550}{24} = 260{,}000{,}000 \text{ B.t.u.}$$

If employed in gas engines using 13,000 B.t.u. per h.p-hr. the power will be

$$\frac{260{,}000{,}000}{13{,}000} = 20{,}000 \text{ h. p., or say, } 14{,}000 \text{ kw.}$$

If steam boilers and turbines are used instead of gas engines the power will be less so to be on the safe side, we will take the round figure of 10,000 kw.

We will also assume that electric furnaces utilizing 10,000 kw. for a whole year, can produce 6,300 tons of 100 per cent acid. Nitric acid capable of furnishing theoretically 8000 tons of ammonium nitrate as indicated below:—

	NH_3 +	HNO_3 =	NH_4NO_3
Molecular weights	17	63	80
In short tons	1700	6300	8000

Allowing 25 lbs. of sulphate of ammonia or 6½ lb. of ammonia per ton of coal, a total consumption of 1900 tons of coal per day should give.

$$\frac{1900 \times 365 \times 6.5}{20{,}000} = 2250 \text{ tons per annum}$$

It will thus be seen that there is plenty of ammonia to combine with the acid made by the surplus gas, even if a higher yield of acid is allowed per kw-yr. and more power is generated.

I purposely leave out of discussion, questions as to types of

nitrogen fixation furnaces and of yields obtained. I may say, however, that it is not right to assume that yields are limited to those usually obtained from certain well known furnaces which must of necessity work with single-phase current.

The amount of ammonium nitrate will be less than the theoretical figure because the efficiency of the reaction is not 100 per cent, also it is usual to convert a certain amount of the gas into sodium nitrate-nitrite. A safe figure would be 7000 tons and at this rate it can be shown that with electric energy at 5 mils per kw-hr. and ammonia at 13 cents a pound, the ammonium nitrate can be made at less than half the price the Government is now paying.

In order to show how large a business the nitrogen industry has become, the following figures (compiled by Dr. Paul J. Fox) give the nitrogen balance sheet for the United States for 1917.

	Tons of 2,000 lb.	Tons of Nitrogen
IMPORTED SUPPLIES		
Chile Saltpetre 95 per cent $NaNO_3$.	1,742,540	272,880
Ordinary saltpetre, potassium, nitrate	4,609	645
Ordinary saltpetre and gunpowder containing 75 per cent KNO_3..	1,500	210
Ammonium sulphate..............	8,135	1,725
Ammonium chloride..............	1,073	280
DOMESTIC SUPPLIES		
Coke oven ammonia—NH_3........	113,760	93,625
Gas works ammonia—NH_3........	12,500	10,288
Calcium cyanamid at 20 per cent nitrogen	12,800	10,534

NITROGEN EXPORTED

	Tons of 2,000 lbs.	Tons of Nitrogen
Nitric Acid, 15 per cent Nitrogen...	486	73
Picric Acid, 18 per cent Nitrogen...	26,610	4,790
Dynamite, 12 per cent nitrogen	8,962	1,255
Gunpowder and smokeless powder, 13 per cent nitrogen............	223,270	29,025
Ordinary saltpetre................	875	123

In addition to the above, there are also about 8800 tons represented nitrogen in the following items which are the figures for 1917.

	Value
Loaded cartridges.......................	$42,000,000
Fuses....................................	34,000,000
Shells and projectiles......................	74,000,000
All other..................................	202,000,000
Total	253,000,000

It will be noticed that ammonium nitrate is not included in these figures, but I assume it would be about 50,000 tons for 1917.

In Great Britain the consumption of ammoniun nitrate is now probably 400,000 tons a year, and the production here will have to be at least as much. To make this, the theoretical proportion of ammonia required is about 85,000 tons and of nitric acid about 315,000 tons.

It will thus be seen that the coke oven plants in the country could supply all the ammonium nitrate required if they were put onto the job.

Until recently most coke oven ammonia was converted into sulphate, but owing to the war demand for nitrate, more and more of it is being made into aqua-ammonia of about 29 per cent strength. In some cases this is being transported many hundreds of miles prior to conversion into ammonium nitrate and since each ton of ammonia necessitates the transportation of about 2½ tons of water, the bearing of this, on the present railway congestion is at once apparent. Tank cars have to be used and they must return empty, so the freight on the actual ammonia carried is extremely high.

There are many coke ovens of the wasteful bee-hive type in operation, which do not recover by-products and the replacement of these by modern coke-ovens would be a great immediate economic gain and meet the war conditions better than the building of large dams for water power.

In the present emergency coke ovens are of great value because they give coke for making steel, gas for power purposes, ammonia for nitrate manufacture, and toluol and benzol for explosives.

After the war ammonium nitrate will be in demand for fertilizer as well as for safety explosives and other purposes. The high percentage of nitrogen which it contains viz., 35 per cent and the ease with which it can be converted into other compounds makes it especially useful for conveying nitrogen in the fixed form over considerable distances.

It is more profitable to make nitrate than sulphate, because, pound for pound, the nitrate contains nearly twice as much fixed nitrogen and the nitrogen commands a higher price per unit when in the form of ammonium nitrate.

To be presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 26, 1918.

SPLIT-CONDUCTOR CABLE—BALANCED PROTECTION

BY W. H. COLE

ABSTRACT OF PAPER

Primarily, this paper is intended to be a brief history of the principal experiences of the Edison Electric Illuminating Company of Boston in the design and application of selective balanced-protection schemes to parallel connected transmission conductors.

Split-conductor cables are discussed at considerable length, both as to design and operation. Paired ordinary conductors also are discussed, and their relation to so-called split-conductors pointed out.

Special apparatus and devices required in connection with current-balancing schemes are illustrated and discussed.

A partial nomenclature is proposed, to assist in clearing the way for intelligent discussion and a uniform understanding of the general subject of current-balance protection for paired conductors.

A schedule of installations in the Boston system is given in order that the extent of the work described may be visualized, supplemented by a description of the results obtained in actual operation.

No general conclusions are drawn since the paper is of the nature of a report on progress; specific conclusions are drawn, however, in a number of cases where the evidence or experience appears to be reasonably conclusive.

A mathematical discussion of a number of reactive end-impedance devices, by Professor C. A. Adams, is appended.

IT IS not the intention to offer this paper as a complete treatise on the general subject of balanced protection for lines and apparatus. It seems advisable to deal as briefly as possible with some of the experiences of the Edison Electric Illuminating Company of Boston in its pioneer work. Only such detailed description and discussion as appears necessary to explain the scope of the work, will be attempted. Such a recital may be of general interest, and of some value to transmission engineers. It is submitted with the hope that a general discussion will follow, disclosing experiences of other engineers along related lines.

Some years ago our engineers became convinced that line protection devices, which function with respect to time, to value of current, or to direction of power flow, were, in an extensive

and rapidly growing transmission network, exceedingly difficult, and frequently impossible to adjust and maintain in such a relation one to another, as to provide for the automatic disconnection of any faulty element, without simultaneously permitting the disconnection or shutdown of elements not themselves involved in the fault.

Following this recognition of the inadequacy of such protective gear, careful investigation was made of fault discriminating systems then in use abroad. As a result our company determined to make use of one or more methods based on the current balance principle. Up to that time the input-output method commonly known abroad as the Merz-Price system, seemed to be the most popular, although the older idea of balancing conductors in pairs also had seen some application.

Our company at first gave serious consideration to the input-output method, as it appeared to have enjoyed a considerable degree of success. Before we were prepared to install this scheme in connection with transmission cables, however, attention was directed to a reassembly of the older proposition of balanced pairs, which consisted of an arrangement of paired conductors in the form of a special cable. Since the pairs were to operate parallel connected at each end, and therefore with substantially no potential between the members of a pair, the amount of insulation between them could be comparatively light, and, consequently, one belt of primary insulation only, was required for each two paired conductors. This resulted in a cheaper and more compact transmission unit, for a given capacity, than two separate or independent primary insulated conductors, of equivalent capacity. It also appeared to be a less costly proposition than the "input-output" scheme, especially when the cost of duct space for the necessary pilot cables was considered.

Weighing the advantages of the two methods, led to a decision to make one or more installations of the special form of paired-conductor cable. This type of cable has now become quite well known as the "split-conductor" type. This name may not be the most expressive, since the arrangement is obviously not so much the splitting of conductors, as it is the assembly in an economical manner, of two separate conductors to be operated as paired conductors in a balanced protection scheme. It may be, however, desirable to perpetuate it when referring to any arrangement of paired conductors assembled

and operated with a common primary dielectric. The nomenclature used herein is, therefore, based on an assumed division or splitting of conductors into an equivalent arrangement.

While paired conductors have been arranged or proposed in forms other than the "concentric twin," Fig. 1 and Fig. 2, such as the so-called "D twin," Fig. 3, and the "sector twin" Fig. 4, no form other than the "concentric twin" will be speci-

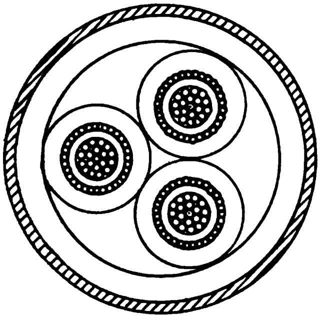


FIG. 1

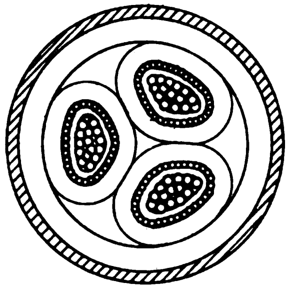


FIG. 2

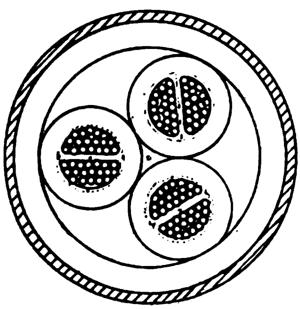


FIG. 3

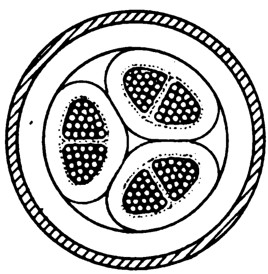


FIG. 4

fically considered herein, since this form, at least up to the present moment, seems to embody more desirable features than any other.

As a matter of historical interest, it may with propriety be stated here, that the generic type of balanced protection hereinafter discussed, had its birth about 17 years ago, when it was suggested by Mr. E. M. Hewlett of Schenectady, N. Y. A form of balanced protection for paired conductors also was proposed

by Mr. Leonard Andrews of Hastings, England, in 1902, embodying the first use of the differential reactor hereinafter described. Balanced protection schemes are, therefore, relatively old suggestions.

In order that the nomenclature used in this paper may be clear in meaning, it seems advisable to define some of the terms particularly applicable to "split-conductor" cable, and to balanced protection, as follows:

1. *Split-Conductor* refers to a conductor divided into two strands separated from one another by comparatively thin insulation, the strands assembled in various shapes and surrounded by insulation commensurate with the operating voltage of the system in which it is to operate.

2. *Conductor Member* refers to one of the conducting strands of a split-conductor.

3. *Primary Dielectric* refers to the dielectric surrounding an assembled group of conductor members, also to the dielectric surrounding an assembly of split-conductors.

4. *Secondary Dielectric* refers to the dielectr c between the conductor members. It is also somet mes called "split-insulation."

5. *Impedance Differential* refers to the degree of unba'ance between the impedances of the current paths of a split-conductor line. It is usually expressed as the percentage, which the difference between the impedances, bears to the impedance of one of the paths.

6. *Resistance Differential* refers to the degree of unbalance between the resistances of the members of an indiv dual cable section. It is usually expressed as the percentage, which the difference between the res stances, bears to the resistance of one of the members.

7. *Normal Differential Current* refers to the vector differential current in an unfaulted split-conductor line. It is a direct measure of the "Impedance Differential." Its value is directly proportional to the total current and, therefore, reaches its maximum when a through short-circuit occurs.

8. *Tripping Differential Current* refers to the minimum value of differential current to which the relays are responsive.

9. *Through Short-Circuit* refers to a short-c rcuit, current to which is fed through any line under consideration. Such current flow is usually termed "through short-circuit current."

10. *Balancing or Differential Transformer* refers to the transformer devices used for comparing the currents in the conductor members, whereby a current proportional to the vector differential current is derived and circulated through the relay circuit.

11. *End-Impedance or End-Reactor* refers to an impedance device used to insure the creation of tripping differential current at either end of a faulted line, when the fault is at or near the opposite end of the line.

12. *Split-Con act Switch* refers to a special form of switch or breaker having three fixed contacts per pole. These contacts are bridged by a three point yoke or blade. Two of the contacts are the term ni of the two members of a split-conductor. The normal closing movement of

the yoke or blade is such that the two conductor-member contacts are connected together, prior to their connection through to the bus. The switching sequence is exactly the reverse when disconnecting the line from the bus. The special function of the two conductor member contacts with the associated portion of the yoke or blade, is to introduce impedance into one path to an end fault, in order to insure a tripping current differential at the remote end of the line.

13. *End Fault* refers to a primary fault which occurs at or so near the end of a line as to require external devices, such as end-impedances or split-contact switches, to assist in creating tripping differential currents at the opposite end of the line, to insure disconnection at that end.

14. *Middle Fault* refers to a fault which occurs toward or at the middle of the length of the line, and which does not require end-impedance devices to insure tripping differential current at both ends of the line.

15. *Secondary Fault* refers to a fault in the secondary dielectric permitting current flow from one conductor member to the other.

16. *Arithmetical Balancing* refers to any method of balanced protection, wherein the effect of arithmetical difference only, between the compared currents, is operative to cause operation of the disconnecting gear.

17. *Vectorial Balancing* refers to any method of balanced protection, wherein the effect of the vectorial difference between the compared currents, is operative to cause operation of the disconnecting gear.

18. *Primary Capacitance* refers to the ordinary capacitance of a cable, such as is usually measured between one conductor and the other two bunched with the sheath.

19. *Secondary Capacitance* refers to the capacitance of the secondary dielectric, as measured between one conductor member and its mate.

20. *Transposition Joint* refers to any joint in which the members of one or more of the conductors are transposed.

21. *Straight Joint* refers to any joint in which the conductor members are held in the same relative position through the joint.

22. *Self Healing* refers to automatic healing of a punctured dielectric of the saturated type. With balanced protection, it often happens that a line is so rapidly disconnected, the fault current does not attain high values, nor does it carbonize much material. The result is a fairly clean puncture which immediately becomes more or ess completely sealed with hot oil or compound. If the potential is restored, it may take considerable time for the carbon particles to align sufficiently to cause repetition of the breakdown.

Split-conductor Cable Design

In split-conductor cable design, the engineering problems do not differ materially from those arising in ordinary cable design, except with respect to the division of the conductors. The chief additional considerations are, first, the dimensions and type of the secondary dielectric and, second, the permissible resistance and impedance differentials.

In order intelligently to design the secondary dielectric, it is

necessary to predict the probable value and duration of voltage stresses in the secondary dielectric, and when they may be expected to occur.

Cables are subjected to two classes of potential stresses, *i.e.*, those occurring during preliminary high potential tests, and those occurring in service. In general, only those occurring in service need be regarded as factors affecting cable design.

If, while in service, one member of a split-conductor becomes involved in a primary fault, say to earth, it is clear that some portion of the other member will for a finite period of time be at star potential above the faulty one. The rapidity of the change from this difference of pressure to a lesser one will depend upon the position of the fault with respect to the end of the cable; the frequency at which the greater one occurs, upon the type of system, *i.e.*, whether grounded or isolated neutral. The current flowing to the fault causes a differential *IR* drop from the terminals to the fault, and under some conditions this drop may result in establishing the star potential across the secondary dielectric. The linear amount of secondary dielectric affected by the higher stresses is dependent upon the position of the fault with reference to the length of the cable, and also upon the type of protective gear. The cumulative duration of stresses in the secondary dielectric for a given fault, is a function of the time required by the switch gear to disconnect the line from all sources of supply.

Since balanced protection schemes provide for rapid disconnection of faulted lines, it is a reasonable assumption that the higher secondary stresses will be more or less transient, for which a conservative allowance may be made when determining the thickness and grade of the secondary dielectric.

The possible effect of high potential testing of the primary dielectric, upon the secondary dielectric, requires further consideration. As stated above, the probable maximum secondary pressure due to operation, will not exceed the star potential of the system. Primary testing at double operating pressure may, if a primary failure occurs, result in stressing a part of the secondary dielectric with nearly four times the star potential. Occasional secondary failures, consequent upon primary failures due to such high tension testing may, therefore, be expected.

Since high potential testing after installation, if accomplished without failure, causes no abnormal stress in the secondary

dielectric, it is good commercial judgment to ignore the possibility of secondary failures resulting from primary high tension tests, and design for operating star pressure maxima only.

The local heat set up by the arc, upon the occurrence of a primary fault, and its possible effect upon the secondary dielectric, must be recognized. As the outer member usually is very thin, we may expect it to be destroyed rapidly at the point of fault, accompanied by extremely local and intense heating or burning of the adjacent secondary dielectric. On the same basis of reasoning, the secondary dielectric often may be completely destroyed at this point, even though insulating material substantially thicker than required to withstand star pressure is provided. This risk coupled with the fact that the maximum pressure across this dielectric is coincident with the point of fault, makes it extremely probable that if any breakdown of the secondary dielectric does occur, it will occur at this point. It is assumed, of course, that this dielectric is uniform in value throughout its length.

At the beginning, we were forced to consider a choice beween a secondary dielectric of sufficient thickness to withstand successfully at least the star voltage of the system impressed across a dielectric being rapidly weakened by the arc at a fault, and a dielectric of minimum safe thickness from a mechanical standpoint, but of more than ample value for the normal operating voltage between conductor members. A choice of the first meant more expensive cable, while the second involved the danger of failure of the protective apparatus then available, to operate, if a secondary fault should occur. Our reasoning was, that the major portion of the secondary dielectric being substantially an idle investment under all normal conditions, any reduction in its thickness with consequent cheapening of the cable was justified, provided a form of protective gear could be devised whereby a failure of the secondary dielectric under any circumstance would not prevent prompt disconnect on of the line so affected.

The required form of gear subsequently became available, resulting in our standardizing, tentatively at least, a secondary dielectric thickness of 3/64 in. (1.19 mm.) paper for all round type, paper insulated, concentric split-conductors for 15,000 volt working pressure; 1/16 in. (1.58 mm.) paper for the same class of conductors for 25,000 working pressure; and 5/64 in. (3.96 mm.) paper for both 15,000 and 25,000 volt sector type,

paper insulated, concentric split-conductors. The additional thickness in the sector split type is due to what appeared to be mechanical necessity, *i.e.*, relatively sharp corners of the inner member, but it will undoubtedly be reduced as the art of manufacture improves. So far as our experience indicates, these thicknesses, if properly applied and thoroughly impregnated, with the conductor members free from mechanical defects, are ample to meet the conditions imposed by manufacture, installation, and service, with our standard form of protective gear.

For economic reasons it seemed better to assume some risk of a possible secondary failure, if the cable was also simultaneously involved in a primary fault and, therefore, bound to be disconnected, than to specify a secondary dielectric of a thickness calculated to withstand any possible stress to which it might be subjected, knowing that this maximum stress cannot occur except under primary fault conditions in the same line.

The impedance differential in a completed line must be considered jointly with the characteristics of the protective gear at the ends of the line, and its magnitude limited in accordance therewith, since if the maximum normal differential current is permitted to be high, thereby requiring a high setting of the relays, the means for creating the required value of tripping differential current under and fault conditions must be of corresponding magnitude. This limitation is of particular importance, if the end-impedance devices are the reactive type.

In order that the impedance differential of a completed cable line shall be as small as desired, it is first necessary to care properly for the resistance differential in the individual cable sections, by limiting its value, or preferably its variation from a predetermined percentage of the resistance of one of the conductor members. In other words, it is not so important that the conductors agree with each other to a small percentage, as it is that the ratio of their resistances, one to the other, shall not differ by more than a small percentage. Second, it is necessary that the conductor members shall be so transposed in the jointing of the cable sections as to insure that the resistance differential of the completed line is within the desired limits, which also provides for relative changes in temperature and resistance of the conductor members. Third, the reactance differential of the completed line must be kept down to the proper value, and it is ordinarily accomplished by the transposing required for resistance balancing. As it is a much larger differential in the

individual cable sections than the resistance differential, running as high as 40 per cent, it may be the larger component of the impedance differential. It is very necessary that this differential be carefully equalized in a completed line. Its value is fixed by the design of the conductor and, therefore, is out of the control of the manufacturer.

A reasonable specification for resistance differential of a cable section is that it shall not exceed two per cent when the nominal resistance ratio of the conductor members is required to be 100: 100. If it is desired to specify a nominal resistance ratio of, say 100: 102, then the requirement should be that the ratio of resistance of the conductor members shall be not greater than 100:101, nor less than 100:103.

Our experience, without feeder reactors, has shown that no material difficulty need be expected in getting completed lines to balance closely enough to permit the maximum through short-circuit current to flow, without setting up normal differential currents of tripping value. Short lines originating at a power plant are, of course, subject to the heaviest through short-circuits and, therefore, require the most care in balancing. While long lines do not require as close balancing, they are easier to balance than short lines, since the law of averages has a more effective application.

During the war, the copper wire market has been such, that considerable difficulty has been experienced by manufacturers, in securing copper drawn with the accuracy desirable for the production of well balanced split-conductors. This has forced the acceptance of cable not quite up to the pre-war standard, and has required more attention in the matter of transposing conductors, in order to secure the desired balance in the completed lines. It is the aim to secure impedance balance in completed lines to within one tenth of one per cent. With uniformly well balanced cable sections, this result is ordinarily secured with from one to five transpositions. The present practise is to use no less than three, dividing the line into four sections.

Relays for Current-balance Protection

Relays should be of the instantaneous overload type, operative on small amounts of energy, hand resetting, and the moving parts should have small inertia. Secondary devices are necessary to close the contacts in the tripping circuit. An auxiliary break in the tripping circuit should be provided, to be actuated by the

movement of the main switch mechanism. At least two manufacturers in this country have produced a satisfactory form.

The hand-reset feature is very important, since it constitutes reliable means of diagnosing the class of fault causing the operation of the protective gear.

Split-Conductor Protective Gear

Single line diagrams, Figures 5, 6, 7, and 8, show diagrammatically some of the forms of protective gear we have used or considered, the form shown in Fig. 5 now being fairly well standardized.

At the risk of some repetition it may be advisable to explain

Fig. 5

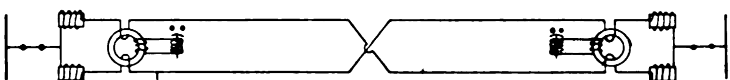


Fig. 6

Fig. 7

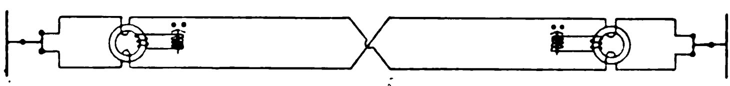


Fig. 8

the functions of the several devices shown, and discuss their different characteristics.

Common to all current-balancing schemes, an apparatus for weighing or comparing the currents in the paired conductors is required. The simple differential transformer is undoubtedly the best device for the purpose so far suggested. It consists of a standard type of current transformer core upon which are wound two primary coils, and one secondary coil. The primary coils are each respectively connected into one of the paired conductors in such a manner that under normal conditions of current flow, the coils will have equal and opposite effects upon the transformer core. If the primary coils are symmetrically disposed with reference to each other and to the core, no core flux will be set up, and consequently no current will flow in the secondary wind-

ing or relay circuit connected thereto. This is the condition that should obtain in a perfectly balanced split conductor under normal conditions. Should a fault occur, however, resulting in a disturbance of the normal current flow, then the balanced-current condition no longer obtains, and the difference of current values in the balancing transformer primaries results in establishing a core flux, and consequently a current flow in the secondary coil and relay circuit. As soon as this secondary current reaches the tripping value, the line is automatically disconnected at one or both ends, depending upon the position of the fault.

If all faults in split-conductor lines were confined to the middle of the length of the line, and to one conductor member only, tripping differential current would be assured at both ends of the line, even though the balancing transformer windings have very little impedance. Balanced current protection for paired conductors would, with such premises, be an ideally simple proposition.

Since it is absolutely necessary that the scope of the protection shall include the entire line, and extremely desirable that line disconnection be effected while the fault current is moderate in value, it is necessary to provide for what are known as end faults.

End fault protection is secured by the use of so-called end-impedance devices. Up to the present, two types have been developed, the reactive type and the non-reactive type. The reactive type has been proposed in several forms, three of which are shown in Figures, 5, 6 and 7. The non-reactive type as embodied in the three point per pole, or split-contact switch, Fig. 8, is the best known form of that type.

The theory of design and application of end-impedances is discussed quantitatively in the appendix to this paper, kindly prepared by Prof. C. A. Adams, but possibly a non-mathematical statement of the action of end-impedances may be helpful.

Assume that a failure occurs on a split-conductor, say at X, Fig. 6, affecting one member only, and for the moment that no impedance devices are in circuit, current will flow to or from the fault in two directions. At the fault end of the conductor, the flow through one primary coil of the balancing transformer will be reversed in direction with respect to the flow through the other primary coil. The current flowing in the first mentioned primary coil will be supplemented by current from the bus at the fault end of the conductor, if this bus is supplied by conductors

other than the faulty one, or possibly by current from synchronous apparatus connected to this bus. As the effect of the current flow in the transformer primaries becomes cumulative under conditions of relative reversal of current, it will be seen that ample secondary or tripping current is assured, with consequent disconnection of the conductor at this end.

From the above, it is clear that end-impedance devices *are not* required in order to insure operation of the disconnecting gear at the end of a line adjacent to an end fault. Since, however, an end fault is so located with respect to the distant end of the line, that the impedance in one current path from the bus at the distant end is substantially equal to the impedance in the other current path, the two components of the total current flow from the distant end will also be substantially equal, or at least they will not differ by more than the normal differential current, due to the inherent impedance differential of the conductor. As the relays must not operate on normal differential currents, it is necessary, in order to produce disconnection of the distant end, that the impedance of one of the current paths be altered with respect to the other, to the extent required to produce a tripping differential current in the balancing transformer at the distant end of the line.

Referring again to single line diagrams Figs. 5, 6, 7 and 8, and assuming an end fault on each, in a position equivalent to that shown in Fig. 6, and that all lines have been disconnected from the bus at the fault end in the manner previously described, the tripping differential current to insure disconnection at the distant end is assured as follows: In Fig. 5, the impedance in one current path to the fault becomes increased with respect to the other, due to the inclusion in the first path of both reactors. In Fig. 6 the impedance of one path is increased, due to the inclusion therein of three of the reactors as against one reactor only in the other path. In Fig. 7 the impedance of one path is increased, as the coils of the reactor at the fault end become cumulative in effect, due to relative reversal of current in one of them. In Fig. 8 the impedance of one path is increased (to an infinite value) by the actual opening of one path to the ault at the so called split-contacts.

Discussion of End-impedance Devices

While any of the types or forms shown, may be designed to provide adequate end-fault protection, they differ enough in their

characteristics to warrant considerable study before a choice is made. Choice should be governed by such considerations as, strength of design, space required for their installation, effect on regulation, effect on switchgear and switchboard design, rapidity of action, cost, etc.

The form shown in Fig. 5 is a special arrangement of simple reactors, rather than a distinct form. It was developed as a result of a desire to reduce, for a given duty, the reactor capacity necessary in the forms shown in Figures 6 and 7, and as well to insure that tripping current differentials would be set up, should both the conductor members be simultaneously involved in a fault.

The form shown in Fig. 6 has had but little application and is shown and described merely for purposes of comparison. It is obviously the least desirable of all forms shown, since the capacity required for a given duty, the space required for installation, and the effect upon regulation, are all of an order twice that inherent to the scheme shown in Fig. 5. The arrangement compares favorably, however, with that shown in Fig. 7, except as to iron losses and effect upon regulation.

The form shown in Fig. 7 is best described as the differential type. It was suggested by Mr. Leonard Andrews of Hastings, England, about 16 years ago, and has had some application abroad, not only in the form illustrated, but also when provided with a super-imposed secondary winding of proper impedance, thereby combining in one device, the function of anend-impedance with that of a balancing transformer. Its chief claim for consideration, is its non-inductive characteristic, and absence of core loss, under conditions of normal current flow. At first glance it might be thought that this form of reactor could be designed with a minimum amount of iron in its core, since under normal conditions there is no flux therein, and thus it could be smaller and cheaper than other reactive forms of end-impedance. This type of reactor is, however, not only differential itself, but cooperates differentially with its mate at the opposite end of the line, so that it is the vectorial difference between their respective effects that produces the required differential current for tripping purposes, at the end remote from an end fault. Any attempt, therefore, to reduce the size and cost of this form of reactor, by designing it for high saturation in the core when functioning for end-faults, is ineffective, since the volt-ampere capacity must be so increased, as to offset any hoped for gains due to reduction of iron in the core.

The form shown in Fig. 8, is merely the combination of a main and auxiliary switch, the auxiliary contacts when opened separating the conductor members from each other. This isolation of the conductor members from each other is, in effect, equivalent to the introduction of infinite impedance into one path to an end-fault, when the opening occurs as a result of such a fault. If the opening is due to regular switching, the members are merely simultaneously disconnected from the bus at that end.

As before stated, the selection of end-impedance devices should be made with several considerations in mind.

From the standpoint of strength of design, the balance appears to be in favor of the reactive type, since it has no moving or wearing parts requiring adjustment or renewal.

The space requirement usually is a local consideration. American practice in line cell construction, however, seems to favor the reactive type, as the reactors in most cases may be installed in what would otherwise be unused space. The additional space required for split-contact switches usually is more difficult to obtain.

The reactive type, in some forms, admittedly has some effect upon voltage regulation. In any form the effect is not serious, particularly when we consider the growing use of current limiting reactors. The form shown in Fig. 5, in the capacities used by the Boston company, has about the same effect upon regulation as would a one per cent feeder reactor. The reactive form shown in Fig. 7, as well as the split-contact type shown in Fig. 8, has no effect upon regulation.

The relative effect of the two types of end impedances upon switch gear design, is of great importance. It is obvious that standard types of switches are adequate with reactive schemes of end protection; no departure from standard practise or design is necessary. The non-reactive type however, as embodied in the split contact switch, requires a special design of switch. For small capacities this may not be a serious matter. For the large capacities, such as are common in American practise, a serious factor is introduced, *i. e.*, the necessity of designing the three-point or split-contact switches, so that all three breaks per pole have the same breaking capacity. This means that if a switch is of a type requiring two pots per pole for standard work, it will require an additional pot of equal capacity in order to corvert it into a split-contact switch.

It may be thought that since the so-called split-contacts each

carry only half the full line current, each might be designed for half duty, when compared with the third or so-called non-split contact. This assumption may be correct if the breaking of through current only is considered. If we assume a case when this type of switch is called upon to perform a dual duty, such as clearing an end fault, it can be shown that one of the split-contact breaks may have a duty exceeding that of the main or non-split-contact break.

Three-point or split-contact switches must be so designed that when being closed both conductor members shall be connected together before either one of them is connected to the bus. This must be assured, for if by chance one of them should be closed on a bus before the other, the instantaneous current flow through one conductor member only, will result in the simultaneous opening of the switches at both ends. In order to prevent such an occurrence, the relative position of the contacts and yoke or blades is made such, that the split-contacts will be connected together just prior to their joint connection to the bus or non-split-contact. The time interval between the two closures must be extremely short, in order to make the switch as effective as possible when rupturing current, since in opening, it is of great importance that all breaks per pole occur as nearly simultaneously as possible, in order to divide the breaking duty equally.

When a three-point switch operates on an end fault, the first break takes place at the main or non-split contact, closely followed by a break at the split-contacts. One of the split-contact breaks handles the component of the fault current flowing over the corresponding conductor member. The other split-contact break must handle the same component, plus any current flowing to the fault from the bus at the fault end of the line via the main break. The third or non-split contact break handles only the current flowing to the fault from the last mentioned bus. Thus it will be seen that one of the split-contact breaks may have a far greater duty to perform than any other.

Close analysis of the relative effect of the two types of end-impedances upon switch gear and switchboard design seems to indicate the superiority of the reactive form of end-impedances.

It has been claimed that the split-contact switch provides for more sensitive control, by virtue of its function of inserting infinite impedance into one path to an end-fault, thereby concentrating the fault current in one path only, at the end remote from the fault, thus creating at that end a greater tripping

differential current than would be obtained if reactive impedance devices of permissible size were used. This advantage is real only in a limited degree, and only if the fault current is so small, that when it is divided between the two paths to the fault, as must be the case if reactive end-impedances are used, the differential current is less than that required to operate the relay. Such small fault currents are encountered only when the fault is one to earth, in a system having small capacitance to earth. In such systems the use of the split-contact switch may be justified, particularly as such a system will ordinarily be of small magnitude. In systems of large magnitude, the cable systems also are usually extensive, and fault currents to earth are ample to insure tripping differential current with reactive end-impedance devices of reasonable size, even if the system is operated with an isolated neutral. Whenever the system is of such size or type as to justify the use of reactive end-impedance devices, disconnection of lines under end-fault conditions is effected more rapidly with such devices than would be the case with split-contact switches, since tripping differential currents are set up simultaneously at each end. With split-contact switches, a tripping current differential exists at the fault end, only, until the operation of the switch at the fault end, effects the introduction of impedance and the setting up of a tripping current differential at the opposite end of the line.

On the whole, it would therefore appear, that considering the rapidity of action, at least on large systems, the reactive form of end-impedance device has a distinct advantage over the non-reactive type.

The matter of relative cost of the two types is at present unsettled, as the development in this country has proceeded along reactive lines. It is believed, however, that taking all features into consideration, the cost differential will be in favor of the reactive type.

In order to check the soundness of our conclusions as to end-impedance devices, it has been thought advisable to secure some actual operating experience with the more promising forms. As a result, we have placed in operation during the past two years two installations of the split-contact type of end-impedance, Fig. 8, also we have on order for installation during the current year, two installations of the differential type of end-impedance. Fig. 7.

Balanced Protection for Ordinary or Independent Conductors

While the experience of the Boston company in the balanced protection of paired ordinary conductors has not been extensive, the work to date has been of great importance in establishing a basis for future practise. It is believed, therefore, that a few references to this phase of the subject are permissible at this time.

As it is obvious that there is no material difference in operating principle between the older and well known arrangement of paired independent conductors, and the so-called split-conductor scheme, any choice between the two arrangements tends to be an economic one. It is incorrect, as is sometimes done, to refer to the earlier method as a combination of ordinary conductors on the "split-conductor plan", since such arrangements were proposed many years previous to the split-conductor suggestion.

In general the protective apparatus already described is applicable to any combination of paired conductors, but when the paired conductors are situated each in different cables or lines, they are each insulated for the primary voltage of the system, and are, therefore, independently suitable for operation if for any reason the companion conductor is out of service. It follows, therefore, with any arrangement of paired conductors, other than actual split-conductors, that each of the paired conductors ought to and usually does have its own switches independent of those for its parallel associate. The use of individual switches for each conductor, moreover, results in a combination equivalent in most respects to the split-conductor arrangement, employing split-contact switches in place of end-impedances or reactances, so that end-faults are adequately provided for.

With a paired independent conductor line, it is not possible to secure and maintain the degree of balance inherent to a split-conductor line, for even were it possible to start out with a fair balance, conditions local to one conductor or the other would soon affect the balance to a considerable degree; for example, should one line of a pair be out for repair and the other remain in service, it is obvious that when put in parallel again, the difference in conductor temperature, which might then exist, would be the cause of a material impedance differential. If a through short-circuit should occur under such circumstances, the normal differential current would attain a very high value.

It is frequently desirable to consider paralleling, with balanced protection, conductors which differ materially in length, cross

section, or reactance. In such cases it is impossible to operate without correspondingly large normal differential currents. Since the end fault condition in such lines will be cared for by the independent switches for each conductor, the remaining necessity is that of compensating for the large normal differential currents set up by through short circuits, and with the same devices provide for selective disconnection, should faults occur that result in differential currents materially smaller than those due to through currents of maximum values.

If the ordinary type of overload relay is used in this class of balanced protection, the settings might need to be so high as to preclude operation on section faults, or in any event to prevent obtaining the maximum benefit of balanced protection.

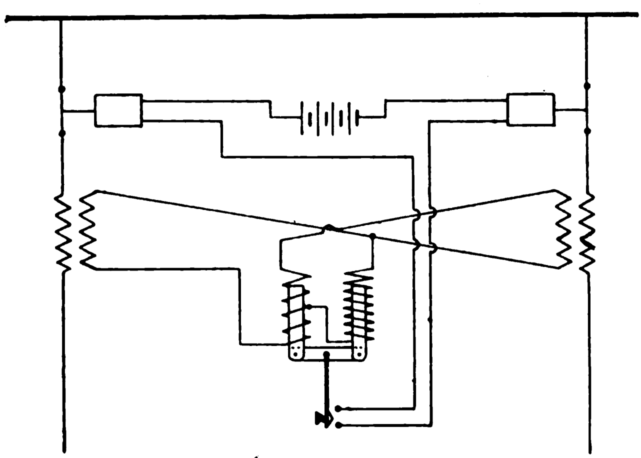


FIG. 9

In order to obviate the necessity of the high relay settings referred to, and at the same time to insure non-operation on through short circuits, relays have been developed both in this country and abroad, variously called, "biased relays", "percentage-balance relays", and "ratio-balance relays".

We have experimented for nearly four years with ratio-balance relays in the forms shown in Figs. 9 and 10. One pair of cables has been equipped with the form shown in Fig. 9, and has been in operation for over three years. In all cases of actual trouble, as well as under artificial fault conditions, the relays have functioned as predicted. While in the case of a single pair of lines, both are disconnected upon the occurrence of a fault in one, the other may be immediately put to work with straight instantane-

ous overload protection, since the relays may be arranged to so function for either line, when the companion line is out of service.

The form shown in Fig. 10 is particularly well adapted for use with groups of three or more parallel lines, in which case the relays may be electrically interlocked, so that the faulty line only is disconnected. When so applied, all the discriminative features of balanced current protection may be secured without the use of pilot wires, special cables, reactors, or split-contact switches.

The relays shown in Figs. 9 and 10 are so constructed, that the restraining force is proportional to the vector sum of two compared currents, and the operating force is proportional to the

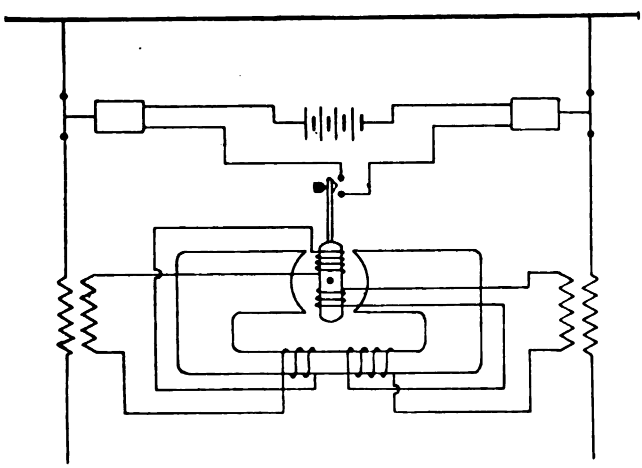


FIG. 10

vector difference between the compared currents. With proper winding ratios, the restraining effect will predominate until the normal relation between the compared currents is altered, due to a fault involving one of the conductors, to such an extent as to cause the ratio between the vector sum and vector difference of the compared currents, to exceed a predetermined limit. After this limit is reached, the operating force becomes predominative and the relay operates to disconnect the faulty conductor.

In order to choose the proper winding ratio for ratio-balance relays, it is necessary to know what the normal differential current will be under extreme conditions, and the percentage it bears to the simultaneous vector sum of the compared currents. Once adjusted for this normal relation, they cannot operate on

any value of through current, since the restraining force will then always be in excess of the operating force.

It is expected that upon the return of more normal conditions, further progress will be made in the application of ratio-balance protection to many of our standard transmission lines, by means of the devices described, or that may hereafter become available.

SCHEDULE OF SPLIT-CONDUCTOR CABLE INSTALLATIONS

The following table shows the installations of split-conductor cable now included in the Boston Edison Company system:

Line No.	Length in feet	Size	Voltage	Date placed in service
34-82	32,032	3x4/0	15,000	9- 4-13
31-76	24,054	3x1/0	25,000	8-31-14
26-76	29,273	3x.10 sq. in.	25,000	1-10-15
26-77	29,305	3x 10 sq. in.	25,000	5-16-15
47-76	41,478	3x4/0	15,000	11-14-15
47-75	41,478	3x4/0	15,000	11-30-15
31-77	24,069	3x.10 sq. in.	25,000	5-2 -16
16-99	7,599	3x4/0	15,000	8-25-16
* 49-98	4,742	3x4/0	15,000	9-10-16
* 49-95	4,742	3x4/0	15,000	11- 4-16
† 23-75	23,652	3x3/0	25,000	11-26-16
† 23-76	23,652	3x3/0	25,000	11-26-16
54-99	33,488	3x4/0	15,000	4- 7-17
* 43-59	32,205	3x3/0	25,000	4- 1-17
‡ 8-53	6,510	3x4/0	15,000	7- 8-17
‡ 8-54	6,510	3x4/0	15,000	7- 8-17
‡ 30-87	9,439	3x4/0	15,000	7-15-17
‡ 30-96	9,436	3x4/0	15,000	7-15-17
49-51	24,889	3x4/0	15,000	8-12-17
39-51	23,480	3x4/0	15,000	8-12-17
51-69	8,789	3x4/0	15,000	10-21-17
* 57-69	9,189	3x4/0	15,000	10-21-17
* 9-69	6,926	3x4/0	15,000	10-21-17
52-77	3,758	3x4/0	15,000	10-21-17
52-78	3,772	3x4/0	15,000	10-21-17
‡ 8-87	7,384	3x4/0	15,000	11-18-17
* 58-80	6,742	3x4/0	15,000	11-18-17
§ 53-53	7,754	3x4/0	15,000	
§ 53-54	7,754	3x4/0	15,000	
§ 53-70	3,516	3x4/0	15,000	
§ 21-72	10,202	3x4/0	15,000	
§ 21-73	10,202	3x4/0	15,000	
101-102	50,044	3x4/0	15,000	5-1-17
101-103	49,272	3x4/0	15,000	5-1-17
§101-104	48,733	3x4/0	15,000	
§101-105	48,691	3x4/0	15,000	
§103-59	11,552	3x3/0	25,000	
§ 20-59	21,841	3x3/0	25,000	

*Line not yet operating as split-conductor line.
†Lines at present operating in series with standard overhead lines.
‡Lines at present operating in series with standard conductor cable.
§Lines under construction to go in service 1918.

The total length of the above cables is 747,590 ft., or approximately 142 miles.

Lines 26-77 and 31-77 are connected in series at present and operate as one line, which is protected with the English form of split-contact switch gear, Fig. 8. Line 26-76 is equipped with a modification of the form of gear shown in Fig. 6. Lines 26-77 and 31-77 are to be separated during the current year and the new terminals are to be protected by the form of gear shown in Fig. 7, the opposite terminals remaining as before, protected with split-contact switch gear. The remaining lines are protected, or will be upon their completion as split-conductor lines, with the form of gear shown in Fig. 5.

Results of Operation

Not including the preliminary testing of lines under artificial fault conditions, to determine their characteristics, and to check the theory of operation, a sufficient number and variety of actual faults have occurred to demonstrate that full reliance may be placed on the system of protection, provided the fundamental requirements are met in its design, installation, and operation.

Line 34-82 has now been in service nearly five years and with the possible exception of one case, to be referred to later, the balanced protection apparatus in connection therewith has functioned in accordance with theory, each time the line has been involved in a fault, or when subjected to the stress of feeding through short-circuit current to faults external to the line. This line up to the present has been involved in one secondary fault (due to defective stranding of the outer member of one conductor), one end fault, and at least six through faults, three of which were bus short circuits at one terminal of this line.

Line 47-75 has been in service about two and one half years and has been involved in one fault, a primary fault in a joint, one conductor to earth. This fault created so little disturbance that the station operator was inclined to feel doubtful as to the need of the switches opening. Tests were made to locate the fault, but after a while the operators concluded that none existed, and the line was placed back in service. Outside of some static discharges of small volume at the line terminal, which soon ceased, the line appeared as good as ever. In order to clear up the action of the relays, it was deemed advisable to inspect the line. Men were sent over the line, opening manholes and examining the cable therein, with the result that a joint was found

with a hole blown through the lead sleeve, and fresh compound expelled in considerable quantity. The line was then promptly taken out of service, the faulty joint opened, whereupon it was found that one conductor had broken down to earth (corresponding to the relays that operated), but the fault had become healed due to hot compound running back into the wound after the line was disconnected. From all outward appearance, this line probably would have operated for years in its then condition. This case is described at some length, as such an occurrence may have been the cause of relays operating on our first line, No. 34-82, on one occasion without the cause ever having been determined. Self healing, therefore, may be expected. This experience, confirmed by results obtained in high potential testing, teaches that any case of apparently unnecessary operation should be carefully investigated before conclusions are drawn.

Line No. 31-77 has been in service about two years and has been involved in two faults. The first fault was due to a connector in a split-conductor pothead (English Type) becoming loosened, thereby increasing the impedance of one of the current paths and setting up a tripping differential. This caused several operations of the protective gear at each end before a diagnosis of the fault was made by our operators. The second fault was due to mechanical injury; a laborer driving a pick into the cable, grounding one conductor. The protective gear, (split-contact switch type) operating in the expected manner, isolating the line at both terminals. This line was again made alive, but was automatically disconnected again, since the fault was well established. The second operation resulted in breaking down both inner and outer members at the point of fault. The cable at the point of injury was laid on the solid system, *i.e.* armored, laid directly in the ground and covered with a two-in. (5.08-cm.) spruce plank. Repairs were effected by making a new joint in this location.

Line No. 31-76 has been in operation for nearly four years, but on account of defective joints and some dry spots in the secondary dielectric which were eliminated by high tension tests, it was not finally accepted until about three years after installation. In the course of making the many primary high potential (50,000 volt) tests, the secondary dielectric failed, due to the stresses set up therein as a result of primary failures, at four or five of the dry spots, also in two cases where the outer member was defective, once as to stranding (crossed strands) and once where

a strand had been brazed. This line could not be kept continuously out of service for testing, however, and on one occasion it was put back in service with the positive knowledge that an incipient secondary fault due to the testing above referred to was present in one conductor. No cable was available at the time to replace the faulted section, and as the fault was of very high resistance, it was decided to assume the risk of operating the line with balanced protection. This proved to be poor judgment on our part, since this fault developed in service to such an extent as to cause operation of the protective gear, the operation, of course, being perfectly normal for this type of fault. Unfortunately it occurred simultaneously with one of the outages of line No. 31-77 described above with which it was ring connected, and thereby contributed to an interruption of service. It was also unfortunate in this case that the first mentioned fault in line No. 31-77 had not been diagnosed and remedied at an earlier date. Such experiences, however, are of great importance as affecting future operating practise.

Line No. 54-99 was placed in service April 7, 1917 and within a few weeks afterward broke down between main conductors, resulting in its disconnection by the balancing gear. This fault occurred about one and one half miles from the power house. No disturbance was noticed when it was disconnected. Relays operated on two of the conductors, normally indicating a short between them. The operators put the line back again, however, and it ran about three minutes when it came out again, the same relays operating as before. The line was again made alive from a separate bus and generator at somewhat higher pressure than the system pressure, and remained in for four minutes, coming out for the third time upon the operation of the same two relays. This appeared to satisfy the operator that the line was faulty. The fault was located and removed, and was found to be due to a breakdown between the two phase wires corresponding to the two relays that operated in each of the three cases of disconnection. Since a short circuit of similar type and location, on a feeder protected with overload relays with as short a setting as one half second, usually makes itself known in a more or less violent manner, the performance of the protective devices on this line appears to be a substantial tribute to the efficacy of the balanced method of protection. In this case, it appears that each disconnection of the line was so rapid that the fault current built up to a value not materially greater than sufficient to create a tripping differen-

tial, and as soon as disconnection occurred the puncture healed up, due to the hot cable compound flowing into the wound. Undoubtedly enough carbon was suspended in the puncture path to gradually line up and bridge between phases, thus causing the second and third disconnection. It is a question how long this process could have been continued, but it would seem that two disconnections with the same relay operation in each case would have been conclusive evidence that a fault did in fact exist. Operators will undoubtedly require considerable experience in order to become convinced that quiet disconnection of transmission cables equipped with balanced protection is sufficient evidence of line failure.

As before stated, some trouble has been experienced by the manufacturers in getting perfect saturation of the secondary dielectric. In the case of the above mentioned primary fault in line No. 54-99, the condition of the secondary dielectric was not checked prior to putting the line back in service after repair, as is now our practise. It was soon found that one of the conductors involved in the primary fault had a secondary fault which resulted in the protective gear again operating. This secondary fault was located and removed, and was found to be due to defective stranding of the outer conductor member. The continued attempts to operate under the main fault condition stressed this defective point severely, and to an amount in excess of the stresses caused by the factory or installed test. It is significant that this secondary fault, as well as the two others (three in all) occurring in service to date, were located in defective construction, which permits of the reasonable assumption that none would have occurred if the construction had been perfect, and also that the values of secondary dielectric chosen for our service, are ample if properly incorporated in the construction.

It is perhaps unfortunate, but nevertheless unavoidable, that so much time is required to obtain extensive operating experience. It is obvious, however, that new cables are not expected to fail soon after having been carefully installed and tested.

REFERENCES

E. B. Wedmore, Automatic Protective Switch-Gear for Alternating Current Systems. *The Journal of the Institution of Electrical Engineers,* (London), p. 158, Jan. 15, 1915.

P. H. Chase, Discussion of "Split Conductor" Method of Relay Protection. PROC. A. I. E. E., p. 1399, Sept. 1916.

E. B. Wedmore, The "Split-Conductor" Protective System for A-C. Distribution. *The Beama Journal* (London). July 1917.

APPENDIX

BY COMFORT A. ADAMS

The purpose of this appendix is to show the relative amounts of reactance required to produce a given current differential in the three arrangements shown in Figs. 5, 6 and 7, under fault conditions, since it is only under these conditions that reactance is required.

In order to simplify the final general analysis assume that the added reactance has in each case a constant value. The effect of saturation will be considered later.

Let x equal the value of a single reactance in the arrangements of Figs. 5 and 6, and of the series or round-the-end reactance of one of the differential reactors of Fig. 7.

Assume that the switch at the fault end has opened—

Let $\dot{I}_1$ be the current from the far end to the fault, by the longer path, $\dot{I}_2$ that by the shorter path, both counted positive in the same direction from supply end to fault end, and $\dot{I} = \dot{I}_1 + \dot{I}_2$ the total fault current.

Let r_1 = the resistance of a single conductor member.

Let x_1 = the combined reactance of the two conductor members considered as a single conductor, in which the current is the vector sum of the two currents in the two members. This is sufficiently accurate for all practical purposes.

Arrangement of Fig. 5. Equating the potential drops from supply to faults by the two paths gives

$$\dot{I}_1 r_1 + 2jx\dot{I}_1 + jx_1(\dot{I}_1 + \dot{I}_2) = \dot{I}_2 r_1 + jx_1(\dot{I}_1 + \dot{I}_2)$$

$$\dot{I}_1(r_1 + 2jx) = \dot{I}_2 r_1$$

$$\frac{\dot{I}_2}{\dot{I}_1} = \frac{r_1 + 2jx}{r_1}$$

$$\frac{\dot{I}_2 - \dot{I}_1}{\dot{I}_2 + \dot{I}_1} = \frac{\dot{I}_d}{\dot{I}} = \frac{jx}{r_1 + jx}$$ where $\dot{I}_d$ is the differential current.

Or in numerical values —

$$\frac{I_d}{I} = q_d = \frac{x}{\sqrt{r_1^2 + x^2}}$$ where q_d is the per cent differential current.

Let $\frac{x}{r_1} = q_x$ then

$$q_d = \frac{q_x}{\sqrt{1 + q_x^2}}$$ or for a given q_d the necessary q_x is

$$q_x = \frac{q_d}{\sqrt{1 - q_d^2}} \tag{1}$$

Arrangement of Fig. 6. Equating potential drops to fault as before —

$$\dot{I}_1 r_1 + 3 j x \dot{I}_1 + j x_1 \dot{I} = \dot{I}_2 r_1 + j x \dot{I}_2 + j x_1 \dot{I}$$

$$\dot{I}_1 (r_1 + 3 j x) = \dot{I}_2 (r_1 \times j x)$$

$$\frac{\dot{I}_2}{\dot{I}_1} = \frac{r_1 + 3 j x}{r_1 + j x}$$

$$\frac{\dot{I}_2 - \dot{I}_1}{\dot{I}_2 + \dot{I}_1} = \frac{\dot{I}_d}{\dot{I}} = \frac{j x}{r_1 + 2 j x}$$

Or in numerical values —

$$\frac{I_d}{I} = q_d = \frac{x}{\sqrt{r_1^2 + 4 x^2}} = \frac{q_x}{\sqrt{1 + 4 q_x^2}}$$

$$q_x = \frac{q_d}{\sqrt{1 - 4 q_d^2}}$$

For purposes of comparison with Fig. 5 this result must be multiplied by two since there are twice as many reactors in this case. The relative magnitude of the reactance required will then be —

$$2 q_x = \frac{2 q_d}{\sqrt{1 - 4 q_d^2}} \tag{2}$$

Arrangement of Fig. 7. Equating the potential drops to fault as before

$$\dot{I}_1 r_1 + j \frac{x}{4} (\dot{I}_1 - \dot{I}_2) + j x_1 \dot{I} + j x \dot{I}_1$$

$$= \dot{I}_2 r_1 + j \frac{x}{4} (\dot{I}_2 - \dot{I}_1) + j x_1 \dot{I}$$

$$\dot{I}_1 \left(r_1 + \frac{3}{2} j x \right) = \dot{I}_2 \left(r_1 + \frac{1}{2} j x \right)$$

$$\frac{\dot{I}_2 - \dot{I}_1}{\dot{I}_2 + \dot{I}_1} = \frac{\dot{I}_d}{\dot{I}} = \frac{j\,x}{2\,(r_1 + j\,x)}$$

Numerically

$$\frac{I_d}{I} = q_d = \frac{x}{2\sqrt{r_1^2 + x^2}} = \frac{q_x}{2\sqrt{1 + q_x^2}}$$

or, for a given per cent differential current.

$$q_x = \frac{2\,q_d}{\sqrt{1 - 4\,q_d^2}} \qquad (3)$$

Conclusion. Referring to equations (**1**), (**2**) and (**3**) it appears that, for the same per cent differential current, the arrangements shown in Figs. 6 and 7 require the same total reactance, and in each case slightly more than double that required by the arrangement of Fig. 5.

As between Figs. 6 and 7, the latter has the advantage, since although the total reactance is the same, it is in two units for Fig. 7 against four units for Fig. 6, which will cost less and take up less space.

Since q_x and q_d are so nearly equal within normal range of values, these formulas supply a simple and convenient method of computing the approximate per cent reactance necessary for any particular differential current. For example, if q_d = 0.20 = q_x (approx.) (Fig. 5), and the normal ohmic drop on the line in question is 0.05, the full-load reactive drop due to the reactors will be 0.20 × 0.05 = 0.01 or 1 per cent of the line voltage. This is an outside figure.

For the case of Fig. 6, this will be twice as great or two per cent.

In the case of Fig. 7, the reactors offer no appreciable reactance under through conditions.

Saturation of Reactor Cores. The effect of the core saturation is to reduce the reactance of each reactor and also to distort the voltage drop across each reactor.

The reduction of reactance works differently in the three cases. In Fig. 5 it reduces the differential current in almost the same proportion; but in Figs. 6 and 7 the differential current is still further reduced by the fact that the reactance in the short-path to ground, which is undesirable as it reduces the current differential, is reduced less than that in the long path to ground, since its core is less saturated. This difference is however not considerable although, as far as it goes, it increases the advantages of the arrangement shown in Fig. 5.

Since the reactance of the reactors absorbs such a small part of the voltage of the system, the saturation of the reactor cores does not influence the current wave shape appreciably but does seriously influence the wave shape of the voltage drop across each reactor. The flux wave is flat topped and the voltage wave very peaked under conditions of core-saturation.

Under fault conditions this distortion results in very high-frequency harmonic voltages between splits or conductor members, which in turn produce high-frequency charging currents between conduction members, which in part reduces the voltage distortion by supplying harmonic exciting currents for the reactors.

This voltage distortion and consequent danger of breakdown of the secondary insulation will be approximately twice as great for Figs. 6 and 7 as for Fig. 5.

Under normal conditions this phenomenon appears to some extent in Fig. 5, but the currents are not large enough to approach the danger point.

These distortion effects can be considerably reduced by the employment of a gap in the magnetic circuit of the reactors.

To be presented at the 34th Annual Convention of the American Institute of Electrical Engineers. Atlantic City, N. J., June 28, 1918.

REACTANCE OF SYNCHRONOUS MACHINES AND ITS APPLICATIONS

BY R. E. DOHERTY AND O. E. SHIRLEY

ABSTRACT OF PAPER

Part I treats of the calculation and application of the armature self-inductive reactance of synchronous machines. A short, reliable method is given in the form of curves, Figs. 20A, B, C, making the calculation from design sheet data a matter of a few minutes. Table I shows a comparison of calculated and test values (obtained from saturation and synchronous impedance curves) for 138 machines, ranging from high-speed turbine generators to the low-speed engine type.

Three points were brought out during the investigation:

(1) That in polyphase machines, the armature self-inductive reactance, just as the armature reaction, is a polyphase, not a single-phase, phenomenon, and therefore the mutual induction of phases in a three-phase machine increases the effective self-induction of each phase by approximately 50 per cent over the single-phase value, while in two-phase machines, in which the mutual induction of phases is zero, the effective self-induction of the phase is the same for two-phase or single-phase operation.

(2) That the variation of armature reactance during the cycle, due to salient-pole construction, is practically eliminated in Y-connected, three-phase machines for the reason that the variation, consisting almost entirely of a third harmonic, is cancelled in such machines. This leaves, in effect, a uniform reluctance for the leakage flux emanating from the tooth tips.

(3) That in the familiar method of obtaining the armature self-induction from the saturation and synchronous impedance curves (*i.e.*, by subtracting the armature reaction, that is, the demagnetizing ampere turns of normal current under sustained short circuit, from the corresponding field ampere turns), a very large error in this test value of self-induction may occur, if, as is usually done, the armature reaction is calculated on sine wave assumptions. A set of curves shown in Fig. 20, which are plotted from results derived in Appendices A and B, give values of the correction factor which applies to calculations based on sine wave.

An approximate, but convenient, method of applying the armature reactance in the calculation of field excitation under load is given in Appendix C.

In Part II it is shown that the initial short-circuit current of synchronous machines is determined not only by the armature self-inductive reactance, as is often assumed, but also by the field self-inductive reactance. Neglecting the field reactance in calculation may give a calculated short-circuit current 50 per cent or more, too high. A formula is derived for calculating the field reactance which, added to the armature reactance, gives the total which determines the initial short-circuit current and which it is proposed should be called, as previously recommended by

Manuscript of this paper was received April 24, 1918.

other authors, *transient reactance*. Table IV shows a comparison on eleven machines of the actual short-circuit currents as determined by oscillograph, and as calculated by methods proposed in the paper.

The calculations of *transient reactance* apply strictly to salient laminated pole alternators without amortisseur winding. However, such experience as is recorded in Table IV, if analyzed in connection with theoretical considerations, affords a basis for estimating the *transient reactance* of turbine generators with massive steel rotors and of salient-pole machines with amortisseur windings. These points are summed up in "Summary and Conclusions".

An attempt is made to describe the apparently complicated physical phenomena of sudden short circuits in terms as free as possible from mathematics. One interesting and important point which the authors establish from the physical interpretation of the problem is that there is a very significant rise in flux at the bottom of the pole at short circuit. This may be (if the rotor is entirely laminated) 30 or 40 per cent of normal flux, and apparently explains why, in machines with solid steel rotor rims, the field attenuation factor, α_f, changes radically after the first few cycles after short circuit.

Introduction

As the march of time obliges designers to become ever more efficient in the use of materials, it becomes more and more necessary that they should extend design calculations to further completeness. The self-inductive reactance of synchronous machines stands in very important relation to this problem. In the predetermination of field excitation, of voltage regulation, and of the initial short-circuit current, the reactance is, of course a very essential factor.

Three years ago the authors undertook a survey of the work which had been done on the calculation of armature reactance in the hope of finding, if not in any particular existing formula, at least in a combination of different ones, an expression for reactance that would give reliable calculations—reliable, we mean when applied to the entire range of machines as built by one of the large manufacturing companies. The result was probably very much the same as that which others have obtained who may have a similar attempt. Particular expressions applied well to particular classes of machines, but none was found, and no combination of existing expressions was found, that would apply generally. Moreover, on account of the many factors in the problem, the equation for reactance is a very clumsy thing to handle—too clumsy, indeed, to make practical the calculation of reactance on each new, proposed design. It is nevertheless desirable to have this calculation. By methods described later an expression was finally arrived at which does apply generally

to all classes of machines from two-pole turbo-generators to 136-pole water-wheel driven generators, of any voltage and frequency, three-phase, two-phase or single-phase, and the results are put in convenient shape for the designer's use, thereby making the calculation for each machine a practical matter. A tabulated comparison of calculated and test results (taken from ordinary commercial tests for open and short-circuit characteristics) is shown in Table I.

The importance of predetermining the initial short-circuit current of synchronous machines need not be dwelled upon. Mechanical stress involved, the rupturing capacity of circuit breakers, heating, etc., all make it a matter of supreme importance. The theory underlying short circuits has been thoroughly worked out by Steinmetz, Berg and Bouchert; and Diamant[1] has extended the mathematics involved and investigated the possibility of determining experimentally the attenuation factors, r/L, which determine the rate of decrease of the transients. That is the complete equation for the instantaneous values of short-circuit current has been available for some time. The difficulty in its use has been that the factors involved could not be predetermined, and, to some extent, there has been uncertainty regarding what constitutes the reactance which determines the short-circuit current. The authors will show that this reactance includes the field self-induction as well as that of the armature, and will derive a formula by which the total reactance may be calculated. To the author's knowledge this has not been previously worked out. Table IV gives a comparison of calculated and actual values of short-circuit current.

In the process of obtaining the above results, another closely related problem was incidentally solved, namely, the effect of the harmonics in the no-load flux wave upon the field excitation required for normal voltage. This is given in Appendix A.

The object of the paper, therefore, is (1) to present a reliable, general formula for the calculation of self-inductive armature reactance of synchronous machines. (Given in Part I).

(2) To give a simplification of the general equation, permitted by standard construction, accompanied by working curves making it possible to calculate the per cent armature reactance from the design sheet in less than five minutes. (Given in Part I).

1. TRANS., A. I. E. E. 1915, Vol. XXXIV, p. 2237.

TABLE I

COMPARISON OF CALCULATED AND TEST VALUES AT ARMATURE SELF-INDUCTIVE REACTANCE

SALIENT POLE MACHINES

	Kv-a.	ϕ	Freq. N	R.p.m.	Voltage	x_{pa} Calc.	x_{pa} Test
1	33	3	600	1200	440	13.4	8.06
2	50	3	60	300	2300	16.0	13.9
						12.0	10.4
3	125	3	62½	750	2300	13.5	11.0
4	180	3	60	900	220	10.9	12.3
5	240	3	60	150	440	17.1	15.2
6	265	3	25	150	400	28.1	34.8
7	300	3	60	1200	4000/2300	15.6	13.0
8	320	3	50	200	5000	12.5	15.5
9	345	3	60	720	4000	13.1	10.7
10	370	3	50	187½	500	14.5	20.0
11	375	3	60	150	2300	14.7	13.2
12	375	3	60	150	3100	14.4	11.4
13	400	3	25	300	2300	15.2	21.0
14	400	3	60	300	4000	27.5	30.1
15	400	3	60	200	600	18.9	19.3
16	400	3	60	150	6600	24.6	21.4
17	450	3	25	750	13200	8.8	3.1
18	475	3	50	214	440	16.4	15.3
19	500	3	60	600	2300	7.6	7.5
20	500	3	60	150	2300	16.0	13.7
21	500	3	60	100	240	14.4	13.9
22	540	3	60	164	6600	31.0	27.3
23	560	3	25	750	480	6.6	5.8
24	560	3	60	720	13200	15.1	12.0
25	560	2	60	720	2400	18.6	23.2
26	570	3	25	750	13200	9.0	8.9
27	580	3	60	720	2300	13.4	15.6
28	585	3	60	90	2000	14.4	15.6
29	605	3	60	150	2200	16.8	18.1
30	625	3	60	150	2300	10.7	10.0
31	625	3	60	100	480	21.1	15.8
32	675	3	60	90	6600	36.8	33.5
33	700	3	25	750	2200	10.1	6.00
34	700	3	25	750	13200	13.2	16
35	700	3	25	500	6600	11.4	7.3
36	700	3	60	900	4000	13.8	12.7
37	700	3	40	300	600	10.5	10.2
38	750	3	60	300	600	22.5	23.6
39	750	3	60	150	2300	18.4	19.5
40	780	3	60	120	2300	21.7	21.0
41	800	3	60	90	2300	41.9	44.0
42	850	3	25	500	6600	5.6	8.4
43	900	3	60	360	600	20.5	18.6
44	900	3	60	150	2300	22.1	18.2
45	1000	3	60	450	2300	22.5	19.8
46	1000	3	60	200	600	21.2	16.8
47	1200	3	25	500	4400	16.3	16.9
48	1250	3	25	750	13200	10.5	10.5
49	1250	3	50	500	5500	12.3	12.8
50	1250	3	60	300	2300	12.0	10.5
51	1250	2	62.5	375	5400	12.1	10.2

TABLE I. (*Continued*)

COMPARISON OF CALCULATED AND TEST VALUES AT ARMATURE SELF-INDUCTIVE REACTANCE

SALIENT POLE MACHINES

	Kv-a.	ϕ	Freq. N	R.p.m.	Voltage	x_{pa} Calc.	x_{pa} Test
52	1350	3	25	375	6600	11.3	12.8
53	1400	3	25	500	2200	10.0	8.9
54	1400	3	25	375	13200	11.5	13.8
55	1400	3	60	514	13200	10.8	10.9
56	1400	3	60	514	13200	12.9	11.9
57	1420	3	25	300	2300	11.0	5.8
58	1425	3	60	240	2200	21.1	22.0
59	1460	3	60	720	2300	11.3	8.6
60	1500	3	25	500	2300	7.9	10.2
61	1500	3	60	514	2200	14.5	15.0
62	1500	2	62.5	750	2300	14.3	20.0
63	1625	3	25	750	3300	10.5	10.5
64	1800	3	60	300	2300	10.6	12.6
65	2000	3	60	720	2200	13.8	14.0
66	2000	3	60	100	2300	8.4	8.0
67	2100	2	60	360	2400	9.6	7.2
68	2100	3	60	514	4600	15.7	15.7
69	2125	3	40	200	600	15.3	14.1
70	2100	3	25	750	3300	9.1	15.0
71	2200	3	25	375	2300	9.1	9.7
72	2250	3	25	214	2300	5.9	4.4
73	2250	3	60	514	6000	15.3	16.9
74	2250	3	60	450	3450	17.8	17.1
75	2500	3	25	250	2200	5.8	4.0
76	2800	3	25	300	2300	5.5	4.9
77	2000	3	60	514	13200	18.9	19.2
78	2800	3	60	450	4000	15.0	18.0
79	3000	3	50	500	6000	13.1	17.4
80	3000	3	60	450	2300	8.25	11.0
81	3125	3	25	750	14000	16.1	15.8
82	3500	3	50	500	11500	19.2	20.0
83	4000	3	60	600	2300	13.8	11.2
84	5000	3	25	300	2300	8.9	10.2
85	5000	3	25	88.3	6600	13.9	13.8
86	5650	3	60	360	6600	19.8	21.3
87	6000	3	50	500	3450	7.65	6.1
88	6000	3	50	500	16500	21.6	26.5
89	6600	3	25	375	14000	9.5	11.0
90	6250	3	60	600	6600	16.8	13.9
91	6750	3	50	600	11000	19.3	25.6
92	7050	3	40	400	600	6.6	8.5
93	7050	3	50	375	6600	13.9	13.0
94	7500	3	60	600	13800	13.3	16.9
95	8750	3	50	500	600	15.1	19.4
96	8750	3	50	600	6600	6.9	7.1
97	9000	3	25	57.7	11000	13.9	16.4
98	10000	3	60	514	6600	6.81	7.9
99	10000	3	62.5	55.6	6600	20.4	19.4
100	12000	3	25	375	6600	13.2	12.6
101	15000	3	50	375	6600	10.2	12.3

TABLE I. (*Continued*)

TURBINE ALTERNATORS

	Kv-a.	ϕ	Freq. N	R.p.m.	Voltage	x_{pa} Calc.	x_{pa} Test
1	3750	3	60	3600	2300	9	14.5
2	4375	3	60	1800	2400	5.9	2.4
3	4375	3	60	1800	4500	6.7	3.9
4	4375	3	60	1800	2300	7.7	8.2
5	4375	2	60	1800	2300	6.25	5.1
6	5000	3	25	1500	5500	10.6	10.2
7	5000	3	25	1500	6600	6.4	6.8
8	5000	3	25	1500	6600	7.3	5.5
9	5000	3	60	3600	2300	9.0	8.9
10	5625	3	25	1500	2300	6.1	3.3
11	6250	3	25	1500	6600	10.1	10.6
12	6250	3	60	3600	2300	8.6	12.2
13	6250	3	60	1800	6600	8.6	7.8
14	7500	3	25	1500	2300	7.1	6.9
15	7500	3	25	1500	6600	10.5	7.1
16	7500	3	25	1500	6600	10.5	11.8
17	7500	3	60	1800	4000/2300	9.4	5.0
18	7500	3	60	1800	4160/2300	9.1	8.2
19	7500	3	60	1800	2300	7.6	9.1
20	7812	3	25	1500	2300	7.7	7.8
21	7812	3	60	1800	2300	10.1	9.5
22	7812	3	60	1800	4000/2300	10.6	8.6
23	7812	3	60	1800	4000/2300	11.8	10.9
24	9375	3	25	1500	13200	8.9	9.3
25	9375	3	60	1800	4000/2300	8.0	8.6
26	9375	3	60	1800	5000	11.0	9.2
27	10000	3	25	1500	6600	9.9	9.8
28	10000	3	40	2400	10000	12.5	13.8
29	10000	3	60	1800	6600	14.2	9.2
30	12500	3	25	1500	11000	8.4	8.2
31	12500	3	60	1800	4000/2300	9.1	7.4
32	12500	3	60	1800	11000	13.0	17.1
33	12500	3	60	1800	13200	19.2	13.5
34	12500	3	60	1800	2300	10.2	8.6
35	18333	2	37.3	2240	4469	6.3	7.4
36	18750	3	60	1800	2300	13.8	12.0
37	25000	3	60	1800	13200	14.4	15.4

(3) To derive a formula for calculating the field self-induction which combines with the armature self-induction to determine the initial short-circuit current; and further to describe in part the physical phenomena which exist in the machine at short circuit. (Given in Part II).

(4) To derive a formula for calculating armature reaction, taking into account the effect of field distribution. (It is necessary to determine the armature reaction accurately, before the armature self-induction can be determined from standard open and short-circuit tests). (Given in Appendix B).

(5) To show the effect of flux distribution (*i.e.* the effect of the harmonics in the no-load flux wave) on the field excitation. (Given in Appendix A).

(6) To give a simplified method of calculating field excitation which has worked out well in connection with the *per cent* armature reactance. (Given in Appendix C).

PART I.

Armature Reactance

Of all the work which had been done on armature reactance, that of Gray[2] was, in our opinion, the most complete. While it involved certain familiar factors, such as that for the slot reactance and the leakage from tooth tip to tooth tip which have appeared in other reactance formulas, it yet took into account for the first time, we believe, the variation of tooth-tip reactance in salient-pole machines for different positions of the pole, which is very important. Fechheimer introduced a factor to account for the effect of fractional pitch armature coils. The results given in this paper have been obtained by a modification and extension of the work done by Gray, Fechheimer[3] and others. A bibliography which may be of historical interest is included at the end of the paper.

There were three difficulties of fundamental importance which it was necessary to overcome before consistent results were obtained. The first was the influence of the harmonics in the no-load flux wave upon the effective armature magnetomotive force or armature reaction. The armature self-inductive reactance, as obtained from the saturation and synchronous impedance curves (the open-circuit and short-circuit characteristics) involves the difference between the actual field m.m.f. on short circuit and the effective armature m.m.f. for the corresponding current. Since the armature reaction is calculated, this difference must also, to that extent, be considered as calculated. The familiar, approximate formula usually employed in determining effective armature m.m.f. per pole,

$$A = \frac{2.12\, N_q\, I}{K_p\, K_d} \quad \text{(three-phase)}$$

is based upon sinusoidal distribution of flux and armature m.m.f.

2. Gray's Elec. Machine Design, 1st Edition, page 215.
3. Trans. A. I. E. E., Vol. 31, p. 578.

N_q = series turns per pole per phase
I = amperes per turn
K_p and K_d = coefficients, greater than unity, which take account of the effect of fractional pitch coils and the distribution of coils respectively.

Often the flux distribution is very different from a sine wave. In such cases the actual effective armature m.m.f. may be different by 8 to 10 per cent from the value based on sine-wave assumptions—*i.e.*, as calculated from the above formula. This error in armature m.m.f., of course causes a much larger percentage

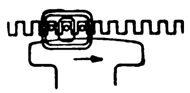


FIG. 1—TOOTH-TIP LEAKAGE FLUX

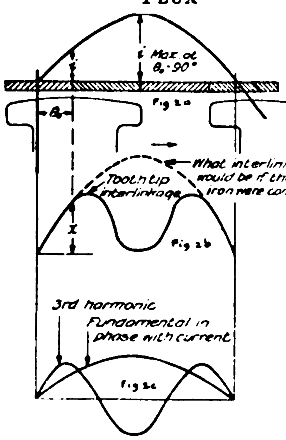


FIG. 2—TOOTH TIP-INTERLINKAGES AT ZERO POWER FACTOR

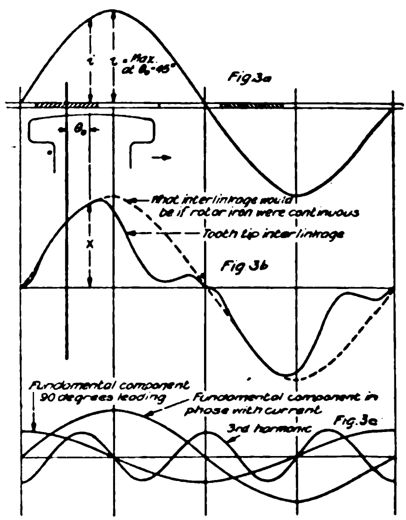


FIG. 3—TOOTH-TIP INTERLINKAGES AT 80 PERCENT POWER FACTOR

error in the difference between field and armature m.m.f's. on short circuit, which difference, correctly determined, represents the armature self-induction. It is therefore obvious that the effect of flux distribution must be taken into account. This has been done by a single factor, K_ϕ, which is derived in Appendix A, and is given in Fig. 20.

The second difficulty was met in accounting correctly for the variation in the permeance of the "tooth-tip" leakage path for different positions of the pole. Gray's method of accounting for this neglected certain serious factors, as will be shown later. Fig. 1 shows *tooth-tip*, leakage of one-phase belt at a time when

the pole-face iron forms part of the leakage path. It is obvious from Fig 1, that as the pole moves on, a part, and finally all, of the path between tooth-tips will be air. At zero power factor the current in any particular phase group is a maximum when that phase group is half way between poles, when the angle, θ_0, between the center of the phase group and the center of the pole Fig. 2A, is 90 deg.; and is zero when the group is over the middle of the pole, when θ_0 is zero. This produces a curve of leakage interlinkages shown as x in Fig. 2B. The dotted curve indicates what the leakage curve would be if the iron of the rotor were continuous. Now for any other power factor, say cos $\theta = 0.7$, if the distortion of the magnetic field is neglected, the current will be a maximum when

$$\theta_0 = \theta = 45 \text{ deg.}$$

and zero when $\theta_0 = -45$ deg. But distortion can not be neglected. By shifting the flux toward the trailing tip (in a generator) the position of the pole is relatively advanced, thereby increasing the total angle between the center line of the pole and the position of the phase group when the current is a maximum, to

$$\theta_0 = \theta + \theta_d$$

where θ_d is the distortion angle, shown in Fig. 18. For illustration let the current be a maximum when θ_0 equals 45 deg. Fig. 3A. This would correspond in certain machines to a power factor of about 80 per cent and would produce a curve of leakage interlinkages, x, shown in Fig. 3B. The dotted curve indicates the leakage which would exist if the rotor iron were continuous.

An analysis of the curves of interlinkages shown in Figs. 2B and 3B discloses an interesting and very important fact: that these curves as shown in Figs. 2C and 3C, are made up principally of a fundamental and a large third harmonic, all other harmonics being negligible; and that since in a Y-connected 3-phase machine the third is eliminated, the fundamental alone remains for consideration is such machines—which probably, comprise 95 per cent of all that are now being manufactured. And even in two-phase or Δ-connected three-phase machines the effective results are much the same. Experience has been that the short-circuit characteristic and saturation curves show practically the same value of reactance in the same machine

whether it is connected Y or Δ. The probable explanation of this is that in Δ-connected or two-phase machines the current, especially under short circuit, is not a sine wave, as it is for Y-connected machines, but instead, contains a negative third harmonic, as shown in Fig. 4. This operates to lower the peaks in Fig. 2B, that is to decrease the third harmonic in the reactance. And although, of course, the effect of the remaining, decreased third harmonic in the reactance voltage is to restrict the flow of current at certain points of the cycle, and therefore produce a current of smaller average value than would exist if the third harmonic were eliminated, nevertheless the form factor of the resulting peaked wave shown in Fig. 4, (Plate LI) is correspondingly higher, that is, the ammeter will read correspondingly higher, and therefore show in this case about the same reading for sustained short-circuit current as would be shown in the case of a Y-connected machine. In general, then, it is necessary to consider only the fundamental in the tooth-tip leakage.

The salient pole construction, which is the cause of the large third harmonic mentioned above, is therefore, in effect replaced by a continuous rotor of uniform reluctance, which is greater than that of iron but lower than that of air, and which is different for different power factors, being lower at high power factor. This is obvious from a comparison of Figs. 2B and 3B.

At high power factor, an interesting circumstance arises. In addition to that component of the fundamental of the tooth-tip leakage which is in phase with the current and which therefore produces a reactive voltage lagging behind the current by 90 deg., there is also a cosine term, leading the current by 90 deg., which produces a reactive voltage in phase with the current. This has important significance in the matter of field excitation required under load, as pointed out later.

Gray's value for the tooth-tip factor is based on a root mean square of instantaneous values of a curve somewhat similar to that shown in Fig. 2B., the principal difference in the curves being that Fig. 2B., takes account of the distribution of the coils in the phase belt. Being based on the root-mean-square and neglecting the spread of the phase belt, Gray's value for this factor is approximately 100 per cent higher than that given in this paper, and does not take account of the in-phase component of reactive voltage, or of the effect of the distortion of the magnetic field—both of which occur under watt load.

The third difficulty involved the conception that on a polyphase machine, the self-induction of the armature is a polyphase, not a single-phase phenomenon; and that therefore the reactance per phase of the same machine, connected three phase in one case, two phase in another, would be approximately 1.5 times the single-phase value in the former, 1.0 times the single-phase value in the latter. In other words the magnetic field of self-induction is a rotating polyphase field just as is the field of armature reaction, and the resulting overlapping of phase produces greater leakage interlinkages in any particular phase of a three-phase machine than would exist by the m.m.f. of the phase alone. For instance, in a three-phase machine, when the

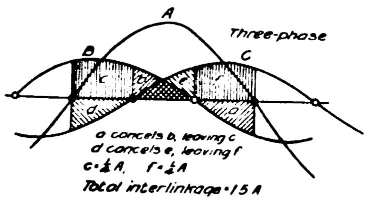


Fig. 5A

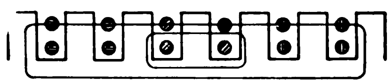


Fig. 6 A

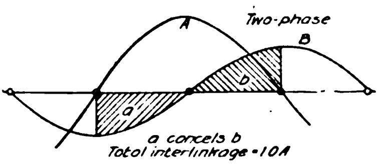


Fig. 5B
Mutual Interlinkages—Polyphase Machine

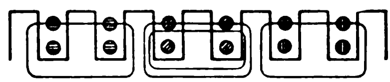


Fig. 6B
Slot Leakage in Three-Phase Machine

current is maximum in phase A, it is one-half maximum in B and C. The increase of interlinkages in A, due to B and C, is shown in Fig. 5A. For two-phase, the current is zero in B when a maximum in A, and moreover they are displaced in space so that the mutual interlinkages must be zero at all times as shown in Fig. 5B.

Unless this fact is taken into account when considering three-phase machines, the calculated value, which is the reactance per leg, will be only two-thirds as large as the test value, unless some of the factors in the reactance equation are overburdened to make up the difference. Then, obviously, the formula would not apply to two-phase machines; it would give calculated results 50 per cent too high. This is just what had happened

during this investigation. By using Gray's formula modified by Fechheimer's "Short Pitch Factor", calculations checked fairly well on certain classes of three-phase machines, but not on two-phase. The factor of tooth-tip leakage being 100 per cent too high and the other factors being approximately right, made up for neglecting the increase of 50 per cent on the total, and naturally made two-phase calculations too high. By using the proper value of tooth-tip leakage, and allowing for the poly-phase action, the equation applies equally well to three-phase, two-phase and single-phase machines.

There is, however, one term of the reactance equation which is not affected by the mutual induction of phases, namely, the slot leakage. Fig. 6A shows the three different phases of a two-slot per pole per phase machine, and the slot leakage which exists when the current is a maximum in the middle group, one-half maximum in the other two. Fig. 6B is equivalent to Fig. 6A, and it is clear that the interlinkages of the middle group have not been increased by the m.m.f. of the other two groups. The same reasoning can be applied to other instants, when the currents have different values, to show that there is no mutual induction. This operates to reduce the ratio of three-phase to single-phase (terminal to neutral) reactance to a value slightly lower than 1.5.

It is not obvious at first that this overlapping of phases occurs in the case of tooth-tip leakage. Consider, however, that the elimination of the third harmonic in this term, in effect produces a path of uniform reluctance (similar to a path of air only having lower reluctance) for the tooth-tip leakage. In other words, the case is similar to that of the end winding whose leakage path is air, or to that of the induction motor, rotating main field. And, as in the two latter cases, the poly-phase field is greater than that produced by one phase alone by about 50 per cent, as shown in Fig. 5A.

Turning to the question of applying the reactance in the calculation of field excitation required under load, experience has demonstrated that the use of the zero power factor value of reactance (that is, the value determined from open-circuit and short-circuit test) gives reasonably reliable calculation of field current for loads of any power factor. Why this could be true when the reactance at 80 per cent power factor is, by reason of the different pole position when the current is a maximum, about 60 per cent higher than that at zero power factor, was a perplex-

ing question. It is answered by the appearance of the in-phase component of reactive voltage in the tooth-tip factor. The $r\,I$ drop is in phase opposition to the current; and the voltage consumed by it is in phase with the current. The $x_2\,I$ drop, produced by leakage interlinkages in phase with the current, is 90 deg. behind the current; and the voltage consumed by it is 90 deg. ahead of the current. The $x_1\,I$ drop, produced by leakage interlinkages in phase with the current, in 90 deg. behind the current; and the voltage consumed by it is 90 deg. ahead of the current. The $x_2\,I$ drop produced by leakage interlinkages 90 deg. ahead of the current, as shown in Fig. 3c, is in phase with the current; and the voltage which it consumes is in phase opposition to the current. Fig. 7 shows these relations for approximately 80 per cent power factor, and makes clear why the internal voltage E_1', determined by the use of the zero power factor reactance, is practically the same as E_1 which is actually required by the much higher reactance, corresponding to 80 per cent power factor, the effect of the latter being mitigated by the in-phase component $x_2\,I$. There is a difference in phase between E_1' and E_1 which, by using the zero power factor value of reactance, will give a calculated excitation slightly too low, but for practical purposes the difference is considered negligible.

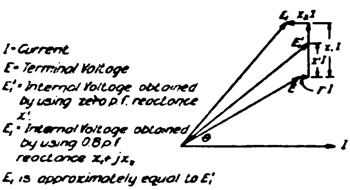


FIG. 7—VOLTAGE DIAGRAM AT 80 PERCENT POWER FACTOR

In the interest of simplicity and convenience in calculation, therefore, it is proposed that for the present at least, the zero power factor value of reactance be used. The method of calculating field excitation which is given in this paper follows that assumption. In the future it may become desirable to make correction for the distortion angle and power factor; hence the problem remains open to this further refinement.

CALCULATION OF ARMATURE REACTANCE

A = effective armature reaction in ampere turns per pole

a = conductors per slot; for a double layer winding a is twice the number of turns per coil.

C = air-gap coefficient based on Carter's fringing coefficient.

c = number of circuits in multiple in armature winding.

C_a = reduction factor for Φ_a at zero power factor, depending on ratio of pole arc to pole pitch.

C_i = reduction factor for Φ_i at zero power factor, depending on ratio of pole arc to pole pitch.

C_1 = 1.5 C_i for three phase
= C_i for two phase.

C_2 = 1.5 C_a for three phase
= C_a for two phase.

d_1 = distance from top of copper to bottom of copper in the armature winding Fig. 8.

d_2 = distance from top of copper to armature face Fig. 8.

e = voltage per phase

f = frequency in cycles per second.

g = average air gap.

I = current per phase in amperes

i = instantaneous value of current in conductor

K = reduction factor for Φ_s Φ_i and Φ_a depending on pitch of armature coils.

K_d = factor for effect of distribution of armature coils in calculation of induced voltage, depending on slots per pole and phase.

TABLE II—VALUES OF K_d

s		1	2	3	4	5	6	7	8up
K_d	Three-phase	1.0	1.035	1.043	1.044	1.045	1.046	1.047	1.047
K_d	Two-phase	1.0	1.083	1.098	1.103	1.106	1.108	1.109	1.11

K_i = factor for Φ_i depending on slots per pole and phase.

K_p = factor for effect of short pitch of armature coils in calculation of induced voltage.

$$K_p = \frac{1}{\text{Sin } (90° \times p)}$$

where p is pitch of coil in per cent of full pole pitch.

K_ϕ = factor for effect of form of flux wave shape in calculation of induced voltage. Values of K_ϕ are given in Fig. 20.

L_e = length of one phase belt of end connections.

l = gross stacked length of armature core (iron + ducts)

N_s = total number of stator coils

n = number of phases.

p = per cent pitch of armature winding (= 0.8 for 80 per cent pitch)

q = number of poles

s = slots per pole and phase.

w_s = width of slot

w_t = width of tooth at armature face.
For w_s and w_t see Fig. 8.

x_a = reactance in ohms

x_{pa} = armature reactance in per cent.

α = ratio of pole arc to pole pitch

τ = pole pitch at armature core surface.

Φ = flux per pole in maxwells.

Φ_a = effective tooth tip interlinkages per phase belt per ampere conductor for unit length of armature, when the entire phase belt is between the poles.

ϕ_a = effective instantaneous value of Φ_a.

Φ_e = effective end connection interlinkages per phase belt per ampere conductor for unit length of phase belt of end connections.

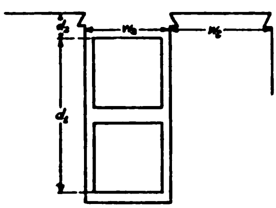


FIG. 8—SLOT DIMENSIONS.

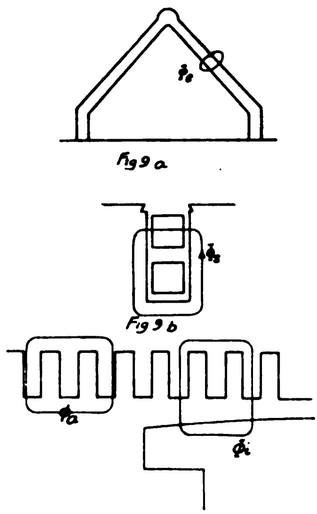


FIG. 9C
ARMATURE LEAKAGE FLUX DIAGRAMS

Φ_i = effective tooth tip interlinkages per phase belt per ampere conductor for unit length of armature, when the entire phase belt is under a pole.

ϕ_i = effective instantaneous value of Φ_i.

Φ_s = effective slot interlinkages per phase belt per ampere conductor for unit length of armature.

ψ = electrical angle by which the armature coil span is less than 180 deg.

Theory. The armature leakage flux in an alternating-current generator may be divided into three factors.

(1) End connection leakage flux, represented by $\Phi_e L_e$, the effective flux per ampere conductor interlinking one phase belt of end connections. (Fig. 9 A).

(2) Slot leakage flux represented by Φ_s, the effective flux per ampere conductor through the slot interlinking unit length of one phase belt. (Fig. 9 B).

(3) Tooth tip leakage flux represented by

(*a*) Φ_i, the effective flux per ampere conductor, crossing the air gap into the pole face and returning across the gap and interlinking unit length of one phase belt, when the entire phase belt is under the pole. See Fig. 9C.

(b) Φ_a, the effective flux per ampere conductor emerging from the tooth tips and interlinking unit length of one phase belt when the entire phase belt is between the poles. See Fig. 9c.

The expression for reactance has been derived by a number of writers, and may be expressed in our notation by the following equation:

$$x = \frac{2\,\pi\,f\,s^2\,a^2\,q\,l}{10^8\,c^2}\,L_0 \tag{1}$$

Where L_0 is the sum of the interlinkages per ampere conductor in the phase belt reduced to the equivalent of unit length of the armature[4].

The factor $\frac{2\,\pi\,f\,s^2\,a^2 q\,l}{10^8\,c^2}$ will be replaced by M in the following derivation.

End Connection Leakage. The theory of end connection leakage can only be approximated, but the value of $\Phi_e\,L_e$ is evidently between the following two limits.

The first limit assumes that the length of the path around the phase belt varies directly as the width of the belt.

Then

$$\Phi_e = K'\,\frac{n}{\tau}$$

The second limit assumes that the length of the path is independent of the width of the belt.

Then Φ_e would be constant.

The length of the phase belt in either case will be $p\,\tau\,K''$.

The actual variation of Φ_e with the width of the phase belt is therefore between the reciprocal of the zero power and the reciprocal of the first power of $\frac{\tau}{n}$. The analysis of tests for entire armature reactance on over 100 machines, as well as

4. Electrical Machine Design, Gray Edition 1913, p. 21.

exploring coil tests for end leakage reactance on a 125-kv-a. generator indicates that Φ_e varies approximately as $\sqrt{\dfrac{n}{\tau}}$ that is

$$\Phi_e = K''' \sqrt{\frac{n}{\tau}}$$

Then
$$\Phi_e\, L_e = K \sqrt{n}\, p \sqrt{\tau}$$

For a three-phase double-layer winding, and dimensions in centimeters, $K \sqrt{n} = 2.8$ was found to give satisfactory results.

The factor for a two-phase two-layer winding will be reduced in the ratio of $\dfrac{\sqrt{2}}{\sqrt{3}}$ and by the factor 0.667 on account of the polyphase effect as explained in the first part of this paper, and will be 1.6 in round numbers.

The end connection reactance for a double-layer winding will be

$$x = \frac{2.8\, M\, p \sqrt{\tau}}{l} \qquad \text{Three-phase} \quad (2)$$

$$= \frac{1.6\, M\, p \sqrt{\tau}}{l} \qquad \text{Two-phase} \quad (3)$$

For a single-layer winding the value of $\Phi_e\, L_e$ will be doubled as proposed by Gray.

Slot Leakage. Three-phase pitch between 66.66 and 100 per cent. The expression for slot reactance with short-pitch armature winding was derived by Fechheimer[5], and is used with a correction in the number of A and B coils, which is $\dfrac{N_s\, \psi}{180}$ instead of $\dfrac{N_s\, \psi}{2 \times 180}$ as given in his paper.

Omitting the factors for the top of the partially closed slot and modifying for our notation, the expression for inductance of open slots becomes

$$L = 0.4\, \pi\, a^2 \left[\left(\frac{19}{96}\frac{d_1}{w_s} + \frac{3}{8}\frac{d_2}{w_s}\right)\frac{N\,\psi}{3 \times 60} + \right.$$
$$\left.\left(\frac{7}{96}\frac{d_1}{w_s} + \frac{3}{8}\frac{d_2}{w_s}\right)\frac{N_s\,\psi}{3\times 60} + \frac{N_s}{3}\left(1 - \frac{\psi}{60}\right)\left(\frac{d_1}{3\, w_s} + \frac{d_2}{w_s}\right)\right] \qquad (4)$$

5. Fechheimer "Synchronous Motors" A. I. E. E., Vol. XXXI, 1912, p. 580.

Simplifying

$$L = 0.4\,\pi\,a^2 N_s \left[\left(1 - \frac{\psi}{320}\right)\frac{d_1}{3\,w_s} + \left(1 - \frac{\psi}{240}\right)\frac{d_2}{w_s}\right] \qquad (5)$$

$$x = M\,\frac{0.4\,\pi}{s}\left[\left(1 - \frac{\psi}{320}\right)\frac{d_1}{3\,w_s} + \left(1 - \frac{\psi}{240}\right)\frac{d_2}{w_s}\right] \qquad (6)$$

The derivation of this expression in the above paper omits the allowance for the out of phase component of the reactive voltage induced in one phase by the half coils of another phase in the same slot. The final expression however is correct as it can readily be seen by reference to Fig. 10, the reactance component of E_2, the voltage induced in phase one by phase two,

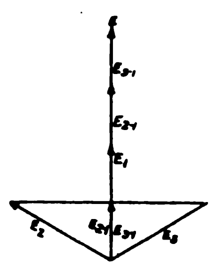


FIG. 10—MUTUAL INDUCTION IN SHORT-PITCH ARMATURE WINDINGS

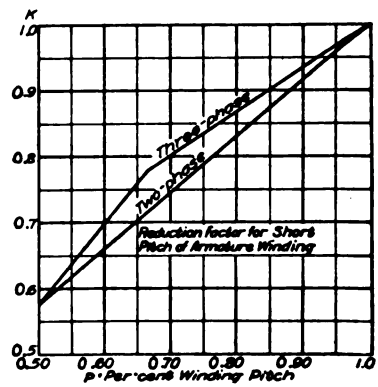


FIG. 11

will be exactly balanced by the reactive component of E_3 the voltage induced in phase one by phase three.

For simplicity and with very little decrease in accuracy, we may replace the two short-pitch factors in equation (6) by an average value K from Fig. 11.

Then $$x = M\,\frac{0.4\,\pi}{s}\,K\left(\frac{d_1}{3\,w_s} + \frac{d_2}{w_s}\right) \qquad (7)$$

Pitch below $66\frac{2}{3}$ per cent.

Following Fechheimer's method with the correction for number of half coils, the expression for reactance where the pitch is less than $66\frac{2}{3}$ per cent will be

$$x = M\,\frac{0.4\,\pi}{s}\left[\left(1.187 - \frac{\psi}{160}\right)\frac{d_1}{3\,w_s} + \left(12.5 - \frac{\psi}{120}\right)\frac{d_2}{w_s}\right] \qquad (8)$$

Replacing the two factors by an average value K from Fig. 11

$$x = M \frac{0.4\,\pi}{s} K \left(\frac{d_1}{3\,w_s} + \frac{d_2}{w_s}\right) \tag{9}$$

Two phase.

Pitch between 50 and 100 per cent.

The expression for two-phase slot reactance with short pitch derived by a similar process is

$$x = M \frac{0.4\,\pi}{s} \left[\left(1 - \frac{\psi}{240}\right)\frac{d_1}{3\,w_s} + \left(1 - \frac{\psi}{180}\right)\frac{d_2}{w_s}\right] \tag{10}$$

Using an average value of reduction factor from Fig. 11

$$x = M \frac{0.4\,\pi}{s} K \left(\frac{d_1}{3\,w_s} + \frac{d_2}{w_s}\right) \tag{11}$$

Tooth-Tip Leakage. The tooth-tip leakage may be most conveniently calculated by dividing it into two components; the effective leakage flux around the part of the phase belt opposite the iron of the pole face (maximum value Φ_i); and the effective leakage flux around the part of the phase belt between the poles (maximum value Φ_a).

The values of Φ_i and Φ_a for a complete phase belt were determined by Gray.[6] His method has been followed for Φ_i except in determining the area of the path across the air gap, which he gives as the tooth width multiplied by the length of armature and the air gap coefficient based on Carter's fringing coefficient. The leakage flux will fringe in almost exactly the same way as the field flux, hence the area of the air gap should be the length of armature times the tooth pitch divided by the air gap coefficient, Fig. 12B expressed in our notation.

$$\Phi_i = 0.4\,\pi \frac{w_s + w_t}{2\,g\,C} \tag{12}$$

for one and two slots per pole and phase.

Following Gray's derivation, it will be found that for higher numbers of slots per pole and phase

$$\Phi_i = 0.4\,\pi\, K_i \left(\frac{w_s + w_t}{2\,g\,C}\right) \tag{13}$$

A curve for K_i plotted against slots per pole and phase is given in Fig. 12.

Gray's expression for Φ_a has been adopted without change,

6. Electrical Machine Design, Gray Edition, 1913, p. 220.

and simplified by assuming $w_s = w_t$, which gives a definite value of Φ_a for any given value of slots per pole and phase.

Φ_a plotted against slot per pole and phase is shown in Fig. 12.

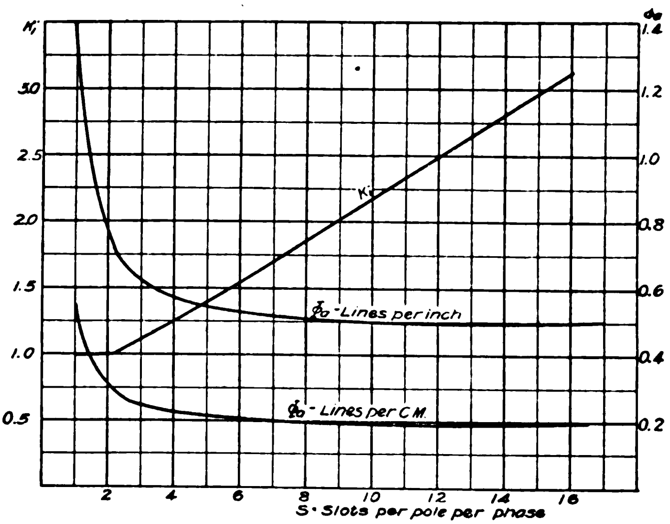


FIG. 12A

Modification of Φ_i and Φ_a by Definite Pole Construction. It should be noted that Φ_i, would be the leakage flux per ampere conductor in the phase belt, if the phase belt were opposite the

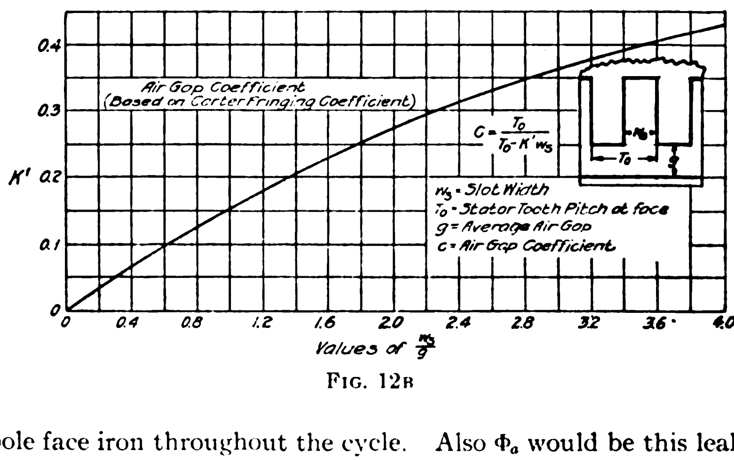


FIG. 12B

pole face iron throughout the cycle. Also Φ_a would be this leakage flux if there were no iron in the tooth-tip leakage path. The actual condition is a path partly iron and partly air and varying

in proportion of air and iron during the cycle. This condition may be approximated by calculating the interlinkages in the iron part of the path, and also the interlinkages in the air part of the path. The ratio of the former to Φ_i, and of the latter to Φ_a will be reduction factors to be applied to Φ_i and Φ_a in calculating the reactances due to tooth-tip leakage. The actual tooth-tip leakage will be obtained by adding the two parts as calculated in this way.

The general theory of the variation of Φ_i and Φ_a at zero power factor has been discussed in the first part of this paper, and it will now be applied to the actual calculation.

The instantaneous values of leakage flux $\Phi_i\, i$ vary from zero when the current is zero (Fig. 13A position a) to a maximum approximately where the pole begins to leave the phase belt (Fig. 13 position b) and then decreases to zero where the phase belt is entirely between the poles. (Fig. 13A position c). The instantaneous value of leakage flux $\phi_a\, i$ is zero when the current is zero, and remains zero until the pole begins to leave the phase belt, after which it increases gradually to a maximum where the current is a maximum at 90 deg. Obviously the curves for $\phi_i\, i$ and $\phi_a\, i$ are symmetrical about the 90 deg. axis. Fig. 14.

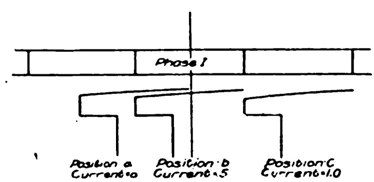


FIG. 13A—DIAGRAM OF POLE POSITIONS

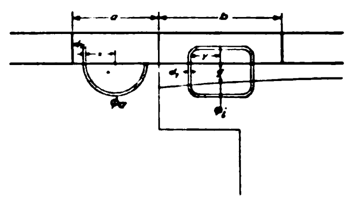


FIG. 13B—TOOTH-TIP LEAKAGE

The values of $\phi_i\, i$ and $\phi_a\, i$ for complete distribution of the armature winding were calculated by the following methods.

Tooth-tip Leakage $\phi_a\, i$

For one ampere in the phase belt and unit length of phase belt the m. m. f. at x (See Fig. 13B) is

$$\text{m. m. f.} = \frac{2\,x}{\frac{\tau}{n}} = \frac{2\,n\,x}{\tau} \qquad (14)$$

where n = number of phases

and τ = pole pitch

$$d\,\phi = 0.4\,\pi\,\frac{2\,n\,x}{\tau}\quad\frac{d\,x}{\pi\,x} \tag{15}$$

$$= 0.4\,\pi\,\frac{2\,n}{\pi\,\tau}\,d\,x \tag{16}$$

Let effective interlinkages due to $d\,\phi = d\,\phi_a$

Then $$d\,\phi_a = \frac{2\,x}{\frac{\tau}{n}}\,d\,\phi \tag{17}$$

$$= \frac{2\,n\,x}{\tau}\,0.4\,\pi\,\frac{2\,n}{\pi\,\tau}\,d\,x \tag{18}$$

$$\phi_a = \int_0^{a/2} d\,\phi_a = 4\,\pi \times \frac{2\,n^2}{\pi}\,\frac{x}{\tau^2} + C\Big]_0^{a/2} \tag{19}$$

$$= 0.4\,\pi\,\frac{n^2}{2\,\pi}\left(\frac{a}{\tau}\right)^2 + C \tag{20}$$

For $a = 0$ $\phi_a = 0$ Hence $C = 0$

$$\phi_a = 0.4\,\pi\,\frac{n^2}{2\,\pi}\left(\frac{a}{\tau}\right)^2 \tag{21}$$

for one centimeter of phase belt.

$$\phi_a = 3.2\,\frac{n^2}{2\,\pi}\left(\frac{a}{\tau}\right)^2 \tag{22}$$

for one inch of phase belt.

For the special case of three phase and where a is expressed as a per cent. of τ.

$$\phi_a = 4.6\,a^2 \text{ for one inch of phase belt.} \tag{23}$$

Tooth-tip Leakage $\phi_i\,i$

Referring to Fig. 13B

$$\text{m. m. f.} = \frac{2\,y\,n}{\tau} \tag{24}$$

Let g_b = average air gap over b.

$$d_\phi = 0.4\,\pi\,\frac{2\,y\,n}{\tau}\,\frac{d\,x}{2\,g_b} \tag{25}$$

Let effective interlinkages due to $d\,\phi = d\,\phi_i$

$$d\,\phi_i = \frac{2\,y\,n}{\tau}\,d\,\phi \tag{26}$$

$$= \frac{0.4\,\pi\,2\,n^2\,y^2}{g_b\,\tau^2}\,d\,y \tag{27}$$

$$\phi_i = 0.4\,\pi\,\frac{2\,n^2}{g_b\,\tau^2}\int_0^{b/2} y^2\,d\,y \tag{28}$$

$$= 0.4\,\pi\,\frac{2\,n^2}{3\,g_b\tau^2}\,y^3 + c\,\Big]_0^{b/2} \tag{29}$$

At $y = 0 \quad \phi_i = 0$ hence $c = 0$

$$\phi_i = 0.4\,\pi\,\frac{n^2\,b^3}{12\,g\,\tau^2} \tag{30}$$

Where b and g are expressed as a per cent. of τ

$$\phi_i = 0.4\,\pi\,\frac{n^2\,b^3}{12\,g_b} \tag{31}$$

per centimeter of phase belt.

$$= 3.2\,\frac{n^2\,b^3}{12\,g_b} \tag{32}$$

per inch of phase belt.

For three-phase and one-inch length of phase belt.

$$\phi_i = 2.4\,\frac{b^3}{g_b} \tag{33}$$

The area of leakage path across the air gap in actual machines will be reduced due to the effect of armature slots and this will be allowed for by the introduction of the air gap coefficient, C, given in Fig. 12B.

$$\phi_i = 2.4\,\frac{b^3}{C\,g_b} \tag{34}$$

The derivation of the actual values of $\phi_i\,i$ and $\phi_a\,i$ will be illustrated by an example for three phase where α is $66\frac{2}{3}$ per cent.; the minimum gap is 1.5 per cent of the pole pitch; and the ratio of maximum to minimum gap is 2 to 1, giving an average gap of approximately 2 per cent of the pole pitch. Average values of air gap coefficient will be assumed. The armature

winding will be assumed to be completely distributed. Table III gives the calculation of $\phi_i\, i$ and $\phi_a\, i$ as the current varies through 90 deg. calculated from the formulas.

$$\phi_i = 2.4 \frac{b^3}{C\, g_b} \qquad \text{from} \quad \textbf{(33)}$$

$$\phi_a = 4.6\, a^2 \qquad \text{from} \quad \textbf{(23)}$$

TABLE III.

θ	b	g_b	C	b^3	$C g_b$	ϕ_i	a	a^2	ϕ_a	i	$\phi_i\, i$	$\phi_a\, i$
0	0.333	0.0165	1.09	0.036	0.018	4.76	0	0	0	0	0	0
15	0.333	0.0177	1.09	0.036	0.0193	4.43	0	0	0	0.26	1.15	0
30	0.333	0.0188	1.08	0.036	0.204	4.2	0	0	0	0.50	2.1	0
45	0.25	0.0196	1.08	0.0156	0.0212	1.75	0.083	0.007	0.032	0.71	1.24	0.023
60	0.167	0.020	1.08	0.0047	0.0216	0.52	0.167	0.028	0.128	0.87	0.45	0.111
75	0.083	0.023	1.07	0.006	0.0246	0.06	0.250	0.063	0.29	0.97	0.06	0.28
90	0	0.027	1.07	0	..	0	0.333	0.110	0.505	1.0	..	0.505

Dimensions are expressed as per cent of pole pitch.
Average gap = 0.02

$$\Phi_i = 2.4 \frac{0.333^3}{1.08 \times 0.02} = 4.08$$

$$\Phi_a = 4.6 \times 0.333^2 = 0.505$$

Fig. 14 shows the above values of $\phi_i i$ and $\phi_a i$, and the total tooth-tip leakage over 180 deg. These waves were analyzed for harmonics and the equation is given in the figure. The fundamentals only are plotted.

It will be noted that the third is the most important harmonic, and since the third harmonics cancel in the terminal voltages of star connected three-phase machines, the effect of only the fundamental needs to be considered. It is obvious from the equation in Fig. 14 that the higher harmonics may be neglected without introducing any appreciable error.

C_i will be the ratio of the maximum value of the fundamental of $\phi_i\, i$ Fig. 14 to the value of Φ_i that is,

$$C_i = \frac{0.89}{4.08} = 0.218$$

Similarly

$$C_a = \frac{0.22}{0.505} = 0.435$$

In the derivation of C_i and C_a a sine wave of armature current has been assumed. This is very nearly the case with a star-connected machine, and as has been explained before the short-circuit current, the current as read on an ammeter is practically equivalent to that with star-connection for the same field current. The values of C_i and C_a as derived for star connection may therefore be used for delta connection, even though the current is not a sine wave in the latter case.

The values of C_i and C_a were calculated for complete distribu-

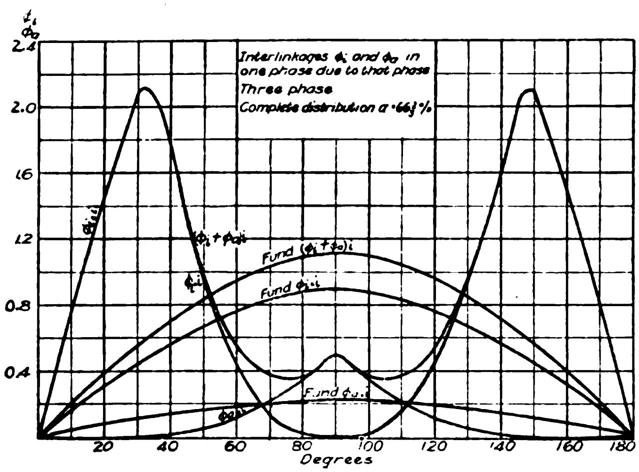


FIG. 14

$$\phi_i = .89 \sin \theta + 1.22 \sin 3\theta + .29 \sin 5\theta - .15 \sin 7\theta - .18 \sin 9\theta - .06 \sin 11\theta$$

$$\phi_a = .22 \sin \theta - .13 \sin 3\theta + .06 \sin 5\theta - .04 \sin 7\theta + .04 \sin 9\theta - .01 \sin 11\theta$$

$$\phi_i + \phi_a = 1.11 \sin \theta + 1.09 \sin 3\theta + .35 \sin 5\theta - .19 \sin 7\theta - .14 \sin 9\theta - .07 \sin 11\theta$$

tion of the armature winding with various ratios of pole arc to pole pitch for three phase and two phase by the method shown in Table III, and are given in Fig. 15.

The variation of C_i and C_a with distribution of the armature winding was investigated and it was found the results obtained by the application of these factors C_i and C_a to Φ_i and Φ_a would give practically correct results when applied to the case of one slot per pole and phase. It is evident that there is just one leakage path around the coil in the case of one slot per pole and phase, and this path will be partly through the pole face iron and partly

through air, but the length of path in each will vary as the pole moves along. The total interlinkages were calculated over 90 deg. for this condition and the waves analyzed. It was found that the ratio of the fundamental of the total interlinkage wave in this case to the value of $\Phi_i + \Phi_a$ was practically the same as the ratio of $C_i \Phi_i + C_a \Phi_a$ to $\Phi_i + \Phi_a$, that is the values of C_i and C_a derived for complete distribution apply also to one slot per pole and phase.

It is very important to note that while the values of C_i and C_a do not change with distribution, Φ_i and Φ_a do show a large variation.

The polyphase effect in tooth-tip reactance has been discussed in the first part of this paper and is taken into account by increasing the values of C_i and C_a by 50 per cent for three phase. For two phase the values of C_i and C_a are used as calculated for one phase of the winding.

Let C_1 be the factor by which Φ_i is multiplied to obtain the effective interlinkages per ampere, and C_2 be the factor for Φ_a Then

$$
\begin{aligned}
C_1 &= 1.5\ C_i \text{ for three phase} \\
&= C_i \quad \text{for two phase} \\
C_2 &= 1.5\ C_a \text{ for three phase} \\
&= C_a \text{ for two phase}
\end{aligned}
$$

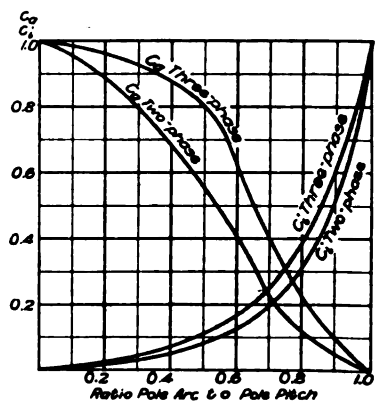


FIG. 15—VALUES OF C_i AND C_a

It will be noted that the values of C_i and C_a are not the same for two phase as for three phase.

The factors C_1 and C_2 for both three phase and two phase are plotted against ratio of pole arc to pole pitch in Fig. 16.

The reduction factor for tooth-tip reactance is the same as that for the slot reactance above the conductor, but may be taken as the average factor K for Fig. 11.

Reactance Formulas. The various factors of the reactance formula may be combined to give the following equation.

For dimension in centimeters.

$$
x_a = \frac{2\pi f\ s^2\ a^2\ q\ l}{10^8\ c^2} \left\{ \overset{\text{End}}{\frac{2.8\ \sqrt{\tau}}{l}} + \right.
$$

Slot Tooth Tip

$$K\left[\frac{0.4\pi}{s}\left(\frac{d_1}{3\,w_s} + \frac{d_2}{w_s}\right)0.4\pi\,C_1\,K_i\,\frac{w_s+w_t}{2_g\,C} + C_2\,\Phi_a\right]\Bigg\} \quad (35)$$

Two Phase
End

$$x = \frac{2\;\pi f\,s^2\,a^2\;q\,1}{10^2\,c^2}\left\{\frac{1.6\;p\;\sqrt{\tau}}{1} + \right.$$

Tooth Tip

$$K\left[\frac{0.4\;\pi}{s}\left(\frac{d_1}{3\,w_s}+\frac{d_2}{w_s}\right)+0.4\;\pi\,C_1\,K_i\,\frac{w_s+w_t}{2\,g\,C}+C_2\Phi_a\right]\Bigg\} \quad (36)$$

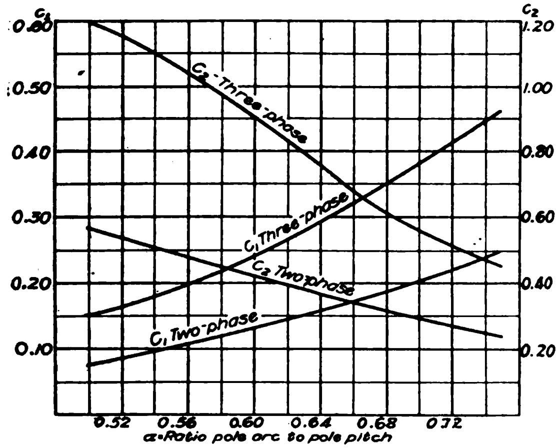


FIG. 16—VALUES OF C_1 AND C_2

REDUCTION FACTORS FOR TOOTH-TIP LEAKAGE

$C_1 = 1.5\ C_i$ for three phase
$= \quad C_i$ for two phase.
$C_2 = 1.5\ C_\alpha$ for three phase
$= \quad C_\alpha$ for two phase.

For dimensions in inches.

Substitute 4.5 for 2.8 in factor for three-phase end leakage; 2.5 for 1.6 in case of two-phase: and 3.2 for 0.4 in slot leakage and first factor of tooth-tip leakage. Use Φ_a in lines per inch.

Where x_a = armature reactance per phase in ohms.
f = frequency in cycles per second
s = slots per pole and phase
a = conductors per slot; for a double layer winding a is twice the number of turns per coil.
q = number of poles

l = gross stacked length of armature core (iron + ducts)
c = number of circuits in multiple in armature winding
p = per cent pitch of armature winding (= 0.8 for 80 per cent pitch)
τ = pole pitch at armature core face
K = reduction factor depending on pitch of armature coils
d_1 = distance from top of copper to bottom of copper in the armature winding. Fig. 8
d_2 = distance from top of copper to armature face Fig. 8
C_1 = 1.5 C_i for three phase
= C_i for two phase
C_2 = 1.5 C_a for three phase
= C_a for two phase
See Fig. 16 for C_1 and C_2
K_i = factor for Φ_i depending on slots per pole and phase
w_s = width of slot Fig. 8
w_t = width of tooth at armature face Fig. 8
g = average air gap
C = air gap coefficient based on Carter's fringing coefficient
Φ_a = effective tooth interlinkages from Fig. 12

Reactance in Per Cent. It is very convenient to have the reactance expressed in per cent and in terms of quantities easily available from the design of the machine.

Three Phase

$$\text{Per cent reactance } x_{pa} = \frac{I\,x_a}{e} \tag{37}$$

(x_{pa} = 0.20 means 20 per cent reactance)
Where I = current per phase
x_a = reactance per phase
e = voltage per phase

$$\text{Armature reaction } A = \frac{2.12\,s\,a\,i}{2\,K_p\,K_d\,K_\phi\,c} \tag{38}$$

$$\text{Flux per pole } \Phi = \frac{e \times 10^8\,K_p\,K_d\,K_\phi\,c}{2.22\,s\,a\,q\,f} \tag{39}$$

Let L_0 = bracket expression from reactance formulas (**35**) or (**36**)

$$x_a = \frac{2\,\pi\,f\;s^2\,a^2\,q}{10^8\;c^2}\;L_0 \tag{40}$$

From (**38**)

$$I = \frac{2\ K_p\, K_d\, K_\phi\, A}{2.12\ s^1\ a} \tag{41}$$

and from (**39**)

$$e = \frac{2.22\ s\ a\ q\ f}{10^8\ K_p\, K_d\, K_\phi\ c}\ \Phi \tag{42}$$

$$x_{pa} = \frac{I\ x_a}{e} = \frac{2.67\ K_p^2\, K_d^2\, K_\phi^2 A\ l}{\Phi}\ L_0 \tag{43}$$

Two Phase

Replacing the factor 2.12 in the armature reaction formula by 1.41 the formula for two phase becomes

$$x_{pa} = \frac{4.\ K_p^2\, K_d^2\, K_\phi^2\ A\ l}{\Phi}\ L_0 \tag{44}$$

Where K_d, and L_0 are based on the two-phase curves and formulas. See reactance formula (**36**).

Working Formulas. The expression for

$$\Phi_i = 0.4\ \pi\ K_i\ \frac{w_s + w_t}{2\ g\ C}$$

for dimensions in centimeters and

$$3.2\ K_i\ \frac{w_s + w_t}{2\ g\ C}$$

for dimensions in inches may be simplified by assuming $w_s = w_t$ and then a series of curves for Φ_i plotted against the values of $w_{s/g}$ may be derived for different values of slots per pole and phase. The curves are given for dimensions in inches in Fig. 17. The working formulas for reactance with lengths expressed in centimeters are as follows:

Three Phase

$$x_{pa} = \frac{2.67\ (K_p\, K_d\, K_\phi)^2\ A\ l}{\Phi} \left\{ \overbrace{\frac{2.8\ p\ \sqrt{\tau}}{l}}^{\text{End}} + K \left[\overbrace{\frac{3.2}{s}\left(\frac{d_1}{3\ w_s} + \frac{d_2}{w_s}\right)}^{\text{Slot}} + \overbrace{0.39\ (C_1\,\Phi_i + C_2\ \Phi_a)}^{\text{Tooth Tip}} \right] \right\} \tag{45}$$

Two Phase

$$x_{pa} = \frac{4.\ (K_p\, K_d\, K_\phi)^2\ A\ l}{\Phi} \left\{ \overbrace{\frac{1.6\ p\ \sqrt{\tau}}{l}}^{\text{End}} + K \left[\overbrace{\frac{3.2}{s}\left(\frac{d_1}{3\ w_s} + \frac{d_2}{w_s}\right)}^{\text{Slot}} + \overbrace{0.39\ (C_1\,\Phi_i + C_2\,\phi_a)}^{\text{Tooth Tip}} \right] \right\} \tag{46}$$

Note that factor 0.39 is introduced in the tooth-tip leakage because Φ_i and Φ_o in the curves are expressed in lines per inch length of core.

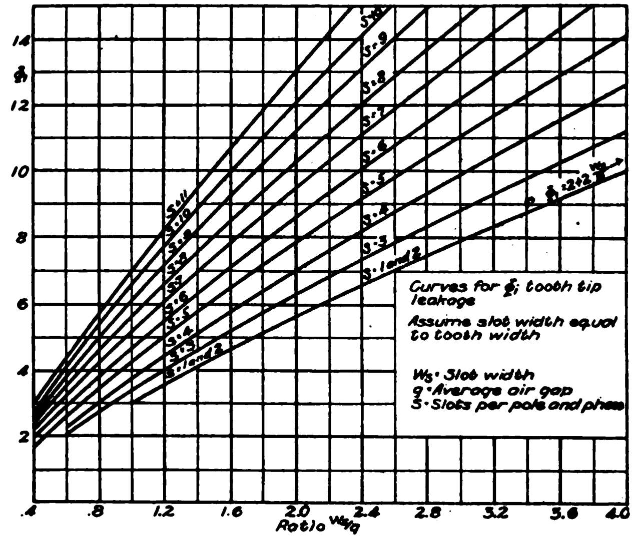


FIG. 17

For dimensions in inches.

$$x_{pe} = \frac{2.67\,(K_p\,K_d\,K_\phi)^2\,A\,l}{\Phi}\left\{\overbrace{\frac{4.5\,p\,\sqrt{\tau}}{l}}^{\text{End}} + \right.$$

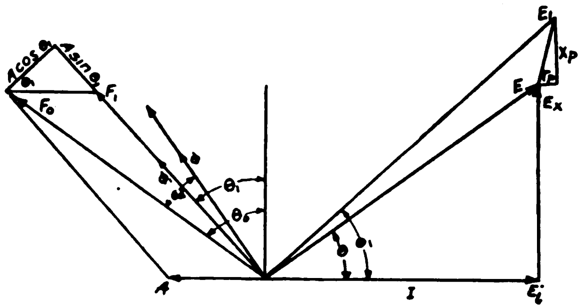


FIG. 18—EXCITATION DIAGRAM

$$\left. K\left[\overbrace{\frac{3.2}{s}\left(\frac{d_1}{3w_s}+\frac{d_2}{w_s}\right)}^{\text{Slot}} + \overbrace{C_1\,\Phi_i + C_2\,\Phi_t}^{\text{Tooth Tip}}\right]\right\} \qquad (47)$$

Two Phase

$$x_{pa} = \frac{4.0\ (K_p\ K_d\ K_\phi)^2\ A\ l}{\Phi} \left\{ \overset{\text{End}}{\frac{2.5\ p\ \sqrt{\tau}}{l}} + \right.$$

$$\left. K \left[\overset{\text{Slot}}{\frac{3.2}{s} \left(\frac{d_1}{3\ w_s} + \frac{d_2}{w_s} \right)} + \overset{\text{Tooth Tip}}{C_1\ \Phi_i + C_2\ \Phi_a} \right] \right\} \tag{48}$$

See Fig. 11 for K, Fig. 17 for Φ_i, Fig. 12 for ϕ_a and Fig. 16 for C_1 and C_2.

Reactance Curves. The formula may be still further simplified by assuming average values of d_1, d_2 and w_s for a certain narrow range of slot pitches. A series of five curves covering the ordinary ranges of slot pitches, and giving values of end, slot and tooth-tip factors as a function of the slots per pole and phase are given in Figs. 20A, 20B, etc. The curves will give the

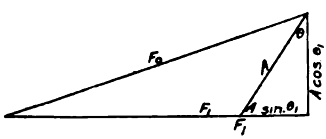


FIG. 19A—M. M. F. DIAGRAM FOR OVER-EXCITED MACHINE

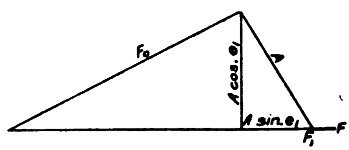


FIG. 19B—M. M. F. DIAGRAM FOR UNDER-EXCITED MACHINE

accuracy necessary for most calculations of reactances and make the work quite simple.

Reactance of Non-Salient Pole Generators. The reactance of turbine generators, the most common type of non-salient pole machines, was calculated by assuming the ratio of pole arc to pole pitch to be the ratio of the unslotted pole centre to pole pitch. Since practically all machines of this type now have five or more slots per pole and phase, the expression for Φ_i was simplified by substituting in equations (35) and (36) the value of K_i expressed as a function of the slots per pole and phase.

That is $$K_i = 0.625 + 0.156\ s \text{ (above } s = 4) \tag{49}$$

With a ratio of pole arc to pole pitch of 33 per cent, an approximate average for turbine generators, the value of C_1 is 0.075 for three phase and 0.035 for two phase, and C_2 is 1.32 for three phase and 0.60 for two phase.

Three Phase

$$C_1 \Phi_i = 0.075 \; (0.625 + 0.156 \; s) \; 0.4 \; \pi \; \frac{w_s + w_t}{2 \; g \; C} \tag{50}$$

in lines per centimeter.

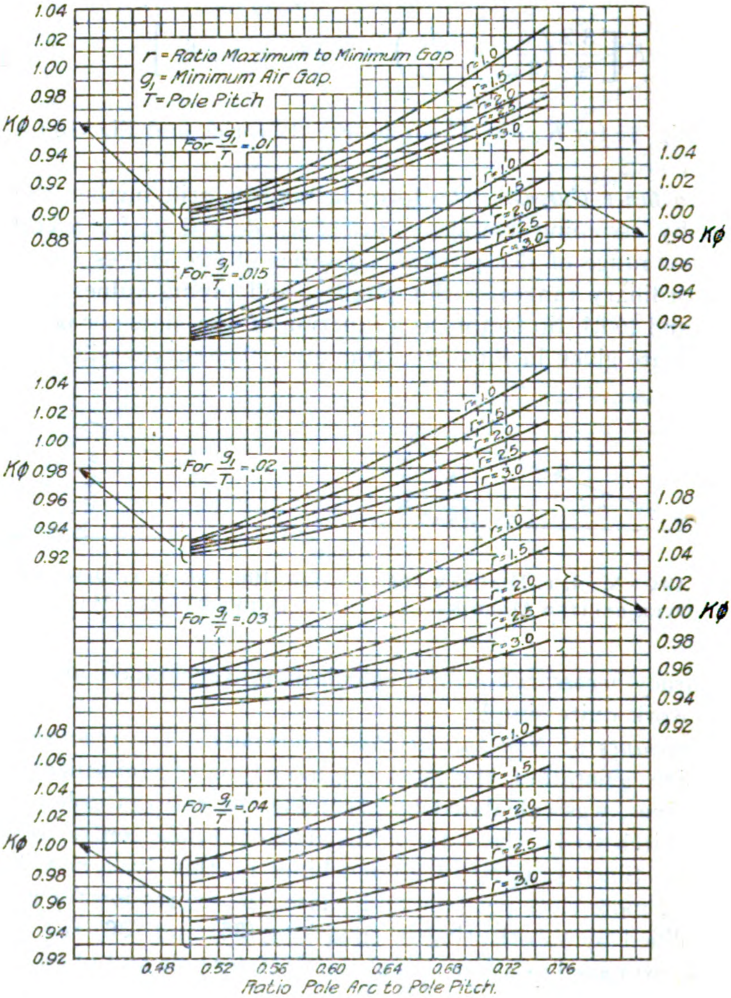


FIG. 20

$$C_1 \; \Phi_i = 0.075 \; (0.625 + 0.156 \; s) \; 3.2 \; \frac{w_s + w_t}{2 \; g \; C} \tag{51}$$

Allowing the approximation $w_s = w_t$ and simplifying we have in lines per inch.

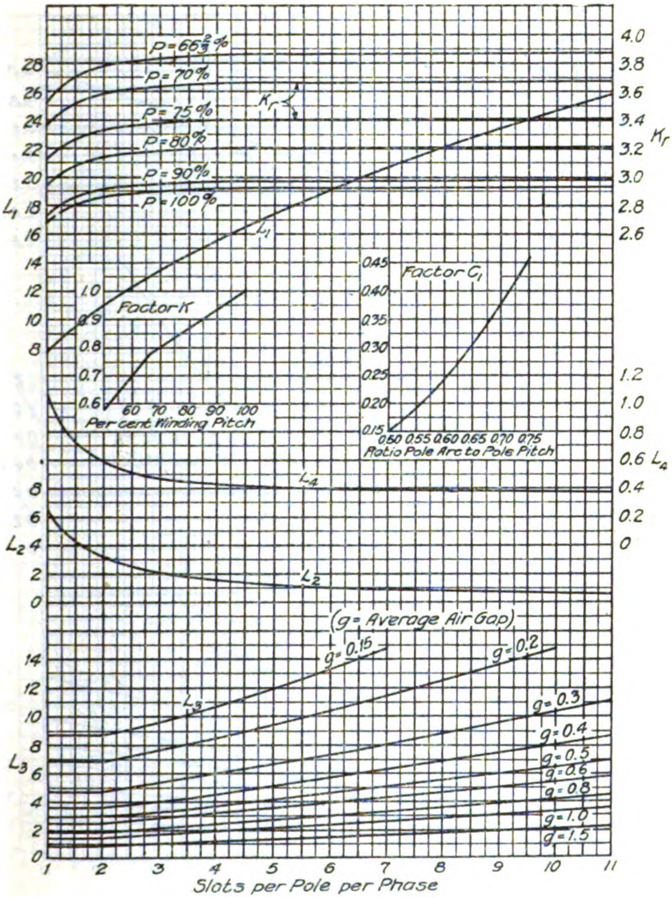


FIG. 20 A

REACTANCE OF THREE-PHASE ALTERNATORS

$$\text{Percent Reactance } X_p = \frac{K_r\, K_\phi{}^2\, A\, l}{\phi}\left[\frac{p\, L_1}{l} + K\,(L_2 + C_1\, L_3 + L_4)\right]$$

(For 20% Reactance $X_p = 0.20$)

K_r, K, L_1, L_2, L_3, L_4 and C are given by curves.
A = $A\ T$ armature reaction per pole.
ϕ = magnetic flux per pole in maxwells.
l = gross length of armature in inches (iron + ducts).
p = arm. winding pitch (0.8 for 80 per cent)
K_ϕ = flux distribution factor (see curves)

Curves for Slot Pitch 0.9 to 1.1. Two-Layer Windings.
For Single-Layer Windings multiply L_1 by 2.0 and use $K = 1.0$.

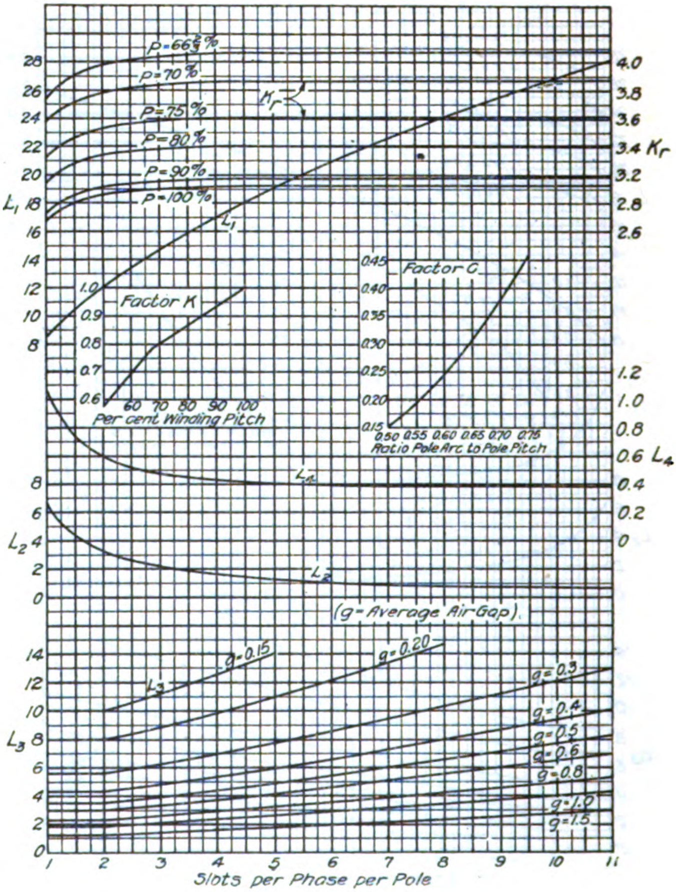


Fig. 20 b

Reactance of Three-Phase Alternators

$$\text{Percent Reactance } X_p = \frac{K_r\, K_\phi^2\, A\, l}{\phi}\left[\frac{p\, L_1}{l} + K\,(L_2 + C_1\, L_3 + L_4)\right]$$

(For 20% Reactance $X_p = 0.20$)

K_r, K, L_1, L_2, L_4 and C are given by curves.

A = $A\,T$ armature reaction per pole.

ϕ = magnetic flux per pole in maxwells.

l = gross length of armature in inches (iron + ducts).

p = arm. winding pitch (0.8 for 80 per cent)

K_ϕ = flux distribution factor (see curves)

Curves for Slot Pitch 1.1 to 1.3. Two-Layer Windings.

For Single-Layer Windings multiply L_1 by 2.0 and use $K = 1.0$.

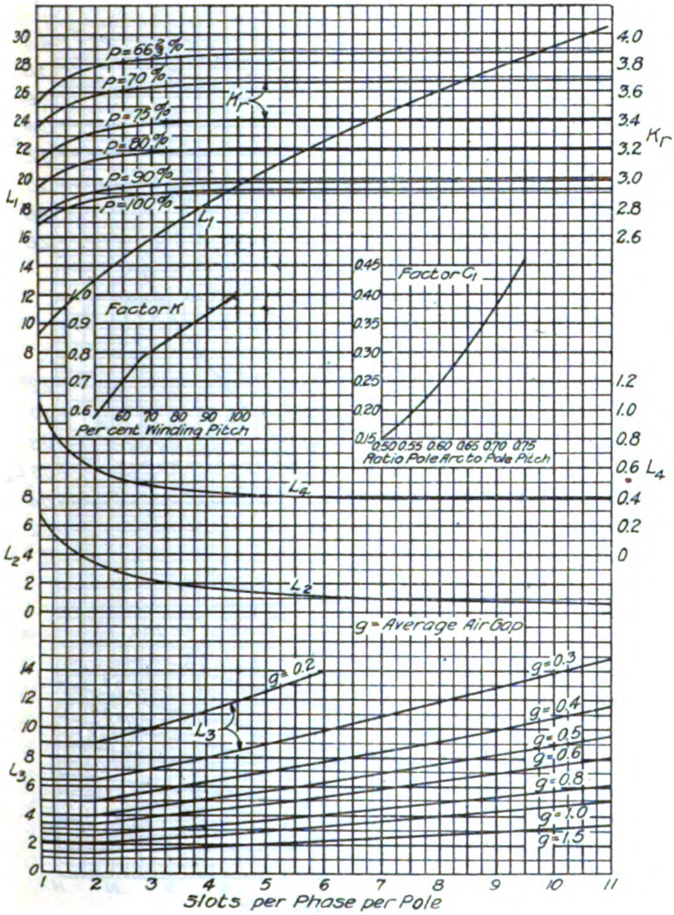


FIG. 20c

REACTANCE OF THREE-PHASE ALTERNATORS

$$\text{Percent Reactance } X_p = \frac{K_r\, K_\phi^2\, A\, l}{\phi}\left[\frac{p\, L_1}{l} + K\,(L_2 + C_1\, L_3 + L_4)\right]$$

(For 20% Reactance $X_p = 0.20$)

K_r, K, L_1, L_2, L_3, L_4 and C are given by curves.

A = $A\ T$ armature reaction per pole.

ϕ = magnetic flux per pole in maxwells.

l = gross length of armature in inches (iron + ducts).

p = arm. winding pitch (0.8 for 80 per cent)

K_ϕ = flux distribution factor (see curves)

Curves for Slot Pitch 1.3 to 1.5. Two-Layer Windings.

For Single-Layer Windings multiply L_1 by 2.0 and use $K = 1.0$.

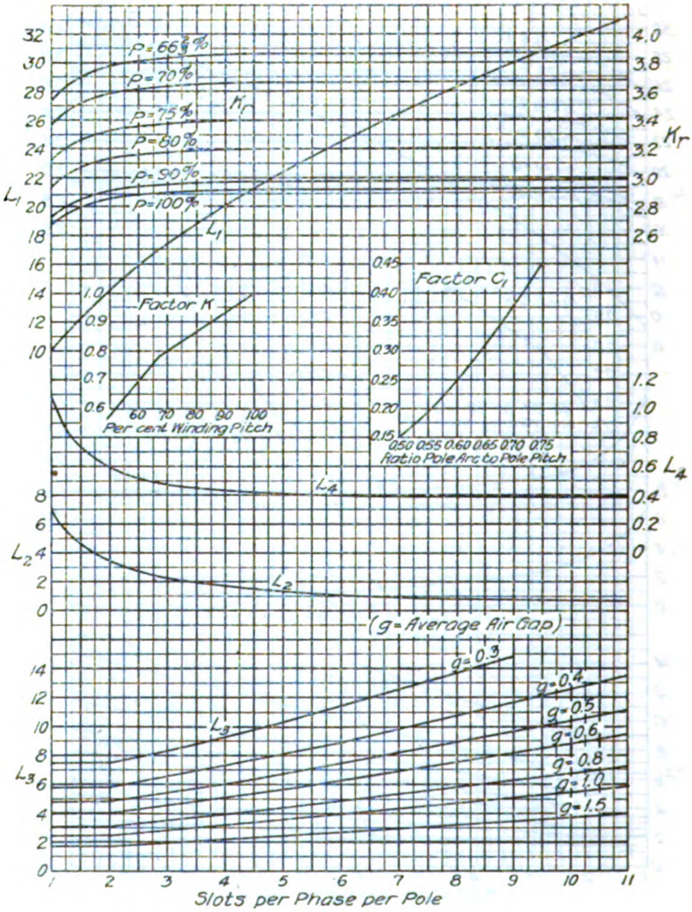


Fig. 20D

Reactance of Three-Phase Alternators

$$\text{Percent Reactance } X_p = \frac{K_r\, K\phi^2\, A\, l}{\phi}\left[\frac{p\, L_1}{l} + K\,(L_2 + C_1\, L_3 + L_4)\right]$$

(For 20% Reactance $X_p = 0.20$)

K_r, K, L_1, L_2, L_3, L_4 and C are given by curves.

A = $A\ T$ armature reaction per pole.

ϕ = magnetic flux per pole in maxwells.

l = gross length of armature in inches (iron + ducts).

p = arm. winding pitch (0.8 for 80 per cent)

K = flux distribution factor (see curves)

Curves for Slot Pitch 1.5 to 1.9. Two-Layer Windings.

For Single-Layer Windings multiply L_1 by 2.0 and use $K = 1.0$.

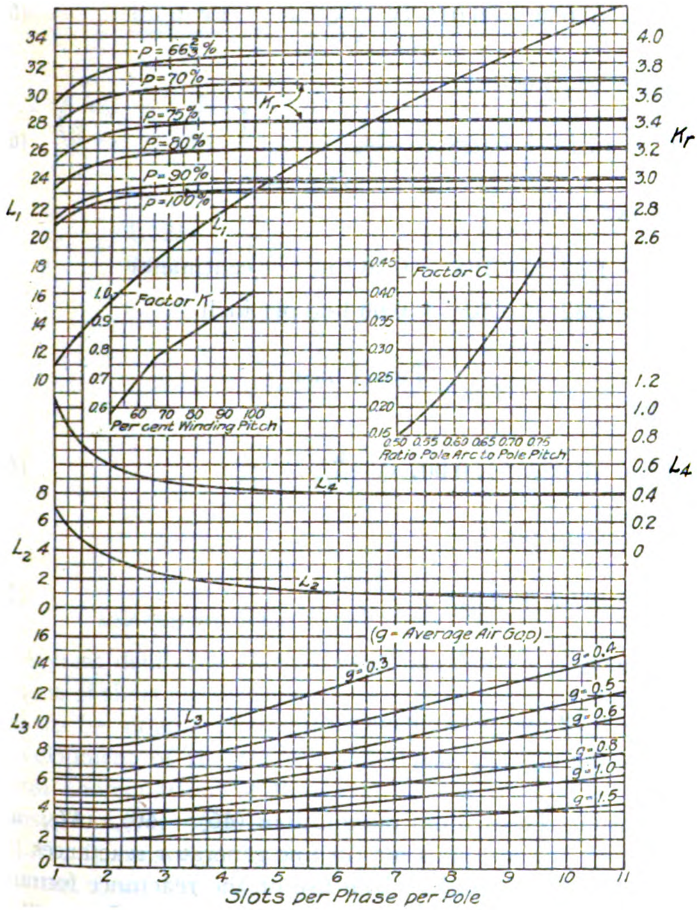


FIG. 20E

REACTANCE OF THREE-PHASE ALTERNATORS

$$\text{Percent Reactance } X_p = \frac{K_r\, K_\phi^2\, A\, l}{\phi}\left[\frac{p\, L_1}{l} + K\,(L_2 + C_1\, L_3 + L_4)\right]$$

(For 20% Reactance $X_p = 0.20$)

K_r, K, L_1, L_2, L_3, L_4 and C are given by curves.

A = $A\ T$ armature reaction per pole.

ϕ = magnetic flux per pole in maxwells.

l = gross length of armature in inches (iron + ducts).

p = arm. winding pitch (0.8 for 80%)

K_ϕ = flux distribution factor (see curves)

Curves for Slot Pitch 1.9 to 2.1. Two-Layer Windings.

For Single-Layer Windings multiply L_1 by 2.0 and use $K = 1.0$.

$$C_1\,\Phi_i = (0.059 + 0.015\,s)\,\frac{w_s}{g\,C} \tag{52}$$

in lines per centimeter.

$$C_1\,\Phi_i = (0.15 + 0.038\,s)\,\frac{w_s}{g\,C} \tag{53}$$

in lines per inch.

$$\begin{aligned} C_2\,\phi_a &= 1.32 \times 0.198 = 0.26 \text{ line per centimeter} \\ &= 1.32 \times 0.50 = 0.66 \text{ line per inch.} \end{aligned}$$

Two Phase

Similarly

$$C_1\,\Phi_i = (0.0275 + 0.007\,s)\,\frac{w_s}{g\,C} \tag{54}$$

lines per centimeter.

$$C_1\,\Phi_i = (0.07 + 0.018\,s)\,\frac{w_s}{g\,C} \tag{55}$$

lines per inch.

$$\begin{aligned} C_2\,\Phi_a &= 0.12 \text{ line per centimeter.} \\ &= 0.3 \text{ line per inch} \end{aligned}$$

Substituting these values in equations (**45**), (**46**), (**47**), and (**48**) will give the formulas for turbine generator reactances.

Application of Formula. The use of the reactance formulas will be illustrated by the application of equation (**47**) to a 6600-kv-a. three-phase 25-cycle 14,000-volt generator.

Example:

$s = 5.25$	$p = 82.5\% = 0.825$	$A = 11,000$	$p = 0.925$		
$\tau = 36.5$	$a = 0.698$	$w_s = 1.125$			
$d_1 = 3.2$	$d_2 = 1.05$	$\Phi = 47.8 \times 10^6$			
$K_p = 1.04$	$K_d = 1.046$	$K_\phi = 1.016$			

$$\frac{2.67\,(1.04 \times 1.046 \times 1.016)^2\,11{,}000 \times 38}{47.8} = 0.0283$$

$$\frac{4.5\,p\,\sqrt{\tau}}{l} = \frac{4.5 \times 0.825\ \sqrt{36.5}}{38} = 0.59$$

$$\frac{3.2}{s}\left(\frac{d_1}{3\,w_s} + \frac{d_2}{w_s}\right) = \frac{3.2}{5.25}\left(\frac{3.2}{3.375} + \frac{1.05}{1.125}\right) = 1.15$$

$$K = 0.885 \qquad K \times 1.15 = 1.02$$

$$\frac{w_s}{g} = \frac{1.125}{0.925} = 1.22 \qquad \Phi_i = 5.1 \qquad C_1 = 0.37$$

$$K\,\Phi_i\,C_1 = 5.1 \times 0.37 \times 0.885 = 1.67$$

$$\Phi_a = 0.54 \qquad C_2 = 0.57$$

$$K\,\Phi_a\,C_2 = 0.54 \times 0.57 \times 0.885 = 0.27$$

$$3.55$$

$$x_{pa} = 0.0283 \times 3.55 = 0.10$$
$$= 10 \text{ per cent}$$

PART II

Transient Reactance and Short Circuits

A great deal of work has been done toward placing the phenomena of short circuits upon a calculable basis, but there is much yet to be done. Diamant's paper[7] reviewed historically the development of the theory underlying short circuits giving in detail the equations of Berg and Boucherot, and with some extensions in the mathematics, gave an expression from which, if all the factors were known, the entire wave of instantaneous current, from the instant of short circuit until the sustained value is approached, might be predicted. The potential usefulness of such an expression is obvious; but it is equally obvious to what extent its usefulness is restricted until all of the factors are known.

This paper offers methods by which the factors may be calculated with practical accuracy on certain classes of machines, (those with salient, laminated poles without amortisseur winding) but there are refinements yet to be made, and extensions to be worked out for other types of machines.

Diamant's equation (**26**), which gives the curve of positive crests, is perhaps the most practical form for the engineer's use. In our notation and simplified for the condition of short circuit

7. Trans. A. I. E. E., 1915, Vol. 34, p. 2237.

occurring at zero voltage (*i.e.* wave completely offset, which must be assumed in most engineering calculations) is,

$$i_c = \sqrt{2}\,(I_0 - I_s)\,\epsilon^{-\alpha_f t} + \sqrt{2}\,I_0\,\epsilon^{-\alpha_a t} + \sqrt{2}\,I_s$$
$$= i_1 + i_2 + \sqrt{2}\,I_s$$

where

I_0 = alternating component[8] of short-circuit current $= \dfrac{E}{x_0}$

E = voltage per leg.

x_0 = total reactance effective on sudden short circuit. This should be called *transient reactance*, as suggested by Durgin and Whitehead[9] and by F. D. Newbury[10].

I_s = sustained value of short-circuit current.

α_f = field attenuation factor, defined later.

α_a = armature attenuation factor, defined later.

t = seconds after short circuit.

For calculation, a convenient form is:

1st term, $$i_1 = \sqrt{2}\,(I_0 - I_s)\,\epsilon^{-\alpha_f t}$$

$$\log_e \frac{\sqrt{2}\,(I_0 - I_s)}{i_1} = \alpha_f\, t$$

2d term, $$i_2 = \sqrt{2}\,I_0\,\epsilon^{-\alpha_a t}$$

$$\log_e \frac{\sqrt{2}\,I_0}{i_2} = \alpha_a\, t$$

Fig. 21 shows the skelton thus derived. Beyond the ordinate A, when i_2 is practically zero, the wave is symmetrical about the horizontal axis. The curve of negative crests between o and c can be obtained by plotting i_2 from the zero line as shown by dotted curve $n\ m$. This may be then taken as the zero of the alternating component; *i.e.* make $a = b$ at all points between o and m.

With the curve shown in Fig. 21, the three questions which the designing engineer is asked, can be more accurately answered than in the past; namely, what is the first rush of current; what is the current at, say, ¼ or ½ second after short circuit, this in-

8. Term proposed by Hewlett, Mahoney and Burnham, PROC. A. I. E. E., Feb. 1918, p. 46.

9. TRANS. A. I. E. E., Vol 31, 1912, p. 1662.

10. *Elec. Journal*, April 1914, p. 196.

terval depending usually upon the operating time of the circuit breaker; and what is the sustained value.

The sustained value is easily taken from the design sheet by the well known method. The value of I_0 is determined by the transient reactance, x_0. And it will be shown later that α_f and α_a depend upon x_0 and the resistance of field and armature circuits.

In the past, the rate of decrease of the current has been predetermined by empirical curves, based upon oscillographic data, such as given by Hewlett, Mahoney and Burnham[11]. After all, for commercial engineering work, such curves will still be used; but in their preparation or in the investigation of special cases, the above equation will be very useful. But before those

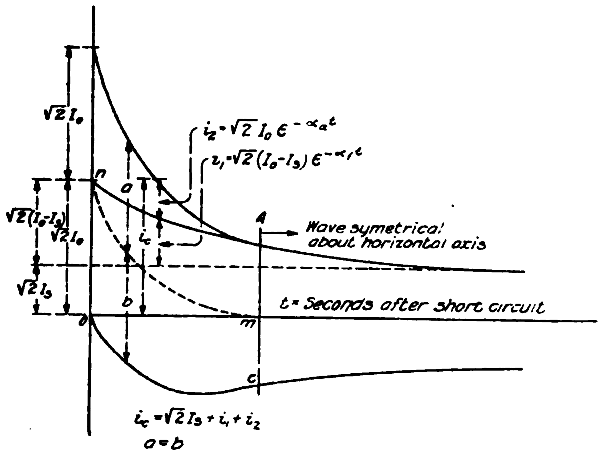


FIG. 21—SHORT-CIRCUIT ARMATURE CURRENT

curves can be used, the generator transient reactance which adds directly to the external reactance, must be known.

The authors would make it clear at this point that they are not in agreement with conception given in Diamant's paper, of what determines the first rush of current, and of what constitutes the factors α_f and α_a. He states that the initial value of the alternating component (crest value), which he calls $(A + B)$, "is equal to the maximum phase voltage before short circuit divided by the armature impedance, with good approximation." As brought out in the following paragraph, this assumption would lead to serious error in many cases. The matter of α_f and α_a will be discussed in subsequent pages.

11. PROC. A. I. E. E., Feb. 1918. p 48.

Turning now to the problem in hand, the reactance which limits the initial short-circuit current in most synchronous machines is not the armature self-induction only; it is rather the combined self-induction of both the armature and field circuits. The calculated short-circuit current, based on the armature self-induction, is therefore higher, in some cases 40 to 50 per cent higher, than the actual short-circuit current. In salient, laminated pole alternators, in which the field leakage flux, that is, the self-induction is large and in which rotor eddy currents are practically negligible, this difference is more pronounced than in the continuous rotor construction of turbine generators, in which the effective field self-induction is largely reduced by the distribution of the field winding and also by the solid steel rotor,[12] or in salient pole machines with low resistance amortisseur winding. Hence in the latter two cases, as might be expected and as experience indicates, the armature self-induction is a reasonably accurate measure of the short-circuit current; and it is proposed, for the present at least, that this basis be used for these cases. This is not the point of view taken by Durgin and Whitehead[13] in 1912. Their conclusion, based on tests of a 12,000-kw., 9000-volt, 25-cycle turbo-generator in the Fisk Street Station of the Commonwealth Edison Company, was that "the short-circuit currents of alternators are limited by reactance much more complex and much higher than the self-inductive reactance of the armature." While the authors agree, as stated above, that this is true for alternators with salient, laminated poles, they submit that it is not true for many turbo-generators, as now ordinarily constructed, or in the case of salient pole machines with low resistance amortisseur windings. Tests in Table IV, confirm this. In the particular 12,000-kw. generator the armature self-inductive reactance was taken as two per cent. The short circuits indicated about three times this value. The armature self-inductive reactance of the above 12,000-kw. generator as determined by the method given in the paper, is 4.3 per cent. This machine has laminated rotor face, which would allow the field self-induction to act. It is the author's opinion, therefore, that the added field self-induction in this case made up most of the difference between the 4.3 per cent and the approximately 6.5 per cent which the test showed. Our point is this: That the conclusion, quoted above, which

12. A. B. Field, TRANS. A. I. E. E., Vol. 31, Part 2, p. 1651.
13. TRANS. A. I. E. E., Vol. 31, Part 2, p. 1657.

was arrived at in Durgin and Whitehead's paper is a little too broad. It does not correctly apply to alternators in general, certainly not to turbine generators with solid steel rotor construction, to single-phase generators with heavy amortisseur winding in the pole face, nor, it would seem, to generators with solid steel poles. In salient, laminated pole machines, however, the field self-induction must be taken into account, and in the following considerations a method is developed by which this can be done.

In order to establish a conception of the effect of armature and field self-induction, consider in part the phenomena which occur in an alternator under short circuit, and the fundamental principle involved. A closed electric circuit, without resistance, must persist magnetically in the same condition as at the instant of closing; that is, must contain, so long as it is closed, the same number of magnetic interlinkages. If the flux tends to increase or decrease, this will produce a current of sufficient magnitude to maintain the interlinkages which existed when the circuit was closed. This law determines, in a large measure, what happens under short circuit; because, although both the armature and field circuits actually contain some resistance, and the currents therefore die down in the familiar transients, Fig. 22, yet its effect during the first cycle or so after short circuit is, for qualitative considerations at least, negligible.

It follows that the flux linked with the field winding before short circuit must persist after short circuit; that likewise whatever flux is caught in the armature circuit at the instant of short circuit, must there persist; and that flux from the rotating field poles can not now enter the armature, *i. e.*, can not change the magnetic interlinkages of the armature circuit.

Consider a simple case of closed circuit in Fig. 23. The closed-ring conductor, without resistance, is moved from position *a* to the center of the magnetic field, ϕ, in *b*. This causes a current I_0 in the ring—a current sufficient to produce around the leakage paths of the ring a flux equal to $-\phi$. This is illustrated hypothetically in *c*. But I_0 in the ring in *b* tends to demagnetize the field. This requires a corresponding increase in m.m.f. of the field coil, to maintain the flux ϕ linked with the coil. The result, as shown on *b*, is to force the flux around the ring.

Suppose the ring is closed around the flux as shown in *d*. Then if the ring is moved to position in *f*, the result will be as

shown: the magnetic flux ϕ will remain linked with the field coil, as in d, and in addition, another magnetic field of equal value will be maintained around the ring by a current I_0, the

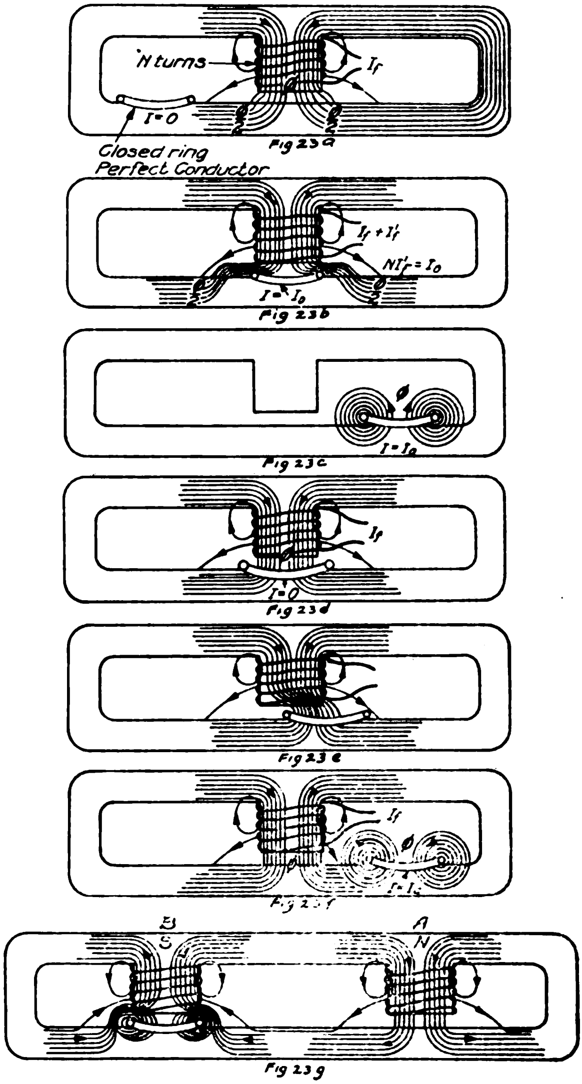


FIG. 23—DIAGRAMMATIC REPRESENTATION OF SHORT-CIRCUIT FLUXES

same value, (assuming equal reluctance of leakage path), which was necessary in b. Force is required to move the ring from d to f, as shown by the distortion in e. The work done in

moving the ring is the energy stored in the new magnetic field around the ring in *f*. If the ring containing zero flux, as in *a*, were moved along in *g*, the current would rise to I_0 when under *A*, decrease to zero between *A* and *B*, and rise to $-I_0$ under *B*; in other words, an alternating current would be produced. If, however, the ring, enclosing the flux ϕ and containing the direct current I_0, as in *f*, were moved as above, the current would decrease to zero under *A*, rise again to I_0 between *A* and *B*, and reach $2I_0$ under *B*. In this case, the resultant current, varying from zero to $2I_0$ is made up of an alternating component and a direct component, the latter being equal to the maximum of the former.

The events occurring in an alternator immediately after a no-load single-phase short circuit have thus been illustrated. The short circuit at maximum voltage is represented by the ring enclosing zero flux, the short circuit at zero voltage, by the ring enclosing the flux ϕ. Fig. 23E illustrates how a large alternating torque is produced when the short circuit occurs at zero voltage.

A three-phase short circuit at no load is different from the single-phase short circuit in this important respect, that it is impossible, three-phase, to have the short circuit occur when zero flux is enclosed in the short-circuited armature winding. The entire flux is at all times enclosed by some parts of the armature winding, and therefore in whatever position the field structure happens to place the flux when the short circuit occurs, there it must remain, a series of polar regions, stationary in space, while each field pole with its equal value of flux moves on These stationary polar regions of normal intensity, of course generate in the rotating field winding an alternating current of normal frequency, and of a maximum value in m.m.f. equal to the direct current which maintains the stator polar regions That is, the flux, once intact through the entire normal magnetic path is now literally torn apart at the air gap, and at 180 electrical degrees from the instant of short circuit, forms two separate magnetic circuits, closing through the leakage paths between armature and field windings. At the end of one cycle the flux is again intact through the normal path. All the work which, during the first half cycle, was done on the field in tearing it apart, in actually establishing normal interlinkages in circuits of much lower permeance than that of the normal circuit, is returned to the rotor during the last half cycle. The torque which accompanies this transfer of energy has been calculated by S. H. Weaver, *G. E. Review*, Nov., 1915.

Return to the question of the self-inductive reactance existing at sudden short circuit. It has been generally understood how the armature self-induction limits the initial short-circuit

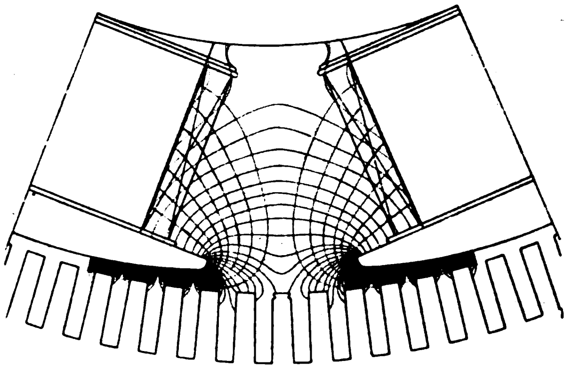


FIG. 24A—DERIVED DIAGRAM OF MAGNETIC DISTRIBUTION OF SALIENT-POLE MACHINE

current; that is, how, if a machine has 10 per cent armature self-induction, normal armature current would cause flux through the armature leakage paths equal to 10 per cent of normal flux,

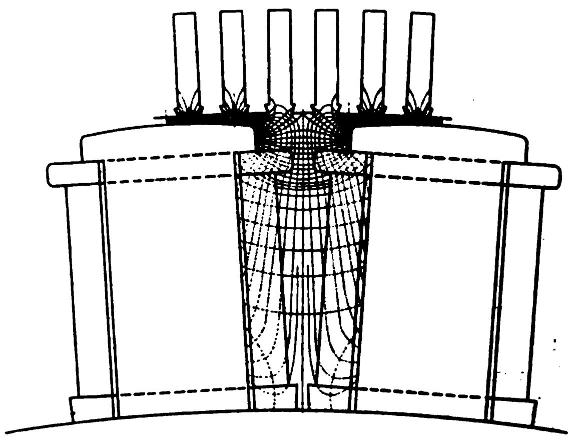


FIG. 24B—DERIVED DIAGRAM OF MAGNETIC DISTRIBUTION OF SALIENT-POLE MACHINE

and that therefore on sudden short circuit, when (neglecting field self-induction) the total normal flux must be absorbed in the leakage paths, the armature current would have to rise to ten times normal. It may not have been so clear how and to

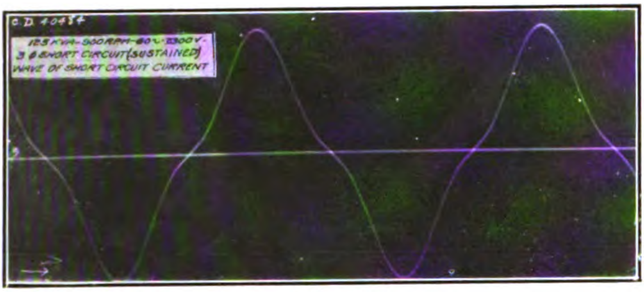


FIG. 4

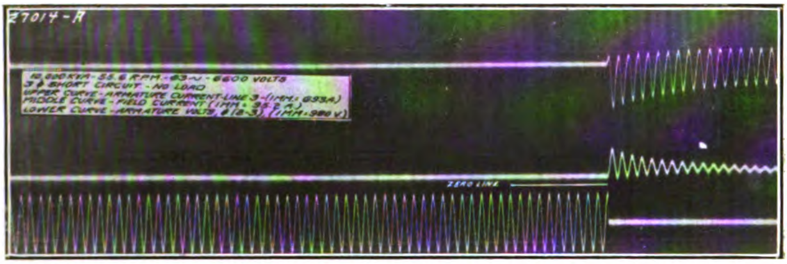
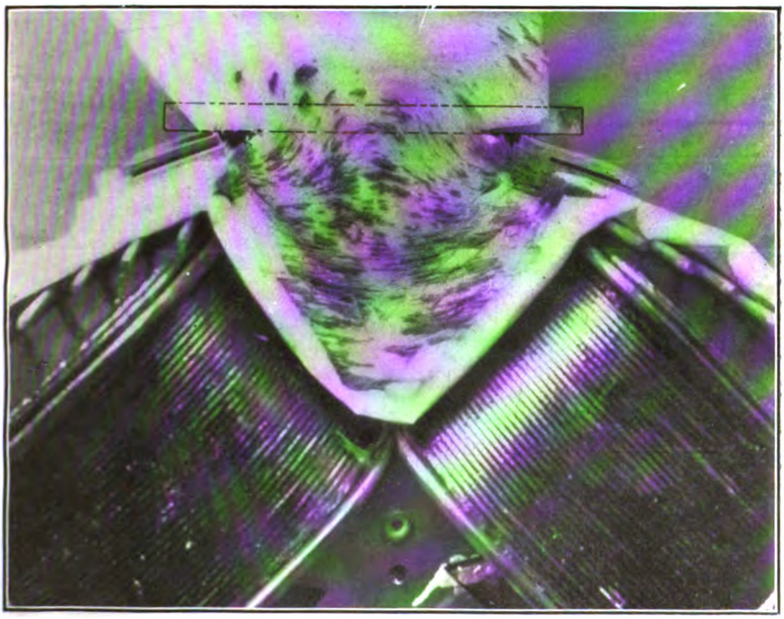


FIG. 22



[DOHERTY AND SHIRLEY]

FIG. 25B.—ACTUAL MAGNETIC DISTRIBUTION OF FOUR-POLE MACHINES.

what extent the field self-induction affected the short-circuit current. From the foregoing considerations and from an approximate knowledge of the field leakage paths, it is possible to determine this point. The leakage paths have been approximately determined by plotting equi-magnetic potential lines as shown in Figs. 24A and 24B. The flux must be at right angles to these lines. The entire flux emanating from field poles of different proportions is thus illustrated in these diagrams. Figs. 25A and 25B show the graphically determined and actual magnetic distribution of a 4-pole rotor with an iron plate bridging the poles, wooden blocks supporting the ends and forming an air gap. It will be noted that a large portion of the leakage flux emanates

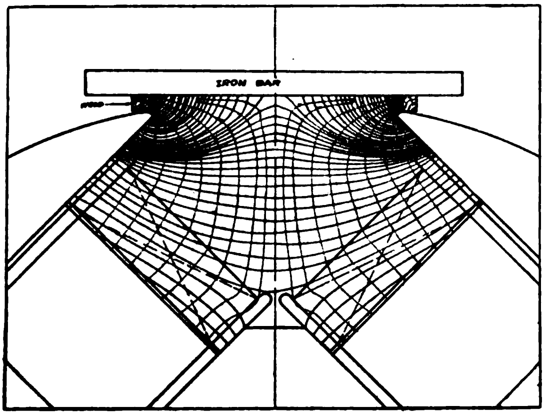


FIG. 25A—DERIVED DIAGRAMS OF MAGNETIC DISTRIBUTION OF FOUR POLE MACHINES

from the under side of the pole tip, which, we believe has not generally been considered in leakage calculations. In Fig. 25B the paths as indicated by the iron filings can not be traced to the pole body and to all of the under surface of the pole tip, on account of the field winding. But it is clear from the direction of the paths at the edge of the field winding that they will terminate approximately as shown in Fig. 25A. These figures of course do not show quite what the distribution would be if the rotor were placed in the stator. Then both the derived and actual distribution would be different, but not greatly different. This experiment was made to confirm the method. Fig. 26 shows the distribution on a machine with a large number of poles.

Fig. 27A represents the distribution of flux at no load on a three-phase generator. *A*, *B* and *C* represent the different phase belts. Fig 27B represents the same machine with open

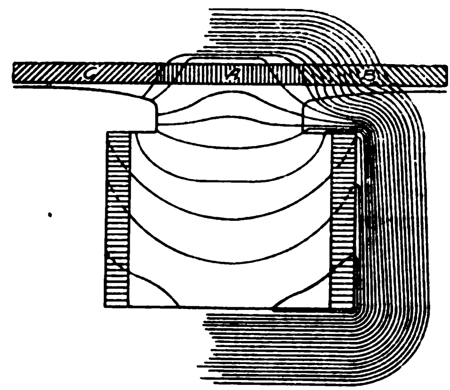


FIG. 27A—DIAGRAM OF MAGNETIC FIELD AT NO-LOAD EXCITATION ON FIELD

field circuit, the field poles rotating synchronously in position shown, and with a three-phase current in the armature. The entire magnetic flux shown in Fig. 27B represents the synchronous reactance. All magnetic lines in Fig. 27B which

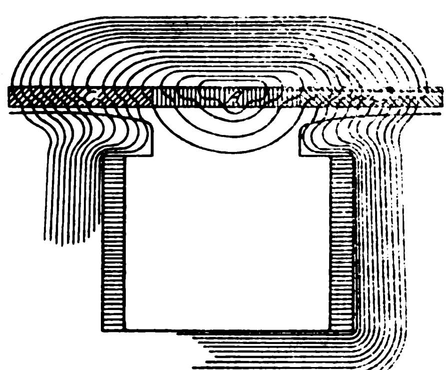


FIG. 27B—DIAGRAM OF MAGNETIC FIELD AT NO-LOAD—THREE-PHASE EXCITATION ON ARMATURE FIELD REVOLVING AT SYNCHRONOUS SPEED

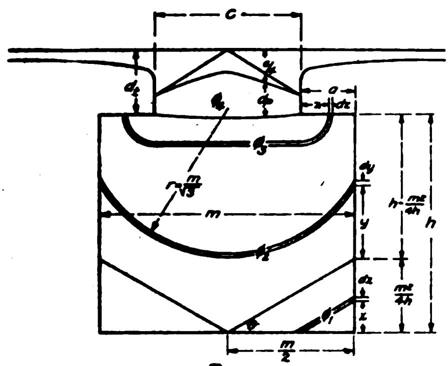
do not link with the field winding represent the armature self-induction, or armature reactance. Likewise in Fig. 27A, that part of the field flux which does not link with the armature winding represents field self-induction, or field reactance. Just

as on sustained short circuit, the field current causes the armature current to flow, and must therefore be greater in m.m.f. than the armature current, by the armature self-induction, that is greater in the ratio,

$$\frac{\text{synchronous reactance at}}{\text{effective armature reaction}}$$

$$= \frac{\text{total flux linked with armature, Fig. 27B}}{\text{mutual flux, Fig. 27B}}$$

so also on sudden short circuit, when the armature current causes the rise in field current, the armature m.m.f. must be



$$\phi = \tan^{-1}\frac{\frac{m}{2}}{h} = \tan^{-1}\frac{m}{2h}$$

$$\frac{m}{2}\tan\phi = \frac{m}{2}\times\frac{m}{2h} = \frac{m^2}{4h}$$

$$a_p = d_t - \frac{c}{4}$$

FIG. 28—DIAGRAM FOR CALCULATING FIELD LEAKAGE

greater than the rise in field m.m.f. by the field self-induction, that is greater in the ratio,

$$\frac{\text{total field winding interlinkages, Fig. 27A}}{\text{mutual flux interlinkages, Fig. 27A}}$$

For convenience in developing the point, assume that all of the field leakage is concentrated between pole tips shown as ϕ_4 in Fig. 28. On sudden short circuit, since the normal flux which is linked with the field winding and which before short circuit entered the armature, can now neither enter the armature nor die out through the field winding, it must be forced through the leakage paths. At the instant at which the armature current in its rise at short circuit passes through normal value, the accompanying rise in field m.m.f. will be approximately equal (actually less) to the effective armature reaction, A, correspond-

ing to normal current. This increase in field m.m.f. must cause a corresponding increase in ϕ_4, Fig. 28; but since ϕ_4 links with the field circuit, and the interlinkages can not increase, it follows that the increase in ϕ_4 represents the portion of the normal flux Φ, which is absorbed on short circuit by the field self-induction when the field and armature m.m.f's. have risen to A. As an illustration, let the flux ϕ_4 corresponding to a field m.m.f., A, be 4 per cent, and let the armature self-induction be 10 per cent. Then as the armature current passes through normal value, 14 per cent of normal flux Φ, will have been absorbed in the leakage paths; and the armature current will rise to 7.15 times normal value.

But ϕ_4 does not, as above assumed, represent all of the field leakage. Other paths of ϕ_1, ϕ_2, ϕ_3, etc., Fig. 28, exist. It is not so obvious how this additional leakage from the sides and ends of the pole, behaves, and how it operates to limit the short-circuit current. It would appear that none of the normal flux, Φ, could sink into leakage paths of ϕ_1, and ϕ_2, since in doing so it would have to cut through the closed field winding; and also, and for the same reason, that the leakage ϕ_1 and ϕ_2 could not increase. Actually, both happen. The condition which must be fulfilled is that *the total effective interlinkages of the field circuit shall not change.* Two lines linking one half of the field turns is equivalent to one line linking all of the turns; four lines linking one fourth of the turns is equivalent to one line linking all, etc., etc. What actually happens, therefore, is that as the short-circuit current rises, some of the normal flux, Φ, sinks into the paths of ϕ_1 and ϕ_2; and to just the extent to which interlinkages are decreased in this process at the top of the winding, new flux, compensating for the loss, appears at the bottom of the winding. So that the total field leakage flux, Φ_l, existing under instantaneous short circuit, is the same (or nearly so) as would exist on open circuit with the same field m.m.f.; and the effective interlinkages which Φ_l represents is the portion of the normal flux, Φ, which would be absorbed in the total field self-induction on short circuit.

Let ϕ'_{ln} be the effective field leakage interlinkages per unit length of machine, corresponding to a m.m.f. on the field equal to normal armature reaction, A. Then, if a rise to normal current in the armature would cause a rise in field m.m.f. equal to A, the ratio

$$x'_{pf} = \frac{l\phi'_{ln}}{\Phi} \qquad (1)$$

would be the per cent field self-induction. Actually the rise in field m.m.f. corresponding to normal armature current is less than A, as already pointed out, by the ratio

$$\frac{\text{mutual flux interlinkages,} \quad \text{Fig. 27A}}{\text{total field winding interlinkages,} \quad \text{Fig. 27A}}$$

$$= \frac{\Phi}{\Phi + \frac{F}{A}\, l\, \phi'_{ln}} = \frac{1}{1 + \frac{F}{A}\,\frac{l\,\phi'_{ln}}{\Phi}} = \frac{1}{1 + \frac{F}{A} x'_{p_f}} \qquad (2)$$

where

F = no-load field m.m.f.

l = length of machine

It is still further reduced by the field self-induction in another way: the portion of normal flux which at short circuit is absorbed in the field leakage paths no longer requires, as it did before short circuit, a corresponding portion of the no-load field m.m.f. F to maintain it across the air gap. But all of F remains at short circuit. Hence the difference between F and the portion of F which is actually needed for the flux which is not absorbed in field leakage paths, is balanced by a correspondingly decreased rise in field m.m.f. Hence, a rise in armature m.m.f. equal to A, produces a rise in field m.m.f. equal to

$$F_a = \frac{A}{1 + \frac{F}{A}\, x'_{p_f}} - x_{p_f}\, F \qquad (3)$$

Hence the effective field self-induction or field reactance, x_{p_f}, expressed as per cent reactance in the same terms as the per cent armature self-inductive reactance is

$$x_{p_f} = \frac{F_a}{A}\, x'_{p_f} \qquad (4)$$

But

$$x'_{p_f} = \frac{l\,\phi'_{ln}}{\Phi}$$

and

$$\phi'_{ln} = A\,L \quad \text{(from equation 35)}$$

Hence

$$x'_{p_f} = \frac{A\,l\,L}{\Phi} \qquad (5)$$

Substituting (3) and (5) in (4) and simplifying

$$x_{p_f} = \frac{a}{F} \frac{\dfrac{F\,l\,L}{\Phi}}{\left(1 + \dfrac{F\,l\,L}{\Phi}\right)^2} \tag{6}$$

Where

A = effective polyphase armature reactance AT corresponding to normal current I_n

F = no-load field AT

Φ = normal flux per pole (in maxwells) entering the armature

$$L = \Bigg[1.42 \frac{h}{m} + 0.16 \left(\frac{m}{h}\right)^5 + 1.6 \log \left(1 + \pi \frac{a}{c}\right)$$
$$+ 5.0 \frac{d_p}{c} \left\{1 + 0.62 \frac{c}{l} \log\left(1 + \frac{\pi\, b}{4\, c}\right)\right\} + 0.55 \frac{d}{l}\Bigg]$$

If length is in inches, the expression for L becomes

$$L = \Bigg[3.6 \frac{h}{m} + 0.04 \left(\frac{m}{h}\right)^5 + 4 \log\left(1 + \pi \frac{a}{c}\right)$$
$$12.8 \frac{d_p}{c} \left\{ 1 + 0.62 \frac{c}{l} \log\left(1 + \frac{\pi}{4} \frac{b}{c}\right)\right\} + 1.4 \frac{d}{l}\Bigg]$$

a, b, c, d, h, m, d_p, see Fig. 28 and Fig. 34

l = gross stacked length of machine (core + ducts)

L = applies for all values of $\frac{h}{m}$ above 0.5, and is the effective leakage lines per unit length of machine, linking the entire field winding when the field m.m.f. is one ampere turn. The expression for L is derived in the following pages.

The terms $4 \log\left(1 + \pi \frac{a}{c}\right)$ and $0.62 \log\left(1 + \frac{\pi}{4} \frac{b}{c}\right)$ are plotted in Fig. 29.

The total per cent reactance therefore which determines the instantaneous short-circuit current (that is, the transient reactance) is

$$= x_{p_0} = x_{p_a} + x_{p_f} \tag{7}$$

where x_{p_a} = per cent armature self-inductive reactance (or armature reactance) calculated by formula given in Part 1.

x_{p_f} = per cent field self-inductive reactance (or field reactance)

(x_{p_0} = 0.2, means 20 per cent reactance)

The above discussion has considered only the alternating component of the initial short-circuit current. Actually the direct component of armature m.m.f. approximately equal to the crest of the alternating component of armature m.m.f. must always exist on a three-phase short circuit, and *may* exist on a

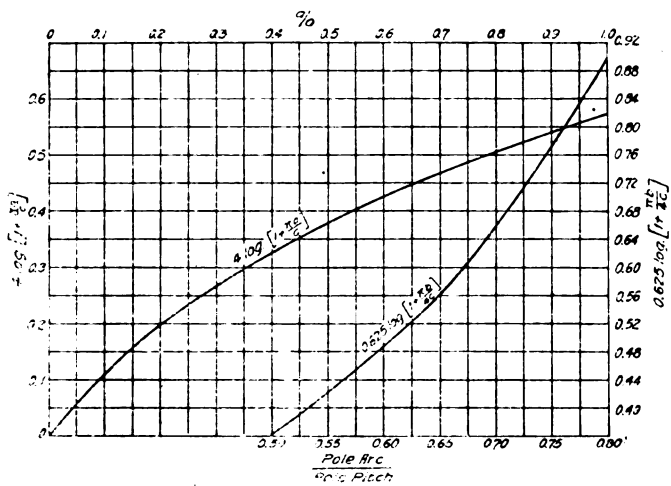


FIG. 29

single-phase short circuit, as already mentioned. That is at 180 degrees after short circuit, it is necessary, in such a case, to put twice normal flux across the leakage paths of the armature and field. It is obvious that this additional flux of normal value would be absorbed by the leakage paths in the same manner as above described.

Hence the effective value of the alternating component of the initial short-circuit current is

$$I_0 = \frac{I_n}{x_{p_0}} \tag{8}$$

Assuming a sine distribution of both the field flux Fig. 27A and

of the armature flux Fig. 27B, the direct component would be equal to the crest of the alternating component; that is

$$I_{dc} = \sqrt{2}\, I_0 \qquad (9)$$

The distribution is, however, usually not sinusoidal. Since the effect of the important harmonics on short-circuit current (alternating component) is practically eliminated, as discussed in Part I, it follows that the magnitude of the alternating component is determined by the fundamental of the flux wave. The direct component, however, is determined by the total flux, that is by the entire flux which the armature encloses, and may therefore be greater or less than $\sqrt{2}\, I_0$ by a small amount. Fig. 30 and Fig. 31 show both cases. In Fig. 30 the direct component is greater than the maximum of the alternating component; in Fig. 31, less.

Fig. 30 also shows the third harmonic voltage, terminal to neutral, existing on short circuit.

The formula for x_{p_0} applies to polyphase short circuits on salient laminated pole machines without amortisseur windings. (The effect of amortisseur windings is discussed later). It will apply to single-phase short circuit, terminal to neutral, if: (1) A, in the formula for x_{p_f} is taken as the maximum value of the pulsating armature m.m.f. of normal current, *i.e.*

$$A_1 = \sqrt{2}\, K_1\, N_q\, I_n \qquad (10)$$

where

N_q = series turns per phase (*i.e.* per leg) per pole.
I_n = normal current per turn
K_1 = factor accounting for short coil pitch, coil distribution, and flux distribution. (2) the armature reactance is taken equal to two thirds of the three-phase value since as pointed out in Part I, three-phase reactance is approximately 50 per cent higher than the single-phase. That is, for single-phase short circuit, terminal to neutral,

$$x_{p_{01}} = \frac{2}{3}\, x_{p_a} + x_{p_{f1}} \qquad (11)$$

where $x_{p_{f1}}$ is field reactance defined in (1). But since the three-phase armature reaction is

$$A_3 = 1.5\, \sqrt{2}\, N_q\, I_n\, K_1 \qquad (12)$$

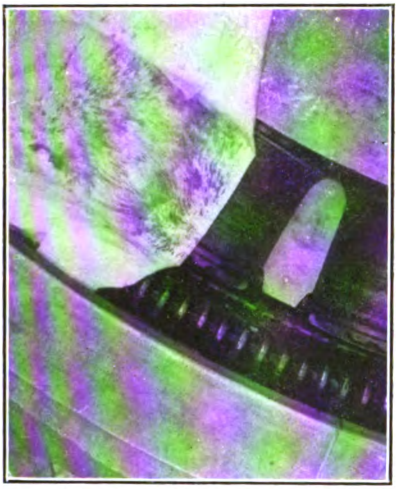


FIG. 26—ACTUAL MAGNETIC DISTRIBUTION OF GENERATOR

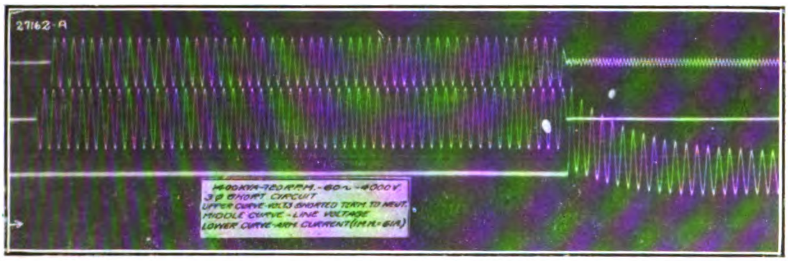


FIG. 30

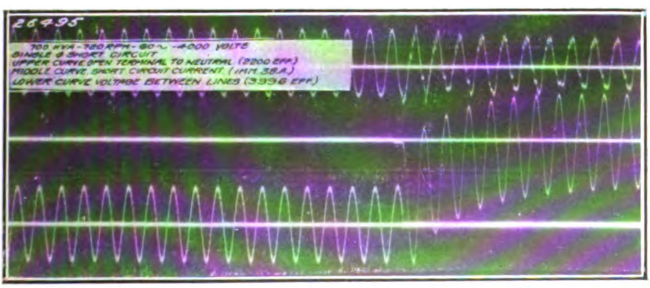


FIG. 31 [DOHERTY AND SHIRLEY]

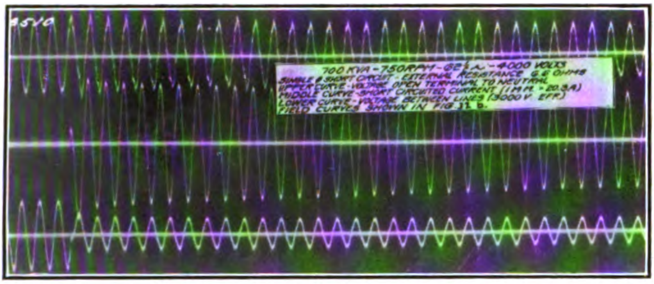


Fig. 32a

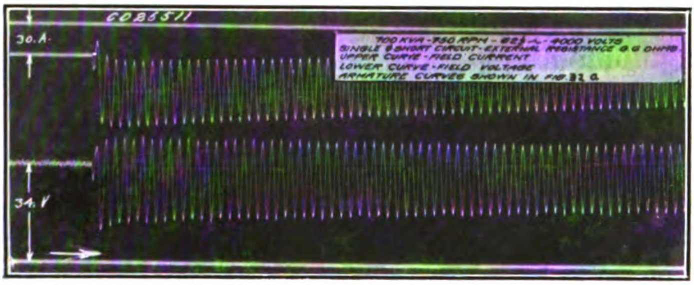


Fig. 32b

[DOHERTY AND SHIRLEY]

Fig. 35b—Field Leakage Path

the equation for x_{p01} can be written

$$x_{p_{01}} = \frac{2}{3}\left(x_{p_a} + x_{p_f}\right) = \frac{2}{3}x_{p_0} \tag{13}$$

That is, on a single-phase short circuit, terminal to neutral, the current should be about 50 per cent larger than on a three-phase short circuit. The authors have not made tests to confirm this point.

The action of the field and armature on single-phase short circuit is as follows: if the short circuit occurs when the armature encloses zero flux, that is, at maximum voltage, Figs. 32A and 32B, the pulsating armature reaction produces in the field, for well known reasons, a direct m.m.f. equal to about one-half (actually less) of the maximum armature m.m.f., and a double frequency m.m.f. whose maximum value equals the direct m.m.f. The double frequency m.m.f. is of course at negative maximum at instant of short circuit, so that at the end of the first half cycle the total field m.m.f. is about equal to the maximum armature m.m.f.; that is, the armature and field absorb in their leakage paths at this instant, the total normal flux in proportion to their respective self-inductions.

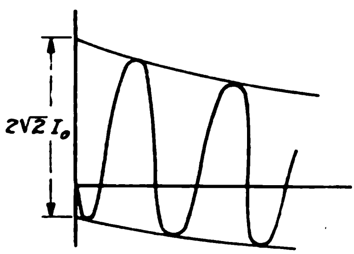


FIG. 33—SHORT CIRCUIT ARMATURE CURRENT

A single-phase short circuit between terminals gives, in most cases, practically the same current as a three-phase short circuit, as experience has shown, and may therefore be calculated by the formula for three-phase until a more accurate method is worked out.

The total self-inductive reactance effective at short circuit expressed in ohms, that is, the transient reactance, is

$$x_0 = \frac{x_{p_0} \times \text{normal voltage per leg}}{\text{normal current per leg}} \tag{14}$$

Table IV shows a comparison of calculated and actual values of the alternating component of short-circuit current in terms of normal current. The actual values were determined by projecting to the axis of zero time the curve of positive crests and also the curve of negative crests, Fig. 33.

The intercept on the vertical axis represents $2\sqrt{2}\,I_0$, *i.e.* two times the crest value of the alternating component.

TABLE IV—SALIENT POLE MACHINES

Machine					Conditions of short circuit			Type of rotor	Amortisseur winding	Ratio—Normal cur. to R. M. S. of alternating component of short-circuit current†	
Kv-a.	Phase	Cyc.	R. P. M.	Voltage	Phase	Load	Voltage			Calc.	Test
10000	3	63	55.6	6600	3	None	6600	Laminated salient pole	None	.27	.25
"	"	"	"	"	3	None	"	"	"	"	.25‡
"	"	"	"	"	3	None	"	"	"	"	.26‡
"	"	"	"	"	3	None	"	"	"	"	.25
"	"	"	"	"	3	7500kw	"	"	"	"	.22
"	"	"	"	"	3	7500kw	"	"	"	"	.24
9000	3	25	57.7	11000	3	None	11000	"	None	.22	.26
"	3	"	"	"	3	"	11000	"	"	"	.26
"	"	"	"	"	3	"	11000	"	"	"	.25
"	"	"	"	"	3	"	8250	"	"	"	.25
"	"	"	"	"	1	"	8250	"	"	"	.26
"	"	"	"	"	1	"	8250	"	"	"	.29
6600	3	25	375	14000	3	None	16000	"	None	.15	.12
"	"	"	"	"	3	"	14000	"	"	"	.13
"	"	"	"	"	3	"	14000	"	"	"	.13
"	"	"	"	"	3	"	8000	"	"	"	.14‡
4300	1	25	300	11000	3	None	11000	"	Low-Res.	.15*	.17
"	"	"	"	"	1	"	11000	"	"	"	.18
"	"	"	"	"	1	"	11000	"	"	"	.16
"	"	"	"	"	1	"	8250	"	"	"	.17
"	"	"	"	"	1	"	8250	"	"	"	.16
"	"	"	"	"	1	"	8250	"	"	"	.19
"	"	"	"	"	1	"	5500	"	"	"	.19
"	"	"	"	"	1	"	5500	"	"	"	.19
1500	3	50	500	6600	3	None	6600	"	None	.16	.12
1500	3	50	500	6600	3	"	4950	"	"	"	.13
1500	3	50	500	6600	3	"	3300	"	"	"	.14
1500	3	50	500	6600	3	"	1650	"	"	"	.15
1400	3	60	720	2300/4000	3	None	4000	"	High-Res.	.18	.21
"	"	"	"	"	3	"	"	"	"	"	.16
"	"	"	"	"	3	"	"	"	"	"	.21
"	"	"	"	"	3	"	"	"	"	"	.21
"	"	"	"	"	1	"	"	"	"	"	.21
"	"	"	"	"	1	"	"	"	"	"	.18
"	"	"	"	"	1	"	"	"	"	"	.21
"	"	"	"	"	1	"	"	"	"	"	.22
"	"	"	"	"	1	"	"	"	"	"	.20
"	"	"	"	"	1	"	3000	"	"	"	.21

SALIENT POLE MACHINES (*Continued*)

Machine					Condition of short circuit			Type of rotor	Amortissuer Winding	Ratio—Normal cur. to R. M. S. of alternating component of short-circuit current†	
Kv-a	Phase	Cyc.	R. P. M.	Voltage	Phase	Load	Voltage			Calc.	Test
700	3	60	720	2300/4000	1	None	3996	Laminated salient pole	High Res	.18	.20
"	"	"	"	"	1	"	3996	"	"	"	.23
"	"	"	"	"	1	"	3996	"	"	"	.16
"	"	"	"	"	1	"	3036	"	"	"	.18
"	"	"	"	"	1	"	3036	"	"	"	.20
"	"	"	"	"	1	"	2106	"	"	"	.17
"	"	"	"	"	1	"	2016	"	"	"	.19
"	"	"	"	"	3	"	3996	"	"	"	.15
"	"	"	"	"	3	"	3996	"	"	"	.14
"	"	"	"	"	3	"	3036	"	"	"	.15
"	"	"	"	"	3	"	2016	"	"	"	.17
"	"	"	"	"	3	"	2016	"	"	"	.15
125	3	60	900	2300	3	None	2300	"	High-Res	.18	.19
"	"	"	"	"	3	"	2300	"	High-Res.	"	.19
"	"	"	"	"	3	"	2300	"	None	"	.18
"	"	"	"	"	3	"	2300	"	None	"	.18
"	"	"	"	"	3	"	2300	"	None	"	.17
"	"	"	"	"	3	"	1150	"	None	"	.20
"	"	"	"	"	3	"	1150	"	None	"	.21

TURBINE ALTERNATORS

Kv-a	Phase	Cyc.	R. P. M.	Voltage	Phase	Load	Voltage	Type of rotor	Amortissuer Winding	Calc.	Test
10000	3	40	2400	10000	3	None	12000	Continuous solid steel	None	.12*	.12‡
"	"	"	"	"	"	"	10000	"	"	"	.13‡
"	"	"	"	"	"	"	5000	"	"	"	.16‡
"	"	"	"	"	"	Full Load 10 P. F.	10000	"	"	"	.15‡
"	"	"	"	"	"	"	10000	"	"	"	.15‡
"	"	"	"	"	"	Full Load O P. F.	10000	"	"	"	.12‡
6250	3	60	3600	2300	3	None	2300	"	"	.10*	.09
3750	"	60	3600	2300/4000	3	None	4000	"	None	.09*	.10
"	"	"	"	"	"	"	4000	"	"	"	.08
"	"	"	"	"	"	"	3000	"	"	"	.11
"	"	"	"	"	"	"	3000	"	"	"	.09
"	"	"	"	"	"	"	2000	"	"	"	.12

†In order to have ratios on same basis, normal current is decreased or increased according as voltage at short circuit is less or greater than normal.
*Transient reactance taken equal to armature reactance.
‡Average from three waves on one film.

Example: 6600-kv-a., three-phase, 25-cycle, 14,000-volt generator.

$$l = 38 \quad A = 10970 \quad \Phi = 47.7 \times 10^6$$

$$F = 15600 \quad \frac{n}{m} = 0.687 \qquad \frac{a}{c} = 0.413$$

$d_t = 2.0 \qquad d_p = 2.0 - \dfrac{11.5}{4} = 0$ (neglect d_p if it is zero or negative) $\qquad \dfrac{d}{l} = 0.407$

$$\text{Ratio } \frac{\text{pole arc}}{\text{pole pitch}} = 0.675 \qquad \frac{b}{c} = \frac{0.675}{0.325} = 2.08$$

$$L = \left[3.6 \times 0.687 + 0.04 \left(\frac{1}{0.687}\right)^5 + 4 \log (1 + \pi \times 0.413) + 1.4 \times 0.407 \right]$$

$$= [2.47 + 0.26 + 3.3 + 0.57] = 6.6$$

$$x_{p_f} = \frac{10970}{15600} \frac{\dfrac{38 \times 15600 \times 6.6}{47.7 \times 10^6}}{\left(1 + \dfrac{38 \times 15600 \times 6.6}{47.7 \times 10^6}\right)^2} = 0.049$$

Armature reactance $= x_{p_a} = 0.10$

$$x_{p0} = x_{pa} + x_{pf} = 0.10 + 0.049 = 0.149$$

That is, the three-phase short-circuit current (*a-c.* component) would be

$$I_0 = \frac{1}{0.149} = 6.7 \text{ times normal}$$

Consider now the question of attenuation factors, α_f and α_a. The attenuation factor of a circuit is the ratio of the effective resistance to the (interlinkages per ampere), that is, the inductance. In the field attenuation factor, α_f, the inductance is not as Diamant gives[14] the exciting inductance of the machine, (using his notation)

$$\frac{\Phi N_f}{I_f 10^{-1}}$$

14. Trans. A. I. E. E., Vol. 34, p. 2247.

where Φ is the flux which the normal field excitation, I_f, would produce in the normal magnetic circuit, as given by the design sheet. It is rather

$$\frac{\Phi' N_f}{I_f 10^{-1}}$$

where Φ' is the flux, linking the field winding, which I_f would produce in the *leakage paths of both field and armature.* The correct attenuation factor of the field to be used in determining the rate of decrease of field flux on short circuit is the ratio of the effective resistance of the circuit to the (transient field interlinkages maintained through the leakage paths of both armature and field on short circuit, per ampere rise in the field circuit). Therefore the question is, what is the value of field interlinkages thus maintained per ampere *rise* in field current on short circuit. It is,

$$\frac{\Phi_t N_f}{I_{f0}} \quad \text{per pole}$$

or

$$\frac{q\, \Phi_t N_f}{I_{f0}} \quad \text{total}$$

That is,

$$\frac{q\, \Phi_t N_f}{10^8 I_{f0}} \quad \text{henrys}$$

Where

Φf_t = transient flux = $\Phi - \Phi_s$
Φ = normal flux per pole
Φ_s = sustained flux per pole

$$\Phi_s = \frac{F\Phi}{\dfrac{A}{x_{p_a}} + F_g}$$

x_{p_a} = per cent armature reactance
($x_{p_a} = 0.2$ means 20 per cent)
F_g = m.m.f. consumed in air gap by normal flux
N_f = turns per pole
q = number of poles

I_{f_0} = *d-c.* component of *rise* in field current at short circuit This corresponds to the reaction of the *alternating component*

in the armature, and for reasons previously stated is less than the armature m.m.f. as shown by equation (3).

$$I_{f_0} = \frac{F_a}{x_{p_0}\, N_f} = \frac{F}{x_{p_0}\, N_f}\left[\frac{\frac{A}{F}}{1+\frac{F\,l\,L}{\Phi}} - x_{p_f}\right] \qquad (16)$$

Hence the transient field inductance is

$$L_{f_0} = \frac{q\, x_{p_0}\, \Phi_t\, N_f^2}{10^8\, F}\,\frac{1}{\left[\frac{\frac{A}{F}}{1+\frac{F\,l\,L}{\Phi}} - x_{p_f}\right]}$$

$$L_{f_0} = \frac{q\, x_{p_0}\, \Phi\, N_f^2}{10^8\, F}\,\frac{\left(1 - \frac{F}{\frac{A}{x_{p_a}} + F_g}\right)}{\left(\frac{\frac{A}{F}}{1+\frac{F\,l\,L}{\Phi}} - x_{p_f}\right)} \text{ henrys} \qquad (17)$$

The field attenuation factor is therefore

$$\alpha_f = \frac{r_f}{L_{f_0}} \qquad (18)$$

where r_f = effective resistance of the field circuit. In laminated pole machines, this may be taken in approximate calculations as the ohmic resistance of the field circuit.

The inductance in the armature attenuation factor , which determines the rate of decay of the *direct component*, is determined as follows: the interlinkages of the armature per pole per phase, maintained through the leakage paths of both the armature and the field, per effective ampere turn rise in the armature m.m.f. on short circuit, is

$$\frac{(K\, N_q)\ \Phi}{\frac{A}{x_{p_0}}} \qquad (19)$$

or, for all poles

$$\frac{q\, x_{p_0}\, \Phi\, (K\, N_q)}{A} \qquad (20)$$

But A, for three-phase, is the product

$$\sqrt{2}\, I_n \times 1.5\ (K\ N_q) \tag{21}$$

where K is a constant depending upon the pitch and distribution of coils, and upon the flux distribution; I_n = normal rated current; and N_q = series turns per pole per phase. Hence, the inductance, that is the interlinkages with $(K\ N_q)$, is

$$L_{ao} = \frac{1.5\, q\, x_{po}\, \Phi\, (K\ N_q)^2}{10^8} \text{ henrys} \tag{22}$$

The factor 1.5 accounts for the mutual induction of phases. For single phase or two phase this factor would be unity. L_{a0} can be arrived at in another and more convenient way.
It is equal to

$$L_{ao} = \frac{x_0}{2\,\pi f} \tag{23}$$

Where x_0 = transient reactance in ohms
f = frequency in cycles per second

$$X_o = \frac{x_{p0}\, E}{I_n}$$

Hence

$$L_{ao} = \frac{x_{p0}\, E}{2\,\pi f\, I_n} \tag{24}$$

The armature attenuation factor is

$$\alpha_a = \frac{r_a}{L_{ao}} \tag{25}$$

where r_a = effective resistance per phase.

It may seem at first thought that since the sum of the inductances of both the field and armature are involved in the attenuation factors, the sum of resistance of both circuits also should enter. However, this is not the case. The armature and field transients involve magnetic energy storage under unstable conditions. In the case of the field transient, immediately after short circuit the energy stored in the field is the product of normal field flux times the large field m.m.f. which now maintains it. The flux can die down only if its energy is dissipated in the circuit with which it links, because, by the very way it is constituted, the energy can decrease only by a decrease in interlinkages, and this can not happen except by resistance

loss in the linking circuit. From a slightly different viewpoint, suppose the field circuit is a perfect conductor. Then the field flux and current would never die out, regardless of the armature resistance. The armature m.m.f. operates only to increase the reluctance[15] of the path through which the flux must pass on short circuit. The armature resistance operates only to decrease the initial rise in current, that is to decrease the amount of flux which on short circuit must be sent through the paths of increased reluctance, and therefore to increase the sustained value of flux, that is, to decrease the magnitude of the transient, but not its time. The same reasoning will apply to the armature transient.

The authors have not attempted to investigate to what extent the results are changed by the increase of the effective resistance of the armature and field circuits. It is their opinion that with a well designed armature conductor, and with laminated field poles, the increase will not be serious. However this is yet to be ascertained.

The relative values of α_f and α_a can be illustrated by taking some rough average figures for salient pole machines. The ratio, armature reactance to armature resistance is, say, 20; field reactance to field resistance, 80; field reactance to armature reactance, 0.5.
Hence

$$\text{Armature} - \frac{\text{resistance}}{\text{transient reactance}} = \frac{1}{30}$$

$$\alpha_a = \frac{2\pi f}{30}$$

$$\text{field} - \frac{\text{res stance}}{\text{transient reactance}} = \frac{1}{240}$$

$$\alpha_f = \frac{2\pi f}{240}$$

That is, the armature transient will die out eight times as fast as the field, in this case.

Diamant found from oscillographic tests that the attenuation factor was larger during the first cycle or so than during the rest

15. An equivalent reluctance could theoretically be produced, with the same effect on the field transient, by a sudden increase, at open circuit, in full reluctance of the armature iron.

of the transient. The authors have found this to be the case; and agree with Diamant that, until this difference, due probably to saturation and eddy currents, can be further studied, the lower value (which corresponds to our calculation) should be used. This is on the safe side, since by this assumption, at any time after short circuit the current will be lower than the calculated value.

The question arises, how much will calculations be thrown out by saturation? From the data available it would seem that in salient pole machines without amortisseur windings, saturation is not a very serious factor. Although at 180 deg. after a three-phase short circuit there must be two times normal flux through the leakage paths; these paths are widely distributed, and therefore very serious concentration is prevented. Yet there is some. The smaller the section of the principal leakage paths (the pole tip and teeth) the greater the saturation. It is still further increased by the action of an amortisseur winding, and by reason of the slots that contain it. But we believe it is safe to assume that in the case of most laminated, salient pole machines, without amortisseur windings, the short-circuit current is practically proportional to the voltage at which short circuit occurs; when thin pole tips and shallow armature slots are used, the short-circuit current may be 10 to 20 per cent greater than indicated by calculations which neglect saturation. In the case of low resistance amortisseur windings, solid steel poles, or in turbine generators, where the leakage paths are much more restricted, saturation makes a greater difference. Hence, for this, as well as another reason given in the following paragraph, the calculation of such machines should be based upon the armature self-induction only. The matter of saturation of leakage paths is one which should be much further investigated.

Another matter of importance is the effect of amortisseur windings, short-circuiting collars around the pole, solid steel poles, etc. Obviously if the amortisseur winding were a perfect conductor, the flux could not change through it and consequently the effect of field self-induction would be nil, since the field m.m.f. would not rise. Moreover, the armature leakage paths would be more restricted. That is the transient reactance would be less than the armature self-induction. But, of course, amortisseur windings and collars are never perfect conductors. On the other hand, many of them are poor conductors; hence in the case of laminated field poles they permit a large part of the

field self-induction to exert itself—the more, naturally the higher the resistance of the amortisseur winding. However, in single-phase generators, or such machines, in which heavy copper amortisseur windings are used, and also in the case of solid steel poles, the resulting increase in short-circuit current can not be neglected. For safety, therefore, it is best in such cases, as proposed above, to neglect the field self-induction in calculating the short-circuit current; that is, to assume that the transient reactance is equal to the armature reactance.

External, as well as internal, field reactance will decrease the short-circuit current. The rate at which the field current rises at short circuit will not be impeded by the external reactance, because the new magnetic interlinkages established in the external reactance represent a corresponding decrease in the interlinkages of the field winding itself, since the net total of the circuit can not change appreciably during the first cycle or so. But, by reason of this decrease in interlinkages of the field winding itself, or what is the same thing, the increase in interlinkages in the external reactance, there remains in the field less flux to be forced across the leakage paths in the machine; hence the short-circuit current will be less by just that amount.

With zero external field reactance the rise in field m.m.f. on short circuit is less than the armature m.m.f. in the ratio

$$G = \frac{1}{1 + \dfrac{F\,l\,L}{\Phi}}$$

Or, in terms of total interlinkages in the machine,

$$G = \frac{q\,\Phi\,N_f}{q\,\Phi\,N_f + \dfrac{F}{N_f}\,L_f\,10^8} = \frac{1}{1 + \dfrac{F\,L_f\,10^8}{N_f^2\,\Phi\,q}}$$

where L_f = coefficient of self-induction of the field winding.

$$L_f = \frac{q\,l\,L\,N_f^2}{10^8} \text{ henrys}$$

F = no-load field m.m.f.

l = gross length of armature core (iron + ducts)

L = bracket quantity in equation (**33**) or (**33a**)

Φ = normal flux per pole (maxwells)

q = number of poles

N_f = turns per pole on field.

If an external reactance, x_{fx}, (coefficient of self-induction, $=L_{fx}$), is placed in the field circuit, the ratio becomes,

$$G = \frac{1}{1 + \dfrac{F\,L_f\,10^8}{N_f^2\,\Phi\,q} + \dfrac{F\,L_{fx}\,10^8}{N_f^2\,\Phi\,q}}$$

$$G = \frac{1}{1 + \dfrac{F\,10^8}{q\,N_f^2\,\Phi}\,(L_f + L_{fx})}$$

Let x_{pf}'' = per cent reactance of the field circuit, assuming the rise in field m.m.f. equals the armature m.m.f. on short circuit. Then

$$x_{pf}'' = \frac{\dfrac{A}{N_f}L_f\,10^8}{\Phi\,N_f} + \frac{\dfrac{A}{N_f}L_{fx}\,10^8}{\Phi\,N_f}$$

$$x_{pf}'' = \frac{A\,10^8}{N_f^2\,\Phi}\,(L_f + L_{fx})$$

The transient reactance of the field circuit is

$$x_{pfx} = G\,x''_{pf}$$

$$= \left[\frac{\dfrac{A\,10^8}{N_f^2\,\Phi}\,(L_f + L_{fx})}{1 + \dfrac{F\,10^8}{N_f^2\,q\,\Phi}\,(L_f + L_{fx})}\right]$$

$$x_{pfx} = \frac{A}{F}\left[\frac{1}{1 + \dfrac{1}{\dfrac{F\,10^8}{N_f^2\,\Phi\,q}\,(L_f + L_{fx})}}\right] \qquad \textbf{(25a)}$$

The total transient reactance of the machine is

$$x_{p0} = x_{pa} + x_{pfx}$$

where x_{pa} = armature reactance

The principal objection[16] to the use of reactance in the field circuit is the high voltage which occurs across its terminals at short circuit. With an external reactance in the field circuit equal to the internal field reactance, this voltage may be of the order of 50,000 volts in the case of large generators.

16. This point has been observed by K. Ito, *Journal Elec. Society* (Japanese), Dec. 1917, p. 891.

Under load the short-circuit current is slightly higher than at no-load. The alternating component, which depends upon the flux per pole, is greater for short circuit under load than at no-load, by the increase in flux which is required under load to overcome the impedance of the armature. Hence for the same load current, the alternating component of short-circuit current will be greater, the lower the power factor, until at zero power factor, it will be increased by the per cent reactance of the machine—10 per cent reactance, 10 per cent increase in the alternating component. At unity power factor, the 10 per cent being added in quadrature, would not cause appreciable increase. The direct component, depending upon the flux linked with the armature circuit, *i.e.* upon the terminal voltage, is the same for any load or power factor, providing there is no external reactance in the armature circuit. External reactance decreases the direct component because it decreases the net magnetic interlinkages of the armature circuit. With 10 per cent external reactance, zero power factor load, the direct component would be 0.9 as large as if the short circuit occurred at no load, the same terminal voltage. The reverse would be true of leading current. For unity power factor, the difference would be negligible, for reasons mentioned above.

Derivation of Formula For Calculating Field Self-Induction

The field leakage paths shown in Figs. 24A and 24B are approximated by the assumed paths shown in Figs. 28, 34 and 35B.

With a m.m.f. equal to NI per pole the effective leakage lines, linking the entire field winding, per unit length of machine will be as follows: the flux element $d\,\phi_1$ is

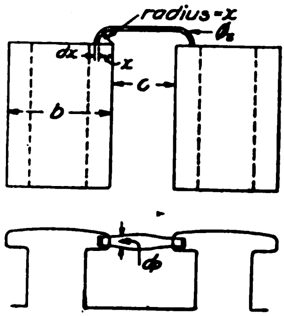


Fig. 34

$$d\,\phi_1 = 2 \times 0.4\,\pi\,\frac{x}{h}\,NI\,\frac{d\,x}{Kx}$$

where $$K = \frac{1}{\tan\theta}\ \text{(approximately)} = \frac{2\,h}{m}$$

$$d\,\phi_1 = 0.4\,\pi\,NI\,\frac{m}{h^2}\,dx$$

Effective flux due to ϕ_1 linking entire winding is

$$\phi_{n1} = \int_0^{\frac{m^2}{4h}} \frac{x}{h}\, d\phi_1$$

$$\phi_{n1} = 0.039\ NI \left(\frac{m}{h}\right)^5 \tag{26}$$

$$d\,\phi_2 = 2 \times 0.4\,\pi \left(\frac{\frac{m^2}{4\,h} + y}{h}\right) 2\ N I \frac{dy}{\frac{2\,\pi}{3}\ \frac{m}{\sqrt{3}}}$$

$$d\,\phi_2 = 1.05\ \frac{NI}{h^2\,m} (m^2 + 4\,h\,y)\,d\,y$$

It is here assumed that the length of path is one third of the arc of a circle whose radius is $\frac{m}{\sqrt{3}}$, that is, the pole sides intercept $\frac{1}{3}$ of the arc.

Effective flux of ϕ_2 linking the entire field winding is

$$\phi_{n2} = \int_0^{h - \frac{m^2}{4h}} \left(\frac{\frac{m^2}{4\,h} + y}{h}\right) d\,\phi_2$$

$$= 0.27\ \frac{N\,I}{h^4\,m} \int_0^{h - \frac{m^2}{4\,h}} (m^2 + 4\,h\,y)^2\,d\,y$$

$$\phi_{n2} = 0.27\ N\,I \left[\frac{16}{3}\ \frac{h}{m} - \frac{1}{12}\left(\frac{m}{h}\right)^5\right] \tag{27}$$

$$d\,\phi_3 = 2 \times 0.4\,\pi \times 2\ N\,I \frac{d\,z}{c + \pi\,z}$$

$$d\,\phi_3 = 5\ N\,I\ \frac{d\,z}{c + \pi\,z}$$

All of ϕ_3 is assumed to be linked with the winding. Hence

$$\phi_{n3} = \phi_3 = 5\ NI \int_0^a \frac{d\,z}{c + \pi\,z}$$

$$\phi_{n3} = 1.6\ N\,I \log\left(1 + \pi\ \frac{a}{c}\right) \tag{28}$$

In determining ϕ_4, it is necessary to find the portion of the pole tip from which ϕ_4 emanates, Fig. 28, and further to find what depth of path in the air between tips should be used. By plotting a large number of diagrams, such as Figs. 24A and 24B for different pole proportions, it was found that this average depth of path could be approximated by the expression

$$d_p = d_t - \frac{c}{4} \tag{29}$$

That is the factor may entirely disappear, which happens in the case of thin pole tips and low pole arc ratio. When $\frac{c}{4}$ is greater than d_t, ϕ_{n4} should be neglected.

$$\phi_{n4} = \phi_4 = 2 \times 0.4\,\pi \times 2\ N\,I\ \frac{d_p}{c}$$

$$\phi_{n4} = 5\ N\,I\,\frac{d_p}{c} \tag{30}$$

$$d\ \phi'_5 = 4 \times 0.4\,\pi \times 2\ N\,I\,\frac{d_p\,d\,x}{\pi\,x + c} \quad \text{(See Fig. 11)}$$

$$d\ \phi'_5 = 10\ N\,I\,d_p \int_0^{b/4} \frac{d\,x}{\pi\,x + c}$$

$d\ \phi'_5$ is integrated between 0 and $\frac{b}{4}$ because toward the middle of the pole the leakage from the end of pole shoe no longer goes from pole to pole, but instead bends upward and enters the stator iron, cutting the armature winding. The

approximation is therefore made that the dividing line falls at $b/4$.

$$\phi_5' = 3.2\, N\, I\, d_p \log \left(1 + \frac{\pi}{4} \frac{b}{c} \right)$$

$$\phi_{n5} = \frac{\phi_5'}{l} = 3.2\, N\, I \frac{d_p}{l} \log \left(1 + \frac{\pi}{4} \frac{b}{c} \right) \qquad \textbf{(31)}$$

where l = gross length of armature core (iron + ducts).

The leakage at the end of the pole is in paths practically straight and downward to the field spider rim as shown in Fig. 26A and 35. At about three-fourths of the way up the pole the leakage bends up and enters the stator iron, part of it, certainly, cutting the armature winding. Of course, depending upon design, the division line may fall at one-half or may be at the top of the pole. But in the authors' judgment, three-fourths represents the average case. On this and the further assumption that the length of path at any height ν, Fig. 35, is $1.3\ \nu$,

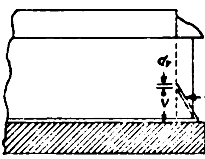


Fig. 35A

$$d\, \phi'_6 = 2 \times 0.4\, \pi \frac{\nu}{h}\, NI \frac{d\, d\nu}{1.3\, \nu}$$

$$d\, \phi'_6 = 1.95 \frac{d}{h} N\, I\, d\, \nu$$

$$\phi'_{n6} = \int_0^{\frac{3}{4}h} \frac{\nu}{h^2}\, d\, \phi_6'$$

$$= 1.95\, N\, I \frac{d}{h^2} \int_0^{\frac{3}{4}h} \nu\, d\, \nu$$

$$\phi_{n6} = \frac{\phi'_{n6}}{l} = 0.55\, N\, I \frac{d}{l} \qquad \textbf{(32)}$$

While one or two rather arbitrary assumptions were made in deriving the above equations, they were made in the light of an experimental and graphical study of the leakage paths, and with the object of avoiding complicated expressions; and it is believed that the error in the final result will not be serious, one reason being that the assumptions involving the most uncercertainties are in the smallest factors, such as ϕ_{n5} and ϕ_{n6}.

Adding equations (**26**), (**27**), (**28**), (**30**), (**31**) and (**32**), the total self-induction of the field is

$$\phi_{ln} = \phi_{n1} + \phi_{n2} + \phi_{n3} + \phi_{n4} + \phi_{n5} + \phi_{n6}$$

$$\phi_{ln} = N\,I\Bigg[\,0.039\left(\frac{m}{h}\right)^{5} + 0.27\left\{\frac{16}{3}\,\frac{h}{m} - \frac{1}{12}\left(\frac{m}{h}\right)^{5}\right\}$$

$$+ 1.6\log\left(1 + \pi\,\frac{a}{c}\right) + 5\,\frac{d_p}{c}$$

$$+ 3.2\,\frac{d_p}{l}\,\log\left(1 + \frac{\pi}{4}\,\frac{b}{c}\right) + 0.55\,\frac{d}{l}\,\Bigg]$$

$$\phi_{ln} = N\,I\Bigg[\,1.42\,\frac{h}{m} + 0.16\left(\frac{m}{h}\right)^{5} + 1.6\log\left(1 + \pi\,\frac{a}{c}\right)$$

$$+ 5\,\frac{d_p}{c}\left\{1 + 0.62\,\frac{c}{l}\,\log\left(1 + \frac{\pi}{4}\,\frac{b}{c}\right)\right\} + 0.55\,\frac{d}{l}\Bigg] \qquad \textbf{(33)}$$

If inches are used, the equation becomes

$$\phi_{ln} = N\,I\Bigg[3.6\,\frac{h}{m} + 0.04\left(\frac{m}{h}\right)^{5} + 4\log\left(1 + \pi\,\frac{a}{c}\right)$$

$$+ 12.8\,\frac{d_p}{c}\left\{1 + 0.62\,\frac{c}{l}\log\left(1 + \frac{\pi\,b}{4\,c}\right)\right\} + 1.4\,\frac{d}{l}\Bigg] \qquad \textbf{(33a)}$$

ϕ_{ln} is the effective leakage lines per pole, linking the entire field winding, per unit length of machine, produced by a m.m.f. of one ampere turn.

The coefficient of self-induction of the field is therefore

$$L_f = \frac{q\,N_f^2\,l\,L}{10^8}\ \text{henrys} \qquad \textbf{(34)}$$

where q = number of poles
N_f = turns per pole
l = gross stacked length of core (iron and ducts)
L = bracket quantity in equation (**33**) or (**33a**)

For the value of leakage, ϕ'_{ln}, per unit length, corresponding to a m.m.f. on the field equal to normal armature reaction, A, substitute A for $N\,I$ in equation (**33**) and (**33a**).
That is

$$\phi'_{ln} = A\,L \qquad \textbf{(35)}$$

Summary and Conclusions

(1) A reliable prediction of the armature self-inductive reactance of synchronous machines can be made by equations (**45**) to (**48**) inclusive, given in Part I. Moreover, working curves for three-phase, based on those equations and given in Figs. 20A 20B, etc., make the complete calculation from the design sheet a matter of four or five minutes. It is proposed that this reactance be called, for brevity, *armature reactance.*

(2) The variation of the effective armature m.m.f. or armature reaction, (under zero power factor conditions) with the no-load flux distribution has been determined. The armature reaction of three-phase machines, as calculated by the familiar formula based on sinusoidal distribution, is the basis of the new calculation. Its result is modified by a single factor, K_ϕ given in Fig. 20, and developed in appendices A and B. The correct calculation of armature reaction is very important in its relation to the armature self-inductive reactance, since the latter is determined from test by the difference between the field and armature m.m.f. on sustained short circuit.

(3) As a corollary of (2), the effect of the harmonics in the no-load flux distribution upon the generated voltage is accounted for also by K_ϕ. See appendix A.

(4) A simplified method of calculating field excitation that has worked out well in conjunction with the reactance given by Figs. 20A, 20B, etc., is shown in appendix C.

(5) The reactance which determines the initial short-circuit current of most synchronous machines is not the armature self-induction only; it is rather the combined self-induction of both the armature and field circuits. The internal field self-induction (which, on short circuit is transient) is often 50 per cent of the armature self-induction; in which case, the calculated short-circuit current, based on the armature self-induction alone, would, of course, be 50 per cent too high.

(6) A method of calculating the transient field self-induction is given in Part II, equation (**6**). It is proposed that this reactance be called, for brevity, *field reactance.*

(7) Equation (**7**), Part II, gives the value of the total per cent reactance, x_{p0} which determines the alternating component of short-circuit current. It is proposed that this reactance be called *transient reactance.*

(8) The attenuation factors α_f and α_a, which determine the

rate of decrease of the field and armature transients, respectively are given in equations (**18**) and (**25**).

(9) External field reactance limits the short-circuit current. Its effect can be calculated by equation (**25a**). The high voltage across the field collector rings (which may be 50,000 volts on large generators assuming external field reactance equal to internal) precludes its use in most cases.

(10) Table IV confirms the following points, which are brought out in the theory given in Part II:

(a) The short-circuit current of synchronous machines can be calculated with practical accuracy.

(b) In the case of turbine generators with solid steel rotors, and also of salient pole machines with low resistance amortisseur windings, the transient reactance should be taken equal to the armature reactance.

(c) In the case of high resistance armature windings, the same value of transient reactance should be used as for laminated salient pole machines without amortisseur windings. That is, the value given by equation (**7**) Part II.

(d) While strictly, the complete calculations apply only to salient, laminated pole machines without amortisseur windings, and the proposals in (b) and (c) are only approximations, nevertheless the results shown in Table IV justify the use of these approximations until the method is further extended.

(e) Saturation at short circuit does not practicably affect value of short-circuit current of salient, laminated pole machines without amortisseur windings, unless the magnetic densities in the pole tips and teeth are high at normal voltage, as in the case of the 1500-kv-a. generator in Table IV. It does affect the short-circuit current of turbine generators, and, probably, of salient pole machines with low resistance amortisseur windings.

(f) The short circuit current is practically the same whether the short circuit on three-phase machines occurs between the two terminals or three.

In conclusion the authors wish to express their thanks to Dr. C. P. Steinmetz and W J. Foster for their interest in reviewing the results of the paper, and to Messrs. E.J. Burnham, E. S. Henningsen, H. K. Humphrey, and J. J. Thalheimer for their assistance.

APPENDIX A

Effect of Harmonics in No-Load Flux Wave

The shape of the wave of flux distribution is determined largely by the shape of the pole face and the ratio of the pole arc to pole pitch. The armature teeth of course add ripples of high order, but usually of small magnitude. The latter are neglected in the following derivation because, even if they are of appreciable magnitude to cause trouble in telephone circuits, etc., they have little effect upon the effective armature m.m.f. or upon the value of terminal voltage. The third, fifth, seventh, and possibly the ninth, all of whose magnitude are determined by the design of the pole, are the terms which introduce errors

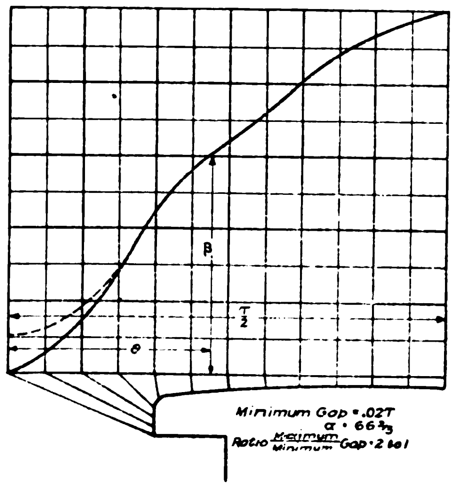


Fig. 36—Flux Distribution Curve

into calculations based upon fundamental sine functions. To determine the effect of these harmonics, the following equations have been developed. They show that the effect of the harmonics upon the terminal voltage, for a given value of flux per pole, can be taken into account by a single factor, K_ϕ.

The flux waves of a number of representative conditions were plotted by the following method. Refer to Fig. 36.

The flux at the middle of the pole was assumed to be 10, and then the flux at any point toward the edge of the pole was taken inversely as the air gap. To obtain the flux beyond the edge of the pole, the distance a b was divided into the same number of spaces as the distance c d and lines drawn joining corresponding

points on these lines. Assuming the flux to be inversely proportional to the length of air gap, the broken curve was derived. Since the effect of the adjacent poles would be to make the flux zero at the point midway between the poles, the solid line was drawn gradually deviating from the broken curve to pass through zero at the middle point.

The calculated flux waves were compared with test flux waves taken by exploring coils on three machines and found to agree very closely. The effect of saturation of the teeth in the middle of the pole was found to make a slight difference in the flux wave shapes on the same machines at different voltages. At ordinary tooth densities, however, the variation will make only one or two per cent difference in the value of K_Φ, for values of voltage from one half to normal.

Flux waves were obtained by the method shown in Fig. 36 for ratio pole arc to pole pitch of 50—66⅔ and 75 per cent for ratios of maximum to minimum air gap from unity to three to one, and for different minimum air gaps varying from one to four per cent of the pole pitch. The flux waves were then analyzed for the harmonics up to the eleventh, and the values of K_Φ were calculated. These values of K_Φ are given in Fig. 20.

The factor K_Φ gives the ratio of the total flux in the irregular wave to the total flux in a sine wave which would give the same effective voltage at the terminals of the machine.

This is exactly true only when the terminal voltage has a form factor equal to that of a sine wave. This is the case with practically all commercial machines as now built, as the distribution of armature winding in slots, short pitch of armature coils and star connections of the machines all operate to eliminate the effect of harmonics in the flux wave.

Let Fig. 36 represent the flux distribution at no load.

$$\beta = (\beta_1 \sin \theta + \beta_3 \sin 3\,\theta + \beta_5 \sin 5\,\theta + \ldots) \qquad (1)$$

Instantaneous voltage generated in unit length of one conductor is

$$e' = \frac{\nu \beta}{10^8} = \frac{\nu}{10^8}(\beta_1 \sin \theta + \beta_3 \sin 3\,\theta + \beta_5 \sin 5\,\theta + \ldots) \quad (2)$$

Where

ν = velocity of flux relative to conductor, in centimeters per second.

β = flux density, lines per sq. cm.

Effective value of voltage is

$$e_{eff.} = \sqrt{\frac{2}{\pi}\int_0^{\pi/2} e'^2 d\,\theta}$$

$$= \frac{\nu}{10^8}\sqrt{\frac{2}{\pi}\int_0^{\pi/2} \beta^2\, d\,\theta} \tag{3}$$

$$\beta^2 = (\beta_1 \sin\theta + \beta_3 \sin 3\,\theta + \beta_5 \sin 5\,\theta + \ldots.)^2$$
$$= \beta_1^2 \sin^2\theta + \beta_3^2 \sin^2 3\,\theta + \beta_5^2 \sin^2 5\,\theta + \ldots.$$
$$+ 2\,(\beta_1\,\beta_3 \sin\theta \sin 3\,\theta + \beta_1\,\beta_5 \sin\theta \sin 5\,\theta + \beta_3\,\beta_5 \sin 3\,\theta \sin 5\,\theta + \ldots.)$$

Since $2\int_0^{\pi/2}(\beta_1\,\beta_3 \sin\theta \sin 3\,\theta + \beta_1\,\beta_5 \sin\theta \sin 5\,\theta + \beta_3\,\beta_5 \sin 3\,\theta \sin 5\,\theta + \ldots.)$ is zero.

$$e_{eff.} = \frac{\nu}{10^8}\sqrt{\frac{2}{\pi}\int_0^{\pi/2}(\beta_1^2 \sin^2\theta + \beta_3^2 \sin^2 3\,\theta + \beta_5^2 \sin^2 5\,\theta + \ldots)\,d\,\theta}$$

$$e_{eff.} = \frac{\nu}{10^8\sqrt{2}}\sqrt{(\beta_1^2 + \beta_3^2 + \beta_5^2 + \ldots.)} \tag{4}$$

$$e_{ff.} = \frac{\nu\,\beta_1}{10^8\sqrt{2}}\sqrt{1 + k_3^2 + k_5^2 + \ldots.} \tag{5}$$

Where

$$k_3 = \beta_3 \div \beta_1$$
$$k_5 = \beta_5 \div \beta_1 \quad \text{etc.}$$
$$\nu = 2\,\tau\,f \tag{6}$$

where

τ = pole pitch

f = cycles per second.

$$e_{ff.} = \frac{2\,\tau\,f\,\beta_1}{10^8\,\sqrt{2}}\sqrt{1 + k_3^2 + k_5^2 + \ldots.} \tag{7}$$

The effective voltage of a complete, full-pitch coil of N_q series turns, (that is, concentrated winding) is

$$E_{eff.} = 2\,N_q\,l\,e_{eff.} \tag{8}$$

where

l = gross stacked length of core (iron + ducts)

$$E_{eff.} = \frac{4\,\tau\, l\, f\, N_q\, \beta_1}{10^8\,\sqrt{2}} \sqrt{1 + k_3^2 + k_5^2 + \ldots .} \qquad (9)$$

If the coil does not span the full pitch τ, the quantity under the radical is reduced. The voltage of the two coil sides are out of phase by angle $(1-p)\,\pi$, for fundamental and by angle

$$(1 - p)\, n\, \pi$$

for n *th* harmonic,

where,

$$p = \frac{\text{coil pitch}}{\text{pole pitch}}$$

Hence the reduction factor will be the cosine of half the angle, that is,

$$k_p = \cos\,(1-p)\,\frac{\pi}{2} \qquad (10)$$

for fundamental and

$$k_{p_n} = \cos\,(1-p)\,\frac{n\,\pi}{2} \qquad (11)$$

for n th harmonic.

[17]Adams has given curves for k_{pn}

If the N_q series turns per pole per phase are distributed in two or more slots, there is an additional reduction. This factor k_{d_n} is given in table prepared also by [17]Adams.

If the three-phase machine is connected Y, there is a still further reduction, but since the reduction for fundamental is same as for all harmonics, namely 0.866, except the 3rd and 9th which are entirely eliminated, it is not necessary to take this into account separately, as is done with k_{p_n} and k_{d_n}. [17]Adams gives also a table showing the product

$$k_{p_n}\, k_{d_n} = k_{r_n} \qquad (.2)$$

for different conditions.

Hence the effective voltage per phase (per leg) is

$$E_{eff.} = \frac{4\,\tau\, l\, f\, N\, \beta_1}{10^8\,\sqrt{2}} \sqrt{k_r^2 + k_{r3}^2 k_3^2 + k_{r5}\, k^2{}_5 + \ldots .} \qquad (13)$$

17. Trans. A. I. E. E., Vol 28, Part 2, p. 798, 799, 805.

Where

N = series turns per phase.

To find value of β_1, refer to equation (**1**)

$$\beta_1 = \beta_1 \sin \theta + \beta_3 \sin 3\,\theta + \beta_5 \sin 5\,\theta + \ldots \qquad (\mathbf{1})$$

$$\beta_{ave.} = \frac{2}{\pi}\int_0^{\pi/2} \beta\, d\,\theta$$

$$\beta_{ave.} = \frac{2}{\pi}\left[\beta_1 + \frac{\beta_3}{3} + \frac{\beta_5}{5} + \ldots\right]$$

$$\beta_{ave.} = \frac{2}{\pi}\,\beta_1\left(1 + \frac{k_3}{3} + \frac{k_5}{5} + \ldots\right) \qquad (\mathbf{14})$$

Total flux per pole is

$$\Phi = \tau\, l\, \beta_{ave.}$$

$$\Phi = \frac{2}{\pi}\,\tau\, l\, \beta_1\left(1 + \frac{k_3}{3} + \frac{k_5}{5} + \ldots\right) \qquad (\mathbf{15})$$

Let

$$K_\phi = \left(1 + \frac{k_3}{3} + \frac{k_5}{5} + \ldots\right) \qquad (\mathbf{16})$$

$$\Phi = \frac{2}{\pi}\,\tau\, l\, \beta_1\, K_\phi$$

$$\beta_1 = \frac{\Phi}{\frac{2}{\pi}\,\tau\, l\, K_\phi} \qquad (\mathbf{17})$$

Substituting in (**13**)

$$E_{eff.} = \frac{4.44\, f\, N\, \Phi}{10^8\, K_\phi}\sqrt{k_r^2 + k_{r3}^2\, k_3^2 + k_{r5}^2\, k_5^2 + \ldots} \qquad (\mathbf{18})$$

It is obvious from (**18**) that if fractional pitch is used and the coils are distributed, k_{rn} renders practically negligible the effect of higher harmonics upon the value of the radical. If the machine is Y-connected, the third and its multiples are eliminated, leaving the value of the radical very near to

$$k_r$$

which is

$$k_p\, k_d$$

In most cases the radical may be taken as $k_r = k_p\, k_d$ with very little error in E_{eff}.

In concentrated windings, full pitch, Δ-connected, the radical cannot be neglected. It can be neglected with little error for machines with two or more slots per pole per phase, or fractional slots per pole per phase, Y-connected armature. In such cases,

$$E_{eff.} = \frac{4.44\, f\, N\, \Phi\, k_p\, k_d}{10^8\, K_\phi} \tag{19}$$

This means that the flux of the fundamental only

$$\Phi_1 = \frac{\Phi}{K_\phi}$$

is useful. That is, the useful flux is

$$\Phi_1 = \int_0^\pi \beta_1 \sin\theta\, d\,\theta \tag{20}$$

This value Φ_1 is what design calculations have usually been based on, *i.e.* calculations which assumed sine distributions of flux.

Therefore, such calculations should be modified only by the factor

$$K_\phi$$

Thus

$$\Phi = K_\phi\, \Phi_1$$

$$\Phi = \frac{E_{eff.} \times K_p \times K_d \times K_\phi \times 10^8}{4.44\, f\, N} \tag{21}$$

where

$E_{eff.}$ = effective voltage per phase. *i.e.* per leg

N = series turns per phase.

f = frequency.

$K_p = \dfrac{1}{k_p}$ = fractional pitch coefficient ordinarily used in design

$K_d = \dfrac{1}{k_d}$ = distributed coefficient ordinarily used in design, and given in Table II.

$$K_\phi = \left(1 + \frac{k_3}{3} + \frac{k_5}{5} + \dots\right)$$

Where

k_3 = percent third harmonic in flux wave
k_5 = " fifth " " " "
k_7 = " seventh " " " "
etc.

If the radical in equation (**18**) is not negligible, (**21**) becomes

$$\Phi = \frac{E_{ff} \times K_\phi \times 10^8}{4.44\, f\, N} \sqrt{k_r^2 + k_{r3}^2\, k_3^2 + k_{r5}^2\, k_5^2 + \ldots} \quad \textbf{(22)}$$

APPENDIX B

Armature Reaction

Much has been written about armature reaction, particularly in Europe and during the last few years[18]. Practically without exception the method of attacking the problem has been to obtain an expression for the wave of armature m.m.f. and an expression for the wave of field m.m.f. and combine the two for the resultant which determines the flux. But at this point difficulty is encountered, especially on salient pole machines. One must know the reluctance at all points, as well as the m.m.f. before the flux can be obtained. On turbine generators where reluctance is practically uniform over the whole pole pitch, the flux can be assumed to be proportional to the m.m.f., and the problem works out well. However, such a method has not succeeded in solving, without questionable assumptions, the case of salient poles.

The method given in the following pages for solving the case of salient poles is based on the principle that *the effectiveness of an armature ampere turn at any instant and at zero power factor, as compared with a field ampere turn, in establishing flux in the mutual magnetic path, depends upon the percentage of the total useful no-load flux that the armature turn would enclose at that instant on open circuit.* The method was worked out during the investigation of armature self-induction covered in the main body of the paper. The authors were met in this investigation by the necessity of determining more accurately the relative strength of armature and field, since the armature self-induction is tested by taking the difference between these quantities on sustained short circuit. This, as in the case given in Appendix A, is one of the few instances where an apparently hopeless case has

18. See bibliography at end of Appendix B.

worked out to the simplest result: the armature reaction is calculated by the familiar formula based on sine wave assumption, and this is modified by a single factor which may be taken directly from the curve shown in Fig. 20

It was found by methods described in Appendix A that the no-load flux wave could be very closely determined from the dimensions of the machine. The flux-wave shape was thus plotted for the variety of conditions within the ranges of normal design. These waves were expressed as a Fourier's series. With the equation of the wave given, the solution easily followed.

Let Fig. 37A represent the no load flux wave. If one armature ampere turn is placed symmetrically about the field magnetic circuit (*i.e.* directly opposite it, when $\theta = O$ in Fig. 37A), the flux wave will be a replica of that produced by one field ampere

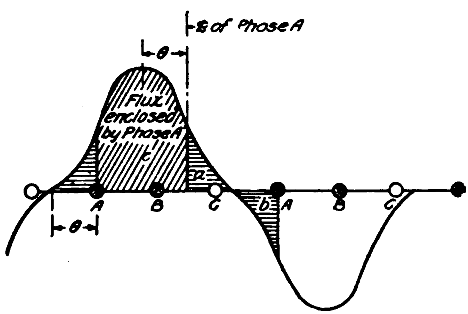


FIG. 37A

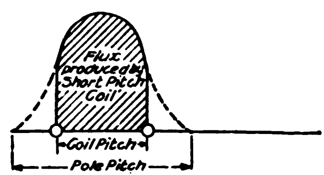


FIG. 37B

turn. If the coil does not span the entire pole pitch, then within the portion which is spanned the flux wave will be a replica of that same portion of a complete wave of a full-pitch turn, as in Fig. 37B.

On sustained short circuit, an armature current is maintained which is just sufficient to create an opposing m.m.f. acting on the mutual magnetic circuit, equal to that of the field; or in other words, sufficient to annihilate the mutual flux. Hence, as mentioned above, the measure of the effectiveness of any turn at any instant is the percentage of the total mutual flux it would enclose on open circuit.

Refer to Fig. 37A. A full-pitch turn, A, encloses all of the flux when in position $\theta = 0$. At any angle θ, observe that the flux enclosed by A is the same as if the two sides of the coil were moved toward the center of the flux wave by the same angle, θ;

because the positive flux, a, is now neutralized by the negative flux b.

The effectiveness of the turns in phase A at the angle is

$$\lambda_\phi = \frac{\text{area } c}{\text{area total wave}} = \frac{c}{c + 2a} \tag{1}$$

As in Appendix A, the equation of the wave is

$$y = \sin\theta + k_3 \sin 3\,\theta + k_5 \sin 5\,\theta + \ldots. \tag{2}$$

$$a = \int_0^\theta y\,d\,\theta \tag{3}$$

From equations (**14**) and (**16**), Appendix A, it follows that

$$c + 2a = \pi \times \text{average ordinate}$$

$$= \pi \times 0.636 \left(1 + \frac{k_3}{3} + \frac{k_5}{5} + \ldots.\right)$$

$$c + 2a = 2\,K_\phi \tag{4}$$

From (**3**)

$$a = \int_0^\theta y\,d\,\theta = \int_0^\theta (\sin\theta + k_3 \sin 3\,\theta + k_5 \sin 5\,\theta + ..)\,d\,\theta$$

$$a = (1 - \cos\theta) + \frac{k_3}{3}(1 - \cos\ 3\,\theta)$$

$$+ \frac{k_5}{5}(1 - \cos 5\,\theta) + \ldots.$$

$$a = K_\phi - \left(\cos\theta + \frac{k_3}{3}\cos 3\,\theta + \frac{k_5}{5}\cos 5\,\theta + \ldots.\right) \tag{5}$$

From (**4**),

$$c = 2\,(K_\phi - a)$$

Hence, from (**1**)

$$\lambda_\theta = \frac{2\,(K_\phi - a)}{2\,K_\phi}$$

$$\lambda_\theta = \left(1 - \frac{a}{K_\phi}\right) \tag{6}$$

$$= 1 - \frac{K_\phi - \left(\cos\theta + \frac{k_3}{3}\cos 3\,\theta + \frac{k_5}{5}\cos 5\,\theta + \ldots.\right)}{K_\phi}$$

$$\lambda_\theta = \frac{1}{K_\phi}\left(\cos \theta + \frac{k_3}{3} \cos 3 \theta + \frac{k_5}{5} \cos 5 \theta + \ldots \right) \tag{7}$$

For the case of a three-phase machine with 60-deg. phase belt, and concentrated winding, and assuming a sine wave armature current (which practically exists on sustained short circuit), the effective m.m.f. of phase A, Fig. 37A is

$$\lambda_\theta \sqrt{2}\, N_q\, I \cos \theta \tag{8}$$

of phase B,

$$\lambda_{(\theta+60)} \sqrt{2}\, N_q\, I \cos (\theta + 60) \tag{9}$$

of phase C,

$$\lambda_{(\theta+120)} N_q\, I \cos (\theta + 120) \tag{10}$$

where,

N_q = series armature turns per pole per phase.
I = r.m.s. amperes per turn.

$$\lambda_{(\theta+60)} = \frac{1}{K_\phi}\left[\cos (\theta + 60) + \frac{k_3}{3} \cos (3 \theta + 180) + \frac{k_5}{5} \cos (5 \theta + 300) + \ldots \right]$$

$$\lambda_{(\theta+60)} = \frac{1}{K_\phi}\left[(0.5 \cos \theta - 0.866 \sin \theta) - \frac{k_3}{3} \cos 3 \theta + \frac{k_5}{5} (0.5 \cos 5 \theta + 0.866 \sin 5 \theta) + \frac{k_7}{7} (0.5 \cos 7 \theta - 0.866 \sin 7 \theta) \right] \tag{11}$$

This neglects harmonics above 7th.
By the same reasoning,

$$\lambda_{(\theta+120)} = \frac{1}{K_\phi}\left[(- 0.5 \cos \theta - 0.866 \sin \theta) + \frac{k_3}{3} \cos 3 \theta + \frac{k_5}{5} (- 0.5 \cos 5 \theta + 0.866 \sin 5 \theta) + \frac{k_7}{7} (- 0.5 \cos 7 \theta - 0.866 \sin 7 \theta) \right] \tag{12}$$

The total m.m.f. per pole (*i.e.* the armature reaction) is

$$A = \lambda_\theta \sqrt{2}\, N_q\, I \cos\theta + \lambda_{(\theta+60)} \sqrt{2}\, N_q\, I\,(0.5\cos\theta - 0.866\sin\theta) - \lambda_{(\theta+120)} \sqrt{2}\, N_q\, I\,(0.5\cos\theta + 0.866\sin\theta)$$

$$A = \sqrt{2}\, N_q\, I\,[\lambda_\theta \cos\theta + \lambda_{(\theta+60)}(0.5\cos\theta - 0.866\sin\theta) - \lambda_{(\theta+120)}(0.5\cos\theta + 0.866\sin\theta)] \qquad (13)$$

Substituting equations (7), (11), and (12),

$$A = \sqrt{2}\, N_q\, I\Bigg[\frac{\cos\theta}{K_\phi}\left\{\cos\theta + \frac{k_3}{3}\cos 3\,\theta + \frac{k_5}{5}\cos 5\theta + \frac{k_7}{7}\cos 7\,\theta\right\}$$
$$+ \frac{(0.5\cos\theta - 0.866\sin\theta)}{K_\phi}\left\{(0.5\cos\theta - 0.866\sin\theta) - \frac{k_3}{3}\cos 3\,\theta + \frac{k_7}{7}(0.5\cos 7\,\theta - 0.866\sin 7\,\theta)\right\}$$
$$- \frac{(0.5\cos\theta + 0.866\sin\theta)}{k_\phi}\left\{(-0.5\cos\theta - 0.866\sin\theta) + \frac{k_3}{3}\cos 3\,\theta + \frac{k_5}{5}(-0.5\cos 5\,\theta + 0.866\sin 5\,\theta) + \frac{k_7}{7}(-0.5\cos 7\,\theta - 0.866\sin 7\,\theta)\right\}\Bigg]$$

Simplifying, this becomes

$$A = \frac{1.5\sqrt{2}\, N_q\, I}{K_\phi}\left[1 + \cos\theta\left(\frac{k_5}{5}\cos 5\,\theta + \frac{k_7}{7}\cos 7\,\theta\right) + \sin\theta\left(-\frac{k_5}{5}\sin 5\,\theta - \frac{k_7}{7}\sin 7\,\theta\right)\right] \qquad (14)$$

If the armature turns are distributed over the phase belt in a number of coils instead of being concentrated in one coil, and if

the coils do not span the full pole pitch, the bracket quantity of equation (**14**) is modified as follows:

1 becomes $k_p\ k_d$

k_5 and k_7 are multiplied by k_{rn}

Adams[19] has given tables foı these reduction factors.

However, since the 5th and 7th are usually not large, and since, as shown by (**14**), their effect is decreased by the order of the harmonic, it may be safely assumed that the further decrease by k_{rn} ɪenders negligible all of the harmonics, in cases where the winding is distributed in more than one slot per pole per phase.

Hence for such cases equation (**14**) becomes,

$$A = \frac{1.5\sqrt{2}\,N_q\,I}{K_\phi\ K_d\ K_p} = \frac{2.12\,N_q\,I}{K_\phi\ K_d\,K_p} \quad \text{(three-phase)} \qquad \textbf{(15)}$$

where

$$K_p = \frac{1}{k_p} = \frac{1}{\sin\ (p\ 90)}$$

$$K_d = \frac{1}{k_d} \quad \text{(See Table II, Part I, for value of } K_d.)$$

N_q = series turns per pole per phase
I = current per turn
K_ϕ = flux distribution coefficient given in Fig. 20.

APPENDIX C

Calculation of Excitation

The reactance as calculated by the method developed in this paper may be used in calculating excitation of alternating-current machines. The method used is an application of the generally accepted principles of excitation calculation, which have been developed by a number of writers in slightly different forms.

The diagram, Fig. 18, illustrates the factors used in the calculation for a generator with lagging current output.

I = armature current.
E = terminal voltage (is taken as unity for calculations on the per cent basis.)
E_i = in-phase component of terminal voltage
= $\cos\ \theta$

19. Trans. A. I. E. E., Vol. 28, Part 2, pp. 798, 799, 850.

E_x = quadrature component of terminal voltage
= $\sin\,\theta$

$\cos\,\theta$ = power factor

r_{pa} = voltage required to overcome resistance drop in per cent of E.

x_{pa} = voltage required to overcome reactance drop in per cent of E.

E_1 = internal induced voltage under load in per cent of E.
= $(\cos\,\theta \pm r_{pa}) + j\,(\sin\,\theta \pm x_{pa})$

From the diagram it can be seen that $\cos\,\theta = \dfrac{E_i + r_{pa}}{E_1}$

Note that for a motor the sign of r_{pa} will be minus.

Also $\sin\,\theta = \dfrac{E_x + X_{pa}}{E}$

This expression holds for the over-excited condition, that is, for a generator with lagging current or a motor with leading current, but the sign of X_{pa} will be minus for the under-excited conditions.

A = ampere turns per pole required to overcome the effective armature reaction.

$$A = \frac{2.12\, s\, a\, I}{2\, K_p\, K_d\, K_\phi\, c} \text{ for three phase}$$

$$= \frac{1.41\, s\, a\, I}{2\, K_p\, K_d\, K_\phi\, c} \text{ for two phase}$$

s = slots per pole and phase

a = conductors per slot

c = circuits per phase

K_p = fractional pitch coefficient

K_d = armature winding distribution coefficient.

K_ϕ = flux distribution coefficient.

Φ = no load flux per pole for normal voltage E.

Φ_1 = full load flux per pole for internal induced voltage E_1

Φ_2 = flux for impedance drop.

F_1 = ampere turns from no load, saturation curve required to produce Φ_1.

F_0 = full-load ampere turns per pole.

The m.m.f. diagram is all that is required for the calculation of the field current and this diagram taken from Fig. 18 for the over-excited condition is given in Fig. 19A, and for under-excited condition in 19B. . The following relation may be derived from this diagram by inspection.

$$F_0 = (F_1 + A \sin \theta_1) + j A \cos \theta_1$$

For the underexcited condition the expression

$$F_0 = (F - A \sin \theta_1) + j A \cos \theta_1$$

The application of this theory can best be illustrated by the application to an example. The formulas necessary for the calculations are given below

$$\cos \theta_1 = \frac{\cos \theta \pm r_{pa}}{E_1}$$

Sign is + for generator
Sign is − for motor

$$\sin \theta_1 = \frac{\sin \theta \pm x_{pa}}{E_1}$$

$$F_0 = (F_1 \pm A \sin \theta_1) + j A \cos \theta_1$$

Sign is + for generator lagging current or motor leading current.

Sign is − for generator leading current or motor lagging current.

$$E_1 = (\cos \theta \pm r_{pa}) + j (\sin \theta \pm X_{pa})$$

See above for signs of same terms.

Example:

Generator—Lagging current.

Assume r_{pa} = 1 per cent = 0.01
x_{pa} = 15 per cent = 0.15
Power factor $\cos \theta$ = 0.80
$\sin \theta = 0.60$
F = 3800 ampere turns (not used in calculations)
F_1 = 4500 ampere turns.
A = 3000 ampere turns.

$$\text{Then } E_1 = (\cos \theta + r_{pa}) + j (\sin \theta + x_{pa})$$
$$= 0.81 + j\, 0.75 = 1.10$$

The value of F_1 is obtained from the no-load saturation curve (test or calculated) at 110 per cent normal terminal voltage and is 4500 ampere turns.

$$\sin \theta_1 = \frac{0.75}{1.10} = 0.68$$

$$\cos \theta_1 = \frac{0.81}{1.10} = 0.735$$

$$\begin{aligned} F_0 &= (4500 + 0.68 \times 3000) + j\,0.735 \times 3000 \\ &= 6900 \text{ ampere turns.} \end{aligned}$$

Motor—Lagging current.
Same data as above except $F_1 = 3400$ ampere turns.

$$\begin{aligned} E &= (\cos \theta - r_{pa}) + j\,(\sin \theta - x_{pa}) \\ &= 0.79 + j\,0.45 = 0.91 \end{aligned}$$

F_1 corresponds to 91 per cent of normal voltage and is 3400 ampere turns.

$$\cos \theta_1 = \frac{0.79}{0.91} = 0.87$$

$$\sin \theta_1 = \frac{0.45}{0.91} = 0.495$$

$$\begin{aligned} F_0 &= (3400 - 0.495 \times 3000) + j\,0.87 \times 3000 \\ &= 3230 \text{ ampere turns.} \end{aligned}$$

The calculation of the above generator may be represented by the following simple calculation.

$$E \left\{ \begin{array}{c|} 0.81 \\ 0.75 \end{array} \right. = 1.10$$

$$F_0 \left\{ \begin{array}{l} \begin{array}{rl} \dfrac{3000}{4500} & = A \\ & = F \end{array} \\[2ex] \left. \begin{array}{l} \dfrac{3000}{1.10} \times 0.75 = \dfrac{2040}{6540} \\[3ex] \dfrac{3000}{1.10} \times 0.81 = 2210 \end{array} \right| = 6900 = F_0 \end{array} \right.$$

Note that $\begin{array}{r|} 0.81 \\ 0.75 \end{array} = 1.10$ and $\begin{array}{r|} 6540 \\ 2210 \end{array} = 6900$

indicate combination of vectors at 90 deg.

This method of calculation of field current is an approximation, and does not allow for the increased field leakage under load. This is probably compensated for by including the added no-load saturation in the armature magnetic circuit which does not exist under load, and the method has been found to give a very close check on test results on a very large number of commercial machines.

BIBLIOGRAPHY

PART I

Wechselstromtechnik IV, p. 8, E. Arnold.

Electrical Machine Design, p. 215, Alexander Gray.

Self-Starting Synchronous Motors, C. J. Fechheimer, TRANS. A.I.E.E. Vol. 31, p. 578.

Leakage Reactance, J. Rezelman, *The Electrician*, April 29, July 22, July 29, 1910.

PART II

Calculation of Sudden Short-Circuit Phenomena of Alternators, Diamant, TRANS. A.I.E.E. Vol. 34, p. 2237.

Short Circuit of Alternators, F. T. Hague, *Elec. Journal*, May 1916.

Alternator Short Circuits, C. M. Davis, *G. E. Review*, Aug. 1914.

Short-Circuiting of Large Elec. Generators and the Resulting Forces on Armature Windings, Miles Walker, Journal of I.E.E.—March 8, 1910.

Turbo Generators, A. B. Field, TRANS. A.I.E.E. Vol. 31, Part 2, p. 1645.

The Transient Reactions of Alternators, W. A. Durgin and R. H. Whitehead, TRANS. A.I.E.E. Vol. 31, Part 2, p. 1657.

Mechanical Effects of Electrical Short Circuits, S. H. Weaver, *G. E. Review*, Nov. 1915.

Power Limiting Reactances, R. F. Schuchardt and E. O. Schweitzer, TRANS. A.I.EE. Vol. XXX, Part 2, p. 1143.

Wechselstromtechnik IV, p. 457, E. Arnold.

Electric Discharge, Waves and Impulses, p. 30, C. P. Steinmetz.

Generator Short-Circuit Waves, F. D. Newbury, *The Electric Journal* April 1914, p. 196.

Sudden Short-Circuit Current of Alternators, K. Ito, Journal of Elec. Society (Japanese).

Analysis of Short-Circuit Oscillograms, O.E. Shirley, *G. E. Review*, Feb. 1917.

APPENDICES A. AND B.

Wave Shape of Generators under Steady Short Circuit, A. E. Clayton, Journal of I.E.E. Vol. 54, 1916, p. 84.

Inherent Regulation of Synchronous A-C. Generators, Alfred Still, Journal of I.E.E. Vol. 53, 1915, p. 587.

Pressure Wave in Electrical Machinery, Smith and Boulding, Journal of I.E.E.. Vol. 53, 1915.

Decomposing Magnetic Fields into their Higher Harmonics, H. Weichsel, TRANS. A.I.E.E. Vol. 34, Part 2, p. 2721.

Wechselstromtechnik III, p. 175, E. Arnold.

Address delivered under the auspices of the Engineering Foundation in the Engineering Societies Building, New York, May 28, 1918.

WAR ACTIVITIES OF THE NATIONAL RESEARCH COUNCIL

BY GEORGE ELLERY HALE, chairman

IT is already a trite saying that this is a war of engineering and science. Yet it is a question whether the average person, or indeed whether even the technically qualified man whose work has been concentrated in a particular field, can realize in how large a sense these words are true. To do so he must survey the vast engineering achievements of the Entente and the Central Powers, and appreciate what countless details they involve and how far their ramifications extend into apparently unrelated fields. At the same time his view must embrace the wide expanse of medicine and hygiene, and the still wider operations of agriculture and the many industries without which the war could not go on. Nor must he be content with this sweeping view, comprehensive though it may seem. For inextricably united with the results achieved by men habitually concerned with the arts, he will find the no less important contributions of investigators in the mathematical, physical, and biological sciences, whose previous efforts have been devoted solely to the advancement of knowledge. Science and the arts have once more united their endeavors, to the advantage of the national defense and to the still greater advantage, let us hope of future national welfare.

In the vast fields of engineering, both military and industrial, the National Engineering Societies have played a truly national part. Merely to enumerate their contributions, in men and in activities of the most varied description, would occupy the entire time at my disposal. I must therefore confine myself to the barest mention of some of the most conspicuous of these activities the prominent part they have played in the work of the Naval Consulting Board, which has contributed in so many important ways to our progress in the war; the gallantry of their members on the western front, where they have proved that the engineer can fight no less effectively than he can build; the countless products, in munitions of war other necessities of national

defense, of the factories they operate; the railways, docks, cantonments, and fortifications they have erected, here and in France; the ships they have sped down the ways to overcome the submarines.

The National Research Council, of whose work I have been requested to speak tonight, has special reasons for gratitude to the engineers of the National Societies. Two years ago, when its organization had just been undertaken, the Research Council was essentially without funds. The Engineering Foundation, established by these societies on Mr. Swasey's endowment for the promotion of research, saw and appreciated the advantage of creating a body for the federation of research agencies, governmental, educational, separately endowed, and industrial. It accordingly placed its entire resources at the disposal of the Research Council, gave it the services of its Secretary and provided an office for the Councils' work in this building. Special contributions from Mr. Ambrose Swasey and Mr. Edward D. Adams enlarged the income available for this purpose, and thus the work of the Research Council was inaugurated. Today when ample funds from other sources have eliminated financial difficulties, the Research Council does not forget the indispensable aid it received from the Engineering Foundation at the most critical period in its existence.

Science in the Civil War

The experience through which we are passing recalls in many vivid particulars, the parallel events of the Civil war. This is true in the field of science, in spite of the popular conception that tends to associate the application of scientific methods in warfare with more modern times. One of the most striking pen portraits of President Lincoln that we possess depicts him on the great tower of the Smithsonian Institution, which he ascended night after night with Joseph Henry, first Secretary of the Institution and charter member of the National Academy of Sciences. From this vantage point lights were flashed to distant stations, in connection with tests of new methods of signalling. It was in such researches for military purposes that the National Academy had its origin.

The period of these experiments was an anxious one. Many months of war, marked by serious and unexpected reverses, had left small room for over-confidence, and taught the necessity of utilizing every promising means of strengthening the

northern arms. With one or two notable exceptions, the great scientific bureaus of the Government, now so powerful, had not come into existence. But the country was not without its leaders of science and engineering, both within and without the Government circle. Davis, fighting Admiral, Chief of the Bureau of Navigation, and founder of the Nautical Almanac; Bache, Superintendent of the Coast Survey, and designer of the defenses of Philadelphia; and Joseph Henry, of whom we have already spoken, clearly recognized the need of a national organization, embracing the whole range of science, to advise the Government on questions of science and art. Joining with them Louis Agassiz the great naturalist; Benjamin Peirce, mathematician and astronomer; and B. A. Gould, founder of the Observatory of the Argentine Republic, they planned the National Academy of Sciences. A bill to incorporate the Academy was introduced in in the Senate by Senator Wilson of Massachusetts on February 21, 1863. This passed the Senate and the House, and was signed by President Lincoln on March 3. This bill, which was subsequently amended to remove the limitation of membership, and to permit the Academy to receive bequests, is given below in its original form:

An Act to Incorporate the National Academy of Sciences.

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled, That Louis Agassiz, Massachusetts; J. H. Alexander, Maryland; S. Alexander, New Jersey; A. D. Bache, at large; F. A. P. Barnard, at large; J. G. Barnard. United States army, Massachusetts; W. H. C. Bartlett, United States Military Academy, Missouri; U. A. William Boyden, Massachusetts; Alexis Caswell, Rhode Island; William Chauvenet, Missouri; J. H.C. Coffin, United States Naval Academy, Maine; J. A. Dahlgren, United States navy, Pennsylvania; J. D. Dana, Connecticut; Charles H. Davis, United States navy, Massachusetts; George Engelmann, St. Louis, Missouri; J. F. Frazer, Pennsylvania; Wolcott Gibbs, New York; J. M. Gilliss, United States navy, Kentucky A. A. Gould, Massachusetts; B. A. Gould, Massachusetts; Asa Gray, Massachusetts; A. Guyot, New Jersey; James Hall, New York; Joseph Henry, at large; J. E. Hilgard, at large; Illinois; Edward Hitchcock, Massachusetts; J. S. Hubbard, United States naval observatory, Connecticut; A. A. Humphreys, United States army, Pennsylvania; J. L. LeConte, United States army, Pennsylvania; J. Leidy, Pennsylvania; P. J. Lesley, Pennsylvania; M. F. Longstreth, Pennsylvania; D. H. Mahan, United States Military Academy, Virginia; J. S. Newberry, Ohio; H. A. Newton; Connecticut; Benjamin Peirce, Massachusetts; John Rogers, United States navy. Indiana; Fairman Rogers, Pennsylvania; R. E. Rogers, Pennsylvania; W. B. Rogers, Massachusetts; L. M. Rutherford, New York; Joseph Saxton, at large; Benjamin Silliman, Connecticut; Benjamin Silliman, junior, Connecticut; Theodore Strong, New

Jersey; John Torrey, New York; J. G. Totten, United States army, Connecticut; Joseph Winlock, United States Nautical Almanac, Kentucky; Jeffries Wyman, Massachusetts; J. D. Whitney, California, their associates and successors, duly chosen, are hereby incorporated, constituted and declared to be a body corporate, by the names of the National Academy of Sciences.

Sec. 2. And be it further enacted, That the National Academy of Sciences shall consist of not more than fifty ordinary members, and the said corporation hereby constituted shall have power to make its own organization, including its constitution, by-laws, and rules and regulations; to fill all vacancies created by death, resignation, or otherwise; to provide for the election of foreign and domestic members, the division into classes, and all other matters needful or usual in such institutions, and to report the same to Congress.

Sec. 3. And be it further enacted, That the National Academy of Sciences shall hold an annual meeting at such place in the United States as may be designated, and the Academy shall, whenever called upon by any department of the Government, investigate, examine, experiment and report upon any subject of science or art, the actual expense of such investigations, examinations, experiments, and reports to be paid from appropriations which may be made for the purpose; but the academy shall receive no compensation whatever for any services to the Government of the United States.

SOLOMON FOOTE,
President of the Senate pro tempore.
GALUSHA A. GROW,
Speaker of the House of Representatives.

Approved, March 23, 1863
ABRAHAM LINCOLN, President.

It will be noticed that the list of incorporators includes many well known names, and that both the Army and Navy are liberally represented. Among the engineers are Brigadier General J. G. Barnard, President of the Board of Engineers for Fortifications and River and Harbor Improvements; Colonel W. H. C. Bartlett, author of a treatise on rifled guns published in the Memoirs of the National Academy; Rear Admiral John Dahlgren, Chief of Ordnance, U. S. N., inventor of heavy guns; Brigadier General A. A. Humphreys, Chief of Engineers, U. S. A., reclaimer of lands inundated by the Mississippi River; Professor D. A. Mahan, Engineer Corps, author of standard treatises on civil and military engineering; Fairman Rogers, Professor of Civil Engineering in the University of Pennsylvania; and General J. G. Totten, Chief of Engineers, U. S. A., distinguished for his development of coast defences.*

*The National Academy of Sciences now elects fifteen new members annually. It has established a separate Section of Engineering, to which three new members were elected at the last Annual Meeting in April.

As the adviser of the Government on questions of science, the Academy was immediately called upon by the War and Navy Departments to report on various problems connected with the war. Among these reports the following may be mentioned:

On the Protection of Bottoms of Iron Vessels from Corrosion

On the Adjustment of Compasses to Correct Magnetic Deviation in Iron Ships.

On Wind and Current Charts and Sailing Directions.

On the Explosion on the United States steamer Chenango.

On Experiments on the Expansion of Steam.

On the Preservation of Paint on Army Knapsacks.

In addition to such formal reports from special committees many members of the Academy contributed individually to the study of war problems. Thus we find in the early records titles of such papers as the following:

F. A. P. Barnard: On the force of fired gun-powder and the pressure to which heavy guns are actually subjected in firing.

Joseph Henry: On materials for combustion of lamps in lighthouses.

J. E. Hilgard: On a chronograph for measuring the velocity of projectiles.

J. E. Hilgard: Note on the changes that have taken place in the bar of Charleston Harbor since the sinking of obstructions in the main channel.

B. A. Gould: Various papers on the stature, proportions, ages, and vision of American soldiers.

W. H. C. Bartlett: On rifle guns.

Most of the work of the members on war problems was, of course, not embodied in published papers, though it formed an important part of the activities of the Government.

This illustration of a national organization of science, including representatives of the Army, Navy and civil branches of the Government, cooperating closely with men of science in civil life, has an interesting parallel in the wars of Napoleon. As a member of the Paris Academy of Sciences, Napoleon fully appreciated the importance of science in connection with war. In organizing his military expedition to Egypt, he drew largely on the membership of the Academy of Sciences, taking with him a most brilliant company of scientific investigators, representing every field of research. While in Egypt, where he invariably signed himself "Le membre de l'Institut, général en chef," Napoleon organized the Institute of Egypt in Cairo, the complete records of which are fortunately preserved in a volume entitled, "Mémoires sur l'Egypte," (Paris An. VIII). From the minutes of the meetings we find that "le citoyen Bonaparte,"

who attended as a member without office, would frequently present requests for the formation of committees to report on such problems as the baking of bread for the armies, and the possibility of making gun powder in Egypt. Other phases of science did not escape his interest, as we recognize from the fact that one of the committees appointed at his request was for the purpose of reporting on the establishment of an astronomical observatory in Egypt. At the same time his architects and archeologists were carrying on extensive studies of the antiquities of Egypt, which are embodied in a superbly illustrated series of folio volumes and mark the first great step in Egyptian archeology, leading to the successful labors of Champollion, Mariette, and Maspero, and the dominance of the French school in Egypt under British administration.

If circumstances demanded, we might easily go still farther back in history to illustrate the connection between science and war, as Alexander the Great took with him on his famous campaigns men of science who determined the positions of captured territory by observations of the stars, and conducted other scientific work. But it is unnecessary to go further into details, since it is clear that any great leader of a state, with sufficient imagination to look at his problems in a large way, must appreciate the importance of utilizing the resources which science has placed at his disposal.

Science in the Present War

One of the most striking results of the present war is the greatly increased emphasis which it has laid on the national importance of science and research. The sharp spur of necessity. felt by the Allies soon after the opening of hostilities, drove them to the instant utilization of scientific research—the policy that lies at the foundation of Germany's military and industrial strength. The superficial notion that science itself is to be condemned because of its barbarous misuse in German hands has no place in the minds of clear-thinking people, who perceive, more plainly than ever before, its significance as a fundamental factor in national progress. Statesmen ignorant of the bearing of science on the problems of war have given place to more enlightened men, and all governments that have felt the pressure of military and industrial necessity are now vying with one another in promoting scientific research.

The activities of the National Academy of Sciences during the

Civil War, as well as the provisions of its charter, indicated its fitness for renewed service in support of the national defence. In April 1916, when the attack on the "Sussex" had greatly increased the tension of our relations with Germany, the Academy received from the President a call for immediate action. The rapid development of technical bureaus and laboratories in connection with the various branches of the Government left no demand for another scientific bureau. But the same function which the Academy had subserved during the Civil War—that of uniting the technical services of the Army, Navy and Civil Departments in close cooperation with scientific investigators from research laboratories all over the country—must, in the President's judgment again be fulfilled. He therefore expressed the desire that the Academy should bring into cooperation governmental, educational, industrial, and other research agencies, primarily in the interest of the national defense, but with full recognition of the duties that must be performed in the furtherance of scientific and industrial progress.

The Academy's connection with the Government, its inclusion of the whole range of science, and its many years of cooperation with the Royal Society of London, the Paris Academy of Sciences, and other similiar academies abroad, pointed to it as the only body in the United States in a position to comply with the President's request. It was clear, however, that membership in the desired organization should not be exclusively confined to the National Academy. Many technical bureaus of the Army and Navy, for example, should be represented by their chief's *ex-officiis*, and in other cases a changing membership, broadly representative of research in its numerous aspects, would also be desirable. The Organizing Committee accordingly recommended the establishment of a new body, resting legally upon the character of the Academy, sharing its privileges, both at home and abroad, and at the same time affording the wide freedom of selection desired.

The National Research Council, comprising the chiefs of the technical bureaus of the Army and Navy, the heads of Government bureaus engages in scientific research, a group of investigators representing educational institutions and research foundations, and another group including representatives of industrial and engineering research, was accordingly constituted by the Academy with the active cooperation of the leading national scientific and engineering societies. On July 24, 1916, President

Wilson addressed a letter to the President of the National Academy expressing his approval of a preliminary report regarding the National Research Council, and promising his cooperation and that of the various departments of the Government. Since that time he has continued to give his support to the work of the Research Council, and has appointed various representatives of the Government to membership in it.

On February 28, 1917, the Council of National Defense passed a resolution expressing its recognition of the fact that the National Research Council, at the request of the President, had organized the scientific resources of the country in the interest of national defense and national welfare, and requesting the Research Council to cooperate with it in matters pertaining to scientific research for national defense. As a result of this action the Chairman of the Council opened offices in the Munsey Building in March, and entered into active cooperation with the Council of National Defense, which was then established in the same building.

Soon afterwards the Research Council was requested to act as the Department of Science and Research of the Council of National Defense, in which capacity it has continued to serve for the organization of investigations on military and industrial problems and, in harmony with the expressed wish of the President, as an agency for securing widespread cooperation in the field of science and research.

A further extension of the duties of the National Research Council occurred in July, when it was requested by the Chief Signal Officer to organize the Division of Science and Research of the Signal Corps. Major (now Lieutenant Colonel) Robert A. Millikan was placed in charge of this Division, which has remained in close contact with the Research Council, engaged in the solution of numerous problems of military importance.

Another important request on behalf of the Government, made by Assistant Secretary of War Stettinius, resulted in the appointment of a Committee of the Research Council to organize and direct extensive researches for the improvement of processes for the fixation of nitrogen, undertaken in cooperation with the Ordnance Department of the Army.

Before describing the war activities of the Research Council, it will be advantageous to consider the full scope of its duties, as set forth in an Executive Order issued by the President on May 11, 1918

Executive Order of President Wilson

The National Research Council was organized in 1916 at the request of the President by the National Academy of Sciences, under its congressional charter, as a measure of national preparedness. The work accomplished by the Council in organizing research and in securing cooperation of military and civilian agencies in the solution of military problems demonstrates its capacity for larger service. The National Academy of Sciences is therefore requested to perpetuate the National Research Council, the duties of which shall be as follows:

1. In general, to stimulate research in the mathematical, physical and biological sciences, and in the application of these sciences to engineering, agriculture, medicine and other useful arts, with the object of increasing knowledge, of strengthening the national defense, and of contributing in other ways to the public welfare.
2. To survey the larger possibilities of science, to formulate comprehensive projects of research, and to develop effective means of utilizing the scientific and technical resources of the country for dealing with these projects.
3. To promote cooperation in research, at home and abroad in order to secure concentration of effort, minimize duplication and stimulate progress; but in all cooperative undertakings to give encouragement to individual initiative, as fundamentally important to the advancement of science.
4. To serve as a means of bringing American and foreign investigators into active cooperation with the scientific and technical services of the War and Navy Departments and with those of civil branches of the Government.
5. To direct the attention of scientific and technical investigators to the present importance of military and industrial problems in connection with the war, and to aid in the solution of these problems by organizing specific researches.
6. To gather and collate scientific and technical information at home and abroad, in cooperation with Governmental and other agencies and to render such information available to duly accredited persons.

Effective prosecution of the Council's work requires the cordial collaboration of the scientific and technical branches of the Government, both military and civil. To this end representatives of the Government, upon the nomination of the National Academy of Sciences, will be designated by the President as members of the Council, as heretofore, and the heads of the departments immediately concerned will continue to cooperate in every way that may be required.

(signed) Woodrow Wilson

The White House
May 11, 1918

Three-fold Nature of the Council Work

If we consider any research problem bearing on the war, we are likely to find that it has a three-fold nature. Take, for example, the question of the supply of optical glass for periscopes, range-finders, field glasses, and other instruments. Obviously such glass, most of which has hitherto been obtained from Germany, is necessary from a military point of view and also in the industries. Therefore researches must be undertaken to determine how it can be made. Back of these lie the more funda-

mental researches on the nature of glasses, crystals, and minerals, undertaken for the purpose of advancing knowledge. Or take the problem of the fixation of nitrogen. Nitrates are needed for the manufacture of explosives and also for use as fertilizers. As we have obtained them in the past wholly from the nitrate bed of Chile, which are open to enemy interference, we must devise processes for their manufacture in the United States. Those involve researches for the direct purpose of accomplishing the necessary reactions, and back of these lie still more fundamental researches on the underlying scientific principles.

In these and many other similiar cases, we observe the three elements which are fundamentally important in the work of the National Research Council. It is clear that a nation unwilling to give place in the industrial world to better informed rivals must adopt every feasible means of promoting research in the industries. It is equally clear that so long as the security of the world is menaced by unscrupulous military powers, research methods must be effectively utilized in perfecting the means of national defense. Fundamental to both is the prime necessity, clearly appreciated and strongly emphasized by far-sighted leaders of American industry, of promoting research in all branches of science, without thought of any industrial application, for the sake of advancing knowledge. As Colonel Carty has said, the pioneers of industrial research are those who see and apply the discoveries of men of science, by whom new territories are opened and explored. Without the knowledge derived form such explorations, the investigator bent upon immediate industrial advantage could make little progress.

Thus any broad plan of promoting scientific and industrial research for national welfare must involve the cordial co-operation of the men, institutions and societies, interested in these three aspects of science: (1) its advancement as the source of new knowledge; (2) its development when applied in the fields of engineering, agriculture, medicine, and other useful arts; and (3) its utilization, so long as predatory military powers threaten national existence, as a means of strengthening the national defense.

Our place in the industrial world, the development of our commerce, the health of our people, the output of our farms, the conditions under which the great majority of our population must labor, and the security of the nation, will thus depend, in large and increasing measure, on the attention we devote to the

promotion of scientific and industrial research. Anyone who doubts this statement would do well to study in detail the causes which account for the industrial and military strength of Germany.

Co-operation with the Government

The final clause in the Executive Order of the President, which provides for the closest co-operation of the National Research Council with the various departments of the Government, is of fundamental importance. Chiefs of Government bureaus, nominated to the President by the National Academy of Sciences, have been appointed members of the National Research Council, in which they constitute its Military Division. This Division consists of the Chief of Operations of the Navy, the President of the Army War College, the Chiefs of Ordnance of the Army and Navy, the Chief Signal Officer of the Army, the Chief Naval Constructor, the Surgeon General of the Army, the Engineer in Chief of the Navy, the Chief of Engineers of the Army, the Director of Naval Intelligence, and the Chief of the Military Intelligence Section, together with the Chief of the Weather Bureau, the Chief of the Bureau of Chemistry, the Director of the Bureau of Mines, the Director of the Geological Survey the Chief of the Bureau of Forestry, the Director of the Council of National Defense, the Director of the Bureau of Standards (Vice Chairman), and the Secretary of the Smithsonian Institution (Chairman).

One of the most valuable results of the cooperation effected through the Military Division is the organization of the Research Information Committees, with offices in Washington, London and Paris. The importance of this step, which should have direct influence upon international cooperation in scientific research, especially if the position of Scientific Attache of our embassies abroad is maintained after the war, is such as to warrant the following detailed statement regarding the organization and work of the Research Information Committee.

1. By joint action the Secretaries of War and Navy, with the approval of the Council of National Defense, have authorized and approved the organization, through the National Research Council, of a Research Information Committee in Washington, with branch Committees in Paris and London, which are intended to work in close cooperation with the offices of the Military and Naval Intelligence, and whose function shall be the securing, classifying, and disseminating of scientific, technical

and industrial research information, especially relating to war problems, and the interchange of such information between the Allies in Europe and the United States.

2. In Washington the Committee consists of, first, a civilian member representing the National Research Council, Dr. S. W. Stratton, Chairman; second, the Chief, Military Intelligence Section; third, the Director of Naval Intelligence; and fourth, a Technical Assistant, Dr. Graham Edgar. Similiar Committees are being organized in Paris and London.

3. The initial organization of the Committee in Paris is:

(a) The Scientific Attache, representing the Research Information Committee, Dr. W. F. Durand, Attache.

(b) The Military Attache or an officer deputed to act for him.

(c) The Naval Attache or an officer deputed to act for him.

(d) A Technical Assistant, Dr. K. T. Compton.

(e) A Military Assistant, Mr. Tod Ford..

4. The initial organization of the Committee in London is:

(a) The Scientific Attache representing the Research Information Committee, Dr. H. A. Bumstead, Attache.

(b) The Military Attache or an officer deputed to act for him.

(c) The Naval Attache or an officer deputed to act for him.

(d) A Technical Assistant, Mr. S. W. Farnsworth.

5. The chief functions of the foreign Committees thus organized are intended to be as follows:

(a) The development of contact with all important research laboratories or agencies, governmental or private; the compilation or problems and subjects under investigation; and the collection and compilation of the results obtained.

(b) The classification, organization and preparation of such information for transmission to the Research Information Committee in Washington.

(c) The maintenance of continuous contact with the work of the offices of Military and Naval Attaches, in order that all duplication of work or crossing of effort may be avoided, with the consequent waste of time and energy and the confusion resulting from crossed or duplicated effort.

(d) To serve as an immediate auxiliary to the offices of the Military and Naval Attaches in the collection, analysis, and compilation of scientific, technical, and industrial research information.

(e) To serve as an agency at the immediate service of the Commander-in-Chief of the Military and Naval forces in Europe for the collection and analysis of scientific and technical research information and as an auxiliary to such direct military and naval agencies as may be in use for the purpose.

(f) To serve as centers of distribution to the American Expeditionary Forces in France and to the American Naval Forces in European Waters of scientific and technical research information originating in the United States and transmitted through the Research Information Committee in Washington.

(g) To serve as centers of distribution to our Allies in Europe of scientific, technical and industrial research information originating in the United States and transmitted through the Research Information Committee in Washington.

(h) The maintenance of the necessary contact between the officer in Paris and London in order that provision may be made for the direct and prompt interchange of important scientific and technical information.

(i) To aid research workers or collectors of scientific, technical and industrial information from the United States, when properly accredited from the Research Information Committee in Washington, in best achieving their several and particular purposes.

6. The chief functions of the Washington office of the Committee are as follows:

(a) To provide means of ready cooperation with the Paris and London offices of the Committee by:

1. Receiving, collating and disseminating information forwarded from these offices.

2. Rendering available such evidence and documents as may be collected by the National Research Council relative to research in the United States, so as to formulate replies to inquiries sent from abroad.

3. Communicating to foreign offices needs for additional information relating to problems originating in the United States.

(b) Classification, cataloging and filing of papers and reports received from various sources at the request of the National Research Council, and record of researches in progress concerning which detailed information may be obtained elsewhere.

(c) Issue of lists of available information and preparation of digests of such information for distribution to properly accredited persons.

(d) Maintenance of contact with various research agencies in the United States.

An appropriation of $38,400 has been made by the Council of National Defense to cover the expenses of the Research Information Committee for the current year.

Vice-Admiral Sims, in Command of the U. S. Naval Forces Operating in European Waters, has been particularly cordial in his welcome of the foreign representatives of the Research Council. Fully appreciating the possibilities of scientific cooperation, he has issued a circular letter to all naval officers and investigators in Europe, directing them to facilitate the work of the Scientific Attache in every possible way, to keep him fully informed of investigations in progress or needed, and to make every proper effort to see that all investigators, whether officers or civilians, shall consult the Scientific Attache in order to avoid unnecessary duplication of work and to utilize scientific and technical information obtained from any source. He has also created a Scientific Division of his staff, and placed Dr. Bumstead at its head. Major-General Biddle, in command at American Army Headquarters in England, has issued similiar orders to ordnance, engineer, gas, signal, aviation, medical and other officers in England. The British Government, on its part, has opened every source of information to Dr. Bumstead, and provided for the closest cooperation in research.

In France, Dr. Durand is also in close touch with our own Army and Navy, and with the French Government and men of science. He has also been appointed the representative of the United States on the Inter-Allied Board of Inventions.

The Ministry of Munitions in Rome has recently requested, through the Italian Ambassador in Washington, that a representative of the National Research Council be sent to Rome as Scientific Attache and head of an Italian branch of the Research Information Committee.

The natural development of the work of the Research Information Committee will lead to the concentration in the office of the National Research Council, where the Washington headquarters of the Committee is established, of all available information regarding research problems under investigation both in the United States and abroad. At the same time a service is being developed for the purpose of bringing properly accredited inquirers into touch with existing sources of scientific, technical, and engineering information in the United States. One of the

most valuable of these is the Information Service of the American Society of Mechanical Engineers, which is furnishing much important material to the National Research Council. A central office from which inquirers may be directed to Government bureaus and to such sources of information as that just mentioned has long been needed, and it is possible that the service of the Research Information Committee, once well organized, will be in increasing demand.

In this same field of supplying information on scientific and technical subjects, the work of the Research Council has already been developed in several different directions. For example, a sub-committee of the Geology Committee, consisting of one geologist and one highway engineer from each of nineteen states extending from Maine to Texas, has collected a very large body of information regarding materials for rapid highway construction along the Atlantic coast. The elaborate report of this Committee, bound in seven volumes, with three atlases, has already been of considerable service, not merely from the standpoint of these interested in highway construction for possible military purposes, but also to the Shipping Board in connection with the problem of building concrete ships, for which the stone quarries described in the report are often adapted. In another field the work of the Botanical Raw Products Committee has supplied extensive data relating to raw products required by industries, especially in cases where imports have been affected by the war. In still another field the Research Council has been called upon to cooperate with the Army War College in supplying information relating to topographical, geographical and related subjects. Without mentioning other cases in which the aid of the Research Council has been sought for the purpose of supplying technical information, it is clear that this section of its work, not only during the war but after its conclusion is likely to undergo extensive development.

How the Research Council Operates—International Operation in Research

When a scientific investigator undertakes any piece of research, his first act is invariably to ascertain just what work has already been accomplished in that field. It goes without saying therefore, that an organization composed of scientific investigators must proceed in the same way in attacking any large problem involving research. Moreover, it must lose no time in

arranging for close cooperation with the scientific men of other nations concerned with the same problem.

The writer, as chairman of the committee appointed by the National Academy to organize the Research Council, made a preliminary visit to England and France in August 1916, in order to learn how the scientific men of these countries were to be utilized in connection with the war. He found the investigators with whom he had cooperated for many years in astronomical and physical researches, actively engaged in the study of war problems. To the superficial observer it might seem strange that a physicist who has never before been engaged in so-called "practical" work, or an astronomer who had spent his life in the study of celestial phenomena, should be able to contribute effectively in time of war. But a moment's consideration of the nature of the problems to be solved, and a slight understanding of the methods in daily use by the physicist and the astronomer, would dispel any such impression. If I were free to betray military secrets, I could show that some of the most vital military questions have been solved by just such men. But enough will be said in the sequel to indicate how both the man of science and the engineer may render invaluable war service.

On the day preceding the entrance of the United States into the war, the following cablegram was sent by the Foreign Secretary of the National Academy of Sciences to the Royal Society of London, the Paris Academy of Sciences, the Accademia dei Lincei of Rome, and the Petrograd Academy of Sciences—leading scientific bodies with which the National Academy has cooperated for many years in the International Association of Academies:

> The entrance of the United States into the war unites our men of science with yours in a common cause. The National Academy of Sciences, acting through the National Research Council, which has been designated by President Wilson and the Council of National Defense to mobilize the research facilities of the country, would gladly cooperate in any scientific research still underlying the solution of military or industrial problems.
>
> HALE, Foreign Secretary.

Steps were also taken to despatch a group of seven scientific investigators, under the chairmanship of Dr. Joseph S. Ames, to France and England for the study of war problems and the arrangement of effective means of cooperation. The members of

the group sailed early in May, 1917, and were most cordially welcomed and given infoımation of great value.

The response of our foreign colleagues to our offer of co-operation was immediate and effective. France sent to the United States an able group of investigators, headed by M. Fabry, the well known spectroscopist of Marseilles, with whom some of us has worked for years in the International Union for Cooperation in Solar Research. This group comprised such men as M. Abraham, able physicist and authority on wireless telegraphy, and the Duc de Gusche, who has no superior in his knowledge of the science of aeronautics. From England the Royal Society and the British Admiralty sent Sir Ernest Rutherford and Commander Cyprian Bridge, R. N., while Italy sent Lieutenant Giorgiò Abetti, of the Osservatorio del Collegio Romano and the Italian Ministry of Munitions. The French members brought with them an invaluable collection of instruments and devices developed in France for military and naval purposes since the outbreak of the war.

Immediately after the arrival of this party in Washington, the National Research Council organized a conference on submarine problems, in which the foreign representatives, with members of the Navy Department and the physicists and engineers who had already studied the subject in this country, participated.* As the result of a two-day's discussion, it became clear that a greatly intensified attack on the problem should be made. The Research Council accordingly brought to Washington about forty leady physicists and engineers, and a second conference, of three days' duration, was held with the foreign naval officers and men of science. This resulted in the selection of several groups of investigators, to take up the problem at the point in its development already attained here and abroad, and to continue its study in cooperation with a board of naval officers appointed by the Secretary of the Navy. While I am not at liberty to mention the results of the investigations thus initiated, I have thought it worth while to describe the mode of procedure in order to illustrate how the Research Council conducts its work.

*Three able groups of investigators were already to work on this problem in the United States under the Bureau of Steam Engineering of the Navy, and both the Naval Consulting Board and the National Research Council had taken an active part in promoting studies of the submarine problem.

Research Conference on Military and Naval Problems

The great value of research conference on war problems, in which representatives of the technical bureaus of the Army and Navy join with members of the scientific bureaus of the civil Departments and with representatives of university, industrial and other research laboratories, is too obvious to require demonstration. Such conference, organized under regulations prescribed by the Military Division of the National Research Council, are held weekly under the immediate auspices of the Engineering and Physics Divisions. The discussions are based upon reports of researches on military and naval problems in progress in the United States, supplemented by the reports received at short intervals from abroad through the Research Information Committee.

Such conferences not only serve to stimulate research in the most effective way, they also insure that principles and methods developed in one branch of the service shall be made available in every other branch. Thus every bureau, whether of the Army or the Navy, is sure to find points of interest and importance in the discussion held at these conferences.

War Problems Involving Scientific Research

It is evident that no detailed account of work on war problems involving scientific research can be given without affording information to the enemy. If you will permit us, however, to omit the most interesting part of what such an account should be, I may hope to afford some idea of the character of the chief work of the National Research Council.

The War Organization of the Council involves the grouping of its various Committees under a series of Divisions, each of which deals with related subjects. Thus the Physics Division comprises the work of committees of Physicists, mathematicians, astronomers and geophysics. This Division works in very close cooperation with the Engineering Division, in view of the impossibility of distinguishing sharply between the problems belonging in their respective fields. Under the Engineering Division, which represents the expansion of the work of our former Engineering Committee due to the rapidly increasing demands upon the Council from Government bureaus and other sources, there are Sections or Committees dealing with mechanical engineering, electrical engineering, metallurgy, and with various special fields of research. The National Advisory Committee for Aeronautics acts as the Aeronautics Section of the Engineer-

ing Division. Permit me to express here the appreciation of the National Research Council of the action of the Engineering Foundation and the National Engineering Societies, which have appointed representatives to serve as members of the Executive Committee of the Engineering Division. We have been fortunate enough to secure the services of Dr. Henry M. Howe as Chairman of this Division, while the Chairmanships of the Sections of Electrical Engineering, Metallurgy and Mechanical Engineering, are held by Professor Comfort A. Adams, Mr. S. L. G. Knox, and Professor Bradley Stoughton, respectively. Lieutenant Colonel Robert A. Millikan, who also heads the Division of Science and Research of the Signal Corps, is Chairman of the Physics Division.

Within the extensive field covered by these two Divisions, numerous problems are constantly presenting themselves. Take such a subject as naval range finders. Here we are dealing with an optical instrument of precision, involving methods employed by the physicist and by the astronomer concerned with the measurement of stellar parallaxes. As existing range finders are marked by several defects, a well-known physicist, who has had extensive experience in the development of new types of instruments, was requested to attack the problem. He has already constructed a new range finder which seems to offer important advantages.

Another problem in physics is the location of enemy guns by sound. As M. Painlevé has stated publicly that apparatus devised by the French physicists for this purpose has been captured by the Germans, there can be no harm in referring to it. Here it is a question of determining the exact time of arrival of the sound wave emitted by the discharge of an enemy gun at three or more points where automatic recording instruments are located. The research problem involved is therefore the development of simple and effective recording instruments and a rapid method of calculation which will permit the observations to be reduced and the position of the enemy gun located in the shortest possible space of time. The National Research Council initiated the sound ranging service of the Army under the Signal Corps (it was subsequently transferred to the Engineer Corps), and secured the development of new forms of recording apparatus.* Major Augustus Trowbridge of Princeton, who is in

***Captain Ernest Weibel, who took part at the Bureau of Standards in developing a new recording instrument, was killed at the front in France while applying the method.**

charge of the service in France, is a well known physicist. The quickest method hitherto devised of reducing the observations is due to an equally well known Princeton astronomer, who is working under the auspices of the National Research Council.

Another field in which the experience of the astronomer in computing orbits is directly applicable is that connected with the dropping of bombs from aircraft and the calculation of the trajectories of projectiles. Here several physicists and astronomers have obtained new and interesting results, useful in military practise. Many submarine problems, the location of invisible aircraft, various questions connected with wireless communication, the visibility of objects on land and sea, and scores of other questions falling within the fields of acoustics, optics, electricity and other branches of physics, afford abundant opportunity for researches of the most varied character. Also lying within the field of the Physics Division is the work of the Committee on Navigation and Nautical Instruments, under the Chairmanship of Dr. L. A. Bauer, which has aided the Shipping Board in the selection and testing of compasses, chronometers, and other nautical instruments.

The Engineering Division has dealt with such problems as the design of two new types of gun, for one of which the Ordnance Department of the Army has made a special appropriation. It is making an extensive study, in cooperation with the Ordnance Bureau of the Navy, of the entire problem of gun-pointing, which involves a great variety of interesting questions and the cooperation of men who have derived the necessary experience from their work in engineering, physics and other branches of science. Another very interesting investigation conducted under the auspices of the Engineering Division has dealt with the best composition and design of helmets and protective body armor for troops. Dr. Bashford Dean, Chairman of this special Committee, and now a major in the Ordnance Department of the Army, is Curator of Arms and Armor at the Metropolitan Museum of Art. Dr. Henry M. Howe, the well known metallurgist, is another active member of this Committee.

It is unfortunate that I cannot go into the very interesting details of the work done under these related Divisions, as it would be so easy to illustrate the importance of securing such widespread cooperation in research as the National Research Council is especially intended to promote.

Passing on to the important Division of Chemistry and Chemical Technology, organized by Professor (now Lieutenant Col-

onel) Marston T. Bogert, and now under the Chairmanship of Dr. John Johnston, we enter a field of the first importance from a military and industrial standpoint. It is impossible here even to touch upon the extensive work of this Division, which includes thirty-four committees dealing with various phases of chemistry. Perhaps the most important work undertaken by the Division is the exhaustive study of processes for nitrogen fixation, conducted under its auspices. The first committee appointed by the National Research Council in May 1916 at the request of the Secretary of War, was charged with the duty of advising the Ordnance Department of the Army regarding the best processes to be adopted in its great nitrate plants. Dr. Arthur A. Noyes, Chairman of this Committee, is also Chairman of a new Committee, recently appointed by the Research Council at the request of Assistant Secretary of War Stettinius to survey the researches now in progress for the improvement of these processes, to plan further investigations relating to nitrogen-fixation, to arrange for the active prosecution of such investigations, and to exercise close oversight over their progress. Working in cooperation with the officers of the Ordnance Department, this Committee has initiated various researches for the purpose of improving existing processes, one of which has proved so successful that it will materially reduce the cost of nitrogen fixation.

It should be said here, once for all, that the policy of the National Research Council is now and has been from the outset invariably to recommend the immediate adoption and utilization for military and naval purposes of the best devices or methods known at the time, with the understanding that research for the purpose of improving such devices should not retard production demanded to meet pressing military needs.

Passing over, for lack of space, the many other activities of the Division of Chemistry, a word should be said regarding the Division of Geology and Geography, under the Chairmanship of Dr. John C. Merriam, which includes Committees on both of these subjects. The important work on road materials done under the supervision of the late Dr. William Bullock Clark, as a part of the work of the Geology Committee (Dr. John M. Clarke, Chairman), has already been mentioned. The Division is working in close cooperation with the Army War College, and has sent a geologist abroad to report on the best services of geologists in connection with the war. A valuable "Handbook of Northern France", prepared by Dr. William M. Davis, Chair-

man of the Geography Committee, has been widely circulated among American officers. The general information thus supplied is being supplemented by lectures on military geography given in the various contonments. In this connection special mention should be made of an important work prepared by Major Douglas W. Johnson, a member of the Geography Committee now in France, entitled "Topography and Strategy in the War."

The Division of Medicine and the Related Sciences, under the Chairmanship of Dr. R. M. Pearce, includes Committees on Anatomy, Physiology, Psychology, Anthropology, Medical Zoology, Toxicity of Preserved Foods, Psychiatry, and other special subjects, and is one of the most active Divisions of the Research Council. Aided by an appropriation of $50,000 from the Rockefeller Foundation, and working in the closest cooperation with the Surgeon General of the Army (through Colonel Russell) and of the Navy (through Dr. Stitt), it has organized many researches of direct military value, assisted the Surgeons General of the Army and Navy in procuring trained investigators to enter the respective services, and initiated many other activities of importance. It is impossible within present limits to describe the numerous medical researches undertaken. About half of them are in the field of physiology alone, and deal with problems of shock, control of hemorrhage and similar subjects. The remaining half are divided, roughly, between problems concerning the acute infectious diseases, the control of vermin, food problems, and the diseases of munition workers.

A word should be said regarding the novel and important work in psychology, due to the initiative of Major Robert M. Yerkes, Chairman of the Psychology Committee of the Research Council. Perceiving the possibility of psychological examination as applied to the Army, and with the active aid of a strong committee, Dr. Yerkes prepared a scheme of psychological examination, and secured permission to test it with troops. Meanwhile a plan for applying tests on a large scale had been prepared and presented to the Surgeon General through the National Research Council. The preliminary tests impressed military officers so favorably that a new Division of the Surgeon General's Office was created, and Dr. Yerkes was commissioned a major in charge of the work. The psychological tests are now being applied to all troops in the Army, and the intellectual rating thus afforded has proved to be very useful in practise. This is one of the most interesting scientific innovations of the war.

As the functions of the Committee on anthropology in connection with military needs might not be grasped without reflection, it should be remarked that this Committee then under the Chairmanship of Dr. William H. Holmes, was the first to point out that under the former height limit of enlistment (five feet, four inches) the taller native American would be discriminated against when compared with the shorter immigrants from many European nations. The figures presented by the Committee convinced the War Department, and the height limit was accordingly reduced to five feet, in accordance with the recommendations of the Committee. A further result of this Committee's activities is the organization of a Division of Anthropometric Measurements in the Surgeon General's office, under the charge of Dr. C. B. Davenport.

The field covered by the Division of Agriculture, Botany, Zoology, Forestry and Fisheries, under the Chairmanship of Dr. Vernon Kellogg, is a wide one, of fundamental importance in connection with the war. Working in close cooperation with the Department of Agriculture, the Bureau of Forestry and Fisheries, and the Food Administration, this Division is accomplishing much valuable work. The information collected by the Botanical Raw Products Committee, under the Chairmanship of Dr. E. M. East, has already been mentioned. The Agriculture Committee has initiated several large investigations involving the cooperation of members of the Department of Agriculture, State Experiment Stations, and investigators in the universities. The last piece of work organized by the Division was undertaken at the request of the Food Administration, which requested the Research Council to appoint a committee to investigate binder twine fibres with special reference to sisal and its substitutes.

I might go on to describe other phases of the Research Council's work, including the activities of the Patent Office Committee, appointed at the request of the Commissioner of Patents; the Section on cooperation with state research committees and with the research committees established at the suggestion of the Research Council in seventy-two eductional institutions; the Committee which has undertaken a census of the investigators and research facilities of the country; etc. Tc do so, however, would greatly enlarge this paper, which is already too long. I will therefore close with a brief statement of the work inaugurated by the Council for the promotion of research in the industries.

Industrial Research and National Welfare

At the outbreak of the war the average statesman of the Allied powers was but little concerned with the interests of research. Necessity, however, soon opened his eyes. He began to perceive the enormous advantages derived by Germany from the utilization of science, and sought to offset them by the creation of appropriate agencies. Thus arose throughout the British Empire a group of Councils for Scientific and Industrial Research. The first of these was established in England by an order in council issued in 1915. Subsequently, Canada, Australia and South Africa followed the example of the mother country, and New Zealand propcses to do likewise. The world-wide movement swept acrcss the Empire, and its benefits will be felt in every country under the British flag. A similar awakening was experienced in France and Italy, but in both of these countries the pressure of the war concentrated attention for the moment upon military problems. At present, the needs of industry are also under consideration, and research organizations are being developed to meet them.

Without entering here into a detailed discussion of these Councils, we may mention certain typical illustrations of their activities from the report of the British Advisory Council for Scientific and Industrial Research for the year 1916-17.

The British Advisory Council has devoted itself during the year mainly to the organization of industrial research, partly because of the prime importance of stimulating and fixing the interest of manufacture in the development of industry through research, and partly because the effect of the war has been to render industrial leaders more susceptible than ever before to the growth of new ideas. In pure science, on the contrary, the war has seriously affected the prosecution of research, because so many investigators have been drawn into military and industrial activities. Thus, while the Advisory Council strongly emphasizes the fundamental importance of pure science, it has been forced to postpone its activities in this field until the arrival of more favorable conditions.

Research for the development of the industries may be conducted in several different ways. In this country a stimulating example has been set by such great corpcrations as the American Telephone and Telegraph Company, the General Electric Company, the Eastman Kodak Company, and the Westinghouse Company, all of which have established large research

laboratories. The value of this example has been enhanced by the remarkable success achieved by these laboratories in matters affecting public welfare, such as the reduction in cost of electric lighting caused by the development of the Mazda lamp and the possibility of transcontinental telephony, not to mention the latest advances in the field of wireless telephony.

Self-interest will sooner or later induce many other corporations to adopt similar methods of improving their products, but the heavy expense of establishing independent research laboratories will sometimes prove an insurmountable obstacle. Other means must then be resorted to. A useful example is that afforded by the American Canners' Association, which has established central research laboratory in Washington, where any member of the Association can send his problems for solution and where extensive investigations, the results of which are important to the entire industry, are also conducted.

The British Advisory Council, aided by a Government appropriation of one million pounds, is actively promoting the organization of Trade Research Associations for the mutual benefit of the members of the great industries. Thus a Provisional Committee representative of the British cotton industry has proposed the establishment of a cooperative Association for Research in Cotton, to include in its membership cotton spinning, doubling and thread making firms, cloth, lace and hoisery manufacturers, bleachers, dyers, printers and finishers, which will conduct researches extending from the study of the cotton plant to the "finishing" of the manufactured article. So long ago as 1835 Baine wrote in his "History of the Cotton Manufacture" —"The manufactory, the laboratory and the study of the natural philosopher, are in close practical conjunction; without the aid of science, the arts would be contemptible; without practical application, science would consist only of barren theories, which men would have no motive to pursue." This spirit, clearly shown in the early cotton industry, is now to be revived for the common benefit.

The woolen and worsted manufacturers of Great Britain are also drafting the constitution of a Research Association, and the Irish flax spinners and weavers are about to do likewise. Research Associations will be established by the Scottish shale oil industry and the photographic manufacturers, while various other British industries are looking in the same direction. Thus a national movement for research, directly resulting from the

war, has already made marked headway. The Research Councils in various parts of the British Empire, actuated by the same spirit, are rapidly extending the advantages which an appreciation of the national importance of research will afford.

The National Research Council, joining with its valued ally and supporter, the Engineering Foundation, is just entering upon an extensive campaign for the promotion of industrial research. In addition to a strong active committee, comprising the heads of leading industrial research laboratories and others prominently identified with scientific methods of developing American industries, an Advisory Committee has been formed to back the movement. This already comprises the following gentlemen: Honorable Elihu Root; Mr. Theodore N. Vail President of the American Telephone and Telegraph Company; Dr. Henry S. Pritchett, President of the Carnegie Foundation for the Advancement of Teaching; Mr. Edwin Wilbur Rice, Jr., President of the General Electric Company; Mr. George Eastman, President of the Eastman Kodak Company, Mr. Pierre S. duPont, President of the E. I. duPont de Nemours Powder Company; Mr. A. W. Mellon, Founder of the Mellon Institute for Industrial Research; Judge E. H. Gary, President of the United States Steel Corporation; Mr. Cleveland H. Dodge, of the Phelps-Dodge Corporation, and Mr. Ambrose Swasey, of The Warner and Swasey Company.

We are indeed fortunate to have the aid of men whose experience and standing are so certain to command public recognition of the claims of scientific and industrial research.

Science is in the air, keen competition is in prospect, and the industries are more favorably inclined than ever before to the widespread use of research methods. Their greatest leaders, moreover, are unanimous in their appreciation of the necessity of promoting research for the sake of advancing knowledge, as well as for immediate commercial advantages. Only thus can the most fundamental and unexpected advances be rendered possible, and continued progress in all directions assured.

In preparing to continue and extend its work in the interest of national defense, industrial development, and the advancement of science, the National Research Council wishes me to assure the Engineering Foundation and the National Engineering Societies of its cordial appreciation of their invaluable assistance in the past and of its sincere desire to utilize every possible means of promoting even closer cooperation in the future.

ANNUAL REPORT OF TRACTION AND TRANSPORTATION COMMITTEE

To the Board of Directors,

The past year has been one of unprecedented difficulties for all railway transportation companies. With labor conditions at their worst, the maximum tonnage ever offered, rolling stock and motive power sadly deteriorated due to a long period of lean income—all added to a winter of unprecedented severity, it is little wonder that the trunk lines practically collapsed under the strain. The collapse of the steam railways, however, did not extend to electrified lines, which once more demonstrated the superiority of electricity as a motive power, particularly in cold weather. This was especially true on the electrified steam railways, such as the Norfolk & Western, and the Chicago, Milwaukee & St. Paul Railways, the latest railways to be electrified; and the Pennsylvania, New York Central, New Haven Railways, and others. These roads, one and all, operated successfully through the hardest weather. The record is one of which all electrical engineers may be proud.

The advantages of electrification are now almost universally recognized. The main features are:

1st, Increased capacity.

2nd, Economy in fuel consumption.

3rd, More reliable service.

4th, Greater safety in operation on mountain grades and through tunnels.

5th, A higher class of service.

The first two advantages were especially emphasized by President Rice in his masterly plea before the Midwinter Convention for the electrification of railways as a war measure, and the Committee urges the most careful consideration of this matter by the Government and the railways.

There are many places where electrification will show an actual economy in operation, especially on lines such as the Norfolk & Western and in the Broad Street Terminal of the Pennsylvania Railroad. In both of these cases, the capacity of the railroads had been reached with steam, and it would have

been utterly impossible to handle the traffic which they have had in the past year with steam locomotives. It is impossible to estimate or to appraise all of the advantages of electrification. Electrification will eventually be adopted because the character of service will be so much better that the railways cannot afford to do otherwise. During the war, however, the amount of electrification work done will probably be limited to places where it is absolutely essential, or especially advantageous, in order to facilitate handling the traffic necessary for the proper prosecution of the war.

In this connection, it is interesting to note that the C. M. & St. P. Ry., in continuing its electrification work with 3000 volts d-c., is electrifying the two engine divisions from Seattle east, which will give them a total of approximately 650 miles of electrified main line. Work is progressing on this new line very rapidly, and it is expected that operations will begin early in 1919. It is reported that this line will be the official route from Chicago to Seattle.

The Pennsylvania Railroad has recently put in electric operation the Chestnut Hill division of the line running into Broad Street Station, practically duplicating the single-phase equipments on the Paoli division.

In street railway circles, the past year has been one of great hardships. The long continued cold weather, combined with the heavy falls of snow, pyramided the troubles and left the street railways in many cases almost as bad off as the steam railways.

A number of things combined to produce this result. First and foremost, is the general labor and material situation which makes it difficult, if not impossible, to secure good maintenance. In addition, the street railways have been working for several years under increasing financial difficulties, so that in many cases the equipments were in no condition to stand the extra strain. This led to a very unusual number of break-downs when the severe weather was encountered, and the long continued cold gave no opportunity to recuperate.

Another feature which contributed in a greater or less degree was the light weight campaign which has taken such a strong hold of all of the railways of the country. Ten years ago there was great need for this campaign. The standard street railway equipment consisted of nothing less than four 40-h.p. motors, which, being of the non-ventilated type, were comparatively heavy. Very few people had paid any particular attention

to the weight of railway equipments, either of motors, car bodies or trucks. Consequently, the weight per passenger was very high on practically all street railways. The idea became prevalent that it costs a railway 5 cents per lb. per annum to haul their equipments around, so that it has been the end and aim of practically every man having anything to do with the design of cars and equipments, to cut the weight. This has been done in a more or less scientific manner, but like all campaigns of this kind, the pendulum has swung past the limit in some respects. probably due to the fact that it is impossible to recognize the limit until it has been passed and trouble encountered. Fortunately, this has not been sufficiently widespread to do more than point to the limit.

Possibilities for trouble with light weight motors have been approached from four angles: First, open ventilation leading to the introduction of snow into the working parts of the motor, with resulting danger to insulation; second, higher armature speeds, leading to more rapid deterioration of armatures and wear of bearings; Third, reduction of weight by increasing the stresses in material; Fourth, the danger from overloading due to lack of sufficient thermal capacity to withstand abnormal loads.

The ventilated motor has come to stay. Its advantages are too many to give up because of the small amount of damage resulting from snow. The logical thing to do when there is danger from this, is to put tight covers over the motor openings in the winter time. The additional margin due to the lower ambient temperature in the winter will, in practically all cases, be sufficient to keep the motor temperature within safe limits.

Higher armature speeds should be approached with conservatism, taking care that the armature is sufficiently substantial to withstand the additional strains, and that bearings are of liberal dimensions, with adequate lubrication.

The use of high-grade steel is recommended, but is preferred as an additional factor of safety, rather than to get minimum weight. It must be remembered that it is not always possible to secure special grades of material for making repairs.

Trouble from overloading with the light weight motors has come as a very disagreeable surprise to operators. It is subject to careful analysis and the reasons are easily understood. The chief reason is that the limitations of the ventilated motor are not generally understood.

The trade has been educated to believe that the continuous

rating of the railway motor is the one that determines its service capacity; and that if its continuous rating is equal to the integrated loads in service, it will be ample to perform the work. This method of selecting motors was quite satisfactory with the old non-ventilated type of motors, where the motor had sufficient thermal capacity to absorb the losses generated in the short applications of heavy loads, without reaching abnormal temperatures. The modern highly ventilated motor, however, has relatively a very high continuous rating, as compared with the one-hour rating. The latter, as is well known, is really the gage of the thermal capacity of the motor and of its capacity for handling heavy intermittent loads. It will be seen at once that when the motor that is selected for its continuous rating is required to develop four times this load for a few minutes under some abnormal condition, a non-ventilated motor would be loaded to only about 60 per cent above its one-hour rating, while the highly ventilated motor would be loaded to two and one-half or three times its one-hour rating, resulting in a much greater rise in temperature. Where this abnormal load is applied at slow speed, as is apt to be the case, the trouble is further accentuated, due to the decreased ventilation secured with fan cooled motors.

The logic of this situation is simply that motors of the ventilated type for a given service will require a higher continuous rating than one of the non-ventilated type. Due regard must be taken to the capacity for short-time over loads in order to avoid reaching a dangerous temperature under abnormal conditions.

There seems to be a distinct tendency towards more conservative selection of equipments for street car service, since it has been definitely established that the cost of maintenance of motors which are worked beyond their capacity, added to the cost of unreliable service, is so high as to off-set any possible saving resulting from the lighter weight. This is also leading to a return to four-motor equipments, simply because of their greater reliability under abnormal conditions. It is hoped, however, that any return along these lines will be taken with the greatest care and with the maximum utilization of the experience that has been secured up to date. Having been at both extremes, it should now be possible to adopt a mean position which will give the very best results.

Great savings in energy can be effected without cutting the

weight of the equipment, by attention to improved methods of operation and, especially, by the further use of field control. It is hoped also that eventually it will be possible to make use of regenerative control for elevated and subway equipments, at least. This has worked out so satisfactorily on mountain grades that its extension to car equipment seems very desirable.

The activity of the Railway Committee in the past year has been at a low ebb, largely due to the fact that everyone in any way connected with transportation business, has been too busy to take up much committee work. It is hoped that another year may see an improvement in this respect, so that a constructive programme may be carried out.

N. W. Storer, *Chairman.*

ANNUAL REPORT OF THE COMMITTEE ON ELECTRICAL MACHINERY

To the Board of Directors,

The Committee on Electrical Machinery submits the following report for the year of 1917-18.

At the first meeting of this new committee called in October to discuss the policy to be adopted, it was found that industrial conditions were such as to make full and regular meetings an impossibility, and that the activities of the Committee would therefore have to be restricted. There was, moreover, a feeling among those present that it would be difficult this year to secure papers from the men in the industry, and it was decided that the Committee limit its activities to the presentation of the subject of "Polyphase Commutator Motors," and to be prepared if necessary to present a critical review of "The Development of Turbo-Alternators". Contrary to our expectations, however, an unusual number of papers were presented for consideration and the Committee was kept busy sifting out the desirable material.

It was clear from the discussion of the three papers on "A-C. Commutator Motors" that little is known about the possibilities of the polyphase commutator machine, and that the available literature in English is so meagre and the methods of attack of the various writers so radically different that it would seem to be the duty of the Institute to find someone who knew the subject and who also knew how to present it to work up the existing material into a comprehensive treatise for publication in the TRANSACTIONS.

We believe that one of the principal functions of the technical committees is to keep the literature of their subject in good shape. The manufacturers will see to it that new developments are brought to the attention of the members. In line with this point of view the Committee has had a critical bibliography prepared on the subject of "Unbalanced Pull in Electrical Machinery" to be attached to a paper to be published in the PROCEEDINGS this summer.

In the April PROCEEDINGS there was published a paper by Mr. Lamme which created considerable discussion. Mr. Lamme had found that the graduates of our universities have not a clear

conception of the operation of single-phase induction motors, and he wrote a paper giving a method of treatment which he had found satisfactory for the graduate students of the Westinghouse Co. Such papers, although they do not present new material, are of special interest to the younger members of the Society and might well be presented under the auspices of the Educational Committee. From the discussion of Mr. Lamme's paper it was obvious that there was decided differences of opinion among teachers and practising engineers as to the relative advantages of vector diagrams, equivalent circuits and complex quantities from an educational point of view.

While the chief function of the technical committees is to secure papers in their respective fields, The Committee on Electrical Machinery considers that it might well be used as a clearing house for suggested changes to the Standardization Rules, a place where suggestions dealing with machinery might be thoroughly thrashed out before being submitted to the Standards Committee.

ALEXANDER GRAY, *Chairman.*

ANNUAL REPORT OF THE LIGHTING AND ILLUMINATION COMMITTEE

To the Board of Directors.

I beg to submit on behalf of the Lighting and Illumination Committee the following report for the year 1917-18.

It is with regret that the Committee finds it necessary to report that again this year it did not seem feasible to hold a session of the Institute for the consideration of papers on illumination. The Committee held a meeting early in the year and proposed the following subjects as suitable ones for presentation before the Institute:

(1) Intensive and Ornamental Street Lighting, as projected for a number of cities in the South and Southwest.

(2) A general discussion of Industrial Lighting Codes;

(3) A discussion of Standardized Methods of Lighting Cantonments, Aviation Fields, etc., provided the report of the I.E.S. Committee on this subject will be available for public presentation.

The Chairman was authorized to make an inquiry regarding the possiblitiy of securing papers on one or more of these subjects and report the results subsequently to the Committee, but following a canvass of the situation it was found impossible to arrange such a program. Consequently, any thought of requesting one of the sessions of the Institute to discuss papers on illumination had to be abandoned. When conditions once again become normal, it will be possible to provide interesting programs on this aspect of electrical engineering, but for the time being it would seem that the Committee can do no more than remain intact and wait.

A brief summary of progress in electric illumination during the past year is appended.

Progress in Electric Illumination

The general trend of practise for direct lighting is very decidedly toward units of low brightness. The extended use of the high-powered incandescent lamps has stimulated the appreciation of good diffusing devices which will give satisfactory light distribution but by their low brightness minimize glare. The enormous increase in commercial activities, particularly in those

lines which are connected with supplies for the Government, has made night work the rule and brought a realization of the importance of proper illumination from the standpoint both of the maintenance of quality and quantity in production and of the health and comfort of the worker. Progress toward this end is evidenced in the revision of industrial lighting codes in several states and by the appointment of a National Committee on Lighting to act as a sub-committee of the Advisory Commission-Council for National Defense for the preparation of suggested regulations to govern industrial lighting, which have subsequently been published in the form of a Code of Lighting by the Committee on Labor with a suggestion that the Code be put into effect in every state in the country.

War conditions have also brought about a more careful consideration of protective lighting and the best way to utilize it. Thus it has been found that in many cases inexpensive reflectors of the ordinary type may be used for lighting open spaces in and around a plant leaving the special flood lighting units for those locations requiring particular treatment. In many cases the use of a large number of properly shaded low-intensity units will avoid dangerous shadows better than high powered sources, even though the light flux from the latter is greater.

A sphere formerly considered impregnably held by the arc lamp has been finally invaded by the incandescent lamp. Motion picture projection work required light flux of extremely great intensity and the small area and high intrinsic brilliancy of the source of light in the arc has enabled it to meet the requirements in a way hard to duplicate. By using a mirror back of the filament and for a condensing lens one of the Fresnel type, it has been found possible to make an incandescent lamp which will give satisfactory results within a certain limited field of motion picture work.

The motion picture theatre has in itself become an arena in which unique lighting effects are being experimented with continuously. Thus in several cases, by the use of several circuits in each fixture, lamps of different colors may be lighted and thereby give a color tone to the whole illumination.

The action of the Government in attempting to save fuel by restricting its use for lighting purposes has shown in many localities the important part played by display lighting in maintaining the illumination of streets and sidewalks.

EDWARD P. HYDE, *Chairman.*

ANNUAL REPORT OF THE COMMITTEE ON TRANSMISSION AND DISTRIBUTION

To the Board of Directors,

The Committee on Transmission and Distribution submits the following report for the year 1917-1918:

Experience during the preceding year indicated that the Committee was entirely too large. At the writer's suggestion the membership was reduced during the present year from 24 to 14 members. We now suggest that the Committees be further reduced to not exceeding 10 members. It would be an act of courtesy on the part of a member to decline the appointment when it is offered him if he can take no part in the work of the Committee.

It has been very difficult to make progress in the problems before us on account of every one, almost without exception, being employed on war work or very urgent duties contributing to the war. Some of the members who were most helpful in the past have gone into the government service and have not been able to continue the work which they started last year. Mr. W. D. Peaslee of the Oregon Agricultural College, who has been investigating the insulator problem from a chemical and microscopic standpoint, is now captain in the United States Army. Professor Harris J. Ryan, who is our strong right arm in insulator matters, is giving practically all of his time to government work. However, some progress has been made.

High-Tension Insulators

Last year we had papers by Messrs. Austin, Peaslee and Ryan pointing out clearly that progress in the design of high-tension insulators must provide for (1) reduction of porosity to the lowest possible limit. (2) joints designed to avoid cracking from expansion and contraction and (3) ample mechanical strength. Mr. G. I. Gilchrest has now made some very careful studies of insulators, both from the laboratory point of view where distribution of electric stresses was considered, and from the practical point of view where troubles and failures by operating companies under wide varieties of conditions were examined. We hope that insulator manufacturers will give careful con-

sideration to the design of insulator that Mr. Gilchrest has evolved. This paper shows very clearly where and why many of the old insulator designs failed. While it is quite probable that a perfect insulator will never be obtained, one has but to compare the insulator of today with that of five years ago to see that great progress has been made.

At least one large transmission company is very hopeful of the wood stick insulator for voltages up to 100,000. This insulator has been in successful service in the West for some years on 60,000-volt service. We had a paper listed for this meeting on the wood stick insulator by Mr. H. H. Cochrane, and hoped to get full details of it and record of the service it has given, but at the last moment Mr. Cochrane asked to have the matter go over until a later date, as some difficulties of impregnation had been encountered.

Lead Sheath Cables

Last year at the annual meeting a whole session of the Institute was devoted to the discussion of dielectric losses in cables. It was shown that cables insulated with mineral-base compounds had greatly reduced dielectric losses over those insulated with vegetable-base compounds. It was further shown that cable ratings under some conditions were more than doubled and on the average could be increased 20 or 30 per cent when the mineral-base insulating compounds are used. A start has been made in the matter of preparing specifications covering dielectric losses in cables. Engineers of some of the principal cable manufacturers have agreed to cooperate. Before such specifications can be formulated, at least two fundamental points must be considered and agreed upon:

First, a standard method of making tests must be established. Very few, even of the cable manufacturers, are equipped for measuring dielectric losses and probably no users of cables have facilities for properly making these measurements. Some very much simpler and more easily workable apparatus than is now available must be developed before commercial routine tests of this kind can be applied to the output of the cable factories. If a portable testing set for measuring dielectric loss were devised it could be used to test newly installed cable as well as to secure experimental data on old feeder cables under various conditions of age, temperature, charred insulation, etc.

Mr. S. M. Farmer recently presented a paper describing a method of determining power loss in three-conductor cables, the

loss being measured directly under three-phase conditions. It is hoped that the method he followed may be developed for factory tests.

In the second place, data must be collected showing the limits of the losses. To secure the data on losses is a difficult matter at this time when men as well as laboratories are occupied to full capacity on war work.

Additional information is being obtained on the characteristics of insulating compounds. For instance, it has been found that cable insulated with mineral-base compound will not withstand a high insulation test when cold, as the insulation resistance is much reduced under such conditions. All this complicates the problem of preparing specifications. Evidently much research work must yet be done, but as before stated, all work of this sort is much hampered by the war.

Mr. E. B. Meyer in a very practical paper gives the experience of one large distributing company in supplying high-voltage cable service to customers in cities where overhead wires are permissible and where the expense of conduit is not justified. The method while adopted as a war expedient has doubtless a wide field as a permanent method of installation. At least it can be used until the demand for energy in the particular locality is large enough to justify the expense of underground conduit.

Mr. W. H. Cole this year presents a paper as result of the experience of Edison Electric Illuminating Company of Boston with split-conductor cables and balanced-relay protection against interruptions caused by short circuit or grounds. It is very gratifying to note the success which has been attained notwithstanding the complexity of the system and the great care which must be exercised in installation of the equipment. All users of underground cables will appreciate Mr. Cole's work in this field.

Suggestions for the Future

For the future work of this Committee we would like to recommend that investigation of the insulator problem be continued. While at present porcelain seems to be the most available material, yet it is not beyond possibility that some other material such as fused quartz or even glass may be used.

Professor Ryan is now supervising some extensive tests at Leland Stanford, Jr. University of aging effect on insulators carried through a large number of temperature cycles corres-

ponding to the daily and seasonal variations experienced in practise. The result of these tests will be of great value.

Further investigation of the fused quartz insulator along the lines suggested last year by Mr. Peaslee will be well worth while as soon as some one can find time to do it.

As indicated above, the matter of dielectric loss in cables should receive most careful attention, both by cable manufacturers and users. Manufacturers must not be hampered by impractical and half-baked specifications. On the other hand, conservatism of manufacturers must not block the road to progress, and as soon as the laws governing dielectric loss can be determined and a practical method of measuring these losses developed, the most efficient cable will be called for and must be produced.

L. E. IMLAY, *Chairman*

ANNUAL REPORT OF ELECTROCHEMISTRY AND ELECTROMETALLURGY COMMITTEE

To the Board of Directors,

The Committee on Electrochemistry and Electrometallurgy submits the following report for the year 1917-1918:

It did not appear to be possible during the past year to secure any papers of interest on electrochemical and electrometallurgical subjects. One paper was submitted to the chairman of this committee for consideration which seemed more suitable for presentation to the American Electrochemical Society than to the Institute.

One of the most vitally important matters to Electrochemistry and Electrometallurgy is the development of water powers and consequently it appeared to be a subject which would properly come under the consideration of this Committee more particularly as regards the matter of legislation since this has probably done more than anything else to hamper their development. Correspondence with the Chairman of the Public Policy Committee brought out the fact that this subject has been referred to the standing Committee on Public Affairs of the Engineering Council.

Both as regards the question of securing papers for presentation to the Institute and as regards all matters pertaining to electrochemistry and electrometallurgy much more valuable results could probably be reached were some scheme worked out by which there could be cooperation of this committee and the American Electrochemical Society. With this end in view a suggestion was made to the Board of Directors of the American Electrochemical Society that it consider the question of approaching the A.I.E.E. with the object of forming a joint committee which would take care of subjects that are of common interest to the A.E.S. and the A.I.E.E.

Such a Committee would be in a good position to take care of various papers which may be of importance to both societies, to bring to the attention of both societies matters in which they have a common interest and finally to make arrangements when possible for joint meetings of the Societies.

At a meeting of the Board of the American Electrochemical Society held April 28, 1918, the Directors acting on this suggestion appointed a Committee "to co-operate with" this Committee. The Committee appointed by the American Electrochemical Society is as follows:—

Dr. Colin G. Fink, Chile Exploration Co., 202nd Street and Tenth Ave., New York.

Howard C. Parmelee Metallurgical & Chemical Engineering, McGraw-Hill Co., Inc., 36th St., and Tenth Ave., New York.

C. G. Schluederberg, Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa.

A letter was received from Professor Karapetoff of Cornell University describing the Research Section of the *Electrical World* which he is conducting and asking for suggestions from this committee. The members of the committee were notified of Professor Karapetoff's request.

Suggestion for Future Activities

In view of the action of the Board of the American Electrochemical Society it is suggested that this Committee should communicate with that appointed by the A.E.S. with the object of discussing the feasibility of cooperation for the benefit of the Institute and the Society. This matter, however, should be left to the Committee for 1918-1919 as the term of office of the present committee is so near an end.

As regards hydroelectric power there is no more important matter for consideration by this Committee and if some scheme for cooperation with the American Electrochemical Society is developed it should be specially studied by this Committee.

Electrochemistry and Electrometallurgy 1917-1918

The entrance of the United States into the Great War had the effect of stimulating enormously electrochemical and electrometallurgical processes.

As regards electrolytic work probably one of the most highly stimulated manufactures is that for the production of chlorine; but the most noticeable stimulation is found in electrometallurgical work.

For years there has been much experimental work carried out in electric furnaces for the melting of non-ferrous metals like brass, bronzes, etc., but no really satisfactory furnace was developed. Then, as one of the consequences of the Great War

graphite crucibles became scarce and expensive with the result that renewed attempts were made on the electric furnace with the result that during the past year various furnaces have been designed of which some show distinct promise of meeting with ultimate success.

There has also during the past year been an enormous increase in the production of electric furnace ferro alloys of all sorts particularly ferrosilicon and ferromanganese. The latter had been produced in electric furnaces before the war but could not compete with the blast furnace.

There has also been a considerable increase in the use of electric steel furnaces and this probably would have been much greater were it not for the great difficulty of getting a sufficient supply of carbon electrodes.

The most interesting electrochemical development of the year, however, is the construction of the cyanamid nitrogen-fixation plant at Muscle Shoals, Ala. Two unique features in this plant are the use of steam for the generation of the electric energy and the use of 60-cycle current for the calcium carbide furnaces. These features will give interesting subjects for study; on the one hand the energy item in the cost of making ammonium nitrate by the cyanamid process and on the other the power factor of carbide furnaces with high-frequency current.

FRANCIS A. J. FITZGERALD, *Chairman*

ANNUAL REPORT OF INDUSTRIAL AND DOMESTIC POWER COMMITTEE

To the Board of Directors,

In presenting our annual report, your Committee on Industrial and Domestic Power desires first to call attention to two of our members who are now in Military Service in the war of 1917, viz: our chairman, Capt. E. H. Martindale now in active service in France, and Lieut. (j. g.) A. M. MacCutcheon, U. S. Navy, also in active service, U. S. S. Louisiána.

We honor their action, and have missed their advice in our work this year, particularly that of our Chairman.

Others of our Committee from whom we have not heard may be similarly employed, and in that event their names should be joined with the above.

During the two years beginning in August, 1914, under the leadership of Mr. David B. Rushmore, work of the committee began to be centered on a study of our activity by industries. This thought was carried out in several meetings of the Institute.

In the Fall of 1916 under Capt. Martindale, it was decided that the major portion of our activities should be devoted to this study. To show the value thereof, it was decided to make an investigation of the machinery, motors, controllers and accessories involved in each process in three industries or parts thereof, and reach conclusions from this as to next procedure. On this basis, the whole committee was divided into three sub-committees headed by Lieut. A. M. MacCutcheon, to investigate a portion of the Metal Working Industry; Mr. R. B. Williamson, to investigate the Cement Industry; Mr. R. M. Goodwillie, to investigate the Passenger Elevator Industry.

The committees have worked hard, collecting much data, but without having their work completed by the end of the last term.

In the Fall of 1917, by reason of his military service, Capt. Martindale was obliged to relinquish active handling of his work as Chairman, the position of Vice Chairman was created, and the writer appointed to the position. The executive duties have been carried out in the way it was believed Capt. Martindale desired.

On account of the absence of our Chairman, and also because

of the many immediate and pressing duties of service of all individually, in connection with the war, it was deemed advisable to maintain our plan just as outlined one year previously, center our thought on its progress and application, and not attempt the preparation or presentation of papers to the Meetings and Papers Committee, and this course has been followed.

The work of the sub-committees in collecting data is well in hand. In the examination of the cement industry for example, Mr. Williamson's committee has first tabulated all the processes involved from the point when the raw product is brought to the cement plant up to the point when the finished product is delivered, barrelled and ready for shipment. Each process may be considered a movement of some kind. Where electric power is applied to this movement, the preferred type of motor, control, and accessories and alternates are indicated. In a similar way, Mr. MacCutcheon's Committee has examined a portion of the metal working industry. Thus mainly the principal data have been collected and compiled. The work has given our whole committee opportunity to thoroughly consider the good features and objections, and in this connection two immediate difficulties; viz: to know when the work is done, and how to present it to the Institute, are not solved. These two points with others brought out by this investigation have been carefully canvassed by the whole committee, and the discussion and recommendations which immediately follow are based upon a majority vote also of the whole committee, with no dissensions.

The work has been carried far enough so that each feels sure that if the data could be gotten together for all processes, kept correct and made available, these would be of immense benefit to the industry as a whole, and particularly at this time. The collecting and preparing of the data presents very serious difficulties as follows:

1. *Tabulation of what industries are within the scope of our committee's activities.* The scope of this committee overlaps that of the committees on mines, steel mills, electrochemical work, marine work, and perhaps others. It is recommended that its scope be plainly defined.

2. *What constitutes an industry.* A tabulation of industries can be prepared by our committee. Such tabulation is bound to be at variance in thought from any tabulation prepared by another committee. We believe that a tabulation should be prepared for the electrical industry, as a whole, and that this can-

not be done short of a careful study of the subject, and the interrelated processes. As Mr. Goodwillie of our committee states, the real essential element is the load characteristics of the machine itself from the starting, accelerating, running, slow-down and stopping conditions which in turn compel considerations of starting friction, inertia, running frictions and load characteristics. It is probably a matter of consideration by some special technical committee, and not by a technical committee devoted to some special work like our own unless the work as a whole for some reason be specially assigned to us. Much study has been put already on this subject by allied associations, and by private enterprises. Much of the benefit of this study can be secured very likely. The list of industries and processes involved will be subject to constant modification as the work proceeds. A plan for caring for these changes is essential.

3. *Magnitude of the work involved in the study.* It is clearly beyond the power of our committee to perform this work alone in any measurable period of time. External assistance is vital and some plan to secure the assistance necessary. Even if it were completed, it is doubtful if the committee alone as it stands could keep it completed.

4. *Preparation and filing for permanent record.* It is apparent that the proper place for recording this data is not in the PROCEEDINGS of the Institute. The expense of doing it in this way would be prohibitive. It would lack availability for use and change as required. A plan for permanently caring for the data should be perfected as will be separately discussed.

5. *Authority for work.* In the execution of this work, time and expense beyond the limits of the committee would be bound to accrue. Definite authorization from the Board of Directors to proceed is necessary, and with it the statement that the work would be continued from year to year, for it is obvious that to start work and continue for a year or so only to change or abandon would involve needless loss of good time and discouragement.

6. The ethics of the study from the standpoint of the professional engineer, from that of patent rights and from the unconscious publicity given to private *enterprise is comparing correct process attainment.* This is considered one of the most difficult phases to cover correctly. As Mr. Dudley states, it is believed that with the exercise of tact with each case, that difficulty should not be encountered. Thus far in its work the sub-committees have been able to steer clear of this as a diffi-

culty. The work must be undertaken strictly from a technical standpoint.

7. *Conflict with other technical committees in other associations.* The whole reason for this work is to avoid duplication of effort. The American Institute of Electrical Engineers is the recognized association devoted to our art from an unbiased technical and professional standpoint. If this work be undertaken by us with carefully prepared plan and be definitely endorsed, the need for similar work by other bodies such as the N.E.L.A., A.I. & S.E.E., etc., should disappear. The committee is confident that help from them will be fully available and given. There should be no conflict with other committees in the Institute. For this, the management must care.

8. *Miscellaneous Difficulties.* Undoubtedly these exist but it is believed that what has been previously stated will cover the major difficulties, and that the minor ones will settle themselves as they arise.

The study which our committee has thus far put on the work has convinced us that it is broader in scope than that of our committee. If it is right to investigate the large group of industrial applications with which our committee is concerned, why should not the whole field be covered. Consideration of this thought has been inevitable, and is crystallized into a number of suggestions or recommendations which are presented as the opinion of a majority of your committee.

We believe that there should be instituted in the office of the Secretary of the Institute in New York City, and under his complete charge, a file of industries using electrical power, the processes involved in each industry, the movements involved in each process, and the electrical apparatus, which is recommended for each movement. We believe that this recommendation should be strictly from the technical standpoint, the apparatus to be described so that it will be technically understood. We believe that the file should be kept up by the machinery of the Secretary's office, and in such form as will make it most available to all who should have access to it. It is assumed that this file should be in card form, and in this connection, Messrs. Weichsel and Dudley of our committee suggest the compilation of pamphlets or a hand book from these files in the belief that they would have ready sale.

It is further suggested by the committee that as Institute Sections are able to take care of them properly, copies of the

files be placed on record at Section Headquarters all under control of the Secretary of the Institute.

Manifestly the data for these files can be secured only through the cooperation of a large number of members, and one perhaps the hardest study would be to enlist this cooperation. Fundamentally, the plan must have the absolute endorsement of the Board of Directors, and then it should be planned to secure this large support. Our committee feels that either the duties of the Industrial and Domestic Power Committee should be broadened to assume the work, or (and this is favored by a majority of the committee) that a new technical committee should be formed to take care of the job. No technical committee alone can get the data unless it have broad powers, and a membership large enough to reach into the activity of the Nation industrially and geographically, in a very comprehensive manner indeed. As a committee, we feel very sure that in some way the individual member must be reached; must be encouraged to furnish data, with proper recognition. We feel that if this action on the part of the individual member in being of service to the Institute and to the industry causes him to recognize his true relation to the Institute of giving to it rather than of receiving from it, it should be of the greatest value.

We further respectfully suggest that in the event of establishing a committee to secure this data that it should work in very close conjunction with the Secretary of the Institute, and with the other technical committees.

Briefly we suggest, that the new committee plan and secure the data, the existing committees censor the data and advise with the new committee, and that the Secretary's office compile and carry the data.

Such a file should be the truest permanent record possible of the application of electrical power and its progress. It should tend to harmonize policies of application of electrical power; not to standardize details. It should prevent an enormous amount of duplication of effort, not only in association work, but in private enterprise as time goes on; should tend to pool, for the good of all, data which are today largely open to all and yet not collected so as to be of mutual benefit.

Your committee sincerely hopes that these views will have your consideration.

We desire to comment on another matter. Our committee's scope today includes domestic as well as industrial power. It

has seemed to some that the domestic end of our activity should be taken from us and given to some other committee whose work is more nearly allied thereto. On a poll of opinion seven were in favor of this action, five did not reply and two were in opposition thereto.

The past year has been crowded with work which your committee feels covers very largely application of earlier progress than it does the invention or perfection of new work. Constant effort to keep existing apparatus operating at the maximum rather than to perfect or develop more efficient operation has marked this period when our Nation for the first time in twenty years was plunged in war. This holds particularly for the production of textiles and clothing generally, steel, coal, and in matters affecting transportation.

In the Chicago territory there has been installed during the past year a 7000-h.p. induction motor having a maximum torque equivalent to 30,000 h.p., which so far as normal rating and maximum torque is concerned is the largest industrial motor which we believe has ever been built.

In the textile industry, the matter of individual drive of spinning frames is progressing, many hundred frames having been equipped during the past year.

The first electrically propelled battleship has been launched during the past year. The application is new.

The year marks the first application of small electric generators, air driven, to air-planes. This extends the use of industrial power to the air along with gasoline engines.

In summary, your committee would value quite specific instructions as to its duty, and scope, and shall be much interested to note any action that may be taken on its recommendations.

A. G. Pierce, *Vice Chairman*

ANNUAL REPORT OF COMMITTEE ON ELECTROPHYSICS

To the Board of Directors,

The Committee on Electrophysics submits the following report for the year 1917-18:

During the year two sessions of the Institute have been devoted to electrophysics under the auspices of this committee as follows:

On the evening of Nov. 9, 1917, Mr. Chester W. Rice presented a paper entitled "An Experimental Method of Obtaining the Solution of Electrostatic Problems with Notes on High-Voltage Bushing Design" which forms a very valuable contribution to our knowledge of the electrostatic field. The paper, published in the November PROCEEDINGS, has a direct bearing on the problem of insulator design and the mathematical appendix gives the solution of two electrostatic problems, the usefulness of which extends beyond the immediate problem of design.

On the evening of Feb. 15, 1918, at the Midwinter Convention, the Institute listened to a most interesting lecture by Dr. A. C. Crehore on "Some Applications of the Electromagnetic Theory of Matter"; the lecture is published in the April, 1918, PROCEEDINGS. Great progress is being made in the theory of atomic structure, to which Dr. Crehore's own work has formed no small contribution, and the placing of these most recent advances by Dr. Crehore before the Institute in non-mathematical form was indeed most opportune. It will be recalled that a similar lecture on "Modern Physics" was given by Prof. R. A. Millikan at the Midwinter Convention in 1917.

The committee has felt that the cooperation and mutual understanding between physicists and engineers, as pointed out in the report of the committee for last year, are of the utmost importance. The close relation has been maintained between this committee and the Technical Physics Committee of the Physical Society. It is believed that joint meetings of the Institute and the Physical Society from time to time should be continued as in the past and such a meeting is planned for the Philadelphia session next October.

The maintenance of a supply of trained physicists and en-

gineers, at all times important, the committee has considered to be particularly important in time of war in order to meet the needs of governmental and industrial service. This matter was discussed on April 11th at a meeting of the committee held in New York for the purpose, and the conclusion was reached that to insure the continuous output of technically trained men from the universities and technical schools of the United States it was most important that steps be taken to provide for the maintenance of adequate teaching staffs, which are in danger of being depleted through the application of the draft and through voluntary enlistment. This matter was brought to the attention of your Board on April 12 and it was the unanimous opinion of the members of the board that immediate action was necessary.

FREDERICK BEDELL, *Chairman*

ANNUAL REPORT OF PROTECTIVE DEVICES COMMITTEE

To the Board of Directors,

The Committee on Protective Devices submits the following report for the year 1917-18.

On account of the decision of a number of companies to cancel all committee work during the period of the war the work of this Committee has been carried on largely by correspondence. One of the members of the Committee entered the military service of the country and a number of others have had increased duties on account of their assistants entering the Army or Navy. These changes have served to considerably retard the work of the Committee.

During the year the Committee has issued the questionnaire on relays, referred to in the previous report, to about fifty of the leading operating companies in the country. Not enough replies have been received from the questionnaire up to the present writing to permit of any summary or resume from the inquiries. It is recommended that the work on the questionnaire be pushed to a conclusion during the coming year.

Several members of the Committee have called attention to the proposed interconnection of transmission and distribution systems throughout the country as a measure of economy and fuel saving, and it is recommended that the Committee investigate this subject in particular to determine what protective features are necessary in such tie lines for the purpose of ensuring continuity of service and stability of operation.

Other subjects which might be taken up by the Committee are the following:

Relays on generators, transformers, synchronous converters, etc.

Lightning arresters for transmission lines.

Preparation of definite recommendaions regarding the standardization of relay nomenclature and rating of circuit breakers.

Detrimental effect of power reactors.

D. W. ROPER, *Chairman*

ANNUAL REPORT OF COMMITTEE ON APPLICATION OF ELECTRICITY TO MINES

To the Board of Directors,

I give below a brief report of the activities of the Committee on the Application of Electricity to Mines.

Owing to the resignation of Mr. H. H. Clark, the writer was appointed as chairman of the Committee on the Application of Electricity to Mines. Pressure of other work has prevented devoting as much time to Committee work as I would like to have given. No meetings of the Committee as a whole have been held, but the chairman has arranged with the Meetings and Papers Committee to take charge of the October meeting which will be held in Philadelphia. We propose at this meeting to present three papers bearing on the subject of the Application of Electric Power to Coal Mining. We also hope at this time to have an informal talk by a member of the Fuel Administration Bureau at the dinner preceding the evening session.

The American Institute of Mining Engineers has a committee on the Application of Electricity to Mining and I do not believe that the best results can be accomplished by the independent action of these committees in two entirely independent societies; and I would suggest as a future activity for the Committee on the Application of Electricity to Mines in the A.I.E.E. that of making some working arrangements with the corresponding committee of the American Institute of Mining Engineers whereby both national societies may get the benefit of the work of the Committees of each. Possibly this can be accomplished by making the annual meeting of these two committees joint meetings, the A.I.E.E. joining with the mining engineers at one meeting and vice versa for the next, the papers being published by both societies.

K. A. PAULY, *Chairman*

ANNUAL REPORT OF INSTRUMENTS AND MEASUREMENTS COMMITTEE

To the Board of Directors,

This committee was appointed during the year of 1917 with the general purpose of promoting interest through the presentation of papers and discussion along the lines covered by the name of the committee. The field for the consideration of instruments and for measurements had not heretofore been made a matter of separate committee work and discussion heretofore has been coupled with the consideration given papers in which both instruments and measurements were incidental to another subject.

Meetings of the committee were held and arrangements were made for the Meetings and Papers Committee to assign one session of the mid-winter convention in February for the presentation of papers on measurements. One afternoon session was assigned and four papers were presented. Attendance at the session and the discussion indicated sufficient interest to justify the continuance of the Committee's activities along these lines.

Two impressions were obtained from the papers and discussion, which while obvious, seem of sufficient interest to report. Two of the papers presented dealt with the investigation and development of a substitute for the standard cell for certain classes of work. This substitute was a thermocouple, eliminating entirely the use of the chemicals required by the standard cell.

One of the papers presented dealt with the measurement of dielectric losses in cables, and while it was a single paper only, it was in reality the latest of a series of several papers on the same subject presented before the Institute in the last few months. The discussion indicated a desire or a necessity for the standardization of methods of measuring the very small energy losses in the dielectric and the specifications covering the purchase and acceptance tests of cables.

No matters of nomenclature or standardization arose requiring the attention of the committee on standards. No matters of policy or coordination with other committees arose requiring the attention or action on the part of the Board of Directors.

Commenting briefly on the progress of the industry falling within the scope of the committee, it can be stated that as might be expected no new development work along purely commercial lines is being undertaken by the manufacturers of apparatus at this time. Developments have undoubtedly taken place, however, in apparatus along the lines of military and naval activity, such as radio and other signal apparatus which will without doubt furnish valuable and interesting material when available at some time in the future.

S. G. RHODES, *Chairman*

ANNUAL REPORT OF EDUCATIONAL COMMITTEE

To the Board of Directors,

Lack of time on the part of the members of the Educational Committee is responsible for the regrettable fact that only one meeting could be arranged during the year.

At this meeting it was decided that two papers, if possible, be prepared; one dealing with electrical engineering education given in colleges at the present time, and the other giving a summary of the educational facilities offered by the large manufacturing and power companies.

The second paper was not completed, the first is given in the synopsis prepared by the Chairman and presented herewith.

The Committee recommends that the particular scope suggested above be considered as an important part of the duties of future Committees so that the members may be able to keep in touch with educational methods and ideas, and assist the leaders of education in shaping their policies.

Synopsis of Electrical Engineering Education given in American Colleges, 1917

The importance of engineering and engineering education, always recognized in this country, has never been more fully realized than today. The engineer is called upon not only to supervise engineering work and to design machines but is more and more involved in administrative work so that—large as his task has been—it will be greater in the future.

It seemed, therefore, opportune to the Educational Committee that a report be presented to the Institute giving a brief summary of the present status of electrical engineering education in this country, as shown by the latest catalogues. A questionnaire was mailed to a large number of colleges giving four-year courses in electrical engineering and the returns are tabulated below.

The studies were divided in ten groups as shown and the figures given represent the percentage of time out of the entire four-year curriculum devoted to each group.

Group 1, includes strictly electrical engineering subjects.

Group 2, includes English, English literature, rhetoric, etc.

Group 3, Foreign languages.

Group 4, Mathematics, excluding descriptive geometry.

Group 5, Physics, including elementary mechanics.

Group 6, Chemistry.

Group 7, General engineering subjects. This heading includes such phases of the engineering curriculum as are usually given to all branches of engineering students. It includes, for instance, drawing, surveying, descriptive geometry, advanced mechanics, applied mechanics, thermodynamics, etc.

Group 8, General subjects. These include prescribed courses in history, law, economics, etc.

Group 9, Electives, technical and general.

Group 10, Physical training, physiology, hygiene, military work, etc.

Table I gives in alphabetical order the colleges which responded. At the bottom of the tabulations are given the total averages.* This should, therefore, indicate what weight is given to the various groups in the average college at the present time.

It is interesting to note that general engineering subjects, that is, subjects which are essentially common to all classes of engineering students, cover 31 per cent of the entire time, and the purely electrical engineering subjects are given 21.6 per cent. Thus the engineering topics occupy approximately one-half of the entire time of the students.

English, including literature and rhetoric, is given only 5.5 per cent, foreign languages 3.2 per cent, general subjects such as economics, history, law, etc., are given 3.4 per cent. Science subjects, mathematics, physics, and chemistry are given 27 per cent of the total time.

Tables II, III, IV, V, VI, emphasize particular studies. So for instance, Table II gives approximately one-half of the entire list in accordance with the prominence of purely electrical studies. Norwich University leads, it devoted 33.5 per cent, or roughly, 50 per cent more than the average time to that subject.

Table III emphasizes English studies. The Agricultural and Mechanical College of Texas leads with 12.5 per cent or more than twice as much as the average. Some leading colleges give no instruction in English. These may, however, to some extent, take care of this feature in a more rigid entrance examination.

*Since deducing the average a couple of colleges have been added which may have very slightly modified the actual value.

Table IV emphasizes foreign languages. It is of interest to note that a large percentage of the colleges give no foreign language course at all.

The average time given to electives is 3.9 per cent which seems rather low. There are, however, a number of institutions which permit of a wide choice. Tufts leads with a percentage of 21.

The preparation of these tables is intended to facilitate a discussion on the very vital subject of electrical engineering education as given in colleges. It is not intended that a "standard" course should be evolved—that would indeed be unfortunate—but it does look as if almost all colleges could to advantage increase some of the non-technical courses, such as English, economics and foreign languages. Some colleges should perhaps adopt a course along scientific lines, neglecting somewhat intruction strictly in engineering and concentrating on mathematics, physics and chemistry. Others may also lay less stress on their engineering subjects and devote more time to English, foreign languages and to problems of economics. The third class, perhaps the largest, should do essentially what the average college is doing today.

ERNEST J. BERG, *Chairman*

TABLE I

INSTITUTION	Electrical engineering	English	Foreign languages	Mathematics excluding descriptive	Physics including el. mechanics	Chemistry	General engineering	General subjects	Electives	Physical Training
Agricultural and Mechanical College of Texas	28.8	12.5	00.0	13.3	4.5	4.5	26.7	3.3	1.7	6.7
Armour Institute of Technology	28.6	2.4	00.0	7.2	8.8	6.4	35.0*	6.8	17.9†	4.8
Bucknell University	23.0	6.0	5.5	9.0	12.6	9.4	23.0	8.0	3.0	0.5
Carnegie Institute	28.3	2.9	4.6	8.0	9.6	4.6	30.0	3.3	0.0	8.7
Case School	25.2	6.4	6.4	10.4	9.1	5.5	25.9	1.3	1.3	8.5
Clemson Agricultural College	13.4	9.7	0.0	9.9	8.0	4.2	34.6	6.9	0.0	13.3
Colorado College	26.5	4.7	7.8	20.3	13.3	4.7	15.7	4.7	0.0	2.3
Cornell University	20.6	0.0	0.0	7.8	9.0	7.0	46.5	3.9	2.6	2.6
Drexel Institute	20.5	7.0	0.0	8.0	10.0	8.0	31.0	4.5	0.0	3.0
George Washington University	25.7	4.3	4.3	8.6	7.1	8.6	33.6	2.8	5.0	0.0
Georgia School of Technology	21.1	8.5	6.1	11.0	10.8	6.0	25.3	2.5	0.0	8.7
Iowa State College	21.7	7.5	0.0	15.0	11.5	6.2	23.7	2.7	7.5	3.6
Johns Hopkins University	20.3	5.5	3.2	9.0	13.6	7.0	26.6	4.7	2.3	7.8
Lafayette College	27.7	3.9	3.9	11.0	6.4	6.4	24.5	11.0	3.9	1.3
Lehigh University	28.6	6.5	7.8	11.0	13.0	6.5	16.2	1.9	1.9	6.5
Maryland State College of Agriculture	25.7	10.8	5.6	10.5	4.5	4.5	21.6	9.3	0.0	7.5
Massachusetts Institute of Technology	23.8	5.2	3.2	8.8	8.2	8.3	25.5	4.2	8.3	4.5
Mississippi Agricultural & Mechanical College	15.3	10.9	0.0	18.8	7.0	4.5	26.1	5.9	6.3	5.2
Montana State College of Agriculture and Mechanic Arts	33.0	3.0	0.0	13.0	16.0	5.2	15.2	7.0	5.2	2.4
New Hampshire College	21.0	4.1	0.0	12.2	10.1	6.1	35.7	0.0*	5.4	5.4
New Mexico College of Agriculture & Mechanics Arts	17.8	2.8	3.8	9.1	8.2	5.8	42.7	2.8	1.0	5.8
North Carolina State College	19.4	7.7	5.3	11.9	8.5	6.0	29.4	5.3	0.0	6.6
Norwich University	33.5	3.9	7.9	0.9	9.9	3.9	13.2	5.9	1.3	10.5
Ohio State University	20.1	2.6	5.2	13.0	13.0	5.2	31.2	0.0	5.8	3.9
Oklahoma Agriculture and Mechanical College	25.0	5.9	0.0	12.5	11.0	5.7	30.9	1.5	3.0	4.5
Oregon Agricultural College	25.0	4.5	0.0	13.0	9.5	4.5	30.0	0.0	5.7	7.5
Pennsylvania College	15.0	9.2	3.9	12.4	14.4	9.2	23.5	11.8	0.0	0.7
Pennsylvania State College	22.0	7.5	7.5	12.5	7.5	5.5	30.0	7.5	0.0	0.0
Polytechnic Institute of Brooklyn	28.0	6.6	3.7	10.7	9.4	7.0	28.0	2.8	1.9	1.9
Purdue University	20.4	7.9	6.6	14.4	9.6	3.5	21.5	5.9	7.6	2.6
Rensselaer Polytechnic Institute	23.3	5.9	6.2	13.0	9.2	6.2	33.8	1.1	0.0	1.3
State Agricultural College of Colorado	25.5	5.0	0.0	10.0	8.5	7.0	21.3	2.5	16.2	4.0
State College of Washington	27.2	6.2	0.0	12.4	6.2	6.8	29.7	4.7	3.1	3.7
State University of Iowa	26.9	6.6	0.0	14.7	8.8	5.9	28.3	6.6	0.0	2.2
State University of Kentucky	12.7	6.2	0.0	10.6	7.8	5.3	53.2	0.0	0.0	4.2

* Armour—Includes shop work

†New Hampshire College—General subjects included in electives

TABLE I—Continued

INSTITUTION	Electrical engineering	English	Foreign languages	Mathematics excluding descriptive	Physics including el. mechanics	Chemistry	General engineering	General subjects	Electives	Physical Training
Stevens Institute of Technology	8.1	3.9	4.2	5.6	6.3	7.5	53.9	2.1	0.0	8.4
Syracuse	25.0	4.0	0.0	16.3	10.2	7.5	35.0	2.0	0.0	0.0
Throop College of Technology	19.5	8.4	7.0	8.2	10.0	6.0	30.0	6.7	0.0	4.2
Tufts College	24.0	4.0	4.0	9.0	6.0	6.0	22.0	2.0	21.0	2.0
Tulane University of Louisiana	12.3	6.4	0.0	12.8	7.9	8.9	48.4*	0.0	2.0	1.3
Union College	22.8	8.6	8.6	17.2	5.6	4.5	20.0	5.5	4.4	2.8
University of Alabama	18.4	3.2	0.0	13.2	10.5	5.3	34.7	4.7	5.8	4.2
University of Arizona	17.3	6.4	5.1	10.9	5.1	5.1	32.7	0.0	7.1	10.3
University of Arkansas	28.5	4.2	4.2	14.6	5.5	5.5	22.9	0.0	10.4	4.2
University of California	15.0	0.0	0.0	11.0	12.0	7.0	40.0	0.0	8.0	7.0
University of Cincinnati	22.4	5.3	5.0	8.0	10.8	10.4	34.2	2.3	0.0	1.6
University of Colorado	30.0	7.1	0.0	14.3	7.1	7.1	31.4	3.0	0.0	0.0
University of Detroit	16.7	3.8	4.2	13.8	18.6	10.5	39.5	2.9	0.0	0.0
University of Florida	13.4	5.2	7.9	11.8	10.5	6.6	41.9†	0.0	0.0	2.7
University of Idaho	21.5	7.8	0.0	15.0	7.8	5.2	36.2	1.3	0.0	5.2
University of Illinois	27.7	4.3	5.7	14.0	9.2	5.7	25.6	8.5	0.0	4.3
University of Kansas	17.0	7.0	7.0	12.0	11.0	9.0	28.0	5.0	2.0	2.0
University of Maine	20.2	6.5	6.5	13.4	8.7	6.0	26.5	4.6	4.0	3.6
University of Michigan	20.7	4.3	11.4‡	12.9	10.0	5.7	25.0	0.0	10.0	0.0
University of Minnesota	20.6	4.1	0.0	13.7	8.2	4.1	33.3	4.1	8.9	2.8
University of Missouri (Rolla)	18.6	6.4	3.5	10.5	7.6	8.5	26.7	1.2	14.8	2.3
University of Missouri (Columbia)	16.16	4.8	0.0	16.1	8.1	4.0	29.8	4.0	13.7	3.2
University of Nebraska	17.8	3.2	0.0	16.0	12.0	4.8	32.8	0.0	10.4	3.2
University of New Mexico	28.0	4.2	0.0	14.0	9.1	5.5	25.9	2.1	11.2	....#
University of North Dakota	21.0	7.0	5.5	11.0	9.5	8.0	33.0	2.0	1.5	1.5
University of Notre Dame	29.6	4.7	6.0	11.8	10.2	0.0	22.0	10.7	1.2	3.6
University of Oklahoma	25.2	6.8	0.0	13.7	14.3	5.3	29.0	3.4	0.0	1.3
University of Pennsylvania	26.0	3.5	4.6	8.0	7.6	6.1	33.5	1.5	0.0	9.2
University of Tennessee	10.4	6.9	6.9	11.6	4.6	5.2	39.4	0.0	3.4	11.6
University of Texas	20.7	9.5	0.0	14.3	15.8	8.0	27.0	4.7	0.0	0.0
University of Utah	14.6	4.6	0.0	12.0	7.7	9.2	41.5	4.5	0.0	1.5
University of Vermont	27.6	4.3	6.5	8.6	5.4	5.4	27.0	2.2	2.2	10.8
University of Virginia	23.4	0.0	0.0	11.8	9.0	5.9	44.0	0.0	0.0	5.9
University of Washington	22.4	1.5	0.0	9.4	9.4	7.9	26.0	0.0	17.2	6.2
University of Wisconsin	21.0	4.0	0.0	11.9	7.9	4.0	31.5	4.0	11.8	4.0
University of Wyoming	16.7	4.8	0.0	14.3	7.9	6.3	38.1	0.0	8.7	3.2
Washington University	22.3	5.0	6.3	11.3	8.8	5.4	32.7	4.4	1.3	2.5
University of West Virginia	21.0	6.0	0.0	11.0	7.0	7.0	44.0	0.0	0.0	4.0
Worcester Polytechnic Institute	20.9	4.8	7.7	11.4	8.8	6.8	32.1	5.3	0.0	2.2
AVERAGE	21.6	5.5	3.2	11.8	9.1	6.3	31.1	3.4	3.9	4.1

*Tulane—Includes shop work
†Florida—includes shop work
‡Michigan—includes cultural electives
New Mexico—Freshman gymnasium not included in percentages.

TABLE II.
ELECTRICAL ENGINEERING

INSTITUTION	Electrical engineering	English	Foreign languages	Mathematics excluding descriptive	Physics including el. mechanics	Chemistry	General engineering	General subjects	Electives	Physical Training
Norwich University......	33.5	3.9	7.9	9.9	9.9	3.9	13.2	5.9	1.3	10.5
Montana State College of Agriculture and Mechanic Art.........	33.0	3.0	0.0	13.0	16.0	5.2	15.2	7.0	5.2	2.4
University of Colorado...	30.0	7.1	0.0	14.3	7.1	7.1	31.4	3.0	0.0	0.0
University of Notre Dame	29.6	4.7	6.0	11.8	10.2	0.0	22.0	10.7	1.2	3.6
Agric. & Mech. College of Texas................	28.8	12.5	00.0	13.3	4.5	4.5	26.7	3.3	1.7	6.7
Lehigh University.......	28.6	6.5	7.8	11.0	13.0	6.5	16.2	1.9	1.9	6.5
Armour Institute........	28.6	2.4	00.0	7.2	8.8	6.4	35.0	6.8	17.9	4.8
University of Arkansas ..	28.5	4.2	4.2	14.6	5.5	5.5	22.9	0.0	10.4	4.2
Carnegie Institute.......	28.3	2.9	4.6	8.0	9.6	4.6	30.0	3.3	0.0	8.7
Polytechnic Institute of Brooklyn.............	28.0	6.6	3.7	10.7	9.4	7.0	28.0	2.8	1.9	1.9
University of Mexico.....	28.0	4.2	0.0	14.0	9.1	5.5	25.9	2.1	11.2	....*
Lafayette College........	27.7	3.9	3.9	11.0	6.4	6.4	24.5	11.0	3.9	1.3
University of Illinois	27.7	4.3	5.7	14.0	9.2	5.7	25.6	8.5	0.0	4.3
University of Vermont...	27.6	4.3	6.5	8.6	5.4	5.4	27.0	2.2	2.2	10.8
State College of Washington...........	27.2	6.2	0.0	12.7	6.2	6.8	29.7	4.7	3.1	3.7
State University of Iowa.	26.9	6.6	0.0	14.7	8.8	5.9	28.3	6.6	0.0	2.2
Colorado College........	26.5	4.7	7.8	20.3	13.3	4.7	15.7	4.7	0.0	2.3
University of Oklahoma..	26.2	6.8	0.0	13.7	14.3	5.3	29.0	3.4	0.0	1.3
Univ. of Pennsylvania....	26.0	3.5	4.6	8.0	7.6	6.1	33.5	1.5	0.0	9.2
George Washington University............	25.7	4.3	4.3	8.6	7.1	8.6	33.6	2.8	5.0	0.0
Maryland State College of Agriculture.........	25.7	10.8	5.6	10.5	4.5	4.5	21.6	9.3	0.0	7.5
State University of Colorado.............	25.5	5.0	0.0	10.0	8.5	7.0	21.3	2.5	16.2	4.0
Case School.............	25.2	6.4	6.4	10.4	9.1	5.5	25.9	1.3	1.3	8.5
Oklahoma Agric. and Mechanical College....	25.0	5.9	0.0	12.5	11.0	5.7	30.9	1.5	3.0	4.5
Oregon Agric. College....	25.0	4.5	0.0	13.0	9.5	4.5	30.0	0.0	5.7	7.5
Tufts College...........	24.0	4.0	4.0	9.0	6.0	6.0	22.0	2.0	21.0	2.0
Massachusetts Inst. of Technology...........	23.8	5.2	3.2	8.8	8.2	8.3	25.5	4.2	8.3	4.5
University of Virginia ...	23.4	0.0	0.0	11.8	9.0	5.9	44.0	0.0	0.0	5.9
Rensselaer Polytechnic Institute.............	23.3	5.9	6.2	13.0	9.2	6.2	33.8	1.1	0.0	1.3
Bucknell University......	23.0	6.0	5.5	9.0	12.6	9.4	23.0	8.0	3.0	0.5
Union College...........	22.8	8.6	8.6	17.2	5.6	4.5	20.0	5.5	4.4	2.8
Univ. of Cincinnati......	22.4	5.3	5.0	8.0	10.8	10.4	34.2	2.3	0.0	1.6
Univ. of Washington.....	22.4	1.5	0.0	9.4	9.4	7.9	26.0	0.0	17.2	6.2
Washington University...	22.3	5.0	6.3	11.3	8.8	5.4	32.7	4.4	1.3	2.5
Pennsylvania State College...............	22.0	7.5	7.5	12.5	7.5	5.6	30.0	7.5	0.0	0.0
Iwoa State College.......	21.7	7.5	0.0	15.0	11.5	6.8	23.7	2.7	7.5	3.6
University of Idaho......	21.5	7.8	0.0	15.0	7.8	5.2	36.2	1.3	0.0	5.2
Georgia School of Technology..........	21.1	8.5	6.1	11.0	10.8	6.0	25.3	2.5	0.0	8.7

(33 Colleges in all)

TABLE III.

ENGLISH

INSTITUTION	Electrical engineering	English	Foreign languages	Mathematics excluding descriptive	Physics including el. mechanics	Chemistry	General engineering	General subjects	Electives	Physical Training
Agricultural and Mechanical College of Texas	28.8	12.5	00.0	13.3	4.5	4.5	26.7	3.3	1.7	6.7
Mississippi Agric and Mechanical College	15.3	10.9	0.0	18.8	7.0	4.5	26.1	5.9	6.3	5.2
Maryland State College of Agriculture	25.7	10.8	5.6	10.5	4.5	4.5	21.6	9.3	0.0	7.5
Clemson College	13.4	9.7	0.0	9.7	8.0	4.2	34.6	6.9	0.0	13.3
University of Texas	20.7	9.5	0.0	14.3	15.8	8.0	27.0	4.7	0.0	0.0
Pennsylvania College	15.0	9.2	3.9	12.4	14.4	9.2	23.5	11.8	0.0	0.7
Union College	22.8	8.6	8.6	17.2	5.6	4.5	20.0	5.5	4.4	2.8
Georgia School of Technology	21.1	8.5	6.1	11.0	10.8	6.0	25.3	2.5	0.0	4.2
Throop College	19.5	8.4	7.0	8.2	10.0	6.0	30.0	6.7	0.0	4.2
Purdue University	20.4	7.9	6.6	14.4	9.6	3.5	21.5	5.9	7.6	2.6
University of Idaho	21.5	7.8	0.0	15.0	7.8	5.2	36.2	1.3	0.0	5.2
North Carolina State College	19.4	7.7	5.3	11.9	8.5	6.0	29.4	5.3	0.0	6.6
Iowa State College	21.7	7.5	0.0	15.0	11.5	6.8	23.7	2.7	7.5	3.6
Penn. State College	22.0	7.5	7.5	12.5	7.5	5.5	20.0	7.5	0.0	0.0
University of Colorado	30.0	7.1	0.0	14.3	7.1	7.1	31.4	3.0	0.0	0.0
Drexel Institute	20.5	7.0	0.0	8.0	10.0	8.0	31.0	4.5	0.0	3.0
University of Kansas	17.0	7.0	7.0	12.0	11.0	9.0	28.0	5.0	2.0	2.0
Univ. of Tennessee	10.4	6.9	6.9	11.6	4.6	5.2	39.4	0.0	3.4	11.6
University of Oklahoma	26.2	6.8	0.0	13.7	14.3	5.3	29.0	3.4	0.0	1.3
Polyt. of Brooklyn	28.0	6.6	3.7	10.7	9.4	7.0	28.0	2.8	1.9	1.9
State University of Iowa	26.9	6.6	0.0	14.7	8.8	5.9	28.3	6.6	0.0	2.2
University of Maine	20.2	6.5	6.5	13.4	8.7	6.0	26.5	4.6	4.0	3.6
Lehigh University	28.6	6.5	7.8	11.0	13.0	6.5	16.2	1.9	1.9	6.5
Tulane University	12.3	6.4	0.0	12.8	7.9	8.9	48.4	0.0	2.0	1.3
University of Arizona	17.3	6.4	5.1	10.9	5.1	5.1	32.7	0.0	7.1	10.3
University of Missouri	18.6	6.4	3.5	10.5	7.6	8.5	26.7	1.2	14.8	2.3
State University of Kentucky	12.7	6.2	0.0	10.6	7.8	5.3	53.2	0.0	0.0	4.2
State College of Washington	27.2	6.2	0.0	12.4	6.2	6.8	29.7	4.7	3.1	3.7
Bucknell University	23.0	6.0	5.5	9.0	12.6	9.4	23.0	8.0	3.0	0.5
Univ. of West Virginia	21.0	6.0	0.0	11.0	7.0	7.0	44.0	0.0	0.0	4.0
Oklahoma Agric. and Mech. College	25.0	5.9	0.0	12.5	11.0	5.7	30.9	1.5	3.0	4.5
Rensselaer Polytechnic Institute	23.3	5.9	6.2	13.0	9.2	6.2	33.8	1.1	0.0	1.3
Johns Hopkins University	20.3	5.5	3.2	9.0	13.6	7.0	26.6	4.7	2.3	7.8
University of Cincinnati	22.4	5.3	5.0	8.0	10.8	10.4	34.2	2.3	0.0	1.6
Massachusetts Institute of Technology	23.8	5.2	3.2	8.8	8.2	8.3	25.5	4.2	8.3	4.5
University of Florida	13.4	5.2	7.9	11.8	10.5	6.6	41.9	0.0	0.0	2.7

TABLE IV.

FOREIGN LANGUAGES

INSTITUTION	Electrical engineering	English	Foreign languages	Mathematics excluding descriptive	Physics including el. mechanics	Chemistry	General engineering	General subjects	Electives	Physical Training
University of Michigan	20.7	4.3	11.4	12.9	10.0	5.7	25.0	0.0	10.0	0.0
Union college	22.8	8.6	8.6	17.2	5.6	4.5	20.0	5.5	4.4	2.8
University of Florida	13.4	5.2	7.9	11.8	10.5	6.6	41.9	0.0	0.0	2.7
Norwich University	33.5	3.9	7.9	9.9	9.9	3.9	13.2	5.9	1.3	10.5
Colorado College	26.5	4.7	7.8	20.3	13.3	4.7	15.7	4.7	0.0	2.3
Lehigh University	28.6	6.5	7.8	11.0	13.0	6.5	16.2	1.9	1.9	6.5
Worcester Polytechnic	20.9	4.8	7.7	11.4	8.8	6.8	32.1	5.3	0.0	2.2
Pennsylvania State College	22.0	7.5	7.5	12.5	7.5	5.5	30.0	7.5	0.0	0.0
Throop College	19.5	8.4	7.0	8.2	10.0	6.0	30.0	6.7	0.0	4.2
University of Kansas	17.0	7.0	7.0	12.2	11.0	9.0	28.0	5.0	2.0	2.0
University of Tennessee	10.4	5.9	6.9	11.6	4.6	5.2	39.4	0.0	3.4	11.6
Purdue University	20.4	7.9	6.6	14.4	9.6	3.5	21.5	5.9	7.6	2.6
University of Maine	20.2	6.5	6.5	13.4	8.7	6.0	26.5	4.6	4.0	3.6
University of Vermont	27.6	4.3	6.5	8.6	5.4	5.4	27.0	2.2	2.2	10.8
Case School	25.2	6.4	6.4	10.4	9.1	5.5	25.9	1.3	1.3	8.5
Washington University	22.3	5.0	6.3	11.3	8.8	5.4	32.7	4.4	1.3	2.5
Rensselaer Polyt. Inst.	23.3	5.9	6.2	13.0	9.2	6.2	33.8	1.1	0.0	1.3
Georgia School of Technology	21.1	8.5	6.1	11.0	10.8	6.0	25.3	2.5	0.0	8.7
University of Notre Dame	29.6	4.7	6.0	11.8	10.2	0.0	22.0	10.7	1.2	3.6
University of Illinois	27.7	4.3	5.7	14.0	9.2	5.7	25.6	8.5	0.0	4.3
Maryland State College of Agriculture	25.7	10.8	5.6	10.5	4.5	4.5	21.6	9.3	0.0	7.5
Bucknell University	23.0	6.0	5.5	9.0	12.6	9.4	23.0	8.0	3.0	0.5
University of North Dakota	21.0	7.0	5.5	11.0	9.5	8.0	33.0	2.0	1.5	1.5
North Carolina State College	19.4	7.7	5.3	11.9	8.5	6.0	29.4	5.3	0.0	6.6
Ohio State University	20.1	2.6	5.2	13.0	13.0	5.2	31.2	0.0	5.8	3.9
University of Arizona	17.3	6.4	5.1	10.9	5.1	5.1	32.7	0.0	7.1	10.3
University of Cincinnati	22.4	5.3	8.0	8.0	10.8	10.4	34.2	2.3	0.0	1.6
Carnegie Institute	28.3	2.9	4.6	8.0	9.6	4.6	30.0	3.3	0.0	8.7
Univ. of Pennsylvania	26.0	3.5	4.6	8.0	7.6	6.1	33.5	1.5	0.0	9.2
George Washington University	25.7	4.3	4.3	8.6	7.1	8.6	33.6	2.8	5.0	0.0
Stevens Institute	8.1	3.9	4.2	5.6	6.3	7.5	53.9	2.1	0.0	8.4
University of Arkansas	28.5	4.2	4.2	14.6	5.5	5.5	22.9	0.0	10.4	4.2
University of Detroit	16.7	3.8	4.2	13.8	18.6	10.5	39.5	2.9	0 0	0.0
Tufts College	24.0	4.0	4.0	9.0	6.0	6.0	22.0	2.0	21.0	2.0
Lafayette College	27.7	3.9	3.9	11.0	6.4	6.4	24.5	11.0	3.9	1.3
Pennsylvania College	15.0	9.2	3.9	12.4	14.4	9.2	23.5	11.8	0.0	0.7
New Mexico College of Agric. and Mech Art	17.8	2.8	3 8	9.1	8.2	5.8	42.7	2.8	1.0	5.8

TABLE V.

MATHEMATICS EXC. DESCRIPTIVE

INSTITUTION	Electrical engineering	English	Foreign languages	Mathematics excluding descriptive	Physics including el. mechanics	Chemistry	General engineering	General subjects	Electives	Physical Training
Colorado College	26.5	4.7	7.8	20.3	13.3	4.7	15.7	4.7	0.0	2.3
Mississippi Agric. and Mech. College	15.3	10.9	0.0	18.8	7.0	4.5	26.1	5.9	6.3	5.2
Union College	22.8	8.6	8.6	17.2	5.6	4.5	20.0	5.5	4.4	2.8
Syracuse University	25.0	4.0	0.0	16.3	10.2	7.5	35.0	2.0	0.0	0.0
University of Missouri (Columbia)	16.1	4.8	0.0	16.1	8.1	4.0	29.8	4.0	13.7	3.2
University of Nebraska	17.8	3.2	0.0	16.0	12.0	4.8	32.8	0.0	10.4	3.2
Iowa State College	21.7	7.5	0.0	15.0	11.5	6.8	23.7	2.7	7.5	3.6
University of Idaho	21.5	7.8	0.0	15.0	7.8	5.2	36.2	1.3	0.0	5.2
State University of Iowa	26.9	6.6	0.0	14.7	8.8	5.9	28.3	6.6	0.0	2.2
University of Arkansas	28.5	4.2	4.2	14.6	5.5	5.5	22.9	0.0	10.4	4.2
Purdue University	20.4	7.9	6.6	14.4	9.6	3.5	21.5	5.9	7.6	2.6
University of Texas	20.7	9.5	0.0	14.3	15.8	8.0	27.0	4.7	0.0	0.0
University of Colorado	30.0	7.1	0.0	14.3	7.1	7.1	31.4	3.0	0.0	0.0
University of Wyoming	16.7	4.8	0.0	14.3	7.9	6.3	38.1	0.0	8.7	3.2
University of Illinois	27.7	4.3	5.7	14.0	9.2	5.7	25.6	8.5	0.0	4.3
Univ. of New Mexico	28.0	4.2	0.0	14.0	9.1	5.5	25.9	2.1	11.2	....*
University of Detroit	16.7	3.8	4.2	13.8	18.6	10.5	39.5	2.9	0.0	0.0
Univ. of Minnesota	20.6	4.1	0.0	13.7	8.2	4.1	33.3	4.1	8.9	2.8
University of Oklahoma	26.2	6.8	0.0	13.7	14.3	5.3	29.0	3.4	0.0	1.3
University of Maine	20.2	6.5	6.5	13.4	8.7	6.0	26.5	4.6	4.0	3.6
Agric. and Mech. College of Texas	28.8	12.5	00.0	13.3	4.5	4.5	26.7	3.3	1.7	6.7
University of Alabama	18.4	3.2	0.0	13.2	10.5	5.3	34.7	4.7	5.8	4.2
Montana State College of Agric. and Mech. Arts	33.0	3.0	0.0	13.0	16.0	5.2	15.2	7.0	5.2	2.4
Ohio State University	20.1	2.6	5.2	13.0	13.0	5.2	31.2	0.0	5.8	3.9
Oregon Agric and Mech. College	25.0	4.5	0.0	13.0	9.5	4.5	30.0	0.0	5.7	7.5
Rensselaer Polyt	23.3	5.9	6.2	13.0	9.2	6.2	33.8	1.1	0.0	1.3
University of Michigan	20.7	4.3	11.4	12.9	10.0	5.7	25.0	0.0	10.0	0.0
Tulane University	12.3	6.4	0.0	12.8	7.9	8.9	48.4	0.0	2.0	1.3
Olkahoma Agric. and Mech. College	25.0	5.9	0.0	12.5	11.0	5.7	30.9	1.5	3.0	4.5
Penn. State College	22.0	7.5	7.5	12.5	7.5	5.5	30.0	7.5	0.0	0.0
Pensylvania College	15.0	9.2	3.9	12.4	14.4	9.2	23.5	11.8	0.0	0.7
State College of Washington	27.2	6.2	0.0	12.4	6.2	6.8	29.7	4.7	3.1	3.7
New Hampshire College	21.0	4.1	0.0	12.2	10.1	6.1	35.7	0.0	5.4	5.4
University of Kansas	17.0	7.0	7.0	12.0	11.0	9.0	28.0	5.0	2.0	2.0
University of Utah	14.6	4.6	0.0	12.0	7.7	9.2	42.5	4.5	0.0	1.5
North Carolina State College	19.4	7.7	5.3	11.9	8.5	6.0	29.4	5.3	0.0	6.6
Univ. of Wisconsin	21.0	4.0	0.0	11.9	7.9	4.0	31.5	4.0	11.8	4.0

TABLE VI.

ELECTIVES

INSTITUTION	Electrical engineering	English	Foreign languages	Mathematics excluding descriptive	Physics including el. mechanics	Chemistry	General engineering	General subjects	Electives	Physical Training
Tufts College	24.0	4.0	4.0	9.0	6.0	6.0	22.0	2.0	21.0	2.0
Armour Inst. of Technology	28.6	2.4	00.0	7.2	8.8	6.4	35.0	6.8	17.9	4.8
University of Washington	22.4	1.5	0.0	9.4	9.4	7.9	26.0	0.0	17.2	6.2
State Agric. College of Colorado	25.5	5.0	00.0	10.0	8.5	7.0	21.3	2.5	16.2	4.0
University of Missouri (Rolla)	18.6	6.4	3.5	10.5	7.6	8.5	26.7	1.2	14.8	2.3
University of Missouri (Columbia)	16.1	4.8	0.0	16.1	8.1	4.0	29.8	4.0	13.7	3.2
University of Wisconsin	21.0	4.0	0.0	11.9	7.9	4.0	31.5	4.0	11.8	4.0
Univ. of New Mexico	28.0	4.2	0.0	14.0	9.1	5.5	25.9	2.1	11.2	....
University of Arkansas	28.5	4.2	4.2	14.6	5.5	5.5	22.9	0.0	10.4	4.2
University of Nebraska	17.8	3.2	0.0	16.0	12.0	4.8	32.8	0.0	10.4	3.2
University of Minnesota	20.6	4.1	0.0	13.7	8.2	4.1	33.3	4.1	8.9	2.8
University of Wyoming	16.7	4.8	0.0	14.3	7.9	6.3	38.1	0.0	8.7	3.2
Massachusetts Institute of Technology	23.8	5.2	3.2	8.8	8.2	8.3	25.5	4.2	8.3	4.5
University of California	15.0	0.0	0.0	11.0	12.0	7.0	40.0	0.0	8.0	7.0
Purdue University	20.4	7.9	6.6	14.4	9.6	3.5	21.5	5.9	7.6	2.6
Iowa State College	21.7	7.5	0.0	15.0	11.5	6.8	23.7	2.7	7.5	3.6
University of Arizona	17.3	6.4	5.1	10.9	5.1	5.1	32.7	0.0	7.1	10.3
Massachusetts Agric. and Mech. College	15.3	10.9	0.0	18.8	7.0	4.5	26.1	5.9	6.3	5.2
Ohio State University	20.1	2.6	5.2	13.0	13.0	5.2	31.2	0.0	5.8	3.9
University of Alabama	18.4	3.2	0.0	13.2	10.5	5.3	34.7	4.7	5.8	4.2
Oregon Agric. College	25.0	4.5	0.0	13.0	9.5	4.5	30.0	0.0	5.7	7.5
New Hampshire College	21.0	4.1	0.0	12.0	10.1	6.1	35.7	0.0	5.4	5.4
Montana State Agric. and Mech. College	33.0	3.0	0.0	13.0	16.0	5.2	15.2	7.0	5.2	2.4
George Washington University	25.7	4.3	4.3	8.6	7.1	8.6	33.6	2.8	5.0	0.0
Union College	22.8	8.6	8.6	17.2	5.6	4 5	20.0	5.5	4.4	2.8
University of Maine	20.3	6.5	6.5	13.4	8.7	6 0	26.5	4.6	4.0	3.6
Lafayette College	27.7	3.9	3.9	11.0	6.4	6 4	24.5	11.0	3.9	1.3
University of Tennessee	10.4	6.9	6.9	11.6	4.6	5 2	39.4	0.0	3.4	11.6
State College of Washington	27.2	6.2	0.0	12.4	6.2	6.8	29.7	4.7	3.1	3.7
Bucknell University	23.0	6.0	5.5	9.0	12.6	9.4	23.0	8.0	3.0	0.5
Oklahoma Agric and. Mech. College	25.0	5.9	0.0	12.5	11.0	5 7	30.9	1.5	3.0	4.5
Cornell University	20.6	0.0	0.0	7.8	9.0	7 0	46.5	3.9	2.6	2.6
Johns Hopkins University	20.3	5.5	3.2	9.0	13.6	7 0	26.6	4.7	2.3	7.8
University of Vermont	27.6	4.3	6.5	8.6	5.4	5 4	27.0	2.2	2.2	10.[illegible]
Tulane University	12.3	6.4	0.0	12.8	7.9	8.9	48.4	0.0	2.0	1[illegible]

ANNUAL REPORT OF THE MARINE COMMITTEE

To the Board of Directors,

The Marine Committee submits the following report for the year 1917-18.

Two merchant vessels equipped with alternating-current lighting and motor service including engine room auxiliaries as well as deck axuiliaries have been completed and put in service. As mentioned in last year's report these equipments followed land practise adopting 250-volt, 60-cycle, three-phase alternating current. The total power provided was 200 kw. divided in two equal units.

These equipments were thoroughly tested and found satisfactory by extensive trial trips of the vessels but the vessels themselves have not been in service a sufficient time to warrant conclusions to be drawn as to service conditions. The vessels are both oil carriers and the heaviest auxiliary loads were those connected with the cargo oil pumps. It is to be expected that the owners will maintain records to show whether the vessels show marked improvement in the loading and discharging of cargo. It was upon this basis and the danger coincident to the use of d-c. motors that the application was made. The two electrically propelled merchant vessels have not yet been completed. Their equipments, however, are now under construction.

Much work has recently been projected on the basis of using oil engine-driven generators and electric motors for ship propulsion. For reasons connected with the low speed of the oil engine and increased efficiency, these plants have been designed for 25 cycles. It is understood that two such vessels may be so equipped.

Present Activities

Your committee has not been able to make further progress this year regarding the full revision of the electrical rules of Lloyd's Register of British and Foreign Shipping. The war conditions have prevented the necessary conferences, but minor matters of installation have been referred to the Lloyd's Register from time to time and approval given in accord with American practise. It was the consensus of opinion of this committee at its last meeting that the present time would not permit of

the preparation of technical papers either for presentation at meetings or for publication in the TRANSACTIONS. There has been, however, a favorable tendency towards the writing of popular articles on marine subjects in order to aid the general public in its conception of the extent of the uses of electricity in the marine field.

Suggestions for the Future

It is the purpose of your committee to continue to make suggested changes in the rules of the various classification Societies, and as experimental equipments emerge into established practise this committee will make the proper recommendations. As was inferred above, the intensive work of the individual members of the committee now prevents the writing of technical papers. It is believed that the time is approaching when it will not only be expedient but necessary to have such papers prepared and published in the TRANSACTIONS. The tendency in the field of ship propulsion seems to be approaching nearer to the use of electric drive due probably to increased interest on the part of ship owners and marine engineers in the efficiency of such systems, and the possibility of obtaining electrical apparatus with less difficulty than other types of propulsive machinery.

The committee desires to call your attention to the desirability of closer coordination of its work with the other technical committees of the Institute. The growth of the shipbuilding industry in this country and allied problems makes this most desirable.

H. A. HORNOR, *Chairman*

ANNUAL REPORT OF THE POWER STATIONS COMMITTEE

To the Board of Directors,

The Committee held two meetings during the year, which, however, were poorly attended. It was early recognized undesirable to request from any engineer any labor for committee work except that which would be absolutely necessary or would become of vital importance to the operation of plants during the war.

At the last meeting, held December 14, 1917, two members in addition to the Chairman undertook to investigate and collect all available information on the broad questions of savings in production and utilization of power. Mr. Gorsuch has collected a good deal of information on what has been accomplished in utilizing waste gases in industries and tying together the electric power distributing lines with such by-product power plants; also studies of fundamental factors affecting economies in operation of power plants and favorable conditions under which they may be secured.

Mr. Putnam and the Chairman undertook to review the present-day relative economic values of new water power developments vs. steam power developments and their dependency and co-ordination, having in view the advisability or not of investing new capital in new water power developments during the time of the war, in contra-distinction of what the economic factors would have been previous to the war and what may be after the war.

The Chairman in collecting these subjects had in view the possibility of eventually securing proper papers for presentation at one of the meetings of the Institute if it were found desirable to cover at such meeting the subjects from the standpoint of the broad national policy during the war period.

In pursuing the study, the Chairman soon found out that the subject of power is a complex one and has many ramifications, so that a comprehensive solution could not be attained without securing the co-operation of representatives of different organizations interested in the application of water powers, steam powers, best methods of securing highest fuel economy by con-

centration of power generation, inter-connection of systems, possibilities of economies in use of wastes and gases from by-product coke ovens, powdered fuel, etc.

The Chairman, on the occasion of the Mid-Winter Convention, took the opportunity of suggesting to the President that it might be advisable to initiate the organization of a National Engineering Commission for considering and discussing plans and ways of advancing the recommendations made in his address. Such a Commission would naturally broaden out to study and report on policies affecting economics of power generation for general power application, steam electrification and special industries requiring continuous use of power. It was believed that such a study and recommendation would be of immense value to the industries, Government and State in shaping their policies in the generation and utilization of power.

A conference to discuss the subject thoroughly and outline plans could not, on account of the pressure of other matters, be arranged to include all who, in the opinion of the Chairman, should be present.

Some individual work was, however, done and considerable material is now available for use if the new administration should decide to carry out the plan.

Respectfully submitted,
PHILIP TORCHIO, *Chairman*

Presented by publication only.

SKIN EFFECT IN TUBULAR AND FLAT CONDUCTORS

BY H. B. DWIGHT

ABSTRACT OF PAPER

A method is presented for calculating the skin effect resistance ratio of a tube, which is a form of conductor to be recommended for high-frequency work. A formula is also developed by means of which the asymptote to the curve of the ratio R'/R may be drawn, and thus the magnitude of the skin effect at extremely high frequencies may be obtained.

The values of the ratio R'/R for tubes of various thicknesses, are plotted in a set of curves (Fig. 3) which may be used for the solution of practical problems.

A similar method is described for the calculation of skin effect in a thin strap. Although the calculations cannot be carried out for as high frequencies as the calculation for tube, it indicates a method of coordinating the test results for straps, which have been published. A set of empirical curves for straps is given in Fig. 7, from which approximate values of R'/R for any case may be read.

IN any conductor carrying alternating current, the magnetic field around the axis of the conductor produces variations in the current density. In an isolated conductor, the current tends to flow more densely in the outermost parts, farthest from the axis of the conductor. The effect is more pronounced as the frequency becomes high, or as the section of the conductor becomes large. The result with a round wire or tube when the frequency is high, is that the current is concentrated in the outer skin of the conductor—hence the name "skin effect". In an isolated flat strap, the current crowds mainly toward the edges of the strap, and term "edge effect" has been suggested[1], although the general term "skin effect" is often used for all cases of the phenomenon.

The formula for the magnitude of the skin effect in a round, non-magnetic wire may be obtained by assuming an infinite series for the current density at any point of the section[2]. The formula may also be obtained by forming a differential equation connecting the current and the dimensions of the section[3]. An

1. Skin Effect Resistance Measurements, by A. E. Kennelly and H. A. Affel, Proc. Inst. of Radio Engineers, May, 1916.

2. Clerk Maxwell, *Electricity and Magnetism*, Vol. II, para. 689.

3. A. Russell, *Philosophical Mag.*, Vol. 17, 1909, p. 524, and A. E. Kennelly, F. A. Laws and P. H. Pierce, TRANS. A. I. E. E., 1915, p. 1953.

alternative method has been described[4] by which the effect of the magnetic field is traced step by step. Successive increments of current and voltage drop are calculated, to keep the voltage uniform over the section, as of course it really is. The increments become smaller and smaller, forming convergent series. This method gives the usual result for round wire, and is useful for the special cases of tubes and straps, which are calculated in this article.

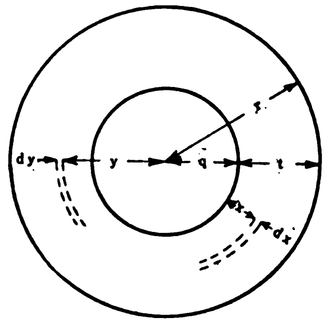


FIG. 1—SECTION OF TUBULAR CONDUCTOR

Assume that a uniform current of density a_0, in absolute electromagnetic units, that is, in absamperes per square centimeter, flows at all parts of the section of the tube indicated in Fig. 1. The current inside the circle dx is

$$I_{(x)} = \pi\, a_0\, (2\, q\, x + x^2) \quad \text{absamperes} \qquad (1)$$

The flux density at dx is

$$\frac{2\, I_{(x)}}{q + x}$$

$$= 2\, \pi\, a_0\, q \left[2\, \frac{x}{q} - \frac{x^2}{q^2} \left(1 - \frac{x}{q} + \frac{x^2}{q^2} - \frac{x^3}{q^3} + \ldots\right)\right]$$

$$= 2\, \pi\, a_0\, q \left(2 \frac{x}{q} - \frac{x^2}{q^2} + \frac{x^3}{q^3} - \ldots\right) \text{ lines per sq. cm.}$$

The flux outside of $d\,x$ and inside the metal of the tube, per centimeter of tube, is

$$\varphi_{(x)}' = 2\, \pi\, a_0\, q \int_x^t \left(\frac{2\,x}{q} - \frac{x^2}{q^2} + \frac{x^3}{q^3} - \ldots\right) d\,x \text{ lines}$$

The reactive drop at $d\,x$ due to a_0 is

$$j\, \omega\, \varphi_{(x)} = j\, \omega\, 2\, \pi\, a_0\, t^2$$

$$\left(1 - \frac{1}{3}\, \frac{t}{q} + \frac{1}{4}\, \frac{t^2}{q^2} - \ldots - \frac{x^2}{t^2} + \frac{1}{3}\, \frac{x^3}{t^2\, q} - \frac{1}{4}\, \frac{x^4}{t^2\, q^2} + \ldots\right)$$

4. Transmission Line Formulas, by H. B. Dwight, 1913, Chap. X.

$$= \frac{j\, m^2\, t^2\, \rho\, a_0}{2} \left(1 - \frac{1}{3}\,\frac{t}{q} + \frac{1}{4}\,\frac{t^2}{q^2} - \ldots - \frac{x^2}{t^2} + \frac{1}{3}\,\frac{x^3}{t^2\, q} - \frac{1}{4}\,\frac{x^4}{t^2\, q^2} + \ldots \right) \text{abvolts} \quad (2)$$

putting

$$m^2 = \frac{4\,\pi\,\omega}{\rho} \quad (3)$$

where ω is equal to $2\,\pi$ times the frequency in cycles per second, and where ρ is the specific resistance of the metal in absolute units. The specific resistance in ohms is $\rho \times 10^{-9}$

Let a current of density a_1 flow, such that $a_1\,\rho$ will be equal and opposite to the above terms in x. Then

$$a_1 = \frac{1}{2}\, j\, m^2\, t^2\, a_0 \left(\frac{x^2}{t^2} - \frac{1}{3}\,\frac{x^3}{t^2\, q} + \frac{1}{4}\,\frac{x^4}{t^2\, q^2} - \ldots \right)$$

absamperes per sq. cm. (4)

By the above process, the total current due to a_1 is found to be

$$\frac{j\, m^2\, t^2\, a_0\, 2\,\pi\, q\, t}{\underline{|3}} \left(1 + \frac{1}{2}\,\frac{t}{q} - \frac{1}{4\times 5}\,\frac{t^2}{q^2} + \ldots \right)$$

absamperes, (5)

and the reactive drop at $d\,x$, due to flux in the metal caused by a_1, is

$$\frac{(j\, m^2\, t^2)^2\, a_0\, \rho}{\underline{|4}} \left(1 - \frac{2}{5}\,\frac{t}{q} + \frac{3}{10}\,\frac{t^2}{q^2} - \ldots - \frac{x^4}{t^4} + \frac{2}{5}\,\frac{x^5}{t^4\, q} - \frac{3}{10}\,\frac{x^6}{t^4\, q^2} + \ldots \right) \text{abvolts} \quad (6)$$

By assuming a current of density a_2, such that $a_2\,\rho$ will neutralize the terms of x in (6), it is found that the total current due to a_2 is

$$\frac{(j\, m^2\, t^2)^2\, a_0\, 2\,\pi\, q\, t}{\underline{|5}} \left(1 + \frac{1}{2}\,\frac{t}{q} - \frac{1}{14}\,\frac{t^2}{q^2} + \ldots \right)$$

absamperes (7)

and the reactive drop at $d\,x$, due to flux in the metal caused by a_2 is

$$\frac{(j\, m^2\, t^2)^3\, a_0\, \rho}{\underline{|6}} \left(1 - \frac{3}{7}\,\frac{t}{q} + \frac{9}{28}\,\frac{t^2}{q^2} - \ldots - \frac{x^6}{t^6} + \frac{3}{7}\,\frac{x^7}{t^6\, q} - \frac{9}{28}\,\frac{x^8}{t^6\, q^2} + \ldots \right) \text{abvolts} \quad (8)$$

By continuing this process, the following results are obtained:

$$I\,Z' = a_0\,\rho\left[1 + b_1\frac{j\,m^2\,t^2}{\underline{|2}} + b_2\,\frac{(j\,m^2\,t^2)^2}{\underline{|4}} + b_3\frac{(j\,m^2\,t^2)^3}{\underline{|6}} + \ldots\right] \text{abvolts} \qquad (9)$$

where I is the total current in the tube, where Z' is the effective impedance due to resistance and inductance caused by flux inside the metal of the tube, and where

$$b_1 = 1 - \frac{1}{3}\,\frac{t}{q} + \frac{1}{4}\,\frac{t^2}{q^2} - \ldots$$

$$= \frac{1}{2} + \frac{q}{t} - \frac{q^2}{t^2}\,logh\left(1 + \frac{t}{q}\right) \qquad (10)$$

$$b_2 = 1 - \frac{2}{5}\,\frac{t}{q} + \frac{3}{10}\,\frac{t^2}{q^2} - \frac{17}{70}\,\frac{t^3}{q^3} + \ldots \qquad (11)$$

$$b_3 = 1 - \frac{3}{7}\,\frac{t}{q} + \frac{9}{28}\,\frac{t^2}{q^2} - \frac{11}{42}\,\frac{t^3}{q^3} + \ldots \qquad (12)$$

$$b_4 = 1 - \frac{4}{9}\,\frac{t}{q} + \frac{1}{3}\,\frac{t^2}{q^2} - \ldots \qquad (13)$$

$$b_5 = 1 - \frac{5}{11}\,\frac{t}{q} + \frac{15}{44}\,\frac{t^2}{q^2} - \ldots \qquad (14)$$

$$b_n = 1 - \frac{n}{2\,n+1}\,\frac{t}{q} + \ldots \qquad (15)$$

If R is the resistance of the tube per unit length, in absolute units,

$$I\,R = \frac{I\,\rho}{\pi\,(2\,q\,t + t^2)}$$

$$= \rho\,a_0\left[1 + c_1\,\frac{j\,m^2\,t^2}{\underline{|3}} + c_2\frac{(j\,m^2\,t^2)^2}{\underline{|5}} + c_3\,\frac{(j\,m^2\,t^2)^3}{\underline{|7}} + \ldots\right] \qquad (16)$$

where $$c_1 = 1 - \frac{1}{20}\,\frac{t^2}{q^2} + \frac{1}{20}\,\frac{t^3}{q^3} - \frac{11}{280}\,\frac{t^4}{q^4} + \ldots \qquad (17)$$

$$c_2 = 1 - \frac{1}{14}\,\frac{t^2}{q^2} + \frac{1}{14}\,\frac{t^3}{q^3} - \ldots \tag{18}$$

$$c_3 = 1 - \frac{1}{12}\,\frac{t^2}{q^2} + \ldots \tag{19}$$

and
$$c_4 = 1 - \frac{1}{11}\,\frac{t^2}{q^2} + \ldots \tag{20}$$

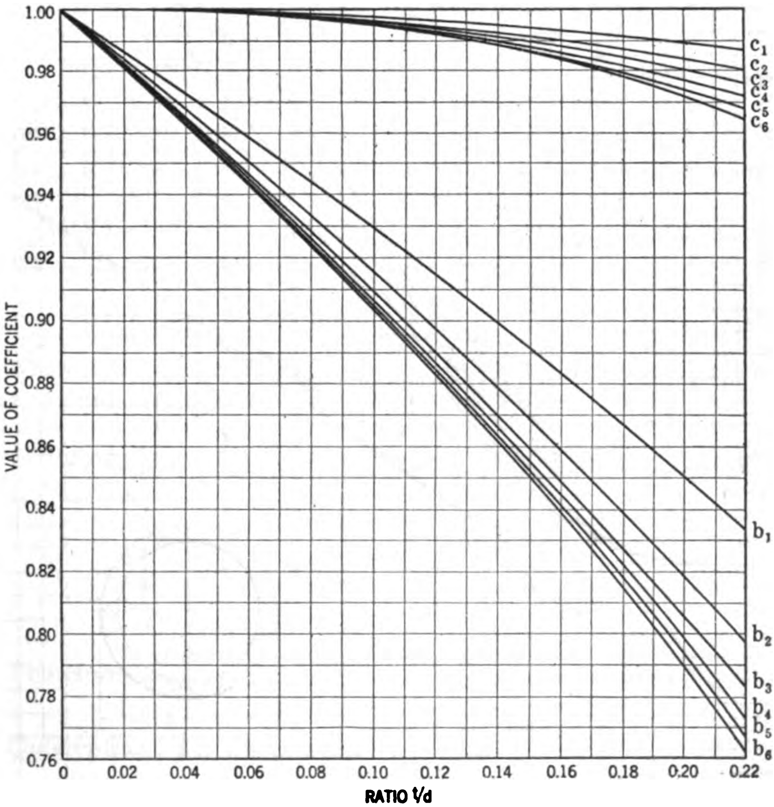


FIG. 2—COEFFICIENTS FOR CALCULATING SKIN EFFECT IN TUBES

The values of the coefficients b_1, b_2, c_1, etc., are shown in Fig.2. These have been used in calculating the curves of $\frac{R'}{R}$ at the lower frequencies shown by the full lines in Fig. 3. The complex quantity $\frac{Z'}{R}$ is found from (9) and (16). The denominator is rationalized, and then the real part of $\frac{Z'}{R}$ is equal to $\frac{R'}{R}$.

The formula for the skin effect in a tube of infinite radius is of the same form as that for a return circuit of two adjacent straps,[5] but different constants are involved.

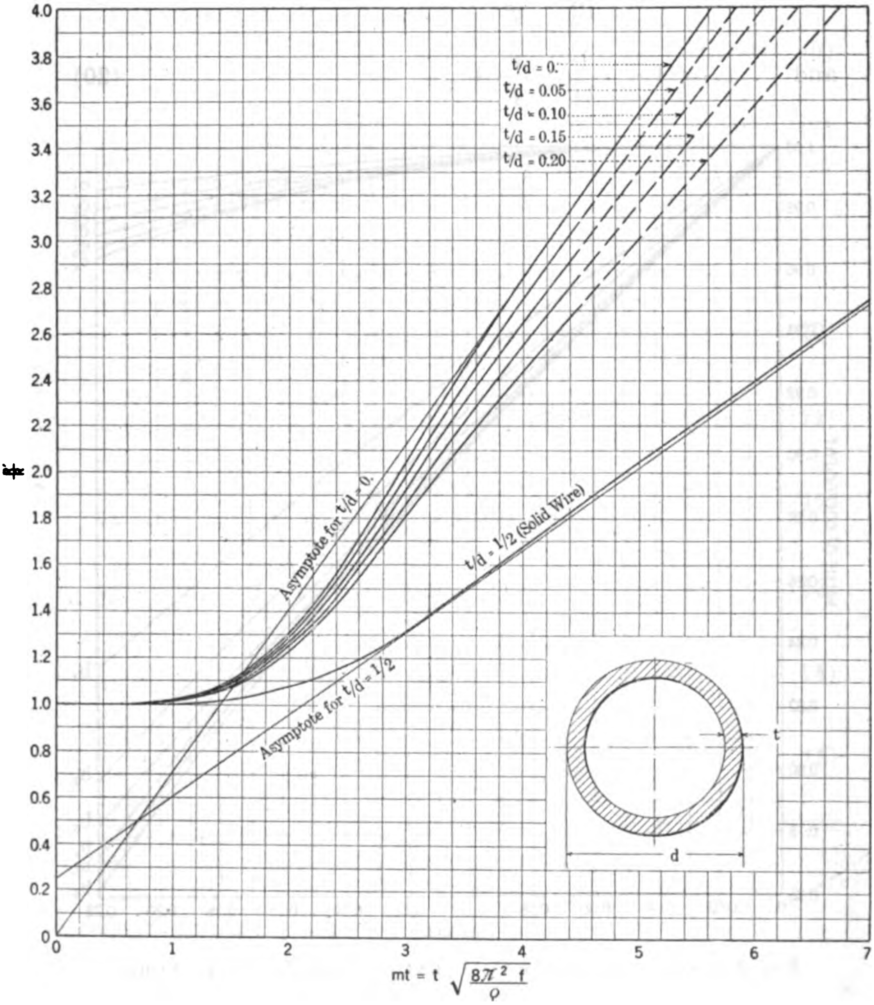


FIG. 3—SKIN EFFECT IN TUBES OF VARIOUS THICKNESSES

The above calculation can be carried to fairly high frequencies and it can be shown by it that the curve of $\frac{R'}{R}$ of a tube be-

---

5. Skin Effect of a Return Circuit of Two Adjacent Strap Conductors, by H. B. Dwight, *The Elec'ric Journal*, April, 1916.

comes approximately a straight line for high frequencies when $\frac{R'}{R}$ is plotted against the square root of the frequency. The angle of slope of the asymptote, that is, the line which the curve approaches as a tangent, when $\frac{R'}{R}$ is large, may be calculated as follows, in a manner similar to that used by Dr. A. Russell in deriving a method for determining the skin effect of a concentric main, that is, a tube containing an insulated return wire.[6]

The skin effect in an isolated non-magnetic tube, in which the current returns at a considerable distance from the tube, is quite different from the skin effect in a tube containing a return wire, and so a new calculation is required, using different constants.

Let i be the current density at radius y, Fig. 1. The drop per cm. of the tube at radius y is

$$e = \rho\, i + \frac{d\,\varphi}{d\,\tau} \tag{21}$$

where ρ is the specific resistance of the metal in the tube, where τ is time, and where

$$\varphi = \int_y^r \frac{2\,I_{(y)}}{y}\,d\,y = -\int_r^y \frac{2\,I_{(y)}}{y}\,d\,y \tag{22}$$

$$I_{(y)} = \int_q^y 2\,\pi\,i\,y\,d\,y \tag{23}$$

Therefore, $$\frac{d\,\varphi}{dy} = -\frac{2\,I_{(y)}}{y} = -\frac{2}{y}\int_q^y 2\,\pi\,i\,y\,dy \tag{24}$$

Differentiate (**24**) with respect to τ

$$\frac{d}{d\,\tau}\,\frac{d\,\varphi}{dy} = \frac{d}{dy}\,\frac{d\,\varphi}{d\,\tau} = -\frac{4\,\pi}{y}\int_q^y y\,\frac{di}{d\,\tau}\,dy$$

Now e is constant over the section. Therefore

$$\frac{d\,e}{dy} = 0 = \rho\,\frac{di}{dy} + \frac{d}{dy}\,\frac{d\,\varphi}{d\,\tau} \qquad \text{from (21)}$$

$$= \rho\,\frac{d\,i}{dy} - \frac{4\,\pi}{y}\int_q^y y\,\frac{di}{d\,\tau}\,dy \tag{25}$$

6. *Philosophical Mag.*, Vol. 17, 1909, p. 524.

When $y = q, \qquad 0 = \rho \dfrac{di}{dy} - 0$

Therefore, when $y = q, \qquad \dfrac{di}{dy} = 0$ **(26)**

From **(25)**, $$y \frac{di}{dy} = \frac{4\pi}{\rho} \int_q^y y \frac{di}{d\tau} dy \qquad \textbf{(27)}$$

Differentiate **(27)** with respect to y. Then

$$y \frac{d^2 i}{dy^2} + \frac{di}{dy} = \frac{4\pi}{\rho} y \frac{di}{d\tau}$$

$$\text{Put} \quad m^2 = \frac{4\pi\omega}{\rho} = \frac{8\pi^2 f}{\rho} \qquad \textbf{(28)}$$

where f = frequency in cycles per second.

Then $$\frac{d^2 i}{dy} + \frac{1}{y}\frac{di}{dy} = \frac{m^2}{\omega}\frac{di}{d\tau}$$

and therefore $$\frac{d^2 i}{dy^2} + \frac{1}{y}\frac{di}{dy} - j m^2 i = 0 \qquad \textbf{(29)}$$

when the current has a sine-wave shape.

The solution of this differential equation may be written

$$i = (A + jB)\, J_0 (m y \sqrt{-j}) + (C + jD)\, K_0 (m y \sqrt{-j}) \qquad \textbf{(30)}$$

where $J_0 (my \sqrt{-j})$ is a Bessel function of the first kind and of order zero, where $K_0 (m y \sqrt{-j})$ is a Bessel function of the second kind and of order zero, and where A, B, C, and D are constants to be determined. Thus

$$i = (A + jB)(\text{ber } my + j \text{ bei } my) + (C + jD)(\text{ker } my + j \text{ kei } my) \qquad \textbf{(31)}$$

that is, $$i = A \text{ ber } my - B \text{ bei } my + C \text{ ker } my - D \text{ kei } my + j(A \text{ bei } my + B \text{ ber } my + C \text{ kei } my + D \text{ ker } my)^7 \qquad \textbf{(32)}$$

From **(26)**, $\dfrac{di}{dy} = 0$ when $y = q$.

---

7. See equations **29, 44** and **77** of the paper by Kennelly, Laws and Pierce, and the paper by A. Russell, referred to in the second paragraph of this paper, reference 3.

Therefore,

$$A\ ber'\ mq - B\ bei'\ mq + C\ ker'\ mq - D\ kei'\ mq = 0 \qquad \textbf{(33)}$$

and $$A\ bei'\ mq + B\ ber'\ mq + C\ kei'\ mq + D\ ker'\ mq = 0 \qquad \textbf{(34)}$$

Let the total current in the tube be the quantity to which all phase relations are to be referred, and let it be equal to $I + j\,0$. Then, since

$$\int y\ ber\ my\ dy = \frac{y}{m}\ bei'\ my, \quad \int y\ bei\ my\ dy$$

$$= -\frac{y}{m}\ ber'\ my, \quad \int y\ ker\ my\ dy = \frac{y}{m}\ kei'\ my$$

$$\text{and} \int y\ kei\ my\ dy = -\frac{y}{m}\ ker'\ my,$$

we have

$$I = \int_q^r 2\pi\, y\, i\, dy$$

$$= \frac{2\pi}{m}\Big[A\ y\ bei'\ my + B\ y\ ber'\ my + C\ y\ kei'\ my$$

$$+ D\ y\ ker'\ my\Big]_q^r$$

$$+ j\frac{2\pi}{m}\Big[-A\ y\ ber'\ my + B\ y\ bei'\ my - C\ y\ ker'\ my$$

$$+ D\ y\ kei'\ my\Big]_q^r$$

The quantities in the brackets are equal to zero when $y = q$, by (**33**) and (**34**). Therefore,

$$I + j\,0 = \frac{2\pi r}{m}\Big[A\ bei'\ mr + B\ ber'\ mr + C\ kei'\ mr$$

$$+ D\ ker'\ mr\Big]$$

$$+ j\frac{2\pi r}{m}\Big[-A\ ber'\ mr + B\ bei'\ mr - C\ ker'\ mr$$

$$+ D\ kei'\ mr\Big]$$

Therefore, $- A\ ber'\ mr + B\ bei'\ mr - C\ ker'\ mr$

$$+ D\ kei'\ mr = 0 \qquad (35)$$

and $A\ bei'\ mr + B\ ber'\ mr + C\ kei'\ mr + D\ ker'\ mr = \dfrac{I\ m}{2\ \pi\ r}$

$$(36)$$

Let Z' be the effective impedance per centimeter of the tube at a certain frequency due to its effective resistance R' and its inductance caused by flux inside the metal. Since the flux does not cause any drop where $y = r$, we have, from (**21**),

$$e = I\ Z' = \rho\ i_{(r)}$$

Therefore, by (**32**),

$$I\ Z' = \rho \left[A\ ber\ mr - B\ bei\ mr + C\ ker\ mr - D\ kei\ mr \right]$$

$$+ j\ \rho \left[A\ bei\ mr + B\ ber\ mr + C\ kei\ mr + D\ ker\ mr \right] \qquad (37)$$

The drop in phase with the current I is

$$I\ R' = \rho \left[A\ ber\ mr - B\ bei\ mr + C\ ker\ mr - D\ kei\ mr \right] \qquad (38)$$

Now, by (**36**), $\qquad I\ R = \dfrac{I\ \rho}{\pi\ (r^2 - q^2)}$

$$= \frac{2\ \pi\ r\ \rho}{\pi\ (r^2 - q^2)\ m} \left[A\ bei'\ mr + B\ ber'\ mr + C\ kei'\ mr + D\ ker'\ mr \right] \qquad (39)$$

Therefore, $\dfrac{R'}{R}$

$$= \frac{m\ (r^2 - q^2)}{2\ r} \frac{(A\ ber\ mr - B\ bei\ mr + C\ ker\ mr - D\ kei\ mr)}{(A\ bei'\ mr + B\ ber'\ mr + C\ kei'\ mr + D\ ker'\ mr)} \qquad (40)$$

The four equations (**33**), (**34**), (**35**) and (**36**) are sufficient to determine the four constants A, B, C, and D.

A similar equation for the skin effect inductance ratio could easily be written down, but would not be of much practical interest in the case of a tube since the flux inside the metal is extremely small compared with the flux outside the tube.

Equation (**40**) could be used to calculate the skin effect resistance ratio of a tube at any frequency if very complete tables of the eight functions, *ber*, etc., were available.

The result is the same as in Dr. Russell's equation (**89**) for a concentric main, except that the inner and outer radii of the tube are interchanged, due to the different position of zero flux. Thus by interchanging the inner and outer radii in Dr. Russell's low-frequency formula (**101**), which is derived from (**89**), a low-frequency formula for an isolated tube would be obtained. In this article, equation (**40**) will be used only for obtaining the slope of the asymptote to the curve of $\frac{R'}{R}$, thus giving approximate values of $\frac{R'}{R}$ for high frequencies by a very simple formula.

For large values of *mr*, approximate formulas for the eight functions *ber*, etc., are given in Dr. Russell's paper referred to above, of which the following is an example:

$$ber\, mr = \frac{\epsilon^{\frac{mr}{\sqrt{2}}}}{\sqrt{2\,\pi\, mr}} \cos\left(\frac{mr}{\sqrt{2}} - \frac{\pi}{8}\right) \tag{41}$$

The values given by these approximate formulas are more accurate as *mr* is larger.

If these formulas are substituted in the well known equation for a solid wire,

$$\frac{R'}{R} = \frac{mr}{2} \frac{(ber\, mr\, bei'\, mr - bei\, mr\, ber'\, mr)}{(ber'^2\, mr + bei'^2\, mr)} \tag{42}$$

we obtain

$$\frac{R'}{R} = \frac{mr}{2\sqrt{2}} \tag{43}$$

which gives the slope of the asymptote of the curve of $\frac{R'}{R}$.

In order to obtain the equation of the asymptote (see Fig. 3), namely,

$$\frac{R'}{R} = \frac{mr}{2\sqrt{2}} + \frac{1}{4} \tag{44}$$

it is necessary to use values of the form

$$ber\, mr = \frac{\epsilon^{\frac{mr}{\sqrt{2}} + \frac{1}{8\, mr\, \sqrt{2}}}}{\sqrt{2\,\pi\, mr}} \cos\left(\frac{mr}{\sqrt{2}} - \frac{\pi}{8} - \frac{1}{8\, mr\, \sqrt{2}}\right) \quad (45)$$

given in Dr. Russell's paper. If further terms of the series in $\frac{1}{mr}$ are added, an asymptotic formula for $\frac{R'}{R}$ is obtained, which is very accurate except at low frequencies.

It is thus evident that the substitution of the formulas similar

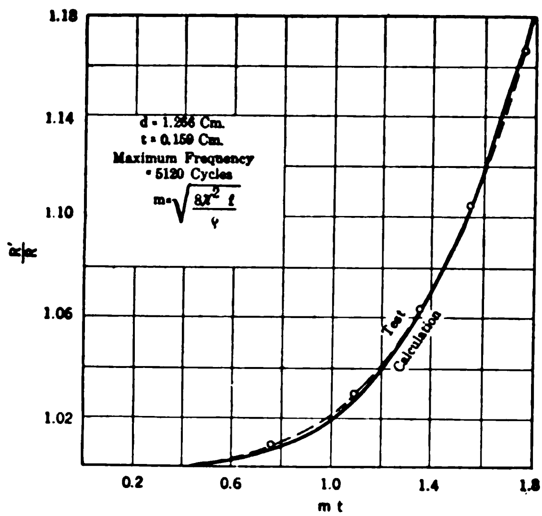


FIG. 4—COMPARISON OF CALCULATION WITH TEST

to (41) in equation (40) will give only the slope of the asymptote of the curve of $\frac{R'}{R}$. The result of the substitution, after a certain amount of trigonometrical work, is

$$\frac{R'}{R} = \frac{mt\,(q + r)}{2\, r\, \sqrt{2}} \; \frac{\{\sinh\,(mt\,\sqrt{2}) + \sin\,(mt\,\sqrt{2})\}}{\{\cosh\,(mt\,\sqrt{2}) - \cos\,(mt\,\sqrt{2})\}} \quad (46)$$

This reduces to
$$\frac{R'}{R} = \frac{mt\,(q + r)}{2\, r\, \sqrt{2}} \quad (47)$$

which gives the required slope of the asymptote. If the inner

radius $q = 0$, we obtain equation (**43**) for a solid wire. If q is practically equal to r, (**47**) becomes

$$\frac{R'}{R} = \frac{mt}{\sqrt{2}} \tag{48}$$

This happens to be the actual equation of the asymptote, which in this case passes through the origin. (See equations **100** and **110** of the paper by Kennelly, Laws and Pierce, reference 3.)

The useful fact is therefore deduced that the asymptote of the curve for $\frac{R'}{R}$ of a thin tube passes close to the origin. It

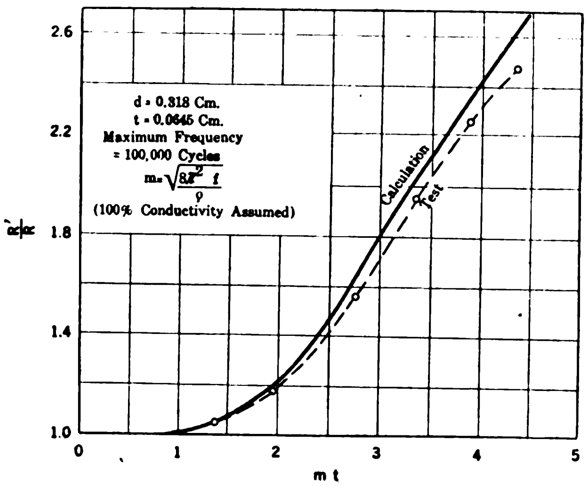


FIG. 5—COMPARISON OF CALCULATION WITH TEST

is therefore possible to plot $\frac{R'}{R}$ closely for tubes at high frequencies, as is done in the dotted lines in Fig. 3. The values found from the high-frequency, Bessel function calculation are in agreement with those found by the low-frequency calculation.

As examples, the results for two tubes tested by Dr. Kennelly, described in the papers mentioned above, are shown in Figs. 4 and 5. From the data given, the d-c. conductivity of the hard-drawn copper tube in Fig. 4 was about 50 per cent, and the curves have been drawn accordingly. The test curve of the soft-drawn copper tube of Fig. 5 was calculated on the basis of 100 per cent conductivity, in the absence of data regarding the

conductivity. If the conductivity were lower, the test curve would be drawn farther to the left.

Skin Effect in Strap. The process of calculating corrective currents to keep the voltage drop uniform over the section of the conductor may be applied to a thin isolated strap as well as to a tube. In this calculation, the strap is assumed to be so thin that the only appreciable action is the crowding of the current to the edges. The test curves shown in Fig. 7 indicate that this assumption is allowable for very thin straps.

Since the magnetic flux around the strap does not lie in circles, the voltage drop at any part of the section is calculated from the effect of the various elements of current in the section. Thus, omitting terms involving the distance to the return conductor, which can be shown not to affect the skin effect resistance ratio when the distance is large, it is found that the reactive drop at dx, Fig. 6, due to a current of uniform density b_0, is

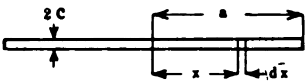


FIG. 6—SECTION OF STRAP CONDUCTOR

$$j\,\omega\,4\,ac\,b_0\left[2-\left(1+\frac{x}{a}\right)\log h\left(1+\frac{x}{a}\right)\right.$$

$$\left.-\left(1-\frac{x}{a}\right)\log h\left(1-\frac{x}{a}\right)\right]$$

$$=\frac{1}{2}j\,p^2\,\rho\,b_0\left[1-\frac{1}{1.2}\,\frac{x^2}{a^2}-\frac{1}{3.4}\,\frac{x^4}{a^4}\right.$$

$$\left.-\frac{1}{5.6}\,\frac{x^6}{a^6}-\ldots\right] \qquad (49)$$

where $$p^2=\frac{4\,\omega}{\rho}\times 4\,ac=\frac{4\,\omega}{R}$$

and where R is the resistance of the conductor per centimeter in absolute units. The resistance in ohms is $R\times 10^{-9}$.

Also, the reactive drop at dx due to current $b_1\,\frac{x^2}{a^2}$

$$=\frac{1}{6}j\,p^2\,\rho\,b_1\left[\frac{1}{3}+\frac{x^2}{a^2}\right.$$

$$-\frac{1}{2}\left(1+\frac{x^3}{a^3}\right)\log h\left(1+\frac{x}{a}\right)$$

$$- \frac{1}{2}\left(1 - \frac{x^3}{a^3}\right) \log h \left(1 - \frac{x}{a}\right)\Bigg]$$

$$= \frac{1}{2} j\, p^2 \rho\, b_1 \left[\frac{1}{3.3} + \frac{1}{1.2}\frac{x^2}{a^2} - \frac{1}{1.4}\frac{x^4}{a^4} - \frac{1}{3.6}\frac{x^6}{a^6} - \frac{1}{5.8}\frac{x^8}{a^8} - \ldots\right] \qquad \textbf{(50)}$$

The reactive drop due to current $b_n \dfrac{x^{2n}}{a^{2n}}$

$$= \frac{1}{2} j\, p^2 \rho\, b_n \left[\frac{1}{(2n+1)^2} + \frac{1}{(2n-1)\,2}\frac{x^2}{a^2} + \frac{1}{(2n-3)\,4}\frac{x^4}{a^4} + \ldots \right.$$

$$\left. \ldots + \frac{1}{1\,(2n)}\frac{x^{2n}}{a^{2n}} - \frac{1}{1\,(2n+2)}\frac{x^{2n+2}}{a^{2n+2}} - \frac{1}{3\,(2n+4)}\frac{x^{2n+4}}{a^{2n+4}} - \ldots\right] \qquad \textbf{(51)}$$

The total current in the strap due to b_0 is $4\,ac\,b_0$. The total current due to $b_1 \dfrac{x^2}{a^2}$ is $\dfrac{1}{3} b_1\, 4\,ac$ and the total current due to

$$b_n \frac{x^{2n}}{a^{2n}} \text{ is } \frac{1}{2n+1}\, b_n\, 4\,ac \qquad \textbf{(52)}$$

Assume that a uniform current i_0, flows in the strap. The reactive drop at $d\,x$ is given by (**49**). Next, assume a current whose resistance drop is equal and opposite to the terms in x of (**49**). The total current in the strap can be found by applying (**52**). By repeating the above process, and carrying the various series out to five terms or more in each case, the following results were obtained. Drop in the strap due to alternating current

$$= I\,Z' = i_0\, r\, [1 + j\,\tfrac{1}{3}\, p^2 - 0.0149994\, p^4 + j\,0.003122\, p^6 + 0.000532\, p^8 - j\,0.0000779\, p^{10} - 0.0000096\, p^{12} + j\,0.00000094\, p^{14} + 0.00000005\, p^{16}] \qquad \textbf{(53)}$$

Drop in the strap due to a direct current of the same amperage

$$= I\,R = i_0\,r\,[1 + j\,0.0965736\,p^2 + 0.0152178\,p^4 - j\,0.00218\,p^6 - 0.000259\,p^8 + j\,0.0000237\,p^{10} + 0.00000100\,p^{12} + j\,0.00000022\,p^{14} + 0.00000008\,p^{16}] \quad (54)$$

Some of the coefficients of the above series can be expressed exactly by algebraic functions. Thus, in (**53**) the term in p^4 is

$$-\left(\frac{1}{4}\log h\,2 - \frac{5}{16} + \frac{\pi^2}{64}\right)p^4 = -\,0.0149994\,p^4$$

In (**54**) the term in p^2 is

$$j\left(\frac{1}{2}\log h\,2 - \frac{1}{4}\right)p^2 = j\,0.0965736\,p^2$$

and the term in p^4 is

$$\left[\frac{1}{8}\log h\,2 - \frac{1}{4}\left(\log h\,2\right)^2 + \frac{5\,\pi^2}{192} - \frac{5}{24}\right]p^4$$
$$= 0.0152178\,p^4$$

The fraction $\frac{Z'}{R}$ can be calculated from (**53**) and (**54**) for various values of p. Then, after rationalizing the denominator, the real part is equal to $\frac{R'}{R}$, the skin effect resistance ratio.

The values of $\frac{R'}{R}$ for various values of p are plotted in Fig. 7. It is seen from this figure that the curve of $\frac{R'}{R}$ of a strap, when plotted on the square root of the frequency, has the same general shape as the curves for wire, tube, and a return circuit of two adjacent straps, and it seems to approach a straight line as an asymptote in the same way that they do.

The calculated curve has been carried only as far as $p = 2$. The other lines in Fig. 7 are test curves published in the papers by Kennelly and Affel, and by Kennelly, Laws and Pierce, referred to in the first part of this article. The test curves, especially at frequencies up to 70,000 cycles, show approximately straight lines, when plotted on the square root of the frequency. The curve, $\frac{a}{c} = 240$, Fig. 7, has the lowest position of any of

the published curves when they are plotted according to Fig. 7. A conductivity of 100 per cent was assumed for this copper strap. If the conductivity were lower, the curve would be slightly higher. The section of this strap was 0.016 by 3.81 cm.

The curve for a round wire of the same sectional area as the strap lies below the curve for strap up to p equal to about 2.1, but at higher frequencies than that, the wire has a greater ratio than the strap. (See Fig. 8). It is to be expected that the curves for thick straps would lie intermediate between the curve

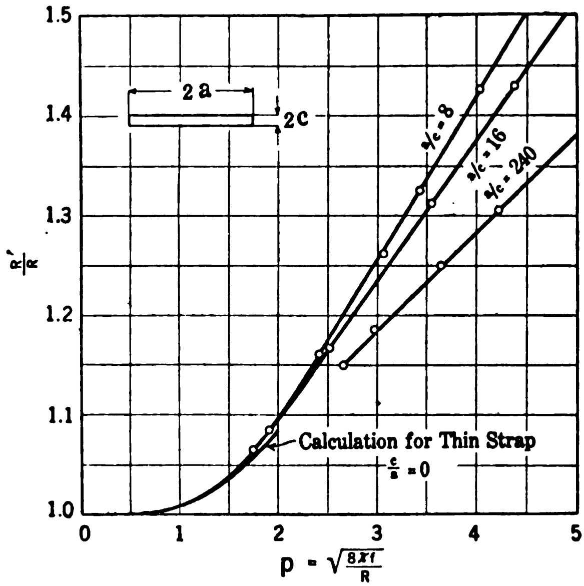


FIG. 7—SKIN EFFECT IN STRAP CONDUCTORS

for thin strap and that for wire. Such curves are represented by test curves $\frac{a}{c}$ = 16 (0.158 × 2.52 cm.) and $\frac{a}{c}$ = 8 (0.1575 × 1.26 cm.)

The curves of Fig. 7 may be considered as an extension in further detail of the article by the writer in the *Electrical World* of March 11, 1916, page 593, giving an empirical curve for skin effect in straps. The calculated curve now given tends to confirm the theory of the former article.

It would appear from Fig. 7 that rolling a strap thinner and wider, but keeping the same sectional area and conductivity

always reduces the skin effect resistance ratio for values of p greater than 2:1. However, it seems probable that the improvement is inappreciable after $\frac{a}{c}$ is more than about 50, and that the strap then approximates to an infinitely thin strap.

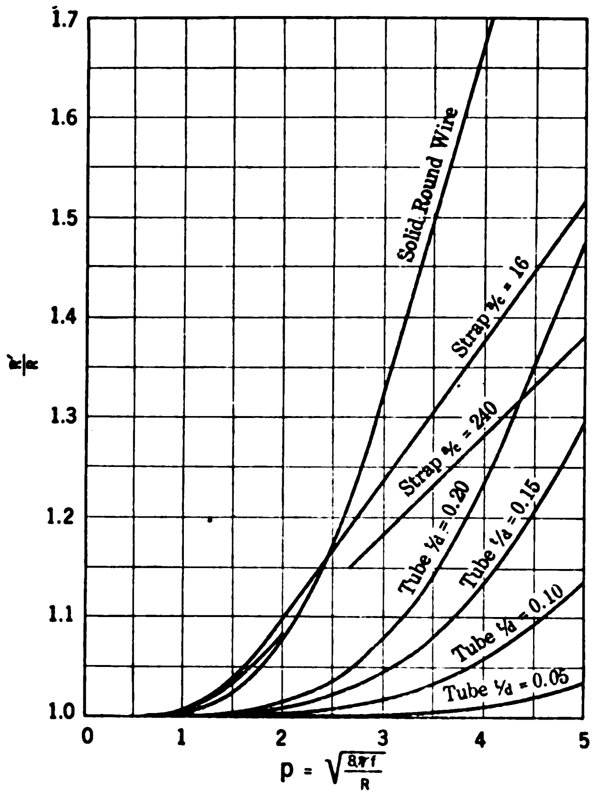


FIG. 8—SKIN EFFECT OF VARIOUS CONDUCTORS OF THE SAME SECTIONAL AREA

Thus a strap a foot wide would have the same skin effect resistance ratio as a thin strap one inch wide, if $\frac{a}{c}$ is in both cases greater than 50, and if both straps have the same resistance per 1000 feet, due to the wide strap being thinner or being made of material of lower conductivity.

By dividing series (53) by (54) a series is obtained which is

convenient for calculations when p is comparatively small. This is

$$\frac{R'}{R} = 1 + 0.0087432\, p^4 - 0.000384\, p^8 + 0.0000189\, p^{12} - \ldots \tag{55}$$

This series is not so convergent as (**53**) and (**54**). It should not be used unless p is small enough to make the last term quite small.

It should be remembered that the discussion on skin effect in strap in this article refers only to a strap which is not close to the return conductor, for in that case the proximity effect changes the value of $\frac{R'}{R}$, generally increasing it when the straps are in an edgewise position, and decreasing it when they are in parallel planes.

A copper strap $\frac{1}{8}$ by 5 inches, carrying current at 60 cycles, may be taken as an example of the calculated formula of skin effect in a thin strap. By changing the dimensions to centimeters, and taking the specific resistance of copper in absolute units to be 1724 at 20 deg. cent., we obtain

$$p^2 = 8\,\pi \times 60 \times \frac{1}{8} \times 5 \times \frac{6.45}{1724} = 3.53$$

Therefore $\quad p = 1.88$ and $\frac{R'}{R} = 1.071$, from Fig. 7.

For a strap $\frac{1}{16} \times 10$ inches,

$$p^2 = 8\,\pi \times 60 \times \frac{1}{16} \times 10 \times \frac{6.45}{1724} = 3.53$$

Therefore the two straps which have the same resistance per foot, have the same value of $\frac{R'}{R}$. Both straps can be considered to be practically the same as an infinitely thin strap, since the thicker one has a width 40 times the thickness.

In order to show graphically what types of conductors are most advantageous for high-frequency work, or for heavy currents in electric furnace circuits, the curves of Fig. 8 have been drawn. These show the skin effect to be expected

from using a given weight of copper in the form of wire, strap, or tubes of various proportionate thicknesses. In the case of a tube

$$\frac{8\,\pi\,f\,A}{\rho} = m^2\,t^2\left(\frac{r^2 - q^2}{t^2}\right) \tag{56}$$

where A is the sectional area, and this equation may be used to obtain the curves of Fig. 8 from those of Fig. 3. From the equation of the asymptote to the curve for a tube, it may be found that a tube in which $\frac{t}{d}$ is less than about 0.025 will always have less skin effect than a flat strap of the same sectional area and conductivity. A tube whose proportionate thickness is greater than the above amount may have a greater skin effect resistance ratio than a strap of the same sectional area at very high frequencies.

The curves of Fig. 8 are in agreement with the conclusion expressed in the paper by Kennelly and Affel, that a thin tube is the most economical form of conductor for high-frequency currents, when the return conductor is a considerable distance away.

As a general principle, it may be stated that a conductor, or a combination of conductors, of a certain proportionate shape and a certain value of $\frac{f}{R}$, will have a definite value of $\frac{R'}{R}$. This is true of tubes and round wires, as is shown by Fig. 3, where the skin effect of a tube of a certain ratio of thickness to diameter, but of any size, is given by a single curve. That the principle is true also of any shaped conductor, or any combination of conductors, may be indicated as follows:

Assume that an irregularly shaped conductor carries current at such a frequency, f, that $\frac{R'}{R} = (1 + a)$. The current will be crowded to certain parts of the section of the conductor, and the exact extent of this crowding may be indicated by plotting on the section, the lines for 95 per cent, 100 per cent and 105 per cent of average current density. In the case considered, the increase in resistance by the factor $(1 + a)$ is caused by a certain location of the current density lines.

Suppose that the conductor heats up so that its resistance increases 25 per cent, and suppose that the frequency also in-

creases 25 per cent. The unbalance of current is equivalent to a circulating current which goes with the main current at parts of the section, and returns at other parts of the section. This circulating current is driven by a voltage proportional to the frequency, and its voltage drop is due to resistance and is accordingly proportional to the resistance. Since both the frequency and the resistance have increased 25 per cent, the circulating current will be unchanged. Therefore, the position of the current density lines and the value of $\frac{R'}{R}$ will be unchanged.

Now, instead of the heat in the conductor raising its resistance, let a conductor of the same shape but 80 per cent as large be substituted, and let the frequency still be 25 per cent higher than at first. The resistance of the conductor is 25 per cent higher than at first. Thus the path for circulating current has 25 per cent higher resistance, and the frequency is 25 per cent higher, so that the circulating current is unchanged. Therefore, the relative position of the current density lines will be the same as before, and the value of $\frac{R'}{R}$ will be the same quantity, $(1 + a)$.

Thus in any conductor or combination of conductors or in any return circuit, of a certain proportionate shape, a given value of $\frac{f}{R}$ corresponds to a certain value of $\frac{R'}{R}$. A tested curve of $\frac{R'}{R}$ plotted on $\frac{f}{R}$ will therefore apply to conductors of different size from those tested, when the shape and relative position are the same.

The practical application of this principle is seen by considering that skin effect and proximity effect values are very often desired to be known for very heavy conductors at commercial frequencies. In such cases the currents are too large and the voltage drops are too small, to be conveniently or precisely measured. Precise results could be obtained by constructing miniature conductors to scale, and measuring the skin effect with conveniently small currents at high frequency.

In this way, the current distribution in three-phase circuits of ventilated busbars or of several cables in parallel, could be studied with exactness.

By considering a ventilated busbar to be approximately equivalent to a solid conductor of the same outside dimensions and the same total resistance, a simple extension of Fig. 7, containing curves for various proportionate thicknesses, would show the approximate skin effect of any isolated, rectangular, nonmagnetic conductor, whether solid or ventilated, hot or cold, large or small, and whether copper or aluminum. Such a set of curves would be useful in designing heavy busbars and conductors, not placed close together.

Presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 26, 1918.

A REVIEW OF ELECTRICAL ENGINEERING PROGRESS

PRESIDENT'S ADDRESS

BY E. W. RICE, JR.

ELECTRICAL engineering now covers such a wide field of scientific and technical activities that your President is presented with an embarrassment of riches in the attempt to select a subject for his address. He has, therefore, decided not to confine himself to any one feature of our Society's work, but to pick out here and there a few of many items to talk about which seemed of present interest and importance.

It is a pleasure to call your attention to the fact that we have added 1235 members of all classes during the year, our total membership being now 9370. This is a most encouraging result in these times of stress and change.

Our Institute is a national asset of increasing value to its members and to the nation. Every person who has the necessary qualifications should identify himself with the Institute, not only for the great benefits which he personally will receive, but in order that the usefulness and power of the great army of electrical engineers may be increased and rendered more available for the highest and most efficient service to our country and to the world.

The engineer is the hope of the nation, not only now, when we are at war, but even more so in the future in the days of reconstruction following the great peace. The engineer may perform valuable service when working alone, but his usefulness and power is manifestly greatly increased when acting in co-operation with thousands of his brothers.

The Institute is not only a democracy but is a democracy of educated men and such men have a heavy responsibility to society at present and will have in the future. They should be leaders and exemplars for those who have not been so fortunate as to have enjoyed their opportunities. The Institute needs its members and the members need the Institute, that the electrical engineer may fulfill his high destiny.

That the work of the Institute is of the highest quality is evidenced by the character of its meetings, its papers and discussions and the splendid work of its various committees. Its value and usefulness has increased every year of its existence and it should continue to gain in strength and usefulness because its methods are in accord with the spirit of the times.

But in order to accomplish this desirable result, it must continue, as at present, to be representative of the electrical engineering profession of the country, and therefore, must continue to expand its membership. This growth will bring with it problems inherent in all great institutions, democratic or autocratic, but I have confidence that all difficulties will be met successfully for the reason that the members of our profession are trained in the scientific view-point and methods of solving problems.

In the early days the progress of the electric science and arts was so rapid that it was relatively easy to find each year plenty of material for a review. Progress has continued and will continue, but naturally a decided tendency to saturation is shown in many directions. In some instances, this saturation can be demonstrated to be due to the fact that limits of perfection have been so closely approached that little remains of possible accomplishment. In other instances the slowing up is due to lack of knowledge, or, especially at the present time, to lack of workers, such workers having been diverted to the work imperatively needed to secure us against the attack of our enemy on the foundations of our existence.

There has been no material improvement for several years in the matter of efficiency in electrical units, such as dynamos, motors, transformers, etc. The efficiencies stated in Past-President Lincoln's address, in 1915, still remain almost exactly of the same values, and for the reasons which he so clearly pointed out.

The efficiency of conversion of mechanical into electrical energy, or the reverse, of electrical into mechanical energy, is still about 90 per cent in the average case, under practical conditions of operation; the efficiency reaching as high as 97 per cent or 98 per cent in the most favorable cases, with the large units, and falling below 90 per cent in unfavorable cases, or in the small units. The efficiency of conversion of electricity from high to low potential, as in transformers, also remains substantially the same, reaching as high as 98 plus per cent in the largest units. It is obvious, as Lincoln pointed out that no material change can be expected where such practical perfection has been reached.

The conversion of mechanical power of falling water into electrical energy by our water-wheels and electric generators has increased from about 87 per cent to 90 per cent in the largest units of 40,000 h. p. This represents about the limit which may be expected.

In the field of thermodynamic engines, represented largely by the steam turbo-generator unit, some improvement has been obtained. Lincoln stated that 75 per cent of Rankine efficiency had been obtained in some large modern steam turbo units in 1915. This has now been increased to about 80 per cent in the largest units of 35,000 to 40,000 kw. and 75 per cent is quite common practise even in such moderate sized units as 10,000 kw. This improvement, while not large, is doubly important because of the great increase in the cost of fuel. It has been realized mainly by bringing the practical design more nearly in accord with the theoretical, by increasing the number of stages or processes of steam extraction, reducing various losses, and by improving many details which, when properly looked after make in the aggregate, gains of practical importance.

Increase in the initial pressure of steam and lowering of terminal pressure, by better condenser arrangements, have also contributed to improvement, as it enables an increase in the range of temperature to be utilized. This makes possible better thermal efficiencies, even with the same per cent of Rankine efficiencies.

The following information illustrates the improvement in efficiency of turbo-electric units beginning with the first 5000 kw. installed in this country, in 1903, and continuing up to the close of 1917:

Year	Size, kw.	Steam Conditions			Lbs. per kw-hr.	Per Cent of rankine efficiency
		Steam pressure	Superheat fahrenheit	Back pressure		
1903	5,000	175 lb.	0	2 in.	24.00	37.8
1908	14,000	200 "	125°	1½ "	13.50	66.1
1911	20,000	235 "	100°	1½ "	13.20	67.0
1913	20,000	200 "	200°	1 "	10.74	75.9
1916	20,000	250 "	250°	1 "	10.00	76.5
1917	35,000	230 "	200°	1 "	10.14	78.7

It is gratifying to note that a percentage of Rankine efficiency of approximately 80 has been reached. This progress reflects

great credit upon the designers of turbo-electric machines and is a record of achievement found only in electrical development.

Concurrently with this improvement in the turbo-electric machines, great advances have been made in the design and operation of steam producing devices—the boilers, and in auxiliaries and other features of the modern power station. As a result the thermal efficiency has been rapidly improved. The thermal efficiency to which I refer may be stated as the ratio of the total energy produced at the terminals of the generator, to the total energy in the fuel burned — expressed as a percentage. It takes account of all losses from the coal under the boiler to the electricity at the dynamo terminals. It is the ratio of the heat units equivalent to one kw-hr., divided by the similar heat units in the fuel consumed to produce one kw-hr. at the generator terminals.

This thermal efficiency is after all, to the electrical engineer, the most important measure of progress. It measures the advance in station fuel economy, and as stated, many factors in addition to the improvement in turbo-generators have contributed to the result. Thermal efficiency may obviously be used to express the results of a single unit, consisting of turbo-generator, with its bank of boilers and other accessories, or it may be used to designate the combined result of all the units in a given power station.

The progress in the case of a combination unit, *i.e.* turbo-generator, with its boilers, auxiliaries, etc. has been as follows:

Year	Size of unit kw.	Thermal efficiency per cent
1903	5,000	10.15
1908	14,000	15
1913	20,000	18
1917–18	35,000	21.6

For comparison, I may state that large gas engines in steel mill practise, under best test conditions, show 25 per cent thermal efficiency, but in actual operation, an efficiency higher than 18 to 20 per cent is rare.

High compression oil engines of the Diesel type, driving electric generators, realize 25 to 26 per cent thermal efficiency when new, but are difficult to maintain at such efficiency.

The figures given must not be confused with the much higher thermal efficiencies often quoted for gas and oil engines, which refer to indicated horse power and not to electrical output.

The steam turbo-electric unit has not reached its limit of thermal efficiency. Calculations show that, with pressures of the order of 500 lb. gage, a thermal efficiency of 26 per cent should be easily realized. For any further substantial improvement, we must look to new methods, such as the use of two fluids, for example mercury and steam, as planned by Mr. W. L. R. Emmet. This method is still under development but its progress has been hampered by the pressure of war work.

As a matter of interest to electrical engineers, I may say, parenthetically, that the steam turbine in this country owes its existence and development almost entirely to the electrical engineer, and this is not surprising as the electrical engineer was familiar with the advantages of rotary machines, and perhaps it is not too much to say, prejudiced in their favor.

While, as stated, the efficiency of electrical units reached about its limit some years ago, those familiar with electrical engineering development are aware that progress has been made and is still possible in the generation, transmission and utilization of electrical energy. The struggle for improvement in efficiency has been transferred from the unit to the aggregate, called the system. We cannot have a system of maximum efficiency without units of maximum efficiency, but individual units of highest efficiency do not, of themselves, insure that the system upon which they are used will be of the highest efficiency, so progress has been made in the direction of improving the system economy or system efficiency.

To obtain the highest efficiency in practical operation, the element of time enters as a powerful factor. Our conception of efficiency should not be limited to a consideration of the relation between the instantaneous value of available heat units in coal and the electrical units produced at the point or points of consumption, but should consider the relation between the total number of heat units in fuel consumed in a given time, say 24 hours, to the total number of electrical units produced and used in the same time. The attempt to improve the efficiency of the system has shown the necessity for utilizing the generator units and transmission and distributing systems, for the maximum possible time.

This has led to the study of such questions as load factors of

generators, of stations, and of the system as a whole, to the study of the diversity factor, to the reduction of idle currents in alternating current systems by the use of synchronous condensers, and to means for the reduction of the constant and no-load losses in all machinery, in transformers, etc.

The resulting improvement has been effected, not only by changes in designs of the units themselves, but also by their method of use, based upon the recognition of the fact that the elimination or reduction of the losses at light load will greatly improve the total efficiency, especially when the time of use of the apparatus under load is a small part of the total time.

Automatic substations for transformers and synchronous converters have come into existence; different power houses of the same system have been tied together electrically; transmission lines of different systems have been interconnected, so that the units may be usefully employed for the maximum period, or lie idle or unloaded for the minimum time.

This general development has led to marked improvement in total energy efficiency, represented by the amount of fuel burned per electrical unit sold or utilized, and has also reduced cost of operation and charges for investment. There is still room for continued improvement in this direction and the progress will be rapid due to the pressure for maximum efficiency in the use of coal and of existing investment at the present time.

Many interesting examples of the methods and devices adopted to improve station and system economy and efficiency may be found throughout the country. In California, large electrical systems have been arranged to be tied together electrically, for exchange of power. In Washington and Idaho, power systems under different management have made similar arrangements. In the South, all important hydro-electric systems have been tied together for exchange of power. The advantage, as I have stated, of such arrangements is better utilization of variable stream flow, improvement in load factor, increased reliability of service, and the net result is to improve the efficiency of the system, not only financially, but in a purely technical sense. One most important advantage is the obvious reduction of the necessary investment in reserve machinery of every description.

In Montana, eight hydro-electric plants successively use the same stream flow, the total effective head amounting to 600 feet, and not only is the natural flow of the stream thus successively utilized, but all the storage water is effectively used by each

plant in series. In this same system, the yearly load factor is stated to reach 75 per cent and the mean monthly load factor to reach 80 per cent.

The interconnection of hydro-electric plants brings about another extremely important saving, based upon the variation of rainfall in amount and time on the different watersheds which are thereby brought to serve a common system. It frequently happens that there will be plenty of precipitation on one watershed, while another watershed may suffer from long continued drought. This condition varies not only in the same year but in different years. Interconnection serves to eliminate these variations by a process of averaging, and where the inter-connected system covers a sufficiently wide area, a remarkable increase in total useful power is made available.

It has frequently happened that thousands of horse power have been wasted over the dams of one system, the watersheds of whose plants happened to have a wet year, and at the same time, a nearby hydro-electric plant, supplied by another watershed, was without water power. The result has been that one system wasted power, while the other was suffering from a power shortage which would frequently be made up by burning a large amount of high grade coal, in the operation of an auxiliary steam plant. This condition has to a large extent been remedied by the interconnections to which I refer.

It has been estimated, and it seems a conservative estimate, that through the saving in reserve equipment, improvement in load factor, and the diversity of different loads, the useful output of groups of large systems may through inter-connection be increased about 25 per cent.

Electric regeneration of power, that is the utilization of the weight of trains running on a down grade due to the force of gravity to generate electricity which is fed back into the electric system to help other trains up grade, is an illustration of the same important improvement in the system efficiency.

I have thought it desirable to call your attention to the improvements obtained in system economy or efficiency because of the important savings in investment, in coal, in transportation, in labor and material, which in the aggregate, have already been realized. It illustrates the wonderful flexibility, value and economy of a general system transmitting energy by electricity, compared with any other possible method.

These advances have been more rapid during the last year,

due to the imperative demands for economy saving and increased efficiency imposed by the war. It is a great satisfaction that the foundation had all been well prepared during the times of peace.

The development of our industry has been so rapid that the need of intelligent and constructive standardization was realized some years ago. The Standards Committee of the Institute, formed in 1898, has been of inestimable value to the profession and to the industry. The standards adopted have been flexible enough to ensure progress and yet to discourage variations which were valueless. The standards promulgated by our committee have so appealed to the profession and to the industry that they have been cheerfully followed, and I am convinced that, as a result, the cost of electrical apparatus to the consumer has been greatly reduced over a number of years and the quality has not, been sacrificed, but has been improved. I consider that the money value of the work so done could be conservatively placed at many millions of dollars.

Sixty-cycle systems have shown, during the past few years, a more rapid growth than 25-cycle, and it is now estimated that 60-cycle systems represent about 70 per cent of the total power supplied in the country. This is undoubtedly due to the lowered cost of transformers, generators, induction motors, and similar apparatus. The relative growth of 60-cycle as compared with 25-cycle systems is reflected in steam turbine installations. In 1910 about 60 per cent of the steam turbine electric energy of the country was supplied from 60-cycle units; in 1917, this had risen to approximately 75 per cent.

This is an instance where standardization is desirable and economical. It will hasten the time so often predicted, when a network of transmission lines, carrying electrical energy, will cover the country. These will be fed by super-power stations, suitably located with respect to cheap reliable supplies of coal for fuel, and water for condensing purposes, and into the same network will also be fed energy from the various hydroelectric installations.

Marked advances have been made during the past year in the application of electricity to the electric furnace. It is estimated that the number of electric furnaces in the United States has been increased about 40 per cent in the past year and that there are now in operation over five times the number that existed five years ago. The world's output of steel from electric furnaces has now grown to approximately four million tons per annum.

Experience has demonstrated that the electric furnaces can utilize the cheapest and most inferior raw material to produce steel of the most uniform and highest quality, with the greatest regularity. The cost of steel so produced, while reasonable, considering its quality, was higher, until recently, than that produced by the open-hearth method. It is now possible to produce electric steel at substantially the cost of that produced by the open-hearth method. This result has been brought about partly by the increased cost of the open-hearth method, due to a variety of well known causes, but largely by a reduction in the cost of electric furnace operation. The marked change which has taken place in the reduction of the cost of operating electric furnaces is based upon greatly increasing the rate at which energy is delivered to the metal, both during the melting and the refining period. This has reduced the time required for an individual heat and also the kilowatt hours required per ton of metal melted, with a net result of increasing the daily output of the furnace.

As a concrete example, I mention the history of a five-ton furnace. It was originally supplied with 800 kv-a. at 80 volts. This was increased to 2000 kv-a. at 150 volts for the melting period and about 1400 kv-a. at 100 volts for the refining period. The time for the heat was reduced from six to three hours, power consumption was reduced from 877 kw-hr. to 588 kw-hr. per ton, and the number of heats per 24 hours was increased from three to five, increasing the net output from 15 to 25 tons.

Electric resistance furnaces of large sizes, for special heat treatment requiring unusual exactness, are being extensively used, producing results greatly superior to oil or gas fire furnaces.

Electric welding, both by the arc and incandescent method, is being rapidly extended and is destined to greater development in ship-building and similar operations.

Electric engineers have been devoting much time to the solution of many war problems. It is not desirable or possible to review such work at present, but when the veil is lifted, we will all be gratified with the result. We must content ourselves with the mere statement that this work has covered means for the detection of the pirate submarine; wireless signalling and telephoning for army and navy, and aircraft devices; searchlights of novel design and great power; improved methods in manufacture of ammunition and ordnance; electro-chemical work of

every description; electric welding; X-ray sets of greater simplicity and, accuracy; and many other lines too numerous even to mention.

The great industrial research laboratories, the educational and governmental research departments have all co-operated enthusiastically and effectively, and the members of their staffs have labored day and night, without regard to pecuniary reward or public applause, sustained entirely by the high purpose of giving their best to the service of the country. I hope the time may come when the story may be told, so that the world may realize the debt which it owes to scientific men and engineers, without whose arduous, unselfish and almost inspired work, our cause, righteous as it is, would have no chance of a victorious conclusion.

In my address at the opening of the mid-winter convention of the Institute, in February, 1918, I called attention to the advantages which it seemed to me would follow a more general electrification of the steam railroads of the country. I merely repeat at the present time that electric locomotives have been so improved and simplified that they are competent to haul the heaviest train that can be held together with the present train construction; to operate at the highest speed permissible by the alignment of the road and independent of its grades; and that the electric locomotives can meet in the most efficient and adequate manner the transportation problems confronting the country, and offer better results than are now obtained or seem possible with steam locomotives.

There can be no question that railroad electrification is not only economical but imperatively needed to improve the present standards of steam operation. Our mountain districts are congested almost entirely by the limitations of the steam railroad systems, and the addition of more tracks, under such conditions, is not the best solution of the problem. The electrified divisions of the steam roads have been free from troubles during the past severe winter and I repeat that the coal famine which the country suffered last winter could have been largely avoided if the steam railroads had been electrified. Moreover, it should not be forgotten that steam locomotives burn about 25 per cent of the entire coal mined in the United States and that 12 per cent of the entire ton mileage movement of freight and passengers carried over our railroad tracks is represented in cars and tenders required to haul coal to supply steam for the locomotives.

It is a truism, which has been frequently stated, that war requires the mobilization of the nation's industries and their devotion to essential work. This is especially true in this country, as it has been necessary in addition to create substantially new industries on an enormous scale, such as the production of ships, ordnance, ammunition, airplanes, chemicals, etc. To operate these industries, it has been necessary to mobilize to the fullest extent our available material and labor, but material and labor can only be converted into war work by the application of power. This power, in view of its great economy and flexibility, must be electrical.

While this country was fortunate in having available a magnificent system of power stations, so great was the magnitude of the demand for increased power, created by the war industries, that it is estimated that there will be a shortage of at least 500,000 kw. of electric power in the Eastern district.

It takes from one to two years to build and equip the large units which are essential for the production of such power. This illustrates the importance of all of the methods which I have mentioned to conserve, utilize and increase the efficiency of existing equipment and investment, as such methods can produce results in a much shorter time.

It is, however, vitally important that the great electrical power producing companies of this country should be helped in every way to meet the heavy demand which is placed upon them. It has been demonstrated that the quickest, most efficient, and altogether best way to meet the demand for power is through the expansion of such existing organizations and installations.

Fortunately, there is general appreciation of the fact and comprehensive schemes are under consideration which will provide for the erection of large steam electric power stations in the mining regions. Favorable locations exist which are within reach by transmission lines of electric power stations now serving large industrial areas. By interconnection, present investment and machinery will be better utilized and a large amount of additional electric power made available, without making any increased demand upon our congested railroad facilities.

It is evident, therefore, that we need to consider and put into effect, every practical method for conserving our existing developments, and also, we should take a courageous view of the future; we should provide, for the future growth at least as liberally as has been the custom of the managers of the great public service

systems in the past. It has been their custom to build from two to three years in advance of existing requirements, in anticipation of the future. I have yet to learn of a single important instance where such foresight has not been amply justified.

I would say in conclusion that the saving in fuel, by such improvements as I have mentioned in various parts of my address, amounts to many millions of tons every year; the saving in material and investment represents millions of dollars, which manifestly represent service of the highest value to the industry and to the country. Such work is just as much the province of the electrical engineer as improvements in the design and efficiency of the electrical units, and requires the same scientific ability, vision and industry.

While I admit to considerable prejudice in favor of things electrical, I think that in no other field of engineering has there been such a remarkable improvement and a condition which so nearly approaches, in the matter of efficiency, to 100 per cent, as has been shown in the field of electricity. This phenomenal record is not the result of accident. It has been due to the enthusiastic devotion of the scientist and engineer and executives to their work. They have not been satisfied with things as they are, or with mediocrity. They have wanted the best; have not been contented with a 75 per cent to 80 per cent efficiency when something better was obtainable. The causes of inefficiency have been scientifically attacked; the losses have been studied and their causes discovered and removed.

Concurrently with the improvements in the efficiency of conversion, the engineer has studied ways and means to reduce the amount of material and the amount of labor required to produce a given effect, and has been equally successful in increasing the effective use of material and labor, and as a result, until interrupted by the war, the cost of electrical machinery and devices of every description has shown a progressive reduction, not only without sacrifice of quality, but with great improvement in quality. This truly marvelous work, we can safely affirm, is the foundation of the phenomenal growth, prosperity and present commanding position of the electrical industry, which is a monument to the broad vision, intellectual honesty, faithful work and the correct economic viewpoint of the electrical engineer and his co-workers.

Presented by Publication only

(*Subject to final revision for the Transactions.*)

A DIRECT-CURRENT GENERATOR FOR CONSTANT POTENTIAL AT VARIABLE SPEED

BY S. R. BERGMAN

ABSTRACT OF PAPER

The standing of this problem in the past is briefly reviewed. As far as is known to the author the type of machine described presents a new solution, the advantages of which are shown to be:

1. The machine is self-excited, *i. e.*, regulates independent of any other source of potential.
2. The machine regulates independent of speed and load and may be compounded.
3. The regulation is inherent, *i.e.*, no external regulating device is used.
4. The regulation is instantaneous.
5. The regulation is approximately independent of the heating.

The theory of the machine is described and diagrams of connections given. Performance curves obtained from tests are shown.

A method is described whereby instantaneous regulation of the voltage is obtained, which method also secures approximately a constant voltage independent of the heating.

Finally there follows a discussion of the efficiency of the new machine as compared with a standard machine of the same speed and output.

A PROBLEM in direct-current engineering, especially met with in traction, is that of producing constant voltage at variable speed. Train lighting from axle-driven generators is one of its oldest applications. The transformation from a variable to a constant voltage can be accomplished by revolving machinery provided that the principle of producing constant potential, independent of the speed, is known. Of late years this problem has received new impetus with the introduction of electric starting and lighting of automobiles and a great number of solutions have been offered, most of them depending upon the presence of a storage battery which greatly simplifies the conditions, since the steadying influence of the battery results in a source of nearly constant potential from which a constant excitation may be drawn. The regulation in the great majority of such charging sets depends upon the load current and is spoiled if the load is disconnected; *i. e.*, if the battery is not in the circuit the generator voltage varies greatly with the speed. Other systems exist employing automatic shunt regulators either oper-

ated magnetically or depending on combinations containing resistors with negative temperature coefficients.

In railway and automobile applications it is desirable to avoid all automatic apparatus if possible and it occurred to the writer some years ago that it would be possible to produce a generator which would regulate on constant potential independent of the speed and load, due to inherent properties of the windings, that is, without the use of any kind of regulator. The conditions to be fulfilled appear as follows:

1. The machine should be self-excited, *i. e.*, should regulate independent of any other source of potential.

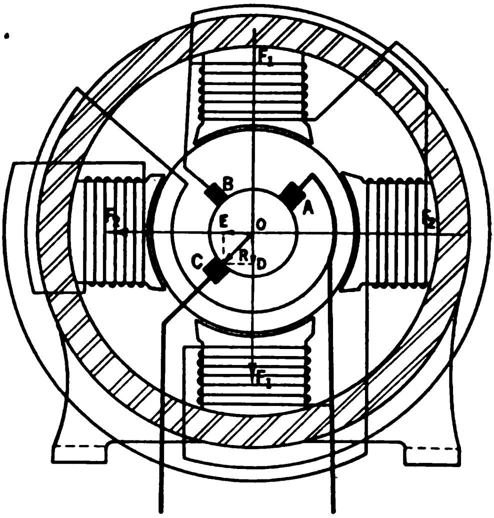


FIG. 1—CONSTANT POTENTIAL GENERATOR

2. The machine should regulate independent of the load and should possess the properties of compounding.
3. The regulation should be inherent.
4. The regulation should be instantaneous, *i. e.*, no over- or under-shooting of voltage should occur at sudden speed variations.
5. The regulation should be independent of the heating, *i. e.*, same potential generated when machine is cold as when hot.

A number of machines fulfilling the above conditions have been built and are now in successful service. In the following pages the principle of operation will be described and illustrated by characteristic curves obtained from tests.

Fig. 1 illustrates, diagramatically, the principle of the machine. The armature is series wound and the field contains twice as many poles as the number of poles for which the armature is wound. In the case illustrated, the armature is wound for two poles and the field contains four poles, symmetrically located. The load is taken from the brushes *A* and *C* and in addition to these load brushes there exists a third brush *B*, placed 90 electrical degrees from the load brushes. The field may be considered

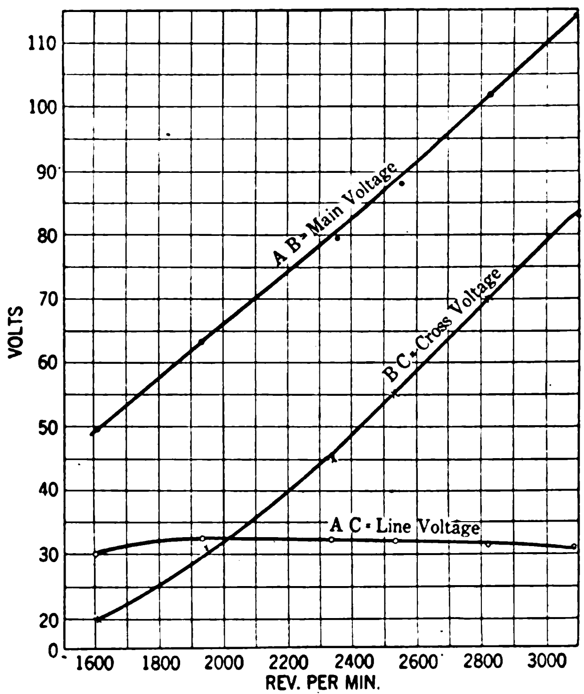


Fig. 2—Constant-Potential Generator
1.5 kw.—32 volts—1600/3200 rev. per min.

to consist of two independent magnetic circuits, one of these circuits *F*1 is saturated and the corresponding flux ϕ_1, will be called the main flux of the machine. The second magnetic circuit *F*2 is not saturated and the corresponding flux ϕ_2 will be called the cross flux. The flux ϕ_1 generates, between the brushes *A* and *B*, an e. m. f. which will be called the main voltage of the machine. The flux ϕ_1 does not generate any e.m.f. between the brushes *B* and *C*. Similarly the flux ϕ_2 generates, between the brushes *B* and *C*, an e.m.f., which will be called the cross voltage.

This flux does not generate any e.m.f. between the brushes A and B. The excitation of the machine is taken from the brushes A and B for which reason the brush B will be called the exciting brush. The excitation consists, as shown, of two multiple branches, one branch exciting the main poles and the second branch exciting the cross poles. The two fluxes ϕ_1 and ϕ_2 are entirely independent of each other, which may be verified by separately exciting the machine, and tests show that if the main excitation is varied only the main voltage is affected, the cross voltage remaining constant; and if the cross excitation is varied only the cross voltage is affected, the main voltage remaining constant.

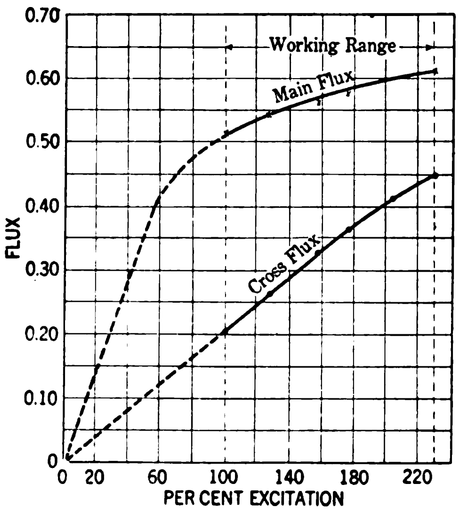


FIG. 3

The directions of the main and the cross fluxes are such that the difference between these fluxes is interlinked, with the line brushes A and C, as shown in the figure, and thus the line voltage AC is the difference between the main voltage AB and the cross voltage BC. Hence, the voltage $AC = AB - BC$. Since the main circuit is saturated the flux ϕ_1 remains constant and the main voltage AB is proportional to the speed. Therefore, the excitation of both the main and the cross fields is proportional to the speed. The cross circuit not being saturated the cross flux ϕ_2 will increase in proportion to the speed and, hence, the cross voltage BC must increase with the square of the speed. As will be shown by saturation curves the cross circuit should not be

entirely unsaturated since then the cross voltage would increase too fast and the machine voltage would decrease with increasing speed. The cross flux should approach saturation and it is possible to choose the saturation of the cross magnetic circuit, so that the line voltage *A C* remains constant. In Fig. 2 is shown the variation of the different voltages with the speed. Since the excitation is taken from the brushes *A* and *B* the variation of the exciting ampere-turns follows the main voltage *A B* and may, therefore, in proper scale, be read from the curve *A B*. From these curves may be determined the amount of flux in each magnetic circuit for any given excitation and in

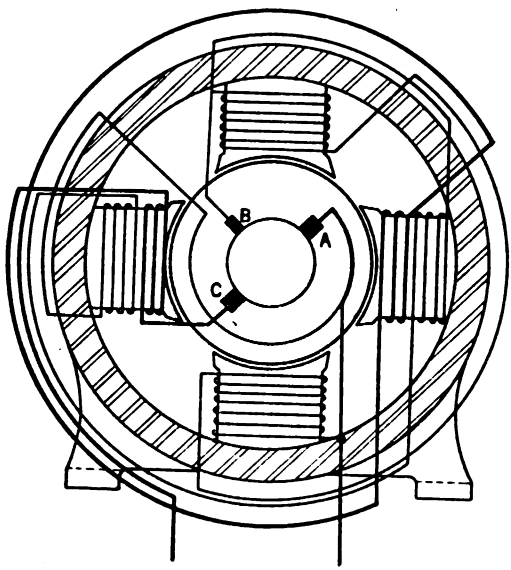


FIG. 4

Fig. 3 are plotted the two fluxes against the excitation. These saturation curves show that over the working range the main magnetic circuit is nearly saturated and the cross magnetic circuit at first is unsaturated but from a certain point, where the curve bends, this circuit starts to approach saturation.

Since the line current is taken from the brushes *A* and *C*, Fig. 1, there exists an armature reaction *O R* in the direction of *A C*. This armature reaction may be resolved in two components, one *O D* in the direction of the main flux and the other *O E* in the direction of the cross flux. The main magnetic circuit being saturated, the additional excitation, due to the armature re-

action, cannot add anything to the main flux. The component *O E* in the direction of the cross flux will, however, interfere with this flux and would disturb the regulation of the machine and in order to overcome this influence a series winding is added to the cross poles. This winding should have an equal and opposite strength to the armature reaction working in this direction, and will be called the compensating winding. The location of this winding is shown in Fig. 4. It will be easily understood that by changing the strength of the compensating winding it is possible to obtain either rising or falling machine voltage with increasing load. By over-compensation the voltage will rise with the load and with under-compensation it will fall. Obviously the strength of the compensating winding can be made

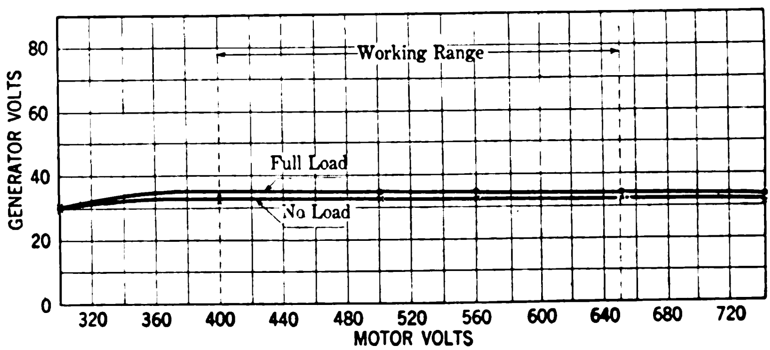


Fig. 5

to compensate for the *I R* drop resulting in a flat compounded generator.

If a generator of the type under consideration is coupled to a motor such a motor-generator is capable of transforming from a variable voltage to a constant voltage. Fig. 5 shows the performance from test of a motor-generator, the motor voltage varying from 400 to 650 and the generator voltage being maintained constant at 32 volts. As will be seen from Fig. 5 the generator voltage rises slightly with the load showing that the machine is over-compounded.

If the motor of a motor-generator of this type is thrown directly on the line a voltmeter connected to the generator shows that the voltage, due to sudden acceleration, exceeds the rated voltage. The amount of this over-shooting of the voltage depends upon the relative values of the time constants of the

two multiple exciting circuits. For example; assuming that the rate of increase of the exciting current is nearly the same in both circuits then the flux in the unsaturated circuit will appear at a later time than in the saturated circuit. Consequently the negative voltage will not develop fast enough, resulting in the phenomenon which may be called over-shooting of the voltage. It is possible, however, to speed up the rate at which the cross magnetizing current increases by insertion of resistance in this circuit. The complete diagram of connections is shown in Fig. 6 in which the permanent resistance is designated as *R*. Tests

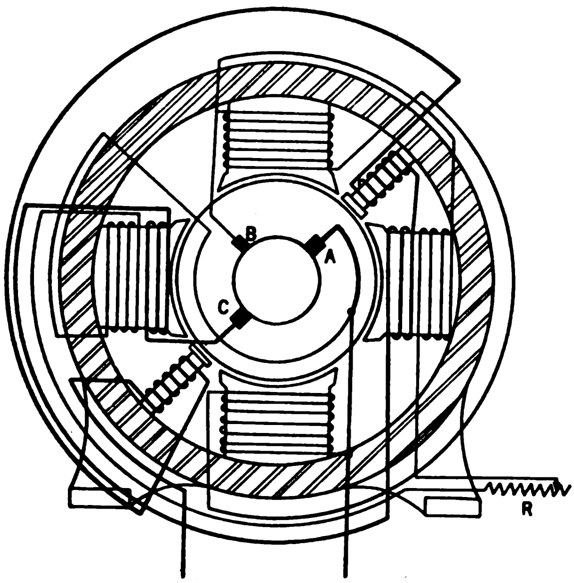


FIG. 6

have demonstrated that if this resistance is properly chosen it is possible to entirely eliminate over-shooting of the voltage. In this figure has also been added commutating poles to assure perfect commutation.

The resistance *R* also serves another purpose, namely, to keep the regulation of the machine approximately the same when the generator is cold as when hot. Assuming for the moment that the generator was built in accordance with Fig. 4, without any resistanoe in the cross circuit, it is obvious that when the generator is cold the current in both exciting circuits will be larger than when hot. The main flux will not be influenced by this increased

excitation, since saturation exists. On the other hand, the increased exciting current of the cross circuit will cause an increase in the cross flux resulting in an increase of the cross-voltage. Since this voltage is negative it follows that when the machine is cold the line voltage will be too low. By inserting a resistance of zero temperature coefficient the variation of the tests cross excitation is limited and test show that if the resistance R is approximately $\frac{1}{3}$ or more of the resistance of the cross exciting circuit, the change of excitation, due to heating, will be limited to such an extent that the variation of voltage is less than 5 per cent, corresponding to a temperature range of 100 deg. cent.

As has already been explained this generator is in reality a combination of two independent generators, one bucking the other, which leads to the conclusion that the machine must become somewhat larger in size and weight than a standard machine of same speed and output. On the other hand, there are several other conditions which tend to make this machine more economical than the standard. When the speed increases all flux densities and hence the stability increases. In a machine of standard make in which the shunt excitation is regulated in order to maintain constant voltage, the flux densities and hence the stability decreases with increasing speed. Therefore, a standard machine requires much larger gaps than this new machine and furthermore, the armature reaction in the standard type must be chosen with due consideration to the speed range. In order to give an approximate idea of the size and weight of this new type of generator, as compared with a standard type of machine, it may be stated that by a sacrifice in the efficiency of approximately 10 per cent, the new machine will have the same weight as a standard type of same rating.

As has already been pointed out this new type of constant potential generator shows some distinct advantages over a standard generator with automatic voltage control. Where reliability of service is concerned this new type is superior since there does not exist automatic devices of any kind for maintaining the regulation. The new machine has also the advantage of being instantaneous in action even at violent and large speed variations. Therefore, it is thought that this new machine will create a field of its own and satisfy a long felt need in certain applications of the electrical industry.

Presented at a Section Meeting of the American Institute of Electrical Engineers, Philadelphia, Pa., April 8, 1918.

INDUCTIVE EFFECTS OF ALTERNATING CURRENT RAILROADS ON COMMUNICATION CIRCUITS

BY H. S. WARREN

ABSTRACT OF PAPER

A brief discussion of inductive interference in general is first given, including reference to the work of the Joint Committee on Inductive Interference in California. Inductive interference due to electrified railroads is then taken up and various possible means for reducing such interference considered. A description is given of four important installations of railroad electrification and the specific means adopted in each case for preventing interference, with the degree of success which has been met with.

INTRODUCTION

TELEPHONE and telegraph companies in the conduct of their business not only have to maintain their lines and service against ordinary forms of interruption, such as lightning disturbances, mechanical failures, etc., to which all overhead electrical lines are inherently subject, but also, they have to see to it that their lines are protected against interference by electric power lines. Such interference may be due to actual or threatened physical contact between wires of the two systems, to passage of current from one system to the other by leakage, or to the class of disturbances known as "inductive."

One important kind of inductive disturbances on telephone and telegraph lines is that arising from installations of alternating current railroad electrification, such installations being principally employed in connection with trunk line railroads carrying heavy traffic.

In approaching the subject of interference to communication systems by such electrified railroads, it seems desirable first to consider some aspects of the general subject of inductive interference from power lines, of which interference from electrified railroads is a special case. In this general discussion of the subject it will be convenient to borrow freely from the work of the Joint Committee on Inductive Interference in California.

Historical

The induction of voltages in an electric circuit by current changes in a parallel circuit was discovered by Faraday in 1831. The property of self induction of an electric circuit was discovered by Joseph Henry at about the same time. The phenomenon of induced electrostatic charges was known already through various experiments. In 1838 Henry, in the course of his researches, observed that a current was induced in an electric circuit when a Leyden jar was discharged through a parallel circuit. This seems to be the first case on record of electric induction between circuits.

Since the time of Faraday and Henry a stupendous amount of electrical research and experimentation has been conducted and as a result of this and of brilliant theoretical work by Maxwell and others, the fundamental laws of electrostatics and electrodynamics has been very fully worked out. The general equations expressing the laws of induction are, however, not suitable for us in the solution of practical problems. On account of the large number of factors involved, many of which cannot readily be evaluated, it is generally necessary in specific cases to resort to simplifying assumptions and approximations.

As to what actually happens when a voltage is set up in an electric circuit by induction we know very little. It is well to bear in mind that we do not even know what voltage really is, or current, or electricity. We use these terms merely to express certain phenomena and relations found by observation, whereby we are able to derive results of much practical value.

Characteristics Affecting Disturbances. Telephonic currents consist of numerous component simple currents varying in frequency from about 200 to 4000 or more periods per second. The signaling currents employed on telegraph lines are also complex, but their most important components are of less than 300 periods per second.

Power circuits are commonly of either 25 or 60 periods per second but, in addition to the fundamental periodicity, harmonics are usually present to a greater or less extent and it is due almost wholly to the harmonics which come within the range of the most important voice wave components, that noise is produced in telephone circuits by induction. By care in designing electric power machinery so as to reduce the proportion of such harmonics, it is probably feasible to avoid much of the noise disturbance to telephone circuits which otherwise would result.

Thus a small expense in eliminating this trouble at its source may obviate a much larger expense later.

Abnormal Conditions. It is important to recognize that the inductive effects of power circuits are liable to be greatly magnified at times of abnormal conditions. The violent changes which occur in the electric and magnetic fields surrounding a power circuit, when one of its conductors breaks or becomes grounded, set up in neighboring communication circuits surges which may represent relatively large amounts of power. A parallel which, under normal operating conditions, causes no disturbance may produce very serious interference under abnormal conditions.

Balanced and Residual Voltages and Currents. In analyzing inductive phenomena it is advantageous to classify power circuit voltages and currents under two general heads: (1) Balanced voltages and currents, that is, those which are balanced or symmetrical with reference to the earth; (2) residual voltages and currents, that is, those which are wholly unbalanced with reference to the earth. The circuit of the residual voltages and currents is comprised of the metallic circuit conductors as a group, constituting one side, and the earth (including earthed conductors), constituting the other side.

At every instant the algebraic sum of the balanced currents, and likewise of the balanced voltages, is zero. Hence, at any instant the algebraic sum of the currents in the line conductors is the residual current and, similarly, the algebraic sum of the voltages to earth of the line conductors is the residual voltage. For example, in an ordinary railway circuit, consisting of overhead trolley wire and grounded rail return, all the voltage and current are residual.

There is no definite relation between these two classes of voltages and currents and entirely different means usually have to be taken in counteracting their respective induction effects. In general, the residuals cause more inductive disturbance than the balanced components as the residuals are all in phase and their effects are cumulative whereas the induction from the balanced voltages and currents in one conductor is partially neutralized by the induction from balanced components in each other conductor of the circuit. Also the residuals usually contain a larger proportion of harmonics than the balanced components and this is another reason why they are likely to cause more disturbance, particularly to telephone circuits.

Causes and Preventive Measures for Residuals. As residual

voltages and currents are largely responsible for inductive interference, it is of great importance in the prevention of such interference that they be suppressed or reduced by all reasonable means available.

Star-connected, three-phase transformer banks with the neutral grounded will set up triple harmonic residuals due to variation of permeability of the iron with varying magnetic density and may also cause residuals by reason of inequalities of the transformer impedances. In general, the most effective measure against this triple harmonic effect is the use of a delta-connected secondary or tertiary winding, thus providing a shunt path for the triple harmonic currents. The magnetic density also should be kept as low as practicable. Residuals due to inequalities of impedances in transformer banks can, of course, be eliminated by equalizing these impedances.

Another condition which may produce large residuals is the use of generators with star-connected armature windings. When such a generator with its neutral grounded is connected to a power line, either directly or by autotransformers, residuals are set up in the line. When such a generator, with its neutral not grounded, is connected to the line through standard transformers, residuals will be impressed on the line if the transformer bank is connected star-star, the line side neutral grounded, and the station side neutral connected to the generator neutral.

Grounding of transformer or generator windings at any points not normally at zero potential, unbalances an electrically connected circuit and thereby causes residual voltages and currents.

Another important cause of residuals, which however may not be so obvious, is the unbalance of a power circuit due to inquality of the capacitances to ground of its several conductors. If the three line conductors of a three-phase circuit are carried throughout in the same relative positions, that is, if the circuit is not transposed, the capacitances to ground will be unequal and a part of what would otherwise be balanced voltage and current becomes residual. These inequalities may be overcome by transposing the power circuits throughout their entire length. To make such transposition effective with respect to interference to telephone circuits it is necessary that the power circuits be transposed at intervals which are short in comparison with the wave lengths of the higher harmonics present. The frequency of transpositions depends somewhat, however, on the inherent unbalance of the conductor configuration. As an indication of

the number of transpositions required for a reasonable degree of balance to ground, it may be said that barrels of 6 to 12 miles, the latter applying to a triangular configuration, are usually adequate.

It is evident that the symmetry of a line which has been thoroughly transposed may be destroyed by connecting to it branches or taps which unbalance the capacitances to earth of the line conductors. Of course, if such a tap is grounded, the residuals resulting may be very large.

At times of accident, when a power circuit is in an abnormal condition, residuals of relatively enormous values are liable to be created. These set up correspondingly large induced voltages in parallel communication circuits. This emphasizes the importance of high grade construction and maintenance of power lines involved in parallels so as to minimize the frequency of such occurrences.

Unbalance of Communication Circuits. Not only are there these two kinds of disturbing voltages and currents on power circuits, namely, balanced and residual, but each of these components may set up, in a neighboring metallic communication circuit, two different effects; (1) an induced voltage between the two conductors of the communication circuit, which directly tends to cause currents through the signaling instruments; (2) an induced voltage between the conductors of the communication circuit and ground, which by reason of unbalances in the communication circuit indirectly causes currents through the signaling instruments. Theoretically, assuming a telephone circuit and all its connected apparatus absolutely symmetrical, electrically, with respect to earth and always so maintained, voltages induced equally in the two sides of such a circuit would not cause noise in the telephone. In practise it is not possible to attain absolute symmetry although in well constructed and well maintained telephone circuits the degree of balance is very high indeed. The telephone is, however, such a very sensitive instrument that no attainable degree of balance can avoid noise when relatively very high voltages are induced between the telephone wires and ground, as is done in many cases by parallel power circuits. It is therefore essential that induced voltages to ground be limited to values which are permissible on communication circuits so maintained.

Transpositions Within Parallels. Interference by induction from balanced currents and voltages can most readily be pre-

vented by means of a coordinated system of transpositions applied to both power and communication circuits within the limits of parallels, the term "parallel" being understood to mean the region within which the two classes of line are in sufficiently close proximity for inductive disturbances to be set up in the communication circuits by the power circuits. It is to be noted that transpositions for this purpose to be applied to power circuits within the limits of parallels, are quite distinct from the transpositions previously referred to as being necessary throughout the entire length of a power circuit in order to equalize the capacitances to ground of the several conductors.

Principal Factors in Determining Interference. Before leaving this general part of the subject I will enumerate the principal factors which determine the amount of induction and whether it is sufficient to constitute interference.

1. *The length of the parallel.*

Other things being equal, the longer the parallel, the greater the induced voltage.

2. *The separation of the two classes of lines.*

In general, other things being equal, the less the separation of the power line and communication line, the greater the induced voltage.

3. *Configuration of the power line.*

The investigations of the Joint Committee on Inductive Interference show that the configuration of the power line has an important bearing on inductive effects, the relative merits of different configurations varying with the separation of the power and communication lines, the spacing of the power conductors, and the relative importance of balanced voltages and currents. While it is not possible to draw a simple general rule for determining the most advantageous configuration the differences in particular cases are marked and deserve special attention as oftentimes substantial benefit can be secured in this way at small additional cost. This is particularly true of multiple circuit lines, the resultant induction depending largely on the relative poling of the power circuits.

4. *The magnitudes and fundamental frequency of the normal operating voltages and currents of the power circuits.*

The effect of electric induction, of course, is proportional to the voltage of the power line, and of magnetic induction to the current on the power circuit.

5. *The magnitudes of residual voltages and currents.*

It has already been explained that residual voltages and currents are a principal cause of inductive interference. Hence, while the amount of residuals on metallic circuits is usually small as compared to the balanced components, the inductive effects of the former are liable to preponderate.

6. *The wave shapes of both balanced and residual voltages and currents, involving the magnitudes and frequencies of all harmonics.*

The effect of wave shape on interference, to telephone circuits particularly, is exceedingly important. Wave shapes in practise on different power systems are found to have extremely wide variations. An unfavorable wave shape, *i. e.*, one having a large proportion of high harmonics, may produce a hundred times as much noise as a pure sinusoidal wave of fundamental frequency.

7. *The unbalances of the communication circuits, their magnitude, character and location.*

Such unbalances are caused by inequalities in resistance, inductance, insulation or capacitance to ground. The last mentioned quantity is balanced approximately by transposing the conductors. The other elements enumerated require proper design, construction and maintenance of these lines, whether in open wire or in cable, together with their connected apparatus.

8. *Terminal apparatus of the communication circuits and the distance of such apparatus from the parallel.*

The sensitiveness of the terminal apparatus is, of course, an important factor in determining the allowable amount of induced voltage. Also if the parallel is at a considerable distance from either terminal of the communication circuit, the induced voltages and currents may become considerably attenuated before reaching the receiving instruments.

9. *The voltages and currents of the power circuits under abnormal conditions.*

It has already been stated that the voltages and currents of power circuits under abnormal conditions, which are liable to be largely residual in character, produce the most severe inductive effects. The values of these quantities under abnormal conditions, in relation to corresponding values under normal conditions, vary a great deal on different power systems.

10. *The number of parallels which may effect cumulatively the same communication circuit.*

In many cases the same communication circuit, especially if

it be on a long trunk line, may be involved in a considerable number of different parallels. In such cases the induction contributed by each parallel must be sufficiently restricted so that the cumulative results from all will not produce disturbances which cannot be endured.

11. *The importance and character of use of the communication circuit.*

It is obvious that the more important circuits, on which interference is most serious, should be afforded a higher degree of immunity from disturbance than circuits of less importance. Also, of course, the character of the communication circuit as, for example, whether it is a telephone or telegraph circuit, is of fundamental importance in considering the question of inductive interference.

12. *The volume of transmission on the communication circuit.*

In case of a long distance telephone circuit, where the volume of transmission is small, a less amount of extraneously induced current will interfere with receiving than on circuits of less length where there is a large volume of transmission.

13. *The relative cost of preventing interference.*

While in all cases means should be employed which will allow adequate communication service to be given, still it is not expected that complete freedom from inductive disturbances can be attained. Any induced voltage, no matter how small, will generally cause some impairment of service. The amount of induced voltage which it is justifiable to allow, depends to some extent on the difficulty and expense involved in further reducing such voltage. After the foreign voltage has been reduced to an amount which can, if necessary, be tolerated, it becomes simply a problem of balancing the value of further improvement against its cost.

It will be seen that the number of elements affecting inductive interference is quite large. Moreover, some of these elements, as for example, the wave shapes of the power circuit voltages and currents, are not ordinarily known, and have induction-producing values varying enormously in different cases. Hence the difficulty of formulating any simple method of determining in advance whether a given construction will or will not produce interference.

The foregoing discussion, while general in its application, is in many respects concerned with induction from power transmission lines. We will next consider specifically some of the inductive effects of alternating current railroad installations.

ALTERNATING-CURRENT RAILROAD ELECTRIFICATION

The reasons why alternating-current railroad electrifications cause large disturbances to neighboring communication lines, principally by electromagnetic induction, will now, I think, be apparent when it is considered that, (1) the railroad trolley current is large, (2) it is all residual current, (3) the railroad circuit, from its nature and use, is more subject to abnormal conditions, such as short circuits, than ordinary power transmission lines.

Classes of Interference. Some of the different ways in which disturbances due to alternating current electrified railroads manifest themselves in the telephone and telegraph plant, may be classified as follows:

1. *Interference with operation.*
   - a. Interruption of service
   - b. False bell ringing
   - c. Noise
   - d. Interference with telegraph signals
2. *Physical injury to plant.*
   - a. Fire hazard
   - b. Magnetization of loading coils
3. *Hazard to employees and to telephone using public.*
   - a. Electric shock
   - b. Acoustic shock

These various disturbances may be of a most serious nature and telephone and telegraph companies are unable by themselves to cope with the problem of protecting their lines and service against them. In order to make this more clear, we will review briefly some of the fundamental characteristics of telephone service and point out some of the distinctive features of the plant required to make this service possible.

The Telephone System. The fundamental electrical problem of telephony is three-fold:

1. The production of an electrical wave which is a faithful copy of the spoken word.

2. The transfer of this wave without appreciable delay over distances which may amount to hundreds or thousands of miles, without excessive change of form by distortion, without the accession of foreign disturbances, and without undue loss of intensity.

3. The production at the receiver of an audible sound wave

which is an adequate counterpart of the electrical wave and, therefore, of the original spoken word.

As speech is carried on telephonically by means of an extremely small amount of energy, it is necessary that a large part of the telephone plant be of a sensitive and delicate construction. This includes the subscribers' sets where occur the delicate transformations from air wave to electrical wave and *vice versa.*

These substation instruments cannot be located at central offices where they would be under the immediate supervision of a trained staff but they must be placed in the subscriber's office, factory, home or wherever they will be most available and convenient for his instant use. There are now over ten million telephone stations in the Bell System. These sensitive nerve ends of the telephone system are distributed throughout the entire country in every conceivable variety of location.

In addition to the delicate substation apparatus, each telephone conversation requires the exclusive use of a connecting circuit. Even though the circuit be hundreds of miles in length it cannot be used for any other telephonic purpose. This exclusive circuit must be low in resistance, capacity and leakage so as not unduly to attenuate the telephone wave. It must be so transposed, balanced and protected that so far as possible it will not pick up electrical disturbances from earth currents telegraph lines or other telephone circuits or itself constitute a source of disturbance to the latter. The network of telephone circuits now comprises more than twenty-two million miles of wires.

In addition to meeting the above basic requirements, the telephone system, in order to realize its potentialities as a utility of the greatest benefit to the public, must include facilities such that at any time on request of a subscriber connections can be made between any two points, without delay or other inconvenience, and the charges for the service must be as low as possible. At present about thirty-two million such telephone connections are made per day in the Bell System.

Prompt, efficient and economical service on the existing scale requires that an immense number of separate circuits be brought together into common central offices and provided with every device and attendance which will facilitate traffic over the system. It requires, for example, that hundreds of wires be crowded into cables, the latest types of which have 2400 conductors within a sheath whose outside diameter is 2 ⅝ inches. It requires

great congestion of wires and apparatus in switchboards in order that many thousands of lines may be brought within reach of a single operator. It requires elaborate and reliable signaling arrangements to economize time and circuits. It requires uniformity in plant and methods throughout the entire system so as to make possible prompt connection between any two points. While it has been found practicable to devise means for transmitting the required signaling currents over the telephone plant safely, the danger of fires from the currents and voltages employed for signaling has been avoided only by the exercise of extreme care, although these currents and voltages are very small compared with the currents and voltages on power lines.

From this brief consideration of the telephone problem, showing that a large portion of the telephone system is inherently of a delicate nature and susceptible to interference, it is clear that telephone apparatus and circuits would be destroyed if but a small fraction of the powerful currents and voltages used by other electric utilities were permitted to enter into the telephone system.

Values of Induced Voltage. In studying the inductive effects of electrified railroads, it has been found advisable to determine approximately the amount of induced voltage in a communication circuit, per mile, per 100 amperes in the trolley, for different horizontal separations between the trolley and the communication circuit, and with different percentages of the trolley current in the rails. For example, it has been determined that, with 60 per cent. rail current (that is, 40 per cent of the trolley current return flowing in the earth as stray current,) the induced voltages per mile, per 100 amperes in the trolley, are in general about 10 volts, 5 volts and 1 volt, at 50 feet, 300 feet and 4000 feet separation respectively. Thus at 50 feet separation, with 1000 amperes in the trolley, a ten mile exposure would result in 1000 volts induced. These are maximum figures in that they are based on the assumption that power is supplied in one direction only. It should be understood that the induction varies considerably in different cases since the induced voltages are affected by all the various conditions which go to determine the course that the stray current takes. Some parallelisms may extend more than ten miles and at times of short-circuit the current may amount to many thousands of amperes, and, in such cases, the induction is liable to be correspondingly more severe unless preventive measures are taken.

The specific effects of these induced voltages will now be touched upon briefly.

Interruption of Service. Induced voltages may be high enough to operate the telephone protective devices and, if the current across the protector is sufficient, the line will become permanently grounded and the telephone service interrupted until the protector is restored to normal condition. If the protector is located in a central office, the time required to make repairs is relatively short, but if it is at a subscriber's premises, considerable time may be required for a repair man to reach the station. In cases where the operation of the protector does not actually ground the line, it may lower the insulation resistance, sufficiently to make the line noisy.

It may also sometimes happen that foreign voltage of a value below that required to break down the protector spark gap will yet be sufficient to puncture the insulation of the wiring at some point.

False Bell Ringing. Voltages of about 8, 20 and 200 volts, depending somewhat on the prevailing earth potentials, are sufficient to ring ordinary grounded bells, standard biased bells, and (by breaking down protector spark gaps) metallic circuit bells, respectively.

An accidental trolley ground on a 25-cycle single-phase electrification through a thickly settled community may ring scores or even hundreds of subscribers' bells, some of which may be located a mile or more from the railroad. Such false bell ringing is apt to be a source of serious complaint by subscribers, and is particularly annoying when it occurs at an unseasonable hour as, for example, 5 o'clock in the morning.

Noise. In order to appreciate the effect of small currents in producing noise in telephone circuits, it must be considered that a very small fraction of a microwatt of power at voice current frequencies will produce an audible sound in a telephone receiver and a few microwatts are sufficient for a telephone conversation in a quiet place. When the current in the telephone receiver caused by induction from outside circuits is large enough to produce an audible sound, it has an important effect on the efficiency of the circuit for transmitting speech, particularly when the circuit is used for talking over long connections so that the energy of the voice currents approaches the minimum which will give a satisfactory conversation. An extraneous sound which is scarcely more than audible to an

untrained ear and might be thought to be of negligible consequence, has in reality, the effect of impairing a telephone circuit by a large percentage, or otherwise expressed, of destroying a material part of the circuit's value for service purposes.

The interfering effect of foreign current of a given magnitude depends very greatly, however, upon the frequency. The maximum effect is for current having a frequency of about 1100 cycles per second. At lower frequencies the effect falls off rapidly, and at 25 cycles is probably only about one two-thousandth as great as at 1100 cycles. This fact explains why the inductive interference to telephone circuits from 25-cycle railway systems is not predominantly noise. Twenty-five-cycle current normally has relatively very small components in the telephone-frequency range and the effect of these high-frequency components is damped out much more rapidly than that of the fundamental by separation of telephone and railroad circuits. Noise from such railways is, however, present to some extent, and is liable to become serious under any conditions which produce a bad wave shape in the power circuit.

Interference with Telegraph Signals. At 25 cycles, an induced current of one milliampere is liable, under some conditions, to interfere with ordinary Morse transmission, while rapid telegraph systems, printers, etc., are more or less impaired by extraneous currents of any value.

Fire Hazard. The use of heavy insulating coverings for wires in telephone switchboards is impossible on account of the necessity of bringing many lines within a limited space. It is not feasible to employ for this purpose such insulation as is considered good practise for electric light and power wires. Thus it is unavoidable that the dielectric strength of the telephone wiring be relatively low.

Investigations of the fire hazard due to foreign voltages impressed on telephone lines indicate that voltages of 200, or even less when backed by considerable power as in the case of induced voltages from alternating-current railways, create a distinct fire hazard.

Although the fire hazard brought about by railroad electrification is due chiefly to the higher voltages induced at times of short circuits on the railroad, it is possible that the repeated electrical stresses of lower voltage, due to normal railroad operation tend to decrease the dielectric strength of the insulation and thereby facilitate breakdown.

Magnetization of Loading Coils. Loading coils in very large numbers are now employed in both open wire and cable telephone circuits. These coils are liable to be permanently magnetized by any induced currents which are materially in excess of telegraph currents. While they are magnetized there is a considerable loss of transmission efficiency and it is ordinarily impossible to demagnetize them without removing them from the circuits. Moreover large currents through the loading coils may permanently reduce the permeability of the iron cores and make them unsuitable for use on long toll cable circuits.

Electric Shock. At times of short circuits on the railroad and sometimes during switching operations, electrical surges may be set up in the telephone circuits, which are of sufficient intensity to produce electrical shocks to persons at the telephone or working on the circuits at the time. While it is improbable that such shocks will be the cause of serious personal injuries, even minor shocks are objectionable and constitute a basis of complaint as the public expects telephone instruments to be perfectly safe at all times.

Acoustic Shocks. Inductive surges such as are capable of producing electric shocks to persons are also liable to cause loud noises in the telephone receivers which may result in acoustic shocks to persons using the telephone at such times. Even the relatively slight clicks which sometimes occur due to battery interruptions may be very annoying to telephone users and acoustic shocks sometimes caused by induced voltages may be much more severe.

Investigations and Experiments. Since 1905, when first notified of the intention to install single phase electrification on a section of the New York, New Haven and Hartford Railroad, the American Telephone and Telegraph Company has done a large amount of work on plans, tests, experiments and studies of various kinds, most of it in conjunction with representatives of railroads and the electrical manufacturing companies, all with the general object of finding means for protecting the telephone and telegraph lines and service against interference from electrification installations. A considerable amount of work has been done in connection with various electrification projects which have not been installed, some of these projects having been abandoned, at least in so far as the specific plans under consideration are concerned, while in other cases the matter is being held in abeyance, awaiting more favorable conditions for undertaking construction.

Means for Preventing Interference from Alternating Current Railway Electrifications

There are various means which have been proposed, some applicable to the railway system and some applicable to the affected communication systems, for preventing or reducing inductive interference. Some of these means have not been found successful or advantageous in practise while others have proved beneficial in varying degrees. It will be of interest to consider briefly some of these proposals.

Separation. The most effective means is to avoid the parallel, wherever practicable, by keeping the communication circuits and electrified railroads sufficiently separated. With the extension of electric traction, and the constantly increasing importance and efficiency of communication circuits, the avoidance of parallels will be increasingly important. However, this first rule for preventing interference, is unfortunately, not one which can be generally adopted in practise. Railroads and communication circuits must serve the same communities and it is necessary that the connecting routes of each be reasonably straight and direct. The field of influence of an alternating-current railroad which uses the running rails as a part of its circuit extends out to a great distance on both sides of the railroad. This makes effective separation from such electrified railroads much more difficult than separation from most other kinds of power transmission circuits.

Neutralizing Transformers. Where communication circuits are subject, under normal operating conditions of the railroad, to induced voltages sufficient to interfere with telegraph service, neutralizing transformers can be resorted to and if properly designed and connected into the disturbed circuits, such transformers will effect neutralization of a large part of the induced voltage. The transformers are provided with a plurality of windings, some of which, called primaries, are inserted in certain of the affected conductors which are grounded at or beyond the limits of the parallel, while the remaining, or secondary windings, are connected serially into other of the conductors in which the induced voltages are to be neutralized. Under favorable conditions the remaining, non-neutralized voltage, is only 5 to 10 per cent of the total induced voltage.

For a more complete description and discussion of neutralizing transformers reference is made to an article by Thomas Shaw in the *Electric Journal*, November, 1914.

Neutralizing transformers, however, have serious disadvantages from the telephone company's standpoint. The primary circuits, of which there are ordinarily from one-third to one-half the number of secondary circuits, are practically lost for telegraph purposes although they can be used for telephoning, at somewhat reduced efficiency. The secondary circuits are also reduced somewhat in telephonic transmission efficiency, but not so much so as the primaries.

Neutralizing transformers have served a useful purpose in the early stages of alternating current railroad electrification where means of restricting the railroad's field of inductive influence were not employed. They continue to have a limited field of usefulness, particularly in making endurable moderate amounts of induced voltages which remain after preventive methods have been applied at the source of disturbance, but they leave the general problem of interference unsolved. They are not applicable to subscribers' lines nor have they been found effective in neutralizing the higher harmonics which cause noise.

Drainage Coils. Drainage coils, bridged across a telephone line, with their mid-points connected to ground, provide a low impedance path for currents induced between wires and ground and thus tend to reduce the voltage. Such coils must be exceedingly well balanced or they will themselves constitute a source of unbalance and thus augment noise. Moreover, they increase the susceptibility to noise resulting from irregularities in series resistance or impedance of the telephone circuits. Also they impair telephonic transmission efficiency.

If telegraph service or direct-current signaling is employed on circuits equipped with drainage coils it is necessary to place condensers in series with the coils. The effect of this apparatus on telegraph service is distinctly detrimental.

Drainage coils have not proved to be adapted for general use on commercial systems but are helpful on private telephone circuits of power transmission companies for reducing high electrostatic charges when such private circuits are carried close to high-voltage wires.

Sectionalization of Telephone Circuit. An affected telephone circuit may be sectionalized by cutting in repeating coils at one or more points. This may be advantageous in certain cases of exposed rural lines where by placing a repeating coil at each end of a parallel it is possible to change to a metallic circuit through the parallel. It is also sometimes useful on private telephone

circuits of power transmission companies as it makes possible the insulation of the telephone sets from the exposed telephone wires. On commercial telephone systems the usefulness of this method is very limited as it introduces large transmission losses, precludes the use of telegraph and brings in difficulties in connection with line signaling.

Shielding Conductor. A copper conductor used as a shield may be strung near the disturbed communication wires and grounded at the ends of the parallel. With a conductor of suitable impedance, the current carried by the conductor will have a neutralizing effect on the induced voltages in the near-by communication wires. The action is similar to that of neutralizing transformers but less effective. In case of an aerial cable, the cable sheath itself can be so used instead of a separate copper conductor. The benefits derivable from this method, however, are very limited.

With a view to increasing the neutralizing action of such a conductor, it has been suggested that a part of the railway current could be diverted into it. The quantitative relations involved are such, however, that great difficulties stand in the way of successful application of this scheme on a commercial scale.

Resonant Circuits. Combinations of coils and condensers, adjusted to be resonant at the disturbing frequency and connected so as to reduce the disturbing current in the receiving instruments, have been employed to some extent. These afford considerable benefit for low-speed telegraph service such as ordinary hand sending. For higher-speed operation, the benefit obtainable in this way becomes rapidly less. Many modern telegraph systems operate at speeds approaching 25 dots per second which makes it impossible to differentiate in this way between the signaling and disturbing currents.

Similar methods have been suggested for reducing noise but are not usually applicable because the harmonics which cause noise are within the range of frequencies required to give good telephonic quality.

Balance and Insulation of Telephone Circuits. It is advantageous to construct and maintain telephone circuits exposed to induction with a high degree of balance and insulation. This includes an adequate transposition system. In all cases of inductive disturbance care should be exercised that these features of the affected lines are properly attended to.

Use of Relay Sets. On direct telephone lines the bell is bridged between the two metallic conductors. On two-party selective lines one bell is connected between each side of the circuit and ground. On four-party semi-selective lines two bells are connected between each side of the circuit and ground. On four-party lines, with full selective ringing, the bells are not connected to ground except at times when an operator is ringing on the line and at such times the connection of the bell to ground is established by means of relays. On all these classes of lines both sides of the circuit are grounded at the central office.

It will thus be seen that an induced voltage between the circuit conductors and ground might ring all grounded bells, but under normal circumstances would not ring bells on a direct line or at stations equipped with relay sets. However, if the induced voltage is high enough to operate the telephone protectors, a path to ground is established through the protector, and bells on direct lines or bells at relay set stations may be falsely rung.

Biasing Bells. In regions where the induced voltages are not too high, false bell ringing can be obviated by biasing the bells, that is, by stiffening the control springs so that increased voltage is required to ring the bells. Obviously, there are very positive limitations to what can be accomplished in this manner.

Measures Applicable to Railroads. The foregoing measures for obviating inductive interference are of a palliative nature and assume a condition of the electrification which produces large inductive effects. Another class of measures for avoiding interference looks to the source of the disturbances, the electrical system of the railroad, and seeks to avoid the conditions which produce large induction. This latter class has, in general, the advantage of benefiting, not one affected circuit only, but all communication circuits within the area affected.

Double Trolley. One radical and probably effective method of preventing inductive interference from single-phase railroads would be the use of a double-trolley circuit completely insulated from ground, thus avoiding the use of the running rails as a part of the railway circuit. This method, however, is distinctly unpopular with railway men, mainly for operating reasons, on account of the complexity of the overhead construction particularly in yards, and at sidings and crossovers. Purely from the cost standpoint this method might have advantages in certain cases, where the conditions of exposure are severe and other methods of restricting the earth currents are expensive to apply.

Frequent Power Supply Stations. One of the most important methods of interference-prevention is the provision of a sufficient number of substations to supply power to the trolley-rail circuit at frequent intervals. If the substations are near enough together, the amount of stray current and the average length of path of such current can be made small. It is particularly desirable that all sections of electrified railroad which are involved in parallels be supplied with power from both directions rather than by stub end feed.

Sectionalization of Trolley System. Considerable advantage may be gained by sectionalizing the trolley, thus decreasing the length of the earth current path as well as reducing the amount of power supplied to a short circuit. However, as each separate section of trolley requires an independent power supply sufficient for its maximum demand, the total transformer capacity required for a given length of electrified road is much increased by any considerable use of trolley sectionalization. Notwithstanding this objection a limited amount of sectionalizing may be used to advantage where the exposure is severe.

Opposing Polarities. On railroad lines having two tracks it is possible to connect the two trolleys for opposing polarities, so that the current flowing in the rails and earth is only the difference in the currents of the two trolleys. An instance of this method applied to a direct current railroad is afforded by the City and South London Railway in England which has been so operated for 20 years. As applied to alternating current railroads of 11,000 volts, this method is considered to have serious operating disadvantages in respect to cross-overs between tracks, and for this reason this plan, which was studied in connection with the revision of the Woodlawn-Stamford electrification of the New Haven Railroad in 1912, was not adopted.

Balancing Transformers. This method, which is now employed on the main line electrification of the New Haven Railroad, is of much benefit in reducing stray currents, particularly where power is supplied from both directions. Its use, however, involves a combined transmission-distribution circuit, tied together by the balancing auto-transformers, whereas the general practise in such matters seems to tend toward a separate transmission line supplying power to the trolley-track circuit through standard transformers.

Booster Transformers. Another important method of controlling railroad currents is by the use of booster-transformers

placed at frequent intervals along the electrified section. These transformers have a substantially even ratio of transformation, the primary winding being inserted serially in the trolley, and the secondary winding inserted serially in the track circuit. In this way the track current is required to be substantially equal to the trolley current at points where the transformers are located. By placing the transformers near together the leakage of current into the earth between transformers is made small.

A modification of this plan is to install a feeder, electrically connected to the rails at intervals, and insert the secondaries of the booster-transformers in series with this feeder. In this way the current is confined to the feeder instead of the rails. The London, Brighton and South Coast Railway embodies an installation of this type. This arrangement is somewhat more effective perhaps than trolley-track transformers but it involves considerable additional expense for the feeder, which must be of high conductivity. One advantage of the feeder-booster, over the track-booster arrangement, is that successful operation of the former is not so dependent on high grade maintenance of the track bonding.

Specific Electrifications

Having now considered various means which are available, or at least, worthy of being considered, for avoiding or reducing inductive interference to communication systems by alternating current electrified railroads, we may now direct our attention to some specific electrification installations and see what has actually been done to prevent such interference and with what degree of success. In so doing I will confine my remarks to the salient features of four single-phase installations: (1) New York, New Haven & Hartford Railroad, Woodlawn to Stamford; (2) New York, New Haven & Hartford Railroad, New Canaan Branch; (3) Norfolk and Western Railroad, Bluefield to Vivian, W. Va.; (4) Pennsylvania Railroad, Broad Street to Paoli, Pa.

Woodlawn-Stamford. The original electric installation of the New York, New Haven and Hartford Railroad Company between Woodlawn, N. Y. and Stamford, Conn., began operation, in part, in the summer of 1907. This is a section of four-track railroad, about 21 miles in length, and all power was supplied by a generating station at Cos Cob about three miles west of Stamford. To move a train at Woodlawn, the current passed for 18 miles over the trolley wires and paralleling feeders from Cos Cob to the locomotive, the remainder of the circuits from

locomotive to Cos Cob, being the running rails and earth. The telephone company's New York-Boston subway is, throughout this section, situated at varying separation averaging about 2000 feet from the railroad, a sufficient distance so that the inductive effect of the trolley current would have been largely neutralized by the inductive effect of the rail current, had these two currents been equal. However due to the long rail path, a large part of the current left the rail and spread into the earth, where its effect in neutralizing the corresponding part of the trolley current was negligible.

After full electric passenger service between Woodlawn and Stamford was inaugurated, the induced 25-cycle voltage on circuits in the New Haven subway, at normal rush hour periods, was as much as 170 volts. On the Shore Line, one of the telephone company's open wire routes between New York and Boston, the corresponding induced voltage was about 300, the higher voltage on the Shore Line being principally accounted for by about a mile and a half of exposure near Greenwich, where the average separation was only about 100 feet. The open wire Shore Line circuits were also affected by noise, which was most intense during periods of train acceleration, the pitch of the noise varying with the speed of the train. The subway circuits being in metal-sheathed, underground cable, and not having any section of very close parallelism, were not made noisy. The Midland Line, another open wire route between New York and Boston, about four miles away from the railroad at the nearest parallel section, sustained corresponding induction of about 40 volts.

Wires of the Western Union Telegraph Company, which were carried on poles located on the railroad right of way, were subjected to much higher voltages. These wires, except a few which were equipped with neutralizing transformers and continued in use by the Railroad Company, were removed to a new pole line which was built a number of miles away.

The conditions as to induction continued substantially as outlined above for a period of four or five years.

Early in 1911 the railroad company made known its intention to extend the electrification to include the Harlem River branch and the New York, Westchester and Boston Railroad. The former is a six-track line used principally for freight, extending about 12 miles from its junction with the main line, near New Rochelle, to the Harlem River. The New York, Westchester

and Boston Railroad, which is partly four-track and partly two-track, was constructed principally for suburban service and extends from West Farms at 176th Street, where it forms a junction with the Harlem River branch, to White Plains, a distance of 16 miles. A branch six miles north of West Farms taps the main line of the New Haven Railroad just east of New Rochelle. These two new lines involved the direct connection, to the Western end of the previously electrified section, of additional electrified line to the extent of about 200 miles of single track railroad. Moreover, it was planned that, after the Harlem River branch was electrified, freight trains, as well as passenger trains, on the entire system west of Stamford should be operated electrically.

This proposed large extension of the electrification, with its resulting increase in load, caused considerable apprehension to us of the telephone company. Our estimates of induced voltages under the new conditions, based on the Railroad Company's estimates as to future train loads, indicated over 1500 volts on the Shore Line and nearly 1000 volts on the subway. These values corresponded to maximum normal railroad loads and the induction would have been still greater at times of abnormal conditions. Voltages of this magnitude are far beyond the endurable limit for the telephone company and the matter was taken up by the companies with a view to determining what could be done to ameliorate the situation.

In January 1912, a joint committee of engineers, comprising a representative each of the New Haven Railroad Company, the Western Union Telegraph Company, and the Telephone Company, was formed to study this question. Several different plans for modifying the railroad distribution system were laid before this committee. After six meetings during the ensuing three months, a plan was decided upon and a sub-committee of engineers was designated to work out the details.

This new distribution system involved quite a comprehensive change in the original installation. It was cut over on January 15, 1914 and has been found to bring about a great improvement with respect to induction.

Besides an additional power supply station at West Farms, which in itself effects a considerable improvement, the new distribution system includes the use of 17 balancing autotransformers of 2000 kv-a. capacity each. These are distributed along the line, in such locations as are most advantageous for

supplying power to the trolley circuit, so as to minimize the length of path of current through the rails.

As full accounts of this distribution system have been published I will not undertake an extended description of it here.

The same system has since been used by the Railroad Company in extending the electrification from Stamford to New Haven.

At present, the induced voltages on the through circuits in the subway seldom exceed 30 volts under normal conditions of railroad operation.

The principal interference now incurred (apart from that due to the New Canaan Branch which is discussed elsewhere) is in connection with railroad short circuits in the vicinity of Stamford. Within the section from 2 miles West of Stamford to 5 miles East of Stamford, local subscribers' lines and trunk lines have been affected by false bell ringing, false flashing of line signals, and grounding of protectors, on about twenty different occasions in the past three years, an average of twenty lines being affected on each occasion. It is of interest to note that these troubles are localized within this seven-mile section of the sixty-mile electrification from Woodlawn to New Haven, and also that it is at approximately the middle of this seven-mile section that the New Canaan Branch, referred to immediately below, joins the Main Line.

New Canaan Branch. In 1907 the New Canaan Branch of the New Haven Railroad, which previously had been operated at 500 volts d. c., was reconstructed and its trolley connected directly to the 11,000-volt trolley on the main line near Stamford.

This branch is about six miles long, single track, and the traffic is light so that under normal conditions of operation no interference with telephone or telegraph lines was produced. However, the telephone circuits have been subjected to a great deal of interference from this branch, due to short circuits. Owing to the conditions of power supply, the short circuit current at New Canaan is about 2500 amperes or ten times the maximum load current. At points nearer the main line the short circuit current is even greater.

The inductive effects of the New Canaan branch were augmented by the large proportion of earth current, due to the relatively high impedance of the single track. Momentarily voltages as high apparently as 1000 were imposed on trunks between New Canaan and other places and voltages up to 500 on many telephone circuits in the New Canaan exchange. These

induced voltages operated protectors, permanently grounded and put out of service many telephone lines and subjected operators to severe acoustic shocks. Due to the recurrence of these surges, the operators in the New Canaan office became so afraid of shocks that the operating efficiency was seriously impaired.

In 1913, after several unusually severe surges from short circuits had been experienced, the matter of finding some means to overcome this interference, which had already been under consideration by the telephone and railroad companies, was taken up with renewed energy. Various plans were proposed and an extended series of experimental tests and measurements were made, as a result of which, means were finally agreed upon as follows: 1. To keep the rails well bonded. 2. To insert a current-limiting reactance in the trolley, near its junction with the main line trolley, so as to restrict the short circuit current. 3. To install 12 series booster track-trolley transformers at intervals of about ½ mile. 4. Readjust the circuit-breaker, at the junction with the main line, for instantaneous operation.

From the tests and from experience with six booster transformers, it is believed that the above mentioned measures will be effective in preventing this interference although the full installation of transformers and the current limiting reactance has not yet been completed.

It is of interest to note that the balancing transformer plan, although generally giving good results on the main line of the New Haven Railroad, does not afford an effective means for preventing inductive interference under such conditions as exist on the New Canaan Branch.

Norfolk and Western Railroad Electrification. This electrified section is between Bluefield and Vivian, West Virginia, a distance of approximately 28 miles. The railroad is double track with numerous yards and sidings and includes some heavy grades. The power house for supplying power to the electrified section is located at Bluestone, about 10.8 miles west of Bluefield. Power is transmitted by duplicate single phase transmission lines at 44,000 volts to five substations, the distances between which, respectively, beginning at Bluefield, are 8.2, 4.6, 6.6 and 4.8 miles. At these substations the voltage is stepped down to 11,000 for delivery to the trolley-track circuit which is electrically continuous throughout from Bluefield to Vivian.

The original plans for this electrification were taken up with the telephone company who proposed some modifications for

the better protection of the paralleling communication circuits. The plans as modified include 23 series booster trolley-track transformers, the average spacings of which are, east of the power house, about a mile and a half, and west of the power house about a mile. Each transformer is 100 kv-a. continuous rating and 400 kv-a. for 2 ½ minutes.

One telephone line paralleling this road has several exposures of about 500 feet separation from the railroad and there are also local circuits of the Bluefield Telephone Company in proximity to the railroad. No trouble has been reported from induction under normal railroad conditions. At times when the electrification wires are down noise has been experienced on some of the above mentioned circuits. Also a telephone trunk circuit which crosses the railroad underground is sometimes thrown out of service when the railroad circuit is in trouble, by reason of the copper block protectors at the crossing becoming grounded. Some trouble has been sustained from corrosion of lead cable sheaths at underground crossings of the railroad in Bluefield but it has not been established whether or not this is due to electrolysis by the alternating railroad currents.

Broad Street—Paoli. Plans for electrifying the Pennsylvania Railroad's four-track main line from Broad Street to Paoli were announced early in 1913. The question of interference with the telephone company's lines was taken up with the engineers representing the railroad company and several plans looking to the prevention of interference were considered. As a result of the best information then available it was decided in 1914 to install series booster trolley-track transformers for the purpose of confining the current to the rails.

The power for this electrification is conveyed by a 44,000-volt 25-cycle transmission line to three substations, one each at West Philadelphia, Bryn Mawr and Paoli, and from these substations supplied to the trolley-track circuit at 11,000 volts. Sixteen pairs of trolley-track transformers, each transformer equipping two tracks and having a continuous rating of 80 kv-a. and 600 kv-a. for one minute, are provided, the spacing between transformers being about one mile.

The regular operation of electric passenger trains between Broad Street and Paoli was begun during the latter part of September, 1915. Extended induction tests were made on the section east of Bryn Mawr in April, 1915 and on the entire line in the following August. Further induction tests were made in the summer of 1916.

In July, 1916, the Railroad Company commenced operating synchronous condensers at Radnor and it has been found that, with these condensers in use, the booster transformers are greatly overloaded at times of short-circuit. This results in the transformer iron becoming heavily saturated and the magnetizing current, consisting largely of the third harmonic, which under these conditions reaches very high values, necessarily flows through the ground. The current wave is badly distorted as a result of this overloading and the induced voltages during short-circuits may actually be higher with the booster-transformers than without them; in fact, it has been found preferable to remove the booster-transformers east of Bryn Mawr, and only those from Bryn Mawr west are now regularly in service.

With all booster transformers in service the maximum induced voltages, (peak values)*, during normal operation are about 10 for subscribers' lines and 25 for trunk lines; at times of short circuit, calculations based upon experimental data show that the maximum voltages may exceed 1000 on subscribers' lines and 1200 on trunk lines. With all booster transformers cut out, the corresponding figures are, for normal operation, 50 volts for subscribers' lines and 125 volts for trunks and, at times of short circuit, 225 volts for subscribers' lines and 900 volts for trunks. These figures assume two condensers in service.

This section of railroad follows through a highly developed suburban area where disturbances on telephone lines are extremely undesirable. Unfortunately, a considerable number of cases of bell ringing due to short-circuits on the railroad circuit have been experienced. Not all short-circuits cause bell ringing however.

During the first three months of electric operation, namely, October, November and December, 1915, the number of short-circuits causing bell ringing averaged 10 per month and the bell ringing troubles over 2000 per month. During the following seven months the average number of short circuits causing bell ringing, fell to 4.5 per month and the bell ringing troubles from 735 in January to 57 in July. Since July, 1916, the number of short-circuits per month which caused bell ringing have been

*On account of the wave shape distortion described above it was found advantageous here to measure peak voltages rather than effective voltages and all values of voltage mentioned in this discussion of the Paoli electrification are peak values.

further reduced and during the year 1917 averaged about 1.5 per month.

The improvement in the bell ringing situation after January 1916 and up to August 1916, was due partly to changes in the railway control circuits and in the operating arrangement of circuit breakers, and partly to the substitution of relay sets for standard party line bells, at subscribers' stations within the regions of heaviest inductive disturbances, and the biasing of bells within the areas of less disturbances. This work required changes in subscribers' apparatus at about 3000 stations.

The synchronous condensers installed in the latter part of July, 1916, increased the maximum voltages impressed on the telephone circuits at times of short-circuit on the railroad, so that while the actual number of short circuits causing bell ringing has been reduced since that date, the average number of bell ringing troubles per month has increased. From August 1916 to December 1917 inclusive, there were 22 short circuits causing bell ringing and 1589 bell ringing troubles, an average of 72; whereas for the five months' period preceding the installation of the synchronous condensers, there were 20 short circuits causing bell ringing, and 356 bell ringing troubles, an average of 18, or one-quarter of the corresponding average number in the later period. During the year 1917, 45 per cent of the bells rung were on direct lines and not grounded.

The high induced voltages causing bell ringing have been experienced over the entire electrified portion of the Main Line except within about three miles of Broad Street. It is fortunate that the region of high induced voltages does not extend to Broad Street, otherwise, owing to the density of telephone development within that district the trouble would be exceedingly difficult to cope with.

In a small percentage of cases where bells are rung the induced voltages are sufficient to leave the telephone lines grounded through the protectors, thus interrupting service.

Methods of signaling on call circuit trunks involving use of the ground have had to be abandoned.

Noise tests made on ten trunks which parallel the electrification indicate that the induced currents from the railroad cause a small amount of noise but not enough to constitute interference with service on these comparatively short lines. However, I may mention the fact that in the construction of the new Philadelphia-Reading toll cable, in which the requirements as

to freedom from noise are more exacting than in the case of the shorter Main Line trunks, the liability of noise, together with other features of interference, was considered a sufficient reason for changing to a different route in order to avoid exposure to the electrified section of the railroad. The route adopted involves charges of $1000 a year more than the route exposed to the electrification, which otherwise would have been followed.

None of the telephone company's circuits affected by the electrification are now used for telegraphy, hence there has been no interference with this type of service. It is expected, however, that telegraph service over the paralleling toll circuits will be required later and to give this service it will be necessary, unless there is some new development, to employ neutralizing transformers with their attendant disadvantages and limitations.

The high voltages induced on the telephone lines at times of short-circuits, also, in the opinion of the telephone company, constitute a considerable fire hazard, although it is fortunately true that no fires have as yet been caused. All terminating trunks at the three directly exposed offices west of Bryn Mawr have been provided with carbon block and heat coil protection. As these trunks are in underground cable, they would not require these protective devices except for the induced voltages. As an additional precaution against fire the telephone company has maintained a special force of night watchmen at several central offices throughout this area where regularly there are no inside men at night, thereby incurring an expense of about $18,000 per year. As a still further precaution, trunks to certain offices west of Malvern were for a time provided with repeating coils at Malvern, but it has recently been necessary to phantom these circuits, which has required the removal of the repeating coils.

A large number of the trunks affected by induction from this electrification are equipped with loading coils. Tests on these coils show that more than 20 per cent of them are magnetized to a greater or less extent but it has not yet been determined whether this trouble has been brought about wholly or in part by induced currents from the railroad circuits or whether it is due to other causes.

Within the area of high induced potentials, telephone subscribers and employees are exposed to the possibility of electric shocks at times of short circuit on the railroad. Fortunately, however, no troubles from such shocks have yet transpired.

Telephone operators and users are also exposed to the possibility of acoustic shocks at times of surges from short circuits, although no serious acoustic shocks, due to this electrification, have been reported.

It will be seen from the foregoing that, notwithstanding all that has been done by the railroad company and the telephone company to reduce interference, there still remain, at times of short circuits, certain hazards of fire and shocks, bell ringing trouble, and other latent interference as described. While this impending interference has not, with the exception of bell ringing, actually materialized into trouble, nevertheless the possibility of such trouble is continually present and the conditions can by no means be regarded as satisfactory.

Looking to further improvement in the situation a number of plans which involve changes in the distribution system of the railroad, have been worked out. The plan which, on the whole, seems entitled to the most favorable regard, involves the installation of additional power supply transformers at Radnor and at a point 16½ miles from Broad Street, and also includes the sectionalizing of the trolley at both these points. This plan further requires the moving of the Berwyn-Malvern aerial telephone cable to a more remote location. By the adoption of this plan it is estimated that the maximum induced voltage at times of short-circuits would be brought down to 250. This would largely reduce, but not wholly avoid, the fire and shock hazards, bell ringing on direct lines, and the other evils involving the operation of protectors. The cost of carrying out this plan, including labor and material only, has been estimated at $140,000.

Another plan, involving more extensive changes in the power supply and distribution system to avoid wholly the interference and hazard, would require a much larger expenditure and perhaps would not be warranted by the existing situation.

A different plan would be to install sufficient additional booster-transformers so that they would not become overloaded at times of short circuits. This would probably require placing the boosters one-third to one-half mile apart and would cost from $85,000 to $150,000. This plan has not been worked out in detail, as the railroad company objects to the introduction of insulating joints in the trolley wires at such frequent intervals.

Conclusion

It may be said in conclusion that means are now known whereby alternating railway currents can be kept sufficiently within control, except under abnormal conditions, to prevent substantial interference to neighboring communication lines, although the application of such means to the extent necessary to produce satisfactory results may involve considerable expense.

Even under abnormal conditions the interference can be greatly reduced by the application of suitable measures, but in some cases there still remains the problem of obtaining a sufficient reduction of interference without incurring a cost which the railroad companies consider excessive.

It is important in each electrification project that the railroad company and the communication companies affected cooperate in determining what interference-preventive measures shall be adopted. Each electrification requires a special study, as the best measures to employ may be quite different in different cases.

I wish to take this opportunity to testify to the broad minded and cordial manner in which the railroad companies and electrical manufacturers concerned have cooperated with us in searching for a satisfactory solution of this problem, a work which, it is probably unnecessary to add, is still in progress.

Address presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 27, 1918.

ENGINEERS AND THE WAR

BY MAJOR GENERAL WILLIAM M. BLACK
Chief of Engineers, U. S. Army

I FEEL some diffidence in addressing you this evening, partly because, although I might have qualified as an electrical engineer in 1880, I know I could not do it now, and then, I feel that a man who has the privilege of talking to a number of men such as are assembled here at this time has a tremendous responsibility, because the task set before us is one that has been unequalled in the history of the world. We represent a creed, not only in our living but in our Government, which we believe is necessary for the salvation of this world, for bringing to this earth the Kingdom of God.

We are fighting a nation that has a creed directly opposite to ours, a creed that leads them into the belief that Might makes Right, and that anything that is for the supposed good of the State is Right. This attitude has led the people of that nation to break every law that humanity has made for the amelioration of the horrors of war, laws that have gradually grown up through the ages. These laws are being ruthlessly sacrificed by these people in strict accordance with the creed which they hold. That creed and that people must simply be swept from off the face of the earth, and that is the task that we have undertaken.

It is an honor to be privileged to address you on a subject of such importance to our country, the duty of the Engineers in war. Although the part played by Engineers in this war is great and the responsibilities of the profession are correspondingly large, this war, like all wars in the past, is and must be a war carried on in accordance with the principles of the art of war, which are unchanging and which have been recognized and taught ever since organized armies were first created.

Do you realize that almost the only absolutely modern method of warfare now in use is the warfare of the air? The invention of submarines was made during the American Revolution, and

submarines were used successfully, though to a limited extent, in our own Civil War. Gas and flame fighting are of ancient origin. Trench fighting is hardly better known today than it was to the veterans of Grant and Lee, of Sherman and Johnson. The advances in human knowledge have caused corresponding improvements to be possible in the weapons of warfare. Increased knowledge of chemistry has produced more powerful explosives and improved methods in metallurgy have enabled these explosives to be utilized, by making possible heavier and more powerful guns. Improvements in the means of transportation have enabled larger bodies to be moved more quickly and more readily and to be subsisted and supplied with greater certainty. The telegraph, the telephone and the wireless have afforded a means of prompt communication and have enabled, larger bodies of men to be given co-ordinated action. With such changes, battles are fought on the same principles and won or lost from the same causes as in the time of Alexander the Great. This war has been called a people's war and so it is in the sense that due to modern facilities the entire resources of the people can be utilized to day as they could not have been utilized in the days of old. It has also been called an Engineer's war because in the quickness of movement and in the works necessitated by these modern inventions the services of Engineers become more conspicuous and perhaps more necessary than in the past. But engineering in warfare has always been essential and it is even doubtful whether the science of engineering does not owe its birth to the works of war. An Engineer myself, I would be the last to belittle the work of our profession. It is a matter of pride that the men of our profession, due to the nature of their employment in time of peace, are, of all the civil professions, most prepared to serve the country in war, but to serve the country adequately in war, the Engineer must add to his peace equipment for professional work. The profession of arms is a profession in itself and it is the profession which deals with the very greatest in magnitude of all the endeavors of men. The effective use of an army which is properly constituted exemplifies the best that men can do in organization, in discipline and in the devotion to duty which causes a man to regard his own life as a thing of small moment toward the attainment of the end sought.

There would be a great amount of effort saved if our people recognized more clearly the existence of the technicalities of

the profession of arms. The Government in Washington is simply deluged with suggestions and so-called inventions for the winning of the war. The records show that about 98 per cent of all of these are without military value and that time and labor have been thrown away by men eager to help, but entirely ignorant of the history and conditions of warfare.

An example with which some of you are familiar is the electrical gun. For years the possibility of such a weapon has been a fascinating line of study to electricians. The principle of the solenoid is the germ. If a series of solenoid coils were to be energized and de-energized in succession sufficiently and rapidly, such a series around a tube can be made to impart a movement of translation and rotation to a projectile. But practical results are today impossible. A six-inch service rifle having a length of 20 feet, fires a projectile weighing 110 lb. with a muzzle velocity of 2600 ft. per sec., or in other words, the projectile leaves the muzzle with a kinetic energy of translation of 115,500 ft-lb. This energy has been stored in the projectile during its travel through the bore of the rifle, or say in 1/65th of a second. The average power expended has therefore been at the rate of 7,607,500 ft-lb. per second or about 14,000 h. p. or 10,500 kilowatts. These figures are simply approximations and neglect entirely the power required for imparting velocity of rotation and for overcoming the friction in the bore. You can easily estimate the weight and dimension of the generating equipment which would be required for even a moderately powerful gun were all the mechanical and electrical problems of its manufacture solved, and making due allowance for the short-load periods. You can understand the impracticability of transporting the electrical plants required for any number of such guns, and the impossibility of distributing this power over shell-swept ground to guns whose position must be constantly shifted, and which must be put in action on a few seconds' notice. I think that you will agree that until new discoveries give a much improved method of storing or generating electricity, smokeless powder will continue to be the most compact and convenient form of stored energy for guns.

And yet there has been a good deal of time and money wasted in trying to perfect such a gun by men whose patriotism is undoubted, and whose ignorance, also, is undoubted. In other words, if a man has an invention or an idea of an invention, by all means let him work on it, but before he goes to Wash-

ington and takes up the time of men busy trying to devise means to beat the Boches, let him make sure that he knows the conditions of war and what he is trying to do to meet those conditions, and then if he is sure of the means, let him present his ideas and inventions to the people in Washington.

We want all we can get and want the best we can get. We want the inventive power of our country if it can be exercised to do good.

There was a proposition made seriously at Washington recently that the United States should provide a fund, of I do not know how many million dollars, and make a Home for Inventors, where any one who thought he had an invention would be able to go, and work it out at the public expense; and recently, although we had a committee of experts there to pass upon these inventions, the results were so utterly unsatisfactory to the inventors that they came in a perfect horde upon the Secretary of the Navy and the Secretary of War, so much so that they had to make a brand new committee of three men, who could be much better occupied, to go ahead and do this same thing; and I am only waiting for the next drive of inventors to show that this Committee will not suit them one particle better than the old one did.

In addition, without doubt, there are many men in our country of the highest patriotism who are sore-hearted because they are not given something to do directly toward the winning of the war. They do not understand, that some condition peculiar to themselves, possibly age, possibly physical condition, possibly mere ignorance of war and its conditions, compel it that the bit that they must do for their country at this time is to continue in their work in civil life and do their part in keeping up the normal life of the country—in itself a service of importance.

The part which Engineers are now playing in the war is a very great one. The records of the American Institute of Electrical Engineers show that out of a total membership of 9443, there are 973 in the service, or 10.3 per cent of its roster. The American Society of Civil Engineers with 8753 active members has 14⅓ per cent in the service. The American Institute of Mining Engineers 10.4 per cent, and the American Society of Mechanical Engineers 10.1 per cent. But these records are not complete. At the outbreak of the present war there were in the Engineers Corps of the Regular Army about 300 officers and approximately 3500 enlisted men. At the present time,

there are about 8000 commissioned officers and 200,000 enlisted men, made up of men formerly engaged in works of an engineering character. It is probable that this does not represent much more than one-half of the number of the profession now serving in the Army.

Let us consider the nature of the work of the Engineer, passing from front to rear of the Army.

First in importance is the work of the sappers. They go before and remove obstacles, clearing away obstructions, building bridges and roads, making the trench systems complete, mining, providing light, water, lines for supply (light railways or roads) and military mapping. In this category enter practically all of the branches of the profession. Further to the rear are found the construction and operation of railways; road and bridge construction; the construction of veritable towns for supply depots, with all their accessories, drainage, sewerage, lighting and water supply; construction of quarters and of hospitals; and furthest to the rear, the construction of the ports of debarkation with their wharves storehouses, railway lines, yards and shops, all with their sanitary systems. Separate from these activities, but necessary for their supply, are the Forestry troops who turn the growing timber into lumber of the dimensions required for the various services. Locomotive and car shop troops are performing essential services. Topographic Corps, Sound Ranging Corps and Camouflage Corps are also among the varied activities of the Engineers.

What preparation is required for the fulfillment of these varied duties? For the actual technical work of construction or installation the civil training of the Engineer should prove sufficient when the plans which embody the military features have been prepared, or when the military technique has been learned and assimilated. A fundamental of this military technique is that the time element is to be considered rather than money cost and that the work must be done with whatever materials are available. This requires clearness of conception of the results required, resourcefulness and organization—factors also required for civil work.

As stated earlier, due to the very small numbers of the personnel of the Corps of Engineers of the Regular Army, reliance had to be placed in the members of the profession in civil life. Confidence in their devotion to country and in their ability has not been misplaced. The results already accomplished prove

this fully. Could more have been done? Undoubtedly, had the profession been better prepared for the call.

Will you permit me to say a few words concerning the general training of our Engineers, based on a professional experience of more than forty years? The conviction has been forced upon me that in educational matters, as in many other affairs of life, we Americans are inclined to go too fast. The basis for any professional career where the highest is to be attained must be a sound general education. Does anyone of you regret the lessons gained in your own experience? Is not the experience of humanity as shown in properly written history of almost equal value? Would Russia now be in the sad condition existing had her people known that the experiments she is trying have always resulted disastrously? Yet is history thus considered in an ordinary technical course? Again, do you not find a knowledge of the general principles of law and of the special rules of the laws of contracts of value? Are these considered essentials? What is the handicap of an Engineer who is unable to express his ideas clearly in spoken and written English? Is this taught thoroughly in our technical courses?

It goes without saying, that the study of pure and applied mathematics is found in all technical courses. But, are these subjects well grasped before their application in special technical courses is studied? Is any faculty of an Engineer of greater value than the ability to form a mental picture of his problems and of its solution? Yet is that study which assists most in this faculty—descriptive geometry—properly apprehended? Is there any branch of the profession which in its application is not based on a knowledge of topographical work, on a knowledge of construction materials and of how these should be used? Is the study of these branches of civil engineering insisted upon sufficiently in the mechanical and electrical courses? In effect would not our professional men be better equipped for their civil work were they not in too great a hurry in their youth to enter directly into life's combat? Does not this war teach that without a long and elaborate preparation down to the last details, an attempted "drive" must fail?

These remarks apply to all Engineers, both military and civil. In the rush of war men cannot always be hand-picked for special jobs and frequently it becomes necessary for an available man to be used for the work immediately necessary, irrespective of his previous training. In this supreme test of

humanity the best man is he who is prepared to meet any emergency—perhaps not in the most finished way—but to meet it.

There are things that the engineers in this country can do. If they do know enough to give us some ideas for helping along in the killing of Boches, for God's sake let us have them. If they do not, what they can do is to help the supply of men for the winning of the war. We are now short of officers of Engineers, very short, and we are going to be very much shorter. We must have educated engineers for this work, and we must not only have the men for the line work of the army, but we must have mechanicians and artisans and laborers for the special work.

All of you men have spheres of influence—do your best in them, and if you can be used otherwise, and the problem comes up in which we need you, you may be sure you will be called upon. There is this problem now, the supply of men, in which you can help, either by your own personal sacrifice, in going out, or by influencing others.

Now as to soldier work. The movements of drill and the construction methods peculiarly military are easily learned. The knowledge of the art of war which will enable these to be applied promptly and properly is more difficult. But most difficult to acquire is the peculiar mental discipline which makes the soldier. The Army is a huge machine which must work co-ordinately in all of its parts. That competition, which in civil life causes one body to advance further and faster than another, is out of place in an Army. *All* must work together and for one common end. Each man must so subordinate his will and desire to the common good as to work willingly and earnestly in the sphere allotted to him. This does not mean that all initiative is to be suppressed. On the contrary each man must use his initiative to the utmost, but in his own allotted sphere of action. Each must learn to obey and obey from the heart. Through such obedience comes the knowledge of how to command when command becomes a duty. All of this is hard to learn. But each man who is called upon to help in this war must learn it, if he would help effectively.

By all means let us have military training in our schools, but let it be true military training and not tin soldier work.

There is another line of technical military knowledge which must also be studied hard. The machinery for the organization, training, supply and leadership of troops; the methods of ob-

taining, accounting for and issuing supplies; of keeping returns of the men; and the channels of command must be studied. To civilians in general this is wholly unknown, but if a man is to be of service in the Army, it must be learned until its use becomes automatic.

What have the Engineers done? War was declared April 6, 1917. By the middle of July, nine regiments of Railroad Engineers had been raised and organized and two had actually started for France. In each regiment were two officers of the Corps of Engineers of the Regular Army, the Colonel and the Regimental Adjutant. The remaining officers were all from the Engineer Reserve Corps, some receiving their commissions only when on the point of sailing. Of course, few of the officers had had any previous military training and the tasks of organization were most difficult. Since then, there have been organized:

Five Corps Regiments consisting of Sapper, Searchlight and Sound Ranging troops; 43 Sapper Regiments and trains; 2 Mounted Battalions and trains; 5 Ponton Trains; 4 Inland Waterway Companies; 40 Railway Regiments and Battalions including all classes of Standard Gage and Light Railway troops necessary for the construction, operation and maintenance of railways; 1 Railway Transportation Corps; 1 Highway Regiment; 1 Gas and Flame Regiment; 1 Gas Training Service; 5 Forestry and Auxiliary Forestry Regiments; 1 Surveying and Printing Battalion; 1 Military Mapping Service; 2 Supply and Shop Regiments; 1 Water Supply Regiment; 1 Quarry Regiment; 1 Mining Regiment; 1 Electrical and Mechanical Regiment; 2 Crane operating Companies; 1 Camouflage Battalion; 18 Truck and Auto Companies and 44 Depot Detachments.

The greater part of these organizations is now overseas. Some are serving with the British Army, some with the French, but the majority is with our own troops, in service both at the front and in the rear.

I wish I could go into greater detail as to the work of these Sound Ranging Corps, because it comprises some new electrical work of the highest character, and the apparatus for it has been perfected in this country. We took the best devised at the beginning of the war, and our physicists went to work and have made marked improvements. Perhaps you do not know what sound ranging is. The artillery is stationed in the

rear of the line. There is almost no direct artillery fire any longer—that is, as a rule, the gun is fired from a point where the target cannot be seen at all. The first thing to be destroyed, invariably, is the enemy's artillery, then the trenches are attacked. The obstructions of wire are torn to pieces, the trenches themselves are practically leveled, and after that is done, in the assault, there is what is termed the barrage fire. That, I suppose you know means a fixed or slowly moving curtain of shells dropped on a certain given line and through which passage is almost impracticable.

On both sides the artillery is carefully camouflaged so it cannot be seen from aeroplanes. To show what care is taken, even the tracks that are made in taking the guns to the front are wiped out, the guns themselves are covered, so that neither from an observation balloon nor an airplane from the enemy's line can the position of the gun be seen, and in order that the flashes of the gun cannot be located, there are dummy guns placed at intervals, and flashes from these guns made by electricity, so that the position of the real guns cannot be known.

In order to determine the position of the real guns, there are delicate instruments which have been devised, which are placed at intervals along the line. These instruments are for the purpose of registering the sound of the gun. There is, first of all the sound of the gun in firing. That is preceded frequently, if the range be great, by the sound of the shell passing through the air, and sometimes by the bursting of the shell itself, before the sound of the gun comes. These are all recorded, and the velocity and the direction of the sound is known. By having these instruments at different points on the line, the position of any one gun can be "spotted," and "spotted" so closely that our own artillery fire can be directed and the gun blotted out. That is one of the improvements of modern warfare rendered possible by the advance in general human knowledge, particularly in electrical knowledge, and these instruments are very exact.

This service of the rear is of great importance and magnitude. Picture to yourselves what is required to transport, house, supply and maintain a million men three thousand miles from home, producing nothing and in their work expending enormous amounts of materials.

Taking the question of storage alone, the provision of space required for an army of 1,000,000 for ninety days aggregates 20,000,000 square feet of floor space of covered storage and

double that amount of uncovered storage space, with the necessary railway tracks for receipt and shipment and for classification yards, aggregating about 650 miles. Add to this an equal mileage of highways, adequate provision for water supply, sewerage and electric lighting and power and you can realize the work involved in this one item. Add to this the constructions which have been built at the Ports of debarkation (at one of which 375,000 square feet of wharf space had to be provided), the hospitals, barracks, shops, and the lighting, water and sewerage systems required, and some conception of the actual new construction work done, can be formed.

It is estimated that the supply of the army requires the transportation to the front of 25 lb. per man per day. This makes heavy demands on the French railway systems, good as they are. These have had to be supplemented in all but the main line trackage, and a large amount of motive power and of rolling stock has had to be supplied and operated.

Among the special services, the work of the Geologists must be mentioned, and in the line of improved apparatus, it may be stated that new instruments and methods for airplane photography have been devised and introduced. Other new auxiliary aids for fighting have been worked out, some of which have already proved their value on the battle field.

Yes, the Engineers are doing their work well. Be it in constructions in the rear, or under fire, be it in the transportation of ammunition to the firing line, the construction of strong points and obstacles, the construction and destruction of bridges in the face of an enemy, or as in recent instances, under the feet of the enemy, or be it with their rifles in beating back an attack, they are doing and *dying*. All glory to our Comrades in arms in France! There is not a red-blooded American who does not envy them.

But is there not a war duty for us also, for us who are held on this side of the ocean? Yes, undoubtedly. To some is allotted a task in supply, to some a task in manufacture, to some a task in organization. But that is not all. The life of the nation must go on. Her civil machinery must function undisturbed. With so many called away from these civil duties, the onus of the work will be the heavier for those who are left. Let us each then do his bit as and where it presents itself knowing that if each does his best, with love of country and forgetfulness of self as guides, the results are sure.

Gentlemen, I have been in the front line trenches, and a more abominable place for a man to be in, apart from any enemy, you can hardly imagine. It is raining there much of the time. The mud is up to your ankles, even though the trench be drained and the best attempt made to keep the drainage in good condition. The men at the front are distant from their supports, and they are scattered, maybe a dozen at a place, directly in the face of the enemy, and they have to stay there, and have to be prepared to resist any attack that comes, with a certain knowledge that should the attack come in force, or should a real earnest effort be made to take the line, they in the front line are almost sure to perish, and they are not minding it. They are staying there in the face, not of possible death, but in the face sometimes of almost certain death. They are doing their work, and our engineers have the most dangerous part of that work to do.

You remember in that first British drive to Cambrai, the Boches got in the rear of the firing line. They struck a part of one of our Engineer Railway Regiments, building light railways to the front to bring up ammunition. These men had not their arms with them. All they could do was to scatter and get in shel holes, and as soon as the Boches went by to rally and go back, and take the arms thrown down by the wounded or dead, and then pass over, and they formed their part of the line and did their share of the fighting.

The evening of the great drive, I had the honor of dining with Sir Charles Douglas Haig. He then told me of one of the battalions of the Sixth Engineers, which had been with the British about six weeks, and in that time they had built twenty bridges with spans, ranging from 16 to 60 feet, and he spoke of the wonderful work they had been doing. Two days afterward these men took their place and held about a mile of the British line, and held it so that the Boches could not get through.

The same happened with one of the railway regiments, again caught in the same way, and they formed a part of that miscellaneous army that Gen. Carey got together of the supposedly non-combatant troops, and the line which they formed prevented the capture of Amiens. They held the Germans. There, again, the engineers played their part valiantly.

Just two days ago a cablegram came across from the other side, about one of the companies, Company C, I think, of the First Regiment, which had been ordered to do particularly

dangerous work in the neighborhood of the Marne. As they started out, two of the officers were killed at once, but the rest went on and did what they were ordered to do.

Then again, you have noted that when the Germans got down to the Marne and were crossing a bridge, some of the engineers waited until the head of the Germans had gotten completely across the bridge, which was filled with the Germans, and they blew up the bridge, and those who got there wished they had not.

Our men are doing their work on the other side wonderfully, and those are our brothers in the profession who have gone over and now wear the uniform.

(General Black then exhibited some moving pictures showing the organization of the Engineers and how they do some of their work, and finally some of the work that they have already accomplished in France, after which he added):

In landing in France, you are struck at once by the number of maimed men you see on the street and by the women in black, and then, as you get to know them, you will see that these people are from their suffering simply the more determined to carry this war through to a successful finish. The same thing is true in England. You are struck by the grim determination to win success at whatever cost. There, due to a difference of temperament, things are taken somewhat differently. I was at a tea in England, and was presented to an English woman of title. She was gowned just as any woman would be for an afternoon reception. In speaking to her I mentioned I had two boys in France. She said, "I did have four, I have only two now," and then seeing me look a little startled, she said, "You know we feel our private griefs must not be allowed to show, that we must not wear mourning, that we must go ahead with our ordinary duties and try to keep up the social life."

We have only just begun to fight. We have not begun to suffer. If you were there, and could see what these people have done, and what they are prepared to do, and then feel as you would feel, how completely they trust us to bring this war to a successful finish, you would feel, too, that it is up to us to equal them in endurance, equal them in sacrifice, and to see that freedom, individual freedom, is preserved for all time n this world. I thank you very much.

Presented by publication only.

(*Subject to final revision for the Transactions.*)

CRITICAL REVIEW OF THE BIBLIOGRAPHY ON UNBALANCED MAGNETIC PULL IN DYNAMO-ELECTRIC MACHINES

BY ALEXANDER GRAY AND J. G. PERTSCH

ALL of the formulas are based on Maxwell's equation

$$\text{pull, in dynes} = \frac{B^2 A}{8 \pi}$$

where B = flux density, in lines per sq. cm.
A = cross section, in sq. cm. of the flux path in the gap between the pulling surfaces.

This formula may be established in a simple manner for the case of a saturated magnetic circuit, as follows:

The magnetic circuit shown in Fig. 1 is symmetrical about the axis $O\,O^1$, and therefore the action of each pole may be considered separately. With n turns per pole and e volts across each coil, the excitation $n\,i$ produces a magnetic flux of ϕ lines.

In Fig. 2, $O\,a$ shows the magnetization curve for each half of the complete magnetic circuit of Fig. 1 when the air gap clearance between the armature and each pole-face is g cm. If now, the armature is allowed to move by a virtual displacement $\Delta\,g$ under the action of the magnetic pull P, so as to reduce the gap, then the curve for the new magnetic circuit becomes $O\,b$ (Fig. 2).

In order to establish the initial flux ϕ at an excitation $n\,i$, the total energy supplied to the coil after closing the switch $s\,s$, is

$$\int_0^t eidt = \int_0^t (n \frac{d\phi}{dt} \cdot 10^{-8})\, idt + \int_0^t iridt$$

In this expression, the stored energy

$$\int_0^\phi ni\, d\phi \quad 10^{-8} = \text{Area } Oac \text{ (Fig. 2), in watt-sec.}$$

If, now, the armature is allowed to move under the action of the pull P, so as to reduce the gap by an amount $\Delta\,g$, the excitation

$n\,i$ being maintained constant, the flux changes from ϕ to $\phi + \Delta\,\phi$ and the change in the stored energy is

$$\int_{\phi}^{\phi+\Delta\phi} \left(n\,\frac{d\phi}{dt} \cdot 10^{-8}\right) i\,dt = ni \times \Delta\,\phi \,.\, 10^{-8} = \text{Area } cabd$$

Likewise, the energy stored in the magnetic circuit under the new conditions, with flux $\phi + \Delta\,\phi$,

$$= \text{Area } Obd.$$

The mechanical work, $P \,.\, \Delta\, g$,is done at the expense of the stored magnetic energy, which is equal to

$$\begin{aligned} &\text{Area } Oac + \text{Area } cabd - \text{Area } Obd \\ &= \text{Area } Oab \\ &= \text{Area } Oa^1b^1 \end{aligned}$$

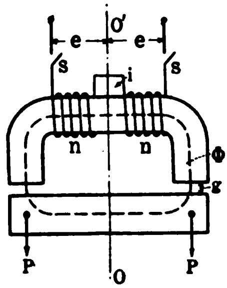


FIG. 1

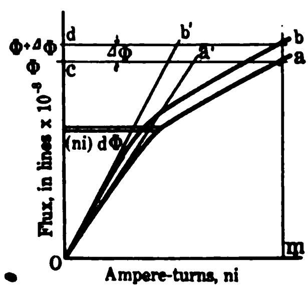


FIG. 2

where Oa^1 and Ob^1 in Fig. 2 represent the magnetization curves for the air gaps g and $g - \Delta\, g$ respectively.

But, for very small values of $\Delta\,\phi$,

$$\text{Area } Oa^1b^1 = \frac{1}{2} \cdot \text{Ampere-turns for gap } \Delta\, g \times \phi \cdot 10^{-8} \ldots \text{watt-sec.}$$

$$= \frac{1}{2}\left(\frac{10}{4\,\pi} \cdot \frac{\phi}{A} \cdot \Delta\, g)\right) \times \phi \,.\, 10^{-8}$$

$$= \frac{\phi^2}{8\pi A} \times \Delta\, g \times 10^{-7} \ldots\ldots\ldots \text{watt-sec.}$$

$$= \frac{\phi^2}{8\pi A} \times \Delta g \text{ ergs} = P \times \Delta\, g$$

or, Pull in dynes, $P = \dfrac{\phi^2}{8\pi A} = \dfrac{B^2 A}{8\pi}$

This formula is thus true whether the magnetic circuit is saturated or not. It will also be seen from the above what is meant by A in the formula, and how problems involving slotted armatures are to be solved.

The subject of magnetic pull in dynamo-electric machines has not received much attention in recent years; in fact, it appeared as if the whole field had been covered by 1911.

Fisher-Hinnen[1], in his "Dynamo Design" published in 1899 derives the following formula:

$$\text{Pull, in dynes} = \left(\frac{B^2 A}{25 \times 10^6} \cdot \frac{x}{g\alpha}\right) C$$

where B = density of flux in the air gap, in lines per sq. cm.
A = section of the pole face, in sq. cm.
x = rotor displacement
g = air-gap clearance
α = ratio of the reluctance of the total magnetic circuit to that of the normal airgap alone.
C is a calculated coefficient, given in the following table:

Value of C	Number of poles	Remarks
2	4	Displacement along axis of field magnets.
2.8	4	Displacement along neutral line.
6	6	Displacement along axis of magnet.
9	8	Displacement along neutral line.
15	12	Displacement along neutral line.
20	16	Displacement along neutral line.

B. A. Behrend[2] in 1900 gives a mathematical discussion of the case of a machine with a very large number of poles and derives the formula

$$\text{Pull} = \frac{B^2 S^1}{8\pi} \cdot \frac{2x}{g} .$$

where $S^1 = \pi \times R \times l = \frac{1}{2} \times$ total air-gap area, and shows that this is exactly one-half the value obtained by assuming that

1. See Bibliography.

half of the poles have a gap $g - x$ and the other half a gap $g + x$.*

If we substitute for $S^1 = \frac{1}{2} \times A \times$ poles, Behrend's expression becomes

$$\text{Pull} = \frac{B^2 A}{8\pi} \cdot \frac{x}{g} \cdot \text{poles.}$$

Edgar Knowlton,[3] about the same time, derives an expression for a machine with a definite number of poles and obtains the formula

$$\text{Pull in lb.} = \left(\frac{B^2 A}{77{,}134{,}000} \cdot \frac{x}{g\alpha} \right) C$$

where B = normal gap density in lines per sq. in.
A = Area of pole-face in sq. inches
x = displacement
g = normal airgap
α = ratio of the total reluctance of the magnetic circuit to that of the normal gap alone.
C = a coefficient, obtained by calculation
= 2 for 4 poles
= 4.7 for 6 poles
= 7 for 8 poles
= 9.6 for 10 poles

Above 10 poles, C = number of poles, and the formula then checks with that of Behrend.

Knowlton's expression is of exactly the same form as that of Fisher-Hinnen, but the constants given by the two authors for 6 and 8 poles differ considerably. Knowlton, moreover, states that it makes but little difference whether the plane of deflection is taken through a pole or between two poles.

Hans Linsenmann[4], in 1902, uses the saturation curve directly (as is done in Fig. 3 of the following article by Rosenberg) for determining, at a given excitation, and with sine distribution of flux under the pole, the variation of the average magnetic pull per pole with the length of airgap. He expresses this variation by an approximate equation and introduces it into the general equations connecting the bending moments of the deflecting forces in large alternators.

* Note that in the following paper Rosenberg formulates a similar statement.

J. Rey[5], in 1904, states that Behrend's and Fisher-Hinnen's formulae are not consistent and sets out to derive a new formula especially applicable to induction motors with eccentric rotor. Rey's formula is

$$\text{Pull in dynes} = \frac{1}{8\pi^2}(B_{eff})^2 S \times C$$

where $B_{eff} = B_{av} \times 1.1$ for a sine distribution of flux
S = rotor surface = $2\pi Rl$
$C = f(\epsilon)$
ϵ = eccentricity, expressed as a fraction, in terms of the single airgap = x/g

and for

$\epsilon = x/g =$	0.05	0.1	0.15	0.2	0.25	0.3	0.4	0.5
$C =$	0.157	0.319	0.488	0.688	0.866	1.084	1.63	2.415

For an eccentricity less than 20 per cent, this formula reduces to

$$\frac{(B_{eff})^2}{8\pi^2} \times S \times \frac{0.488}{0.15} \times \frac{x}{g}$$

$$= \frac{(B_{eff})^2}{8\pi} \times S \times \frac{x}{g}, \text{ approximate within one per cent,}$$

which is identical with Behrend's formula, because with a sine distribution of flux, as in an induction motor, $(B_{eff})^2$ is the average value of $(B)^2$, where B is the density at any point along the gap.

J. K. Sumec[7] derives a similar formula to that of Behrend and Rey, but by certain transformations reduces it to the form

$$\text{Pull in dynes} = \frac{B^2}{8\pi} \cdot S \frac{x}{g} \frac{1}{[1-(x/g)^2]^{3/2}}$$

This is the same as Behrend's formula except for the last term which for an eccentricity as large as 25 per cent, is equal to 1.1 or a difference of only 10 per cent, while for eccentricities below 10 per cent, the correction term differs from unity by only about one per cent.

Sumec's formula, as might be expected, gives the same values as that derived by Rey. It is to be noted that in most of the formulas derived mathematically, the effect of the reluctance of the iron part of the magnetic circuit is neglected; the assump-

tion made that the flux density is inversely as the airgap is therefore only true in unsaturated machines

In 1905, B. Soschinski[8] gives an account of some tests made to check the formulas of Rey and Sumec. Very good agreement was obtained for small airgaps and iron feebly saturated, taking the area and flux density at the top of the teeth but with increasing saturation, the calculated results (iron reluctance neglected) were higher than experimental ones. With larger values of airgap, however, the test values were throughout higher (up to 100 per cent). The latter result was accounted for by the fact that as the armature moves to one side the lines become concentrated at the tooth tip on the side of the reduced gap and spread out on the side of the increased gap.

In the same year (1905), Niethammer[9], by a transformation of Sumec's expression, obtains the following formula which permits taking into account the reluctance of the iron path by determining the flux density from the saturation curve, as is done by Rosenberg. This expression is regarded by him as giving the most reliable results.

$$\text{Resultant pull, in } kg. = \frac{1}{2}\left\{\left(\frac{B_{max}}{5000}\right)^2 - \left(\frac{B_{min}}{5000}\right)^2\right\} p\, A$$

where p = number of pairs of poles
A = mean effective cross sectional area of gap per pole in sq. cm.

In 1907, Picou[10], starting with the relation for stored energy, derives the expression for magnetic attraction, and gives a modification of the formula due to Sumec for unbalanced pull.

C. R. Moore in 1911, gives a graphical method for studying the unbalanced magnetic pull, by using Maxwell's formula and the given saturation curve of the machine, and summing up the pull for various adjacent halves of adjoining pole faces. He takes the airgap density for these half arcs as corresponding to the average airgap lengths across these faces and to the given excitation. Upon plotting the unbalanced magnetic pull for various field excitations he establishes the important fact noted by Rosenberg that, for low saturations the pull increases with the excitation, at a critical density, however, corresponding to about the knee of the magnetization curve, the unbalanced pull reaches a maximum for all eccentricities, and for larger excitations then decreases again. Therefore a machine, which normally

operates at high saturation, may be subjected to greater stresses while it is building up than under normal field excitation.

Miles Walker[14] in his book on "Specification and Design of Dynamo-Electric Machinery" (1915), points out that the amount of the unbalanced pull for a given displacement will depend on the extent to which the iron parts are saturated, and that the effect of increased saturation is to reduce the pull for a given airgap clearance and magnetic induction. He first assumes that all the ampere-turns are expended in the airgap and by the usual method derives an expression for the unbalanced pull exactly similar to that of Behrend (see above). He then shows how the saturated magnetic circuit may be replaced by an equivalent airgap obtained by means of a graphical construction on the saturation curve of the machine, and then uses this equivalent airgap in the formula for the unbalanced pull.

R. E. Hellmund[11], Miles Walker[14] and others have pointed out the effect of series and parallel windings on the unbalanced pull and have also discussed how the unbalanced magnetic pull in the induction motor may be reduced to a minimum, by using in the stator winding two paths in parallel which lie on opposite halves of the frame and thus make it impossible for the flux on these two halves to be very unequal.

BIBLIOGRAPHY

1. J. Fisher-Hinnen, "Dynamo Design," pp. 260–265; (Van Nostrand), 1899.

2. B. A. Behrend, "On the Mechanical Forces in Dynamos Caused by Magnetic Attraction," TRANS. A. I. E. E., Vol. 17, pp. 617–626, Nov. 1900.

3. Edgar Knowlton, "Magnetic Attraction in Dynamos Due to Armature and Field being Non-Concentric," *Electrical World and Engineer*, Vol. 37, pp. 969–970, June 1901.

4. H. Linsenmann, "Mechanical Strength of Large Alternators" *Electrotechnische Zeitschrift*, Vol. 23, pp. 81–84, Jan. 1902 and pp. 103–107, Feb. 1902.

5. J. Rey, "Magnetic Pull Due to Eccentricity of Rotor in Induction Motors", *L'Eclairage Electrique*, Vol. 38, pp. 281–285, Feb 1904; Vol. 41, pp. 257–260, Nov. 1904.

6. F. Niethammer, "Deflection of Stators of Electric Generators", *Electrical World*, Vol. 44, p. 330, Aug. 1904.

7. J. K. Sumec, "Determination of Magnetic Pull on Rotor Due to Eccentricity," *Zeitschrift für Elektrotechnik*, Vol. 22, pp. 727–728, Dec. 1904.

8. B. Soschinski, "Pull Due to Rotor Eccentricity," *Zeitschrift für Elektrotechnik*, p. 153, Mar. 1905.

9. F. Niethammer, "The Lateral Magnetic Pull of Dynamos and Motors," *Zeitschrift für Elektrotechnik*, Vol. 23, pp. 421–422, July 1905; *L'Eclairage Electrique*, Vol. 44, pp. 154–155, July 1905.

10. R. V. Picou, "The Law of Magnetic Attraction," *Bulletin de la Societe Internationale des Electriciens*, Vol. 7, pp. 307–319, May 1907.

11. R. E. Hellmund, "Series vs. Parallel Windings for A. C. Motors," *Electrical World*, Vol. 49, pp. 388–389, Feb. 1907.

12. Charles R. Moore, "A Study of Unbalanced Magnetic Pull," *Electrical Review and Western Electrician*, Vol. 58, pp. 83–86, Jan. 1911.

13. F. W. Carter, "Magnetic Centering of Dynamo Electric Machinery," Proc. of the Inst of Civil Engineers (London) Vol. 187, pp. 311–318, May 1912.

14. Miles Walker, "Specification and Design of Dynamo-Electric Machinery," pp. 57–62, 347, 416, 452; (Longmans, Green), 1915.

Presented by publication only.

(Subject to final revision for the Transactions)

MAGNETIC PULL IN ELECTRIC MACHINES

BY E. ROSENBERG

LIST OF SYMBOLS

A	Pole face area in square centimeters
B	Magnetic induction (flux density in the airgap.)
B_1	Reduced induction with increased airgap
B_2	Increased induction with reduced airgap
B_m	Critical induction, causing maximum unbalanced pull
$\frac{d\,B}{d\,H}$	Differential quotient of induction and magnetomotive force in lines per sq. cm. and Gilbert's ($4\pi/10$ ampere-turns) respectively
D	Rotor diameter
E	Voltage
$\frac{d\,E}{d\,i}$	Differential quotient of voltage and exciting current
f	"Gravity deflection"; deflection of the horizontally placed shaft under the static influence of the rotor weight
F_x	Unbalanced magnetic pull of the machine, due to a displacement x
F_f	Unbalanced magnetic pull of the machine, due to a displacement f
$(F_x)_{neutr.}$	In two-pole machines: Unbalanced pull due to displacement x in the direction of the neutral diameter.
$(F_x)_{axis}$	In two-pole machines: Unbalanced pull due to displacement x in the direction of the field axis
g	"Correct" airgap, taken as an average around the machine
g_i	"Virtual airgap", which, for low induction, would require the same m.m.f. as the magnetic half circuit of the machine
g_2	In two-pole machines: Phantom airgap of the interpole (airgap of an imaginary interpole with same section as the main pole, which would require the same m.m.f. for the passing of a certain flux as the real interpole)

G	Rotor weight
H	Magnetomotive force
i	Exciting current
L	Effective length of rotor (air ducts subtracted)
n	Revolutions per minute
$n_{crit.}$	Critical speed in rev. per. min.
$2p$	Number of poles
q	Ratio between the combined deflection of all the machine parts, caused by unbalanced pull, to the displacement, causing the unbalanced pull
q_r	Ratio between rotor deflection caused by unbalanced pull to the displacement causing the unbalanced pull
q_s	Ratio between combined deflection of the other machine parts to the displacement causing the unbalanced pull
$T_{circ.}$	Ampere-turns per pole required to produce a certain induction (within the straight line characteristic) in magnetic circuit of the machine with "correct" air-gap
T_x	Ampere turns required to produce the same induction in the airgap x
$\tan\alpha$	Gradient of the magnetization characteristic
x	Displacement of rotor and stator centers
X	Final static displacement
$X_{mom.}$	Final momentary displacement
α	Local angle between magnetization curve and horizontal
θ	Angle between point of periphery and symmetry diameter

NOTE: Lengths are given in centimeters, forces and weights in kilograms, inductions in lines per square centimeter, unless otherwise stated.

Designers who figure in inches and pounds and who use "Kapp Lines per square inch" (one Kapp Line being equal to 6000 c.g.s. lines), substitute for $\left(\frac{B}{5000}\right)^2$ the value $\frac{1}{2}\cdot(B \text{ Kapplines/sq. inch})^2$, to obtain the pull in pounds per square inch in formulae (**1**) to (**10a**).

The critical speed, formula (**11**), changes to

$$n_{crit.} = \sqrt{1-q}\cdot\frac{188}{\sqrt{f_{inch}}}$$

if f_{inch} denotes the gravity deflection in inches.

Introduction

THE aim of the present paper is to investigate the occurrence and the effect of unbalanced magnetic pull in electric machines and to derive simple practical formulas for the use of the designer. There is an interesting chapter on the subject in Miles Walker's excellent book "Specification and Design of Dynamo Electric Machinery" in which for a given induction the influence of saturation on the unbalanced pull is considered. The present author has endeavored to carry the matter to its logical conclusion and to find, whether in a given machine there is a "critical induction" which gives a higher unbalanced pull than any other, smaller or larger, induction; further, to find a simple rule for determining this critical induction, and to investigate the permissible deflection of the machine parts in connection with the unbalanced pull, and the influence of the latter on the critical speed. Multipolar and bipolar machines on the one hand, and those with salient poles and cylindrical fields with distributed winding on the other hand, show very striking differences although the maximum of the forces occurring can be covered by the same formula.

A. Magnetic Pull of a Pole and of a Series of Poles

The magnetic attraction on a square centimetre of a pole face is

$$\frac{B^2}{8\,\pi} \text{ dynes}$$

or, with an accuracy of nearly 1 per cent,

$$\left(\frac{B}{5000}\right)^2 \text{ kilograms,}$$

if B, the flux density in the airgaps, immediately adjoining the pole face, is expressed in c.g.s lines per sq. cm. To get the pull per pole, provided the pole extends over a comparatively small part of the periphery, we have to multiply by the area A of the pole face, measured in square centimeters. In general, in electric machines the magnetic pulls of the different poles are equal and are arranged symmetrically around the center. With a stator of the same stiffness in all diameters the magnetic pull would only cause a certain well defined and moderate strain all around the machine. With a split machine, however, especially if the joints are not very well stiffened, the deflection of the frame in the vertical direction will be different from that in the horizontal direction and a distortion may result which actually reduces

sensibly the airgaps in the diameter perpendicular to the plane of the joint. Each half of the stator can be considered as a beam, the support and fixing of the upper half being different from that of the lower half which contains the feet.

It can happen that, through error or accident, only part of the poles are excited. The worst case will be when all the poles in one half of the machine are overexcited, the poles in the other half being without excitation. The mechanical parts of the machine should be strong enough to withstand this condition without being overstrained, although it will not be possible in large machines with small airgaps to prevent entire pulling over of the rotor against the stator core. Considering the resulting force on the one half of the rotor or stator, it is clear that every pole, the center line of which describes an angle θ with the symmetry line (Fig. 1), will contribute a component in the direction of the symmetry line

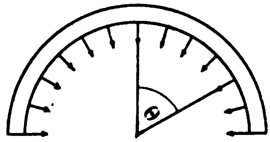


FIG. 1

$$\left(\frac{B}{5000}\right)^2 \cdot A \cdot \cos\theta$$

The average value of the cosine function over a quadrant is $2/\pi$. If therefore the machine has $2p$ poles, only p neighbouring poles being excited, the resulting force, both on the stator and rotor, will be

$$\frac{2}{\pi} \cdot \left(\frac{B}{5000}\right)^2 \cdot A \cdot p \tag{1}$$

If the airgap density B is not constant over the whole pole face, but graded, the mean value of B^2 should be taken.

The unbalanced pull, which appears when only half of the machine is excited, grows with the second power of the airgap density or of the flux. With an air gap density of 5000 it is one kilogram per sq. cm.; with a density of 10,000 it is four kilograms per sq. cm., and with a density of 12,250, six kilograms per sq. cm. The highest possible, not the normal, saturation has to be considered to provide for this case, for it is likely that the accident or error which causes the non-excitation of one half of the poles, will cause the over-excitation of the other half as for instance, when all the exciting coils are connected in series, and wires leading to opposite coils become accidentally short circuited. It must be ascertained whether both the shaft and

frame etc. will withstand the force found by formula (1) without overstraining, or whether the rotor core will pull over hard against the stator core with some lower force which is not sufficient to cause overstrain.

B. Unbalanced Pull Due to Eccentricity

I. *Multipolar Salient Pole Machines.* An entirely different and very general case is that of a machine, the coils of which have an equal excitation, but cause a different flux under the various poles due to differences in the airgap. Many causes contribute to such a condition. Frequently the outer surface of the rotor and the inner surface of the stator are not perfectly cylindrical. Even if they are perfectly cylindrical and concentric while the machine is cold, a noticeable deviation may occur due to the difference in temperature between stator and bedplate when the machine is heated. With a machine of five meters diameter, for instance, a difference of 20 deg. cent. in the average temperatures of frame and bedplate would correspond to approximately one mm. difference in length. If both feet of the hotter frame are rigidly bolted down to points of the cooler bedplate, a distortion of the frame will result; if only one foot is rigidly fixed, a shifting of the stator center will occur.

Another factor is the clearance between shaft and bearings. Even with perfectly new bearings this clearance can be measured by tenths of a millimeter. While the machine is at rest, the oil is squeezed out from underneath the shaft, and all the clearance will be between the top part of the shaft and the upper bearing shell. In this position the machine is erected and centered. When the machine is running, the bearing is flooded with oil and the clearance divides equally around the shaft. Very frequently of course, imperfect erection, a bending of the shaft or slight subsidence of the foundations with consequent distortion of the bedplate may be the cause of a displacement of the rotor and stator centers.

We shall now investigate the case of a rotor and stator each with a cylindrical surface, but with the centers displaced by an amount x (the eccentricity). Without displacement, the radical airgap would have a constant value g. The diameter drawn through the two centers we shall call "symmetry diameter". Assume, first of all, a multipolar machine with salient poles, all the exciting coils giving the same number of ampere-turns. In the symmetry diameter the airgap will in one place be reduced

to $g - x$, in the opposite place increased to $g + x$. At points, the radius through which describes an angle θ with the symmetry diameter, the increase or reduction of the airgap will be, with close approximation, $x \cdot \cos \theta$. At right angles to the symmetry diameter the increase or reduction will be practically zero.

The flux emitted by one field pole returns through the contiguous halves of the neighboring poles. In a bipolar machine the flux of each pole is the same in spite of airgap differences; in a multipolar machine the poles of one half of the machine will carry a greater flux than those of the other half, unless the winding arrangements prevent this (Equalizing connections). These apart, the flux of each pole will be different from that of the others, dependent upon the local airgap. Poles in the diameter at right angles to the symmetry diameter will carry the normal flux, for their neighbor poles in the one half (with reduced airgap) will tend to increase their flux just as much as those in the other half (with increased airgap) will tend to reduce it. The result is that in a multipolar machine each pole may be considered as if it would create its own flux and not be dependent upon adjacent poles.

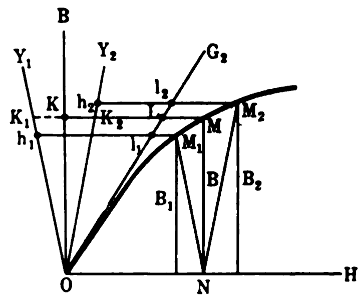


Fig. 2

We assume the machine is excited to give, with the mean airgap, a flux density B lines per sq. cm. of pole face. Where the airgap is reduced to $g - x$, the density will have a greater value B_2, and where the airgap is increased to $g + x$, a smaller value B_1. The first gives a greater magnetic pull in one direction than the second in the opposite direction, and the difference of the two pulls or the "local unbalanced pull" in kilograms per sq. cm. of the pole surface is given by the formula

$$\left(\frac{B_2}{5000}\right)^2 - \left(\frac{B_1}{5000}\right)^2 = \left(\frac{B_2 + B_1}{5000}\right) \cdot \left(\frac{B_2 - B_1}{5000}\right) \qquad (2)$$

It is easy to determine the flux densities B_2 and B_1, if the magnetic characteristic of the machine is given. In Fig. 2 the abscissae H of the curve represent the exciting m.m.f. per pole, while the ordinates B, represent the flux density per sq. cm. of the pole face, with the normal airgap. $O\,N = K\,M$ represents a certain m.m.f., required to produce the induction B. $O\,G$ is

the "airgap line", the portion $K L$ representing the m.m.f. required for the magnetization of the air-gap g, the part $L M$ the m.m.f. for the magnetization of the iron. The real existing air-gap, however, is not g but is $g + x$ in one case and $g - x$ in the other. We draw through O two lines OY_1 and OY_2 such that for any induction B (= $N M$), $K_1 L$ represents the m.m.f. required for the airgap $g + x$ and $K_2 L$ the m.m.f. for the airgap $g - x$. We then draw through N two lines respectively parallel to these new axes $O Y_1$ and OY_2. The points M_1 and M_2 where lines so drawn intersect the magnetization curve then give the actual airgap densities B_1 and B_2 under the poles with the increased and with the reduced airgap, for a given excitation $O N$, because

$$h_1 M_1 = h_2 M_2 = O N.$$

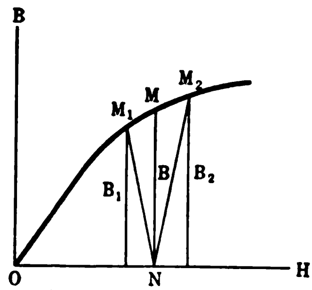


FIG. 3

Fig. 3 is a repetition of Fig. 2 without unnecessary lines.

In general, the abscissae of the lines $O Y_1$ and $O Y_2$ (Fig. 2) will bear to the abscissa of $O G$ for the same flux density very nearly the ratio of the displacement x to the airgap length g. Only if the displacement is very great compared with the average airgap and if the machine has open slots, will the proportionality be markedly disturbed due to the well known crowding of the lines in the airgap near the teeth. Thus, if teeth and slots have equal width and the normal radial airgap is equal to half the width of the slot, then in the extreme case of a machine pulling hard over, the abscissa of the line $O Y_1$ will be equal to about 0.85 of the abscissa of the line $O Y_2$ which latter one, of course, then coincides with $O Y$.

For small displacements, $O Y_1$ and $O Y_2$ can always be drawn symmetrical with reference to the ordinate, and their m.m.f. for a given induction B is approximately $\pm\, x \,.\, B$, where the displacement x is measured in centimeters and the m.m.f. in gilberts

$$\left(\frac{4\,\pi}{10}\ \text{ampere-turns}\right)$$

In formula (2) there appears the sum and the difference of the two inductions B_2 and B_1. In Fig. 4 the part M_1MM_2 of Fig.2 is drawn on a greater scale and also the chord M_1M_2 which cuts

the ordinate of point M in m. The chord describes with the horizontal an angle α. The difference $B_2 - B_1$ is in the figure represented by

$$P_1 P_2 = P_1 m + m P_2 = (M_1 P_1 + P_2 M_2) \cdot \tan \alpha$$

$M_1 P_1$ and $P_2 M_2$ are the abscissae of the x characteristics for inductions B_1 and B_2 respectively, are therefore equal to $x B_1$ and $x B_2$. We therefore hav e

$$B_2 - B_1 = (B_2 + B_1) \cdot x \cdot \tan \alpha$$

and the formula (**2**) can be written in the form

$$\left(\frac{B_2 + B_1}{5000}\right)^2 \cdot x \cdot \tan \alpha$$

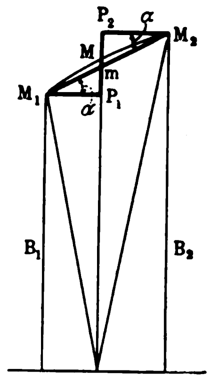


FIG. 4

If x is not very great, the sum $B_2 + B_1$ can be accurately replaced by $2\,B$, and the angle which the chord forms with the horizontal can be replaced by the angle formed by the tangent in point M with the horizontal. Tan α is then the ***gradient*** of the curve in point M and can be expressed as a differential quotient $\frac{dB}{dH}$ or the ratio of an infinitesimal increase in induction to the increase in m.m.f. causing it.

The unbalanced pull in kilograms per sq. cm. is, therefore,

$$4\left(\frac{B}{5000}\right)^2 \cdot x \cdot \tan \alpha = 4\left(\frac{B}{5000}\right)^2 \cdot x \cdot \frac{dB}{dH} \qquad (\mathbf{3})$$

The expression $x \cdot d\,B$ in formula (**3**) represents also the m.m.f. necessary to produce an increase $d\,B$ in induction in an airgap x, while $d\,H$ represents the m.m.f. per pole necessary to produce in the magnetic half circuit of the machine with the correct airgap the same increase $d\,B$ in induction. The ratio $\frac{x \cdot dB}{dH}$ is therefore a ratio of magnetomotive forces and can also be replaced by the ratio of the corresponding ampere turns; ampere turns required to drive through the airgap x an infinitesimal increase $d\,B$ in flux density, divided by the ampere turns per pole required to drive through the magnetic circuit of the machine with the correct airgap the same increase in flux density.

Looking closer into the approximations which we introduced

in formula (**3**), we find: If x is large and if the magnetization characteristic were a straight line (Fig. 5), the sum $B_2 + B_1$ would actually be greater than $2B$. In Fig. 5 the value $\frac{B_2+B_1}{2}$ is shown dotted in. If, however, the characteristic is strongly curved (Fig. 6), $B_2 + B_1$ will be slightly smaller than $2B$. For hard pulling over, when x reaches the greatest possible value, the saturation will in any case be high enough that the curvature of the characteristic is marked. We are therefore certain that, on this score, formula (**3**) does not give too low values for the extreme case.

A very important question is, now: How does the expression of formula (**3**) change with growing excitation? The first part of almost every magnetization curve is a distinct straight line going through the origin of the ordinates. For every straight

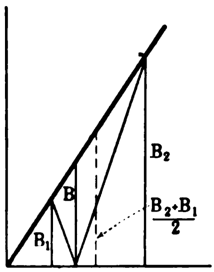


FIG. 5

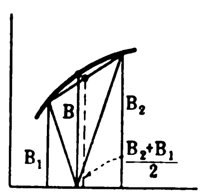


FIG. 6

line characteristic, whether or not going through the origin, the gradient $\frac{dB}{dH}$ is constant, the unbalanced pull therefore will grow with the square of the induction B. As soon, however, as the saturation of the iron parts is noticeable, the quotient $\frac{dB}{dH}$ will constantly diminish, and the question is how to find the induction B for which the product $B^2 \cdot \frac{dB}{dH}$ is a maximum. In Fig. 7 a graphical procedure is shown. For a point M of the saturation curve I the ordinate $B = NM$ is drawn and a line MP perpendicular to the direction of the curve in point M. The angle $N\,M\,P$ is identical with the angle α formed between the curve in point M and the horizontal.

$$\tan \alpha = \frac{dB}{dH}$$

$$N\,P = N\,M \tan \alpha = B \tan \alpha$$

and the area of the triangle $N\,M\,P$ is

$$\frac{1}{2} B \cdot B \cdot \tan \alpha = \frac{B^2}{2} \cdot \frac{dB}{dH}$$

If such triangles are drawn for different points of the curve, the triangle area will be proportional to the function $B^2 \cdot \frac{dB}{dH}$ for the various points.

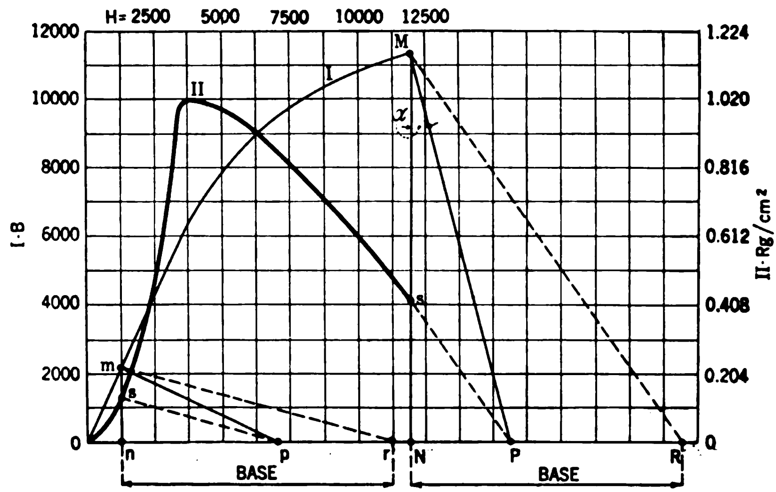


FIG. 7

In order to obtain a curve representing the magnetic pull for every excitation, it is now only necessary to replace all the triangles $N\,P\,M$ by others which have a suitable arbitrary base $N\,R$. Connecting R with M and drawing a line $P\,S$ parallel to $R\,M$, it follows from the elements of geometry that the triangles $N\,P\,M$ and $N\,R\,S$ are equal in area. (The line $R\,S$ of the latter triangle is not drawn in Fig. 7). $N\,S$ is therefore a measure of the function $B^2 \cdot \frac{dB}{dH}$. The scale is determined by the base $N\,R$. If the construction is repeated point for point (for a second point m the construction is shown), we obtain the bold line curve II in Fig. 7, the first part of which, corresponding to the straight

portion of the magnetization characteristic, consists of a parabola with vertical axis. Nearly immediately as the magnetization characteristic begins to bend, the curve of the pull reaches its maximum and from then falls steadily.

This teaches us a thing of the utmost importance. In salient pole machines which are not *entirely* unsaturated, it is not sufficient to calculate the unbalanced magnetic pull for a high excitation. The "critical" excitation for which the unbalanced magnetic pull is a maximum, is reached at the very beginning of the "knee" in the magnetization curve; as generators and motors must be able to stand abnormal changes in voltage, the unbalanced magnetic pull must be calculated for this critical excitation.

In some machines, exciters for instance, and also turbo-generators with cylindrical fields, very high saturation occurs in a small part of the magnetic circuit (saturation plates or teeth), before appreciable saturation occurs in the other parts of the magnetic circuit, and in this case, after a very short initial straight part, a nearly continuous bending of the saturation curve or even two distinct bends can be observed. At even very small induction the curve bends, (the saturation plates becoming saturated), then there is a straight line characteristic and then there is a second bend, (when the saturation of the iron circuit as a whole becomes marked). In this latter case the straight line characteristic does not go through the origin of the coordinates. The first bend may cause a *local* maximum of the unbalanced pull, that is to say, a point higher than the neighbor points, but the important maximum occurs at the second bend of the characteristic. The "short cut" explained later on for the determination of the "critical induction" does not refer to these cases. The character of the unbalanced pull as shown in the bold curve II of Fig. 7 is obtained for most electric machines.

It is not necessary really to construct the whole curve of unbalanced pull, point by point. A few points at the end of the straight line and at the beginning of the curved characteristic are sufficient. What we are interested in is only the ***maximum*** value of the unbalanced pull and this is, with such characteristics, obtained immediately after the straight line part of the saturation curve is ended. The only task is, then, to find an official "terminus" of the straight line part.

A rule which the author suggests and which seems to give sufficiently accurate results, is to draw a tangent to the saturation

curve with a gradient equal to 5/6 of the gradient of the straight part of the curve. In Fig. 8 the simple construction is shown for the characteristic of Fig. 7. Through the origin a line OZ is drawn which has an ordinate of 5000 for the same abscissa for which the straight line characteristic has an ordinate of 6000. TU is the tangent parallel to OZ; the point of contact with the curve is T. V, the point on the extended straight line, which has the same ordinate B_n as T, is what we will call the end point or terminus of the straight line characteristic.

We now see the two factors which enable us to reduce in a machine with fixed principal dimensions the unbalanced mag-

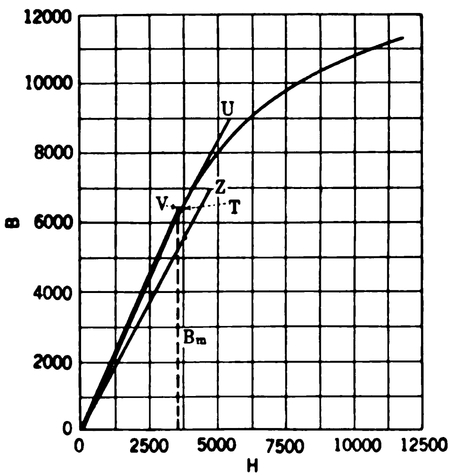


FIG. 8

netic pull caused by a definite displacement. The first is to increase the airgap, because $\frac{dB}{dH}$ for the straight part of the magnetization curve is very nearly in inverse proportion to the airgap. The second means, however, is to reduce the iron section, for example that of poles and teeth, because that causes the saturation characteristic to bend earlier, or in other words it cuts short the straight part of the magnetization curve.

Up to now we have calculated the radial unbalanced pull per sq. cm. for an eccentricity x. The eccentricity changes in a quadrant from x to zero according to the expression $x \cos \theta$.

The radial magnetic pull per sq. cm. in the different parts of a multipolar machine will therefore be very nearly proportional

to $x \cos \theta$ and the component of this pull, working along the "symmetry diameter" will be proportional to $x \cos \theta \cdot \cos \theta = x \cos^2 \theta$.

In machines with a number of poles divisible by 4, we may always consider together two points, separated by a full quadrant one with an angle θ, the other with an angle (90 deg. + θ). The *sum* of the components of these pulls per sq. cm. is

$$4 \cdot \left(\frac{B}{5000}\right)^2 \cdot \frac{dB}{dH} \cdot [x \cos^2\theta + x \cos^2 (90 \text{ deg.} + \theta)]$$

$$= 4 \cdot \left(\frac{B}{5000}\right)^2 \cdot x \cdot \frac{dB}{dH},$$

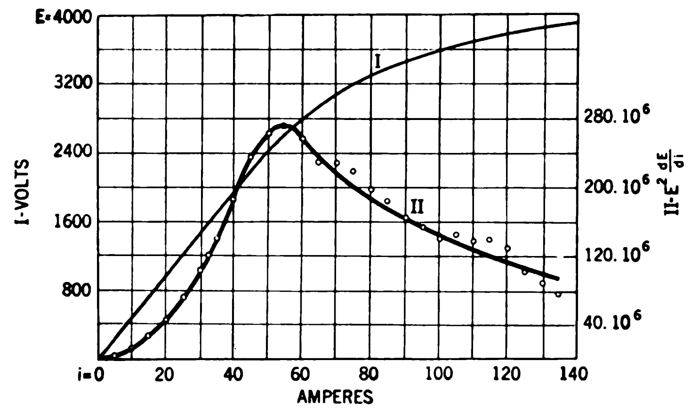


FIG. 9

as $\cos (90 \text{ deg.} + \theta) = -\sin \theta$ and therefore $\cos^2 \theta + \cos^2 (90 \text{ deg.} + \theta) = 1$.

In such machines we can therefore always find two pole pairs which, taken together, develop the same unbalanced pull as a single pole of identical dimensions and induction, the axis of which is parallel and the pole face of which is perpendicular to the symmetry diameter.

In a six pole machine three pole pairs together develop 1½ times the unbalanced pull of an imaginary pole pair with an axis parallel and a pole face perpendicular to the symmetry diameter, as is proved by the following consideration:

In a six-pole machine we can consider at the same time three

places with angles θ, 60 deg. + θ, 120 deg. + θ, and we have $\cos^2\theta + \cos^2(60 \text{ deg.} + \theta) + \cos^2(120 \text{ deg.} + \theta)$

$$= \frac{1 + \cos 2\theta}{2} + \frac{1 + \cos(120 \text{ deg.} + 2\theta)}{2}$$

$$+ \frac{1 + \cos(240 \text{ deg.} + 2\theta)}{2} = \frac{3}{2}$$

The same investigation for a machine with 10, 14.....poles would confirm that *the total unbalanced pull can be calculated in all multipolar machines, as if the airgap were reduced by an amount x in one quarter of all the poles, and increased by x in another quarter of all the poles, and left alone in the remaining two quarters.* As poles of electric machines are symmetrical, there is no difficulty in calculating the pull belonging to one half pole.

The total unbalanced magnetic pull F_x of the whole machine will therefore be obtained, if we multiply the pull per sq. cm. from formula (3) with $\frac{p \cdot A}{2}$, or one quarter of the total pole face area of the machine.

$$F_x = \frac{pA}{2} \cdot 4 \cdot \left(\frac{B}{5000}\right)^2 \cdot x \cdot \frac{dB}{dH} = 2pA \cdot \left(\frac{B}{5000}\right)^2 \cdot x \cdot \frac{dB}{dH} \quad (4)$$

Let us call B_m an induction, situated on the extension of the straight line characteristic, which gives the same magnetic pull as is actually obtained by the "critical" induction. In reality B_m will be slightly smaller than the critical induction, but we shall speak, with a small inaccuracy, of B_m as critical induction.

For the straight line part of the characteristic, going through the origin, we can replace $\frac{x \cdot dB}{dH}$ by $\frac{x \cdot B}{H}$ or $\frac{T_x}{T_{circ.}}$ if T_x stands for the ampere turns required to produce in an airgap x any flux density within the straight line characteristic and $T_{circ.}$ stands for the ampere turns per pole, required to produce in the magnetic circuit of the machine with the correct airgap the same flux density.

The formula takes therefore the following simple form:

$$F_x = 2pA \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{T_x}{T_{circ.}} \quad (5)$$

or

$$F_x = 2pA \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1} \tag{6}$$

if g_1 is the "virtual airgap", that is to say, an airgap which would take, for an induction within the straight line characteristic, as many ampere turns as the real airgap and the iron path of the magnetic half circuit together.

For machines with small airgap, as induction motors, the "virtual airgap" is appreciably higher than the real airgap, say 30 per cent higher or more; while in generators, especially turbo-generators, the virtual airgap is nearly equal to the actual one.

Instead of the total pole face area $2pA$, we may substitute the cylindrical field surface $c \cdot \pi \cdot DL$, if c represents the "pole factor" or ratio of the pole arc to the pole pitch, D the diameter and L the effective length of the field (air ducts, if any, excluded). The formula is then written

$$F_x = c \cdot \pi \, DL \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1} \tag{6a}$$

The factor c is mostly in the neighborhood of 2/3.

The magnetization curve I shown in Fig. 7 is a reproduction of Fig. 347 from Miles Walker's book. The machine in question is an alternator with 40 poles, each with a pole face area of 650 sq. cm. The radial airgap is 0.51 cm.

$$2p = 40 \qquad A = 650 \qquad g = 0.51 \text{ cm.}$$

The curve II of the unbalanced pull shows that the maximum is 1.02 kilograms per sq. cm. for a displacement of 0.1 cm. The scale for the "pull curve" II is marked on the right of Fig. 7 and it can be verified by figuring the pull for any particular induction on the straight part of the magnetization characteristic. Taking the simpler form of the determination of the maximum (Fig. 8) we obtain the critical induction $B_m = 6300$. The straight part of the line goes through a point with an ordinate $B = 4300$ and an abscissa $H = 2500$. The ratio

$$\frac{x \cdot dB}{dH} = \frac{T_x}{T_{circ.}} = \frac{4300 \cdot x}{2500} = 1.72 \cdot x$$

The maximum radial unbalanced pull per sq. cm. is therefore according to formula (3)

$$4 \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{dB}{dH} \cdot x = 4 \cdot \left(\frac{6300}{5000}\right)^2 \cdot 1.72 \cdot x = 10.8 \cdot x$$

or for a displacement of $x = 0.1$ cm., 1.08 kilograms. This method of calculation gives here a result 6 per cent high, which is entirely satisfactory. It is, of course, not necessary to calculate at first the force for one sq. cm. Formula (5) gives us direct the unbalanced pull for the whole machine

$$F_x = 2pA \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{T_x}{T_{circ.}} = 40.650 \cdot \left(\frac{6300}{5000}\right)^2 \cdot 1.72 \cdot 0.1$$
$$= 7100 \text{ kg.}$$

If we had used formula (6) instead of formula (5), we should have obtained at first from the straight line characteristic

$g_1 = \dfrac{2500}{4300} = 0.58$ cm. (*i. e.* 0.07 cm., or 14 per cent greater than the actual airgap)

$$F_x = 2pA \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1} = 40.650 \cdot \left(\frac{6300}{5000}\right)^2 \cdot \frac{0.1}{0.58}$$
$$= 7100 \text{ kg.}$$

It may be mentioned that Walker figures for the machine in question, which works with a normal airgap density of 9160, an unbalanced pull of 5400 kilograms. If we make the calculation in our way for normal saturation, we should also get approximately the same result. But, as our investigations have shown the normal induction does not give the greatest pull, which must be considered by the designer.

The flux density in the most saturated iron parts (sheet steel) corresponding to an airgap density of 6300 is not more than about 11,000 to 13,000 lines per sq. cm.

A salient pole machine with given dimensions of the iron parts and with a given airgap experiences for a certain displacement the greatest unbalanced pull with an excitation, which is, as a rule, well below the normal working excitation. There is a passage in Walker's book, which, although it was most likely correctly understood by its author, is apt to be misleading. Walker says on page 56 that in large alternators with a great number of poles and a small airgap, the flux density in the gap must be limited to a moderate value, such as 50 kilolines per square inch (7750 lines per sq. cm.) to keep the unbalanced pull down. Most designers believe indeed that in a given machine

with fixed iron dimensions the unbalanced magnetic pull is reduced by lowering the saturation. This is contrary to the result of our investigation, (unless the induction be reduced below the end point of the straight line characteristic.) As long as the iron dimensions are not changed, the flux density may be increased to any desired value, and the effect will in fact be a reduction of the magnetic pull at the normal voltage, although the *possible* maximum unbalanced pull which has to be considered for the mechanical design remains the same. The unbalanced magnetic pull can only be reduced by a reduction of the flux, if a reduction of the iron sections goes hand in hand with the flux reduction.

II. *Multipolar Machines with Distributed exciting Winding.* Up to the present we have considered machines with salient poles

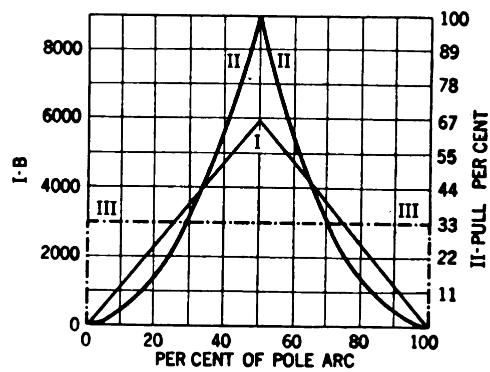


FIG. 10

in which the induction over the whole pole face would be constant but for the displacement of the rotor and stator centres. In the case of induction motors, however, and turbo-alternators, the induction changes gradually from zero to a maximum. At first we shall deal with multipolar machines of this description. The rule already arrived at, that the unbalanced pull is the same as if the airgap had been reduced by the full amount x in one quarter of the poles and increased by x in the opposite quarter holds good, from its derivation, also for machines with distributed winding.

Let us assume that the curve I in Fig. 7 represents the curve of the airgap saturation of such a machine, while the ordinates of the bold line curve II represent the magnetic pull in kilograms per sq. cm. for a displacement of 0.1 cm. The induction over

the pole face may change according to a straight line law, (I in Fig. 10), if the machine is entirely unsaturated and if the exciting winding (in case of turbo-generators for instance) is equally distributed around the whole pole face. These two conditions are hardly ever fulfilled, but the case forms a starting point. For every induction of curve I in Fig. 10 the magnetic pull per sq. cm. can be taken from Fig. 7 and so we obtain as curve representing the local pull over the pole face the bold line parabolas II in Fig. 10. The average value of the pull taken over the whole pole·face is in this case exactly 1/3 of the value corresponding to the maximum induction. The area of the dotted rectangle III in Fig. 10 is equal to the area enclosed by the base and the double parabola. This, however, does not represent by any means, the

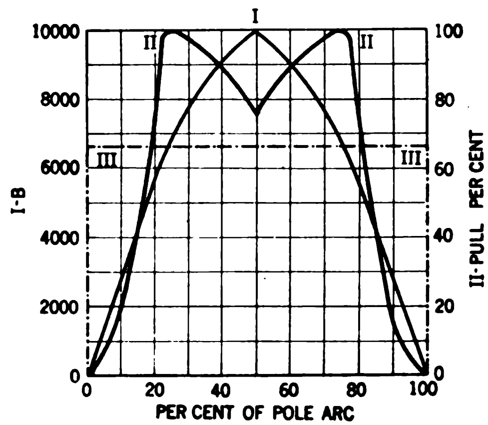


FIG. 11

maximum unbalanced pull possible in this machine. In Fig. 11 the field form I and the "pull curve" II are drawn for the case that the induction in the center of the pole is increased to 10,000. The average value of the pull, line III, is here 66 per cent of the maximum value. A full review of the possible changes in the average value of the pull is given in curve III of Fig. 12. I and II in Fig. 12 are repetitions of the curves I and II in Fig. 7. The ordinate of curve III for any given abscissa gives the average value of all the ordinates of the pull curve II starting from the abscissa *O* up to the abscissa in question. As long as the pull curve II follows the law of a parabola, the ordinates of curve III are exactly 1/3 of those of curve II. The further points are obtained by point to point calculation. With this curve it is quite

easy to give the average value of the pull over one half pole (and therefore over the whole pole), if the total m.m.f. per pole or the highest induction is known. The curve III has a maximum of about 66.5 per cent of the maximum of curve II for a highest induction of 10,500 lines per sq. cm. The curve III is very much flatter in its upper part than the curve II and its ordinates exceed 60 per cent for abscissae varying from H = 6250 to H = 12,000. In a machine with distributed winding there is not by any means the marked falling off of the unbalanced magnetic pull, after a certain comparatively low induction is reached. On the contrary, the magnetic pull remains very nearly constant over a wide range of possible inductions.

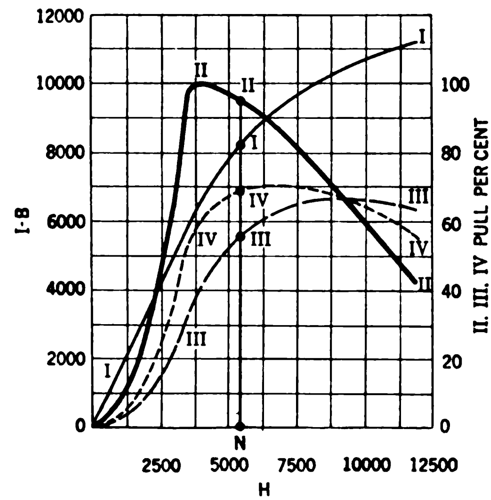


FIG. 12

As a rule, the winding in turbo-generators and induction motors is distributed so as to give a more sinusoidal shape of the flux. For instance, the middle third of a pole face is left without exciting winding, and the winding is only distributed in the first and third part of the pole. If rotor and stator are slotted evenly all around the periphery, as is the case in induction motors, a-c. commutator motors and sometimes also in turbo-generators, the field form can be represented by curve I in Fig. 13, while curve II shows the local pull and the dotted straight line IV the average value of the pull in this case, the area of the dotted rectangle IV being equal to the area enclosed by curve II and the base. It is quite easy to determine the average pull value over the pole

for *any* saturation by combining the curves II and III in Fig. 12. It is shown as an example for the abscissa ON, which corresponds to an induction N I (curve I). Two thirds of the pole have an induction gradually rising from zero to the value mentioned before. The average value of the pull in this part is given by the ordinate N III. One third of the pole face has a constant induction N I, its pull per sq. cm. being represented by the ordinate N II. To get the average over the whole pole, we take 2/3 of N III and add 1/3 of N II, thus obtaining the ordinate N IV. The distance III · IV is one third of the distance III · II. If we know the maximum value of the induction or the total m.m.f. per pole, the ordinate of curve IV gives at once the average value of the pull for this field form. A value of 2/3 of the peak ordinate

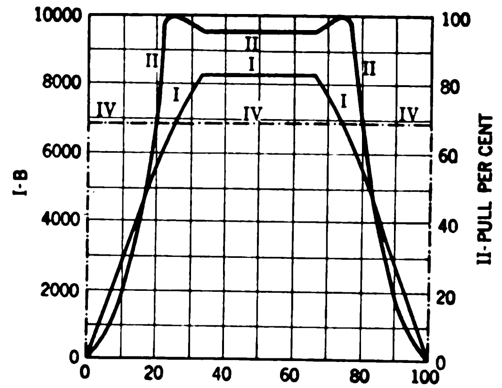


FIG. 13

of curve II is in the example of Fig. 12 already obtained for an induction of approximately 7700, the highest value of IV is 70.5 per cent of the peak value of II and corresponds to an induction of 9300; the line of 2/3 is reached again for an induction of 10,500.

Other magnetization curves were investigated by the author and a slightly higher value was found for the maximum pull of a pole with such distribution of winding.

We can very well say, in a machine with a field slotted all round and a distributed exciting winding, the unbalanced magnetic pull for a given displacement remains very nearly constant with saturations as they are practically applied, and it is approximately 2/3 of the value which would be obtained, if the whole

active cylindrical surface of the rotor were equally excited, so as to give the "critical induction". In semi-closed-slot rotors the active rotor surface is very nearly equal $\pi \cdot DL$, L representing the effective length, after deduction of airducts. For such machines we can therefore use the formula obtained for salient pole machines

$$F_x = c \cdot \pi DL \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1} \tag{6a}$$

the value of c or "pole factor" being roughly 2/3 for closed-slot rotors. In a salient pole machine we have called "pole factor" the ratio of the pole arc to the pole pitch and this value as a rule is also about 2/3. ***The maximum unbalanced pull in a machine with a field slotted all round and with distributed winding is therefore approximately equal to the maximum pull in a machine with salient poles having a pole factor of*** **2/3.**

With open-slot rotors (turbo fields) the surface $\pi D L$ should be reduced by the area of the slot openings. It must be born in mind, that, with the reduction of the surface, goes hand in hand a nearly proportional increase of the value of B_m, the critical airgap saturation, because the saturation curve starts to bend when the *iron* saturation, and not when the airgap induction, reaches a certain value. As B_m appears in the second power, an open-slot rotor will give a slightly higher value for the unbalanced pull than the closed-slot rotor.

To get an accurate result in turbo-generator fields which are not slotted in the middle part of the pole, two magnetization characteristics should be considered: The one for the solid part (say 1/3 of the pole width) with the highest induction, and the other for the slotted part, with induction grading down to zero. The critical induction of the slotted part will be appreciably lower than that of the solid part. The unbalanced pull contributed by the slotted parts, will only be about half of that contributed by the solid part. The latter corresponds to a salient pole with a pole factor 1/3. The factor for the whole machine, if B_m *for the solid part only* is put into the formula, will then be about 0.5.

On the other hand, it is possible to adhere to our accustomed pole factor 2/3, if one constructs a *mean* magnetization curve about the average between that of the slotted and that of the plain part and considers the B_m and g_1 of this mean curve.

The case is similar to that of a salient pole with graded airgap.

III. *Salient Pole Machines with Graded Airgap.* Often in salient pole machines the airgap is not constant but a minimum in the middle of the pole and increased gradually towards the tips. The induction therefore cannot reach the critical value in all parts of the pole face at the same time. The maximum pull will not be reached when the middle part of the pole has the critical induction, because it would mean that all the other parts have a lower induction and therefore contribute an appreciably smaller magnetic pull than the maximum. It will be just as in machines with distributed winding, that the maximum resulting pull is obtained with a higher saturation in the pole center and that the upper part of the curve, representing the magnetic pull dependent on the excitation, is flatter than for a machine with constant induction over the pole face. If in one part of the pole the induction reaches the critical value, this part gives the maximum pull. The parts from there to the corner which have lower values of induction, give a much smaller pull per sq. cm.; the parts towards the center of the pole with higher inductions also give a pull below the maximum, but not falling so rapidly. If the excitation is increased, the point of the critical induction will shift towards the corner, increasing the pull of the corner parts and reducing that of the center parts, so that the total change in pull is not so very marked. If the excitation is lowered, the critical induction will shift more towards the center, increasing the pull there and reducing the pull of the corner parts. In general, it will be possible to change the excitation within comparatively wide limits without very great change of the resulting unbalanced magnetic pull. The calculation of the greatest value of the unbalanced pull can be made, with sufficient accuracy for practical purposes, as if the airgap (without displacement of the rotor center) had a constant mean value.

IVa. *Bi-Polar Salient Pole Machines.* In multipolar machines the flux leaving the field at a place where the airgap is reduced by the eccentricity, also returns into the field in a place where the airgap is reduced. The sum total of all the fluxes taken over one-half of the machine will therefore be greater than the sum total of all the fluxes taken over the other half of the machine.

The same possibility exists in a two-pole machine, if the eccentric displacement is at right angles to the field axis. In Fig. 14 this case is shown. A rotor is assumed with salient poles built in the manner of the Siemens *H* armature, with one exciting

coil, the pole face covering a greater arc than is usual in actual machines, but presenting a distinct neutral zone. The field axis $A A_1$ has a distance x from the stator center. It is clear that in this machine, lines of force leaving the left corner of the upper pole, marked 1, will after passing through the armature, re-enter the field at the left corner of the lower pole, marked 4, while lines leaving the field at the right hand corner 2 of the upper pole will re-enter at the right hand corner 3 of the lower pole. In the half machine on the left side of the axis $A A_1$ the airgaps are throughout smaller than on the right side, and as the two sides represent two parallel magnetic paths with different reluctance, the total flux on the left side will be greater than the total flux on the other side. This case is similar to that of a multipolar machine except for the fact on the one hand that through the neutral zone a

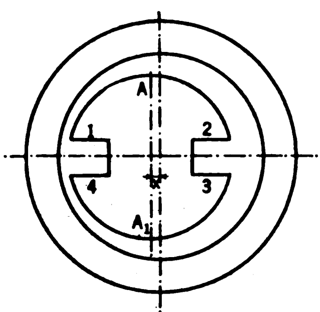


FIG. 14

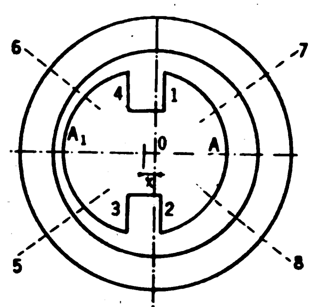


FIG. 15

considerable part of the magnetic pull is cut out, while on the other hand the saturation of the stator and rotor *cores* is hardly influenced by the unequal distribution of the flux in the two machine parts. In order to find out the airgap densities B_2 and B_1 corresponding to a certain displacement x, a magnetic characteristic of the airgap, teeth and pole tips should be drawn, excluding the ampere turns required for the cores, and therefore the critical induction B_m will be higher and the local unbalanced pull resulting from the eccentricity will be greater than it would be if the magnetic characteristic of the whole machine had to be considered.

A different state of affairs exists if the rotor center is displaced with regard to the stator center by an amount x in the direction of the field axis. (Fig. 15). Although the airgap in the left half is smaller than in the right half, the total flux must be equal in both halves, if no flux can return through the neutral zone.

The lines of force pass in series through the smaller and the larger airgaps. The only effect of the eccentricity will be a different distribution of the flux over the pole faces. Let us assume for simplicity's sake that the rotor in a central position the airgap induction over the whole pole face would be equal. Then it is clear, that with the rotor shifted as in Fig. 15 the flux density will be a minimum in the center A of the right pole, where the airgap is a maximum, while it will increase gradually to the corners 1 and 2 of the pole. In the left pole, however, the flux density will be a maximum in the pole center A_1 and will grade down towards the corners 3 and 4. Therefore, considering a square centimeter in the middle A of the right pole and one in A_1 of the left pole, there will indeed act on the rotor an unbalanced magnetic pull directed to the left.

The airgap changes, going in a quadrant from the horizontal to the vertical position, in the left half from the value $g - x$ to nearly the value g; in the right half from the value $g + x$ to nearly g. The average values of the airgaps in the left and right halves are about $g - 0.7\,x$ and $g + 0.7\,x$ respectively.

Within the sector $5 \cdot O \cdot 6$ shown in Fig. 15 the flux density will be greater, and in the corner parts 3 and 4 smaller than the average. In the opposite sector $7 \cdot O \cdot 8$ the flux density will be smaller, and in the corner parts 1 and 2 greater than the average. The sectors mentioned will contribute an unbalanced pull directed to the left, the corner parts a smaller component directed to the right. It is clear that the total unbalanced pull in this case is smaller than in the case of Fig. 14. A two-pole rotating field will therefore, if its center is displaced with regard to the stator center, experience an unbalanced pull which changes twice during a revolution from a maximum to a minimum value.

In practise, a salient pole of a two-pole machine, covers about 120 deg. In the case corresponding to Fig. 14 (displacement parallel to the neutral diameter) the limits for O from 1 to A are 30 deg. and 90 deg. or $\pi/6$ and $\pi/2$. An element of the surface covering an infinitely small angle $d\,\theta$ has an area $\frac{1}{2}\,DL \cdot d\,\theta$. The component of unbalanced pull in the horizontal direction contributed by this element is

$$\frac{1}{2} \cdot DL \cdot d\theta \cdot 4 \cdot \left(\frac{B_m}{5000}\right)^2 \cdot x\,\frac{dB}{dH} \cdot \cos^2\theta$$

$$= \frac{1}{2} \cdot DL \cdot d\theta \cdot 4 \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1} \cdot \cos^2\theta$$

The integration is to be made from + 30 deg. to 90 deg and from − 30 deg. to − 90 deg., or the integration made from 30 deg. to 90 deg. must be doubled. This gives

$$4\,DL \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta \cdot d\,\theta$$

The value of the integral is $\frac{\theta}{2} + \frac{\sin 2\,\theta}{4}$ taken between above limits, or $\frac{\pi}{6} - \frac{\sqrt{3}}{8} = 0.307$. Therefore

$$(F_x)_{neutr.} = 1{,}228 \cdot DL \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1}$$

$$= 0.39 \cdot \pi\, DL \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1} \qquad \textbf{(7a)}$$

The pole factor is here 0.39 instead of the accustomed value of 2/3 We see that the unbalanced pull in case of a two-pole salient pole turbo-rotor with 120 deg. pole angle is only 59 per cent of that of a multipolar rotor of the same dimensions and total pole area, provided that the critical induction and the virtual airgap are the same in both cases.

IVb. *Bipolar Salient Pole Machines with Interpoles.* Bipolar salient pole machines are used mainly for continuous current, while for two-pole turbo-alternators the cylindrical field and distributed winding is the rule.

If there are interpoles in a two-pole constant-current machine, the total flux of one main pole can be different from that of the other main pole, and the case of Fig. 15 requires a revision for such machines. If, for example, the airgap of the north pole is smaller than that of the south pole, there will be a greater flux in the north pole and the difference in flux will return through the iron of the interpoles, creating in the latter ones south magnetism, whereas normally they would be neutral as long as the armature gives no current. The two interpoles together carry the difference between the actual fluxes of the main poles. One interpole carries the "excess flux" viz. the difference between the greater flux and the average flux. In a four-polar machine

which has poles in and at right angles to the symmetry diameter, no m.m.f. is required to drive the excess flux back, for in that case there is, for instance, just as much increase of flux in the upper pole as reduction of normal flux in the lower pole. In the present case, however, a certain amount of m.m.f. is required for the return of the excess flux through the interpole. This will depend mainly upon the section of the interpole and the airgap under the interpole. In a multipolar machine one per cent vertical displacement causes one per cent excess m.m.f., therefore one per cent excess flux in the one pole and likewise one per cent deficit flux in the opposite pole. Here one per cent excess m.m.f. has to drive the excess flux not only through the upper airgap, main pole and half yoke, but also through the interpole. If the magnetic reluctance of the interpole were equal to that of the main pole, the percentage of the excess flux would obviously be one-half of the percentage of excess m.m.f. By introducing

$$\frac{x}{2g_1} \text{ instead of } \frac{x}{g_1} \text{ for } x \cdot \frac{dB}{dH} \text{ in formula (3).}$$

we would obtain the local radial pull for a displacement in the direction of the field axis. In general we may introduce the idea of a "phantom interpole airgap" (Symbol g_2), or the airgap length of an imaginary interpole with the same section and pole tip as the main pole, which would require the same m.m.f. for the passing of a certain flux as the real interpole. While g_1 is only slightly greater than the real main pole airgap g, the phantom gap g_2 will be a multiple of the real interpole airgap, if the section of the interpole is only a fraction of that of the main pole.

Introducing $g_1 + g_2$ instead of g_1, we obtain the element of the unbalanced pull component parallel to the field axis for an induction B_m:

$$\frac{1}{2} DL \cdot d\theta \cdot 4 \cdot \left(\frac{B_m}{5000}\right)^2 \frac{x}{g_1 + g_2} \cdot \cos^2 \theta$$

We assume a pole factor of $^2/_3$ or a pole arc of 120 deg. The total unbalanced pull therefore is given by integrating twice between O and $\pi/3$

$$(F_x)_{axis} = 4\, DL \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1 + g_2} \int_0^{\frac{\pi}{3}} \cos^2 \theta \, . \, d\,\theta$$

The solution of the integral is $\dfrac{\theta}{2} + \dfrac{\sin 2\,\theta}{4}$, taken between

above limits, that is $\frac{\pi}{6} + \frac{\sqrt{3}}{8} = 0.740$

$$(F_x)_{axis} = 2.96 \ DL \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1 + g_2}$$

$$= 0.94 \ \pi \, DL \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1 + g_2} \qquad \textbf{(7b)}$$

If g_2 is larger than $2g_1$ (generally it *is* larger), the pull calculated according to this formula is smaller than the pull due to a displacement at right angles to the field axis, formula (**7a**), which is for interpole machines the same as for other two-pole machines, as long as the armature gives no current.

V. *Bipolar Machines with Cylindrical Field and Distributed Winding.* The case of bipolar cylindrical fields lends itself to easy mathematical, treatment, if we assume an entirely unsaturated sine-shaped field and a rotor and stator slotted evenly around the periphery. $x \cdot \frac{dB}{dH}$ in formula (**3**) is then constant and can be replaced, for a displacement in the direction of the neutral diameter, by x/g_1. The induction at any point, (starting to count the angle θ from the neutral diameter bb_1 in Fig. 16) is $B \sin \theta$. For a displacement x in the direction of the neutral diameter the unbalanced pull is

$$(F_x)_{neutr.} = 2 \int_0^{\frac{\pi}{2}} \frac{1}{2} \cdot DL \cdot d\theta \cdot 4 \cdot \left(\frac{B \sin \theta}{5000}\right)^2 \cdot \frac{x}{g_1} \cdot \cos^2 \theta$$

$$= 4 \ DL \cdot \left(\frac{B}{5000}\right)^2 \cdot \frac{x}{g_1} \cdot \int_0^{\frac{\pi}{2}} \sin^2 \theta \cos^2 \theta \cdot d\,\theta$$

$$= \frac{1}{4} \cdot \pi \, DL \cdot \left(\frac{B}{5000}\right)^2 \cdot \frac{x}{g_1}$$

For a displacement in the direction of the field axis, the conditions are in principle the same as in two-pole machines with interpoles; the phantom airgap of the interpole can be regarded here as equal to the virtual airgap of the main pole. We write

$2g_1$ instead of $g_1 + g_2$ and consider further that, if we start to count the angle from the field axis, the local induction is $B \cos\theta$. The unbalanced pull is then

$$(F_x)_{axis} = 2\int_0^{\frac{\pi}{2}} \frac{1}{2}\cdot DL\cdot d\theta\cdot 4\left(\frac{B\cos\theta}{5000}\right)^2\cdot\frac{x}{2g_1}\cdot\cos^2\theta$$

$$= 2DL\cdot\left(\frac{B}{5000}\right)^2\cdot\frac{x}{g_1}\cdot\int_0^{\frac{\pi}{2}}\cos^4\theta\cdot d\theta$$

$$= \frac{3}{8}\cdot\pi DL\cdot\left(\frac{B}{5000}\right)^2\cdot\frac{x}{g_1}$$

In this case, a displacement in the direction of the field axis would actually cause a greater unbalanced pull than a displacement in the neutral diameter. The upper limit for B in these formulas would be B_m and we obtain pole factors of 0.25 and 0.375 respectively. The relation of these pulls changes, however, if we increase the saturation. As long as the induction is on the straight line characteristic, $B^2\cdot\frac{dB}{dH}$ is highest for the point a in the field axis (Fig. 16); this point contributes, for a displacement in the direction of the axis, the full value of its radial pull as vertical component; for a displacement in the direction of the neutral diameter, the horizontal component of the radial pull is zero. If, however, we increase the saturation above B_m, than the point of the maximum $B^2\cdot\frac{dB}{dH}$ will shift the farther away from the field axis the higher the saturation goes, and the horizontal component of this maximum pull will grow, while the vertical declines.

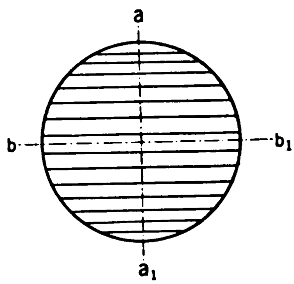


FIG. 16

As here the saturation curve comes into play, we shall abandon the analytical treatment and the assumption of a sine-shaped field and return to a field form like that represented in Fig. 13.

It is clear to us from the beginning that to obtain the highest possible unbalanced pull for a given displacement in the neutral diameter, we shall have to choose the excitation high, so as to obtain the value B_m already for a small angle θ, the cosine of which is near unity.

A four pole machine with winding distributed evenly over the whole surface and with a zero value of field in the horizontal has a field maximum under an angle of 45 deg. to the horizontal. The maximum pole factor for such a machine was 66.5 per cent for the saturation curve of Fig. 12. By far the greatest part of the magnetic pull, caused by a displacement in horizontal direction, is contributed by the rotor zone lying between 0 deg. and 45 deg., and if we are free to choose the induction of the two pole rotor for the zone from 0 to 45 deg. exactly equal to that of the four pole rotor, the unbalanced pull of the two-pole machine will not greatly differ from that of the four-pole machine. In the zone from 45 to 90 deg., it is true, the induction of the four-pole machine will fall again, while that of the two pole machine will rise from 45 to 60 deg. and remain constant from 60 to 90 deg. But the contribution for the total pull from this zone does not count for very much. The maximum value of the unbalanced pull will be reached in the two-pole machine for an appreciably higher peak saturation than in the four-pole machine.

This is born out in Fig. 17. Curve I represents the induction for one half pole (from 0 deg. to 90 deg.). The peak induction (from 60 to 90 deg.) is very high viz. 11,100 lines per sq. cm. or 75 per cent above the critical induction. The latter one is reached already at an angle of 20 deg. Curve II represents the local radial pull per sq. cm. for a constant displacement. Both curves I and II are transferred from Fig. 12, only the horizontal scale being changed. To obtain the component of the radial pull in the neutral diameter, we multiply every ordinate of curve II with $\cos^2 \theta$ and obtain thus curve V. For instance, the ordinate of V for an abscissa 10 deg. is equal to that of curve II multiplied with $\cos^2 10$ deg. = 0.97. For small angles, the curve V therefore hugs closely the curve II. The greater the angle, however, the smaller the ordinates of V become compared with those of II.

The average value VI of the ordinates of V taken from 0 deg. to 90 deg. is, in this case, 31 per cent of the peak value of the pull curve II. This also would not change appreciably for an *equally* distributed winding, which would reach still higher saturation from 60 to 90 deg.

The same calculation carried through for a peak induction of 9700 gives 28 per cent as the average value.

To compare this with multipolar machines we must consider that there the average of 66 to 70 per cent which we took from the curves III and IV of Fig. 12 was to be multiplied with one quarter of the total pole face area, while here we have to multiply with one half of the total pole face area. To bring the result into line with the formula for multipolar machines, the value 0.31 is to be doubled to give the "pole factor". The pole factor is therefore 62 per cent which is very near the value of $^2/_3$ obtained in multipolar machines with distributed winding.

For displacements in the direction of the pole axis we have to

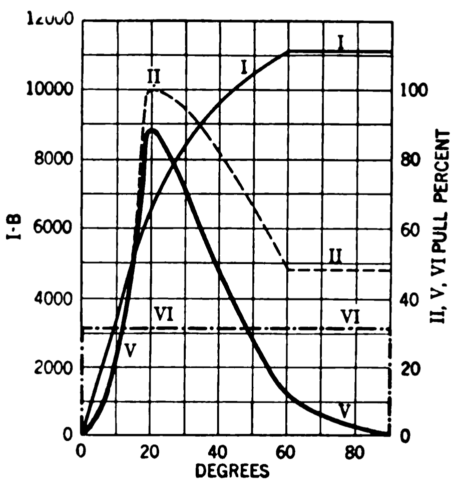


FIG. 17

multiply the ordinates of curve II by $\sin^2 \theta$ instead of $\cos^2 \theta$, and to half the values thus obtained. The parts of the ordinates between the curves V and II in Fig. 17, if halved, represent this value. The average value of the ordinates between V and II equals here just about the average value of curve V ordinates. The result is, therefore, that for the case represented by Fig. 17 the unbalanced magnetic pull for displacement in the direction of the field axis is about half of the pull for displacement in the neutral diameter.

If the middle part of each pole (60 deg. to 120 deg.) remains unslotted, this will have on the unbalanced pull for a displacement in the direction of the neutral diameter, a small effect

which may be deemed to be covered already by the small increase (from 62 per cent to $^2/_3$) which we have made in the value of the pole factor.

The ultimate result is that for multipolar salient pole and cylindrical fields and also for two-pole cylindrical fields the maximum magnetic pull can be figured from the same formula (**6a**) *and, with approximate accuracy, with the same pole factor, viz.,* $^2/_3$.

C. Increase of Eccentric Displacement Due to Unbalanced Pull

A displacement of the rotor from its true central position causes a one-sided magnetic pull. A one-sided magnetic pull, on the other hand, causes a further displacement. There is a force working on the rotor and stator pulling them towards each other in the direction, where the airgap is a minimum. This will cause not only an elastic deflection of the rotor shaft but also of the stator frame, of the bedplate and of the pedestals. If the machine is not excited, all its parts experience a certain deflection due to the weights. The elastic deflections of all parts will change if a one-sided magnetic pull occurs, whatever its direction may be. A magnetic pull on the rotor directed upwards will reduce the apparent weight of the rotor and increase the apparent weight of the stator. The transfer of this weight from the rotor bearings to the feet of the stator frame will also change the elastic deflection of the bedplate. The absolute value of the difference in deflection will be the same, whether the unbalanced pull on the rotor is directed up- or downwards. But if the pull is directed sideways, the deflection of the bedplate and of the bearing pedestals will be entirely different. All parts of the machine contribute their share to the combined deflection. In many cases the contribution of pedestals and bedplate may be negligible. Rotor shaft and frame, however, must be calculated in *every* case, so as to be strong enough not to increase unduly by consequent deflection an existing initial displacement. All the deflections work in the same sense, so as to reduce the airgap in the spot where it was originally reduced and to increase it where it was originally increased.

The force F_x of the one-sided magnetic pull, is proportional to the displacement x. The elastic deflections of all the machine parts caused under influence of the force F_x are proportional to this force, therefore also proportional to the initial displacement x. The sum of these deflections constitute the "first increment"

of the original displacement. This increment in displacement will cause an increment in the one-sided magnetic pull which bears the same ratio to the original pull as the increment in displacement bears to the original displacement. The increment in magnetic pull will cause a second increment in displacement, this a second increment in pull, the latter a third increment in displacement etc. etc. As long as the proportionality between displacement and pull exists, every following increment will have the same ratio to its immediate predecessor as this to its own predecessor. In other words, the initial displacement and its increments are represented by a geometric series which we may write in this form:

$$x + q\,x + q^2\,x + q^3\,x + \ldots .$$

The sum of the series can only have a finite value, if $q\,x$, the first increment of the displacement, is smaller than the initial displacement x, or if q is smaller than unity. The sum of the geometric series of the final static displacement is represented by the formula

$$X = \frac{x}{1 - q} \tag{8a}$$

By substituting into the previous formulas X instead of x, the final static force of the magnetic pull is obtained.

Up to now we have considered static forces and displacements. If, however, a machine is *suddenly* excited, as it is generally the case with an induction motor, the shaft, frame and other parts of the machine will not only deflect so as to increase the initial displacement x to the final value X, but all the parts of the machine being elastic will overshoot the position of the new static equilibrium like a spring or a chord. The combined distance travelled by the parts of the machine from the position of the old into the position of the new equilibrium is $X - x$, and they will overshoot this position by approximately the same amount. The *momentary* displacement therefore will be

$$X_{mom.} = x + 2\,(X - x) = 2\,X - x = x \cdot \frac{1 + q}{1 - q} \tag{8b}$$

The original mechanical displacement must therefore be multiplied with the factor $\frac{1 + q}{1 - q}$ to obtain the displacement occurring at the moment of switching in.

We can now formulate the requirement, that a machine, say an induction motor, should be free from "pulling over," even if the initial displacement of the rotor center is equal to one half of the mean airgap. In other words, the displacement at the moment of switching in, should be less than twice the initial displacement.

$\dfrac{1+q}{1-q}$ must be smaller than 2. This gives the result:

q smaller than $\frac{1}{3}$.

The smaller q, the safer is the machine against pulling over. If, for instance, $q = 0.2$, the displacement at the moment of switching in, will be only $\dfrac{1+0.2}{1-0.2} = 1.5$ of the original mechanical displacement. In other words, the machine would only pull hard over, if the original displacement were two thirds of the original airgap.

This calculation considers only elastic deflections. It is, however, quite feasible that there is, at the moment of switching in, at first a movement of the machine parts through a space without any appreciable resistance being encountered. Suppose, for instance, that there is a clearance in the bearings and that nevertheless the airgap between stator and rotor core is smaller at the top than at the bottom. At the moment of excitation the rotor will travel freely throughout the clearance of the bearing. Only after the clearance is taken up, an opposing force will be caused through the elastic deflection of the machine parts. The rotor travelling through the clearance has obtained a kinetic force which carry it further than according to formula (**8b**).

A "momentary" appearance of the field does not only occur in induction motors but also in generators, if a circuit breaker opens after a short circuit has taken place. While the short circuit lasts, the actual value of the magnetic field is very small; at the moment of the opening of the circuit breaker, the field suddenly rises and it passes certainly also through the value of the "critical" induction.

A measure for the stiffness of the shaft is the deflection, produced by the action of gravity. If the machine has a vertical shaft, we must, for the purpose of this comparison, figure out the deflection which *would* occur, if the shaft were placed horizontally. We shall call this deflection "gravity deflection." Suppose the frame and the other parts of the machine were

infinitely stiff, so that we need not consider any deflection of these parts caused by the magnetic pull. We then could allow such an unbalanced pull as to produce in the rotor shaft a first increment in displacement equal to $\frac{1}{3}$ of the original displacement. Let us call f the gravity deflection caused under the static influence of the rotor weight G. If now we take f as original arbitrary displacement, a first increment $f/3$, produced by the magnetic pull, would be permissible. A deflection of $f/3$, however, is produced by a force equal to $G/3$, provided the second force attacks in the same part of the shaft as the rotor weight. The latter condition would be exactly fulfilled, if we exclude from G the weight of the ends of the shaft protruding over the rotor body. But even if this is not excluded, the inaccuracy is very small. We can therefore say: The gravity deflection of the shaft must be within such limits, that the unbalanced magnetic pull produced by a displacement equal to that deflection is less than one-third of the rotor weight.

The deflections of the stator frame and the other parts may be negligible in small machines and also in turbo-generators and high-speed induction motors. In machines with large diameters, however, as low-speed induction motors and engine type generators, they cannot be neglected. There we may divide the quotient q into two parts

$$q = q_r + q_s,$$

q_r being the ratio of the rotor deflection caused by unbalanced pull to the displacement causing the unbalanced pull, and q_s being the ratio of the combined deflection of the other machine parts (principally the stator frame) to the displacement. Then the force of unbalanced pull caused by a deflection f is

$$F_f = q_r \cdot G \tag{9}$$

If we allow

$$q_r = q_s = \frac{q}{2} = \frac{1}{6}$$

we may say that *the shaft must be strong enough so that a displacement equal to the gravity deflection should not produce an unbalanced pull more than* 1/6 *of the rotor weight, provided that the combined deflection of the other parts of the machine under a certain unbalanced pull does not exceed the deflection of the rotor under the same pull.* Should the deflection of the other machine parts under the influence of a certain pull be doubly as great as that of the rotor shaft, the highest permissible value for q_r would be 1/9.

For the calculation of the deflection of the stator frame the force, of course, must not be taken as attacking concentrated in one point, but must be distributed, the parallel components attacking under an angle θ from the symmetry diameter being proportionate to $\cos^2 \theta$.

From formula (**9**) the permissible gravity deflection can be directly deduced. The machine, the characteristic of which is given in Fig. 7, has a rotor weight of approximately 9000 kg. We want to allow an unbalanced pull of not more than say $\frac{1}{6} \cdot 9000 = 1500$ kg. for a displacement equal to the gravity deflection. If now the unbalanced pull for 0.1 cm. displacement is 7100 kg., the permissible gravity deflection is $\frac{1500}{7100} \cdot 0.1 = 0.021$ cm. and the shaft must be made strong enough so that this deflection is not exceeded.

We can establish a direct formula to give us the permissible gravity deflection from the point of view of unbalanced magnetic pull without at first having recourse to the "arbitrary displacement" x.

We use formula (**6a**) for our purpose which is applicable to all machines, and substitute the gravity deflection f for the arbitrary displacement x.

$$F_f = c \cdot \pi \, D \, L \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{f}{g_1}$$

On the other hand is

$$F_f = q_r \cdot G$$

It follows

or $$f = \frac{q_r \, . \, G}{c \cdot \pi \, D \, L \cdot \left(\frac{B_m}{5000}\right)^2} \cdot g_1 \qquad \textbf{(10)}$$

$$q_r = \frac{f}{g_1} \cdot \frac{c \cdot \pi \, D \, L \cdot \left(\frac{B_m}{5000}\right)^2}{G} \qquad \textbf{(10a)}$$

In the previous example $c \, . \, \pi \, D \, L$ is 40.650 and we obtain

$$f = \frac{1/6 \cdot 9000}{40.650 \cdot \left(\frac{6300}{5000}\right)^2} \cdot 0.58 \text{ cm.} = 0.021 \text{ cm.}$$

Suppose the same frame had to be used also for an induction motor with a radial airgap of 0.2 cm. In the generator the virtual airgap was 0.07 cm. larger than the real airgap. Let us assume the same for the motor, then g_1 is 0.27 cm. The critical value of B_m is likely to be reduced in an induction motor, as there, in general, a lower flux is applied and therefore the teeth section correspondingly reduced. This, as a rule, is already necessitated by the greater number of slots which are usual in induction motors. Let us assume that B_m is 5000 for the motor. The rotor core has a diameter of 385.6 cm. and a gross length of 33 cm. from which 5 ventilation ducts of together 6.5 cm. are deducted, giving a net length of 26.5 cm. The total pole surface $\pi D L$ is therefore

$$\pi \cdot 385.6 \cdot 26.5 = 32{,}100 \text{ sq. cm.}$$

The permissible gravity deflection for $B_m = 5000$ and $q_r = 1/6$, assuming the rotor weight to be again 9000 kg. is

$$f = \frac{1/6 \,.\, 9000}{\frac{2}{3} \cdot 32{,}100 \cdot \left(\frac{5000}{5000}\right)^2} \cdot 0.27 \text{ cm.} = 0.019 \text{ cm.}$$

In the assumed case the permissible gravity deflection is nearly the same as the one for the generator, but everything depends on the sections of yoke, rotor and stator teeth, for they determine the value at which the saturation makes itself felt. If the critical induction of the motor were 6000 instead of 5000, the permissible deflection would be reduced to 0.013 cm., and this would also require a stiffer stator frame to comply with the condition that the deflection of the other machine parts, caused by the unbalanced pull, should not exceed, the rotor deflection caused by the same pull. If the stator cannot be made stiff enough, q_r would have to be reduced, and the permissible deflection would be less than 0.013 cm. Should it be impossible to provide such stiffness in the shaft, then the machine should always be provided with a *winding with equalizing connections.*

We see that in all machines (apart from windings with equalizing qualities) the consideration of the unbalanced magnetic pull fixes a limit to the permissible deflection of the shaft under the influence of the rotor weight, just as in high-speed machines the consideration of the critical speed fixes such a limit.

D. Inequalities in Exciting Coils

If, *in a multipolar machine*, one pole is excited with a greater or smaller number of ampere turns than the others, the induction in the pole in question and in the adjoining parts of the two neighbor poles will be higher or lower than in the rest of the machine, and an unbalanced pull is set up. The induction is, in a working machine, produced by the action of the primary exciting coil and the reaction of the armature coil combined. Whether the ampere turns of one of the primary coils are different from those of the others and all the armature coils give the same number of ampere turns, or whether the primary coils work identically amongst themselves and one of the armature coils different from the others, the effect will be an unbalanced pull which is stationary, if the disturbing part is stationary, and rotating, if the disturbing part is rotating. Specially, if a coil of the rotating part . s accidentally short-circuited, heavy vibrations will be set up. A disturbance in the balance of the machine which only occurs when the machine is excited, accompanied by the necessity of an increase in exciting current in order to obtain a given voltage, is an indication of a partial or complete short circuit of a rotor coil.

In *two-pole turbo-generators* with cylindrical field and distributed winding, the flux density is highest in the middle part of the pole and is gradually reduced to zero, when approaching a line at right angles to the pole axis. As long as this distribution is the same in two opposite poles and the rotor is in a central position, no unbalanced pull occurs. If, however, one of the coils of one pole is short-circuited, the flux distribution in one pole will be different from that of the other pole, and an unbalanced pull will be set up, rotating with the rotor and producing vibrations of the stator.

E. Equalizing Connections

In Fig. 18 the winding of an a-c. machine is indicated, consisting of 8 coils, the four coils of the upper half being connected in series, as are also the four coils of the lower half. The two paths are connected in parallel. The voltage and periodicity impressed on the two parts is alike, and as both parts consist of the same number of identically situated coils, the total flux passing the upper half will be nearly identical with the total flux in the lower half. Any dissymetry would be practically wiped out by a slight increase of magnetizing current in the one and corresponding reduction of the magnetizing current in the other.

An arrangement like this will therefore prove effective against the consequences of a vertical displacement of the rotor from its true central position. It is, however, entirely ineffective with regard to horizontal displacement. If the rotor is displaced to the left from the true central position, the coil 1_a through which the same magnetizing current is flowing as through the coil 4_a, will cause a stronger magnetic field than the latter, and so will the coil 4_b create a stronger field than the coil 1_b. The result therefore will be an unbalanced magnetic pull tending to create an increment of the displacement which is directed to the left of the rotor.

In a two-phase machine it is possible to arrange the windings of one phase with a horizontal symmetry line, the windings of the other phase with a vertical symmetry line, and so to provide to a certain extent for two components of the magnetic pull.

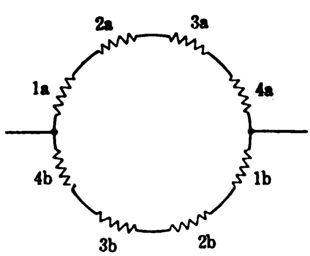


FIG. 18

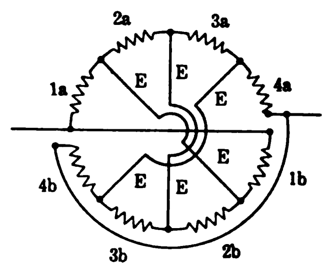


FIG. 19

In a three-phase machine the symmetry lines of the windings can be arranged mechanically under a 60 deg. angle.

A far more effective arrangement, however, is obtained, if equipotential connections $E\,E\,E$ are added to the windings of every phase, strictly equalizing diametrically opposite coils, as shown in Fig. 19. The coil 1_a is equalized against the diametrically opposite coil 1_b. On both coils the identical voltage is impressed, the flux and the magnetic pull on both parts will therefore be very nearly identical. This is true of all the coils around the whole circumference and therefore the unbalanced pull will amount to a negligible figure.

The necessary requirement for magnetic equalization is of course, that the coils which are equalized against each other, contain the same number of identically situated turns. If, however, a turn should accidentally become short circuited or open circuited, the magnetic balance would of course immediately be upset. Equalizing arrangements in the windings therefore do

not free the designer from the duty to design the mechanical parts strong enough so as to deal, in case of accidents, with the force calculated in formula (1), but for ordinary running conditions the unbalanced pull need not be considered.

In multipolar *continuous current machines* with multiple winding but without equipotential cross connections and with as many brush arms as poles, an incomplete magnetic equalization between pole pairs is produced by currents flowing between the brushes of the same polarity. The part of the armature winding moving in a stronger field generates more current, the reaction of which weakens the field. As, however, large continuous currents are required to produce any marked weakening effect by armature reaction, the commutation becomes, as is well known, very seriously impaired, if there are any sensible magnetic inequalities. The tendency to equalization of pole strength refers here to all north poles and equally to all south poles round the machine.

If equipotential cross connections are applied, equally situated armature coils are subjected to the same *alternating* voltage, just as the coils of Fig. 19.

The current flowing through the equalizing or equipotential connections as "wattless" current, is as a rule only small, as comparatively few ampere turns in the armature are sufficient to correct the difference in field strength caused by small differences in the airgap. If, however, by accident an exciting coil is short-circuited, a very large current will flow into the part of the armature winding influenced by the pole in question, and therefore it is not wise to stint in the section of the equalizing connections. Cases are known of four-pole constant-current turbo-generators in which the equalizing connections of the armature have been burnt out due to accidental occasional short circuit of a series field coil.

Equalizing connections between coils excited with continuous current are, of course, useless for our purpose, as the current distribution in such coils is not influenced by the field strength, but solely by the ohmic resistance of the parallel paths. In a machine at rest, excited with direct current, no equalization of flux or pull takes place.

F. Effect of Unbalanced Pull on Rotating Machine

The effect of the unbalanced magnetic pull due to eccentricity may be of a twofold character. If the outer rotor surface and

the inner stator surface are truly cylindrical but not concentric, while the rotor taken by itself is running true, the forces exercised by the unbalanced magnetic pull have always the same direction in space independent of the rotation and are in the case of multipolar machines constant, and only in the case of two-pole rotating fields variable.

If, however, the rotor itself is untrue, no matter whether the field or the armature is rotating, then the place of the smallest airgap is rotating round with the rotor and therefore the force of the unbalanced pull rotating round. As far as the rotor is concerned, the effect will be the same, as if its weight were not balanced mechanically, only with the difference that the unbalance is dependent upon the excitation of the machine. The rotating force acts, however, also on the stator and will set up in low-speed machines visible "breathing"; in high-speed machines sensible vibrations of the frame, the frequency of vibrations being equal to the number of revolutions. We shall not go, in this paper, into the question what "phase difference" exists between the passing of the eccentric part of the rotor and the radial oscillations of a point of the stator, but shall assume in the following section that the stationary parts of the machine are infinitely stiff, so that the above question is eliminated from the following investigation.

G. Influence of Unbalanced Pull on Critical Speed

If the center of gravity of a rotor does not coincide with the axis of rotation, but has a distance x from the same, a centrifugal force will be developed which is proportionate to the displacement x and to the square of the number of revolutions per minute. At a certain speed the centrifugal force will be so great that it would cause (in the absence of friction etc.) a deflection of the shaft equal to the original displacement. The displacement being doubled, the centrifugal force would be doubled causing a double increment in deflection etc. At this speed therefore, the so-called "critical" speed, an original displacement, if not infinitesimal, would cause an infinite final displacement, if there were no resistance to these movements. Although in reality the resistance will keep the displacement down to finite values, the great increase of vibrations at the critical speed is highly objectionable and there is always careful calculation in the design of high-speed machines to make sure that the running speed is well above or well below the critical speed.

In an electric machine, the unbalanced pull due to the rotor displacement will work in the same direction as the centrifugal force and is itself proportionate to the displacement, but independent of the speed. If we call x the displacement in centimeters, the displacement measured in meters will be $\frac{x}{100}$. Let G be the rotor weight in kilograms, then the mass of the rotor in terrestrial units is $\frac{G}{9.8}$, the acceleration of gravity being 9.8 m. per second per second. Let n be the number of revolutions per minute. Then the centrifugal force is

$$4 \cdot \frac{G}{9.8} \cdot \frac{x}{100} \cdot \pi^2 \cdot \left(\frac{n}{60}\right)^2 = x \cdot G \cdot \left(\frac{n}{300}\right)^2$$

The unbalanced magnetic pull for a displacement equal to the gravity deflection f is $q_r \cdot G$ and for a displacement x is

$$\frac{x}{f} \cdot q_r \cdot G$$

Assuming that the stationary machine parts are infinitely stiff (so that $q_r = q$), the same formulas hold good for the rotating machine.

The sum of centrifugal force and unbalanced pull is

$$x \cdot G \cdot \left(\frac{n}{300}\right)^2 + \frac{x}{f} \cdot q \cdot G$$

The elastic gravity deflection, caused by the weight G is f, the elastic deflection caused by one kilogram is f/G. To obtain the elastic deflection caused by the above sum of forces, we have to multiply by f/G.

$$\frac{f}{G} \cdot \left[x\, G \left(\frac{n}{300}\right)^2 + \frac{x}{f} \cdot q\, G\right] = x \cdot \left[f \cdot \left(\frac{n}{300}\right)^2 + q\right]$$

For the critical speed this deflection will be equal to the original displacement x, the expression in brackets therefore will be unity for the critical value of n

$$f \cdot \left(\frac{n_{crit.}}{300}\right)^2 + q = 1$$

$$n_{crit.} = \sqrt{1 - q} \cdot \frac{300}{\sqrt{f}} \tag{11}$$

For the unexcited machine, q is zero and the critical speed is $\frac{300}{\sqrt{f}}$, (f given in cm.), which formula, of course, is well known. The critical speed is reduced, if the machine becomes excited and will be the lowest for that excitation which gives the highest value of the unbalanced magnetic pull. If q is equal to 1/6, the critical speed will be

$$\sqrt{\frac{5}{6}} \cdot \frac{300}{\sqrt{f}} = 0.912 \cdot \frac{300}{\sqrt{f}}$$

If q is equal 1/3, the critical speed will be

$$\sqrt{\frac{2}{3}} \cdot \frac{300}{\sqrt{f}} = 0.817 \cdot \frac{300}{\sqrt{f}}$$

An electric machine without equalizing connections has not *one* definite critical speed, but *a range* of critical speeds according to its excitation. In two-pole turbo-generators it must also be considered that the unbalanced pull in the diameter of the field axis is different from that at right angles to the field axis, and therefore such a machine will even for a definite excitation have different critical speeds in respect to these two diameters, similar to a rotating shaft with rectangular section.

To illustrate the numerical influence of the unbalanced pull on the critical speed, we may take a rotor of 60 cm. diameter and 100 cm. rough length of the iron core. After deduction of air-ducts and slot openings, 80 per cent of the cylindrical surface, that is $0.8 \cdot \pi \cdot 60 \cdot 100 = 15{,}100$ sq. cm. may be considered as effective. We will assume that the rotor belongs to a two-pole machine with distributed field winding, running at 3000 rev. per min., that the actual radial airgap is 1.5 cm., the virtual airgap 1.6 cm., and will consider as alternatives first, a mechanical critical speed of 3600, then of 1800. The weight of the rotor may be 3000 kg. We will further assume that the critical induction for which the unbalanced local pull is a maximum, is 6000 lines per sq. cm.

As the mechanical critical speed is given, we can figure directly the gravity deflection of the rotor shaft

$$f = \left(\frac{300}{n_{crit.}}\right)^2 = \left(\frac{300}{3600}\right)^2 = 0.0070 \text{ cm. and } \left(\frac{300}{1800}\right)^2$$
$$= 0.0278 \text{ cm. respectively.}$$

We can figure $q = q_r$ directly from formula (**10a**). The pole factor c we will take as $^2/_3$.

$$q = q_r = \frac{f}{g_1} \cdot \frac{c\,\pi\,D\,L \cdot \left(\frac{B_m}{5000}\right)^2}{G} = \frac{f}{1.6} \cdot \frac{\frac{2}{3} \cdot 15{,}100 \cdot 1.44}{3000}$$
$$= 3.02 \cdot f$$

that is, 0.021 and 0.083 respectively.

$\sqrt{1-q}$ is $\sqrt{0.979} = 0.989$ and $\sqrt{0.917} = 0.956$ respectively. The influence of the magnetic pull on the critical speed is here small, it lowers the cricical speed only 1.1 per cent in one case, 4.4 per cent in the second case.

If the rotor belonged to a four pole machine with half the virtual airgap, and a mechanical critical speed of 1800, q would be 0.166,

$$\sqrt{1-q} = \sqrt{0.834} = 0.913,$$

which would reduce the critical speed to 0.913 . 1800 = 1640 rev. per min. a value which is already uncomfortably near the running speed of 1500.

In the case of an induction motor with wound rotor and with the same core dimensions, the same rotor weight, a virtual airgap of 0.3 cm. (mechanical 0.2 cm.), and a critical induction of 5000 lines per sq. cm. and a critical speed of 3600 and alternatively 1800, q would be

$$q = \frac{f}{0.3} \cdot \frac{\frac{2}{3} \cdot 15{,}100 \cdot 1}{3000} = 11.2\,.\,f$$ that is 0.078 and 0.31 respectively. In the latter case, the critical speed would be reduced by the magnetic pull to $\sqrt{0.69} \cdot 1800 = 1490$ rev. per min. This could not be accepted for a machine running at 1500 rev. per min. and it would be necessary to stiffen the shaft.

This example has been simplified by assuming the same rotor weight, whether the rotor is destined for 1500 or 3000 rev. per min., for a two- or four-pole machine, generator or motor. In reality, for an exact calculation, differences in weight would have to be considered, also certain minor differences in effective surface, critical induction, pole factor etc., etc.

Conclusions

The main conclusions arrived at in this paper are as follows:

1. The mechanical parts of a multipolar machine, whether its winding is provided with equalizing connections or not, should

be designed strong enough to withstand, as an emergency condition, without overstraining, the magnetic pull produced, if only one-half of the machine is magnetized up to full saturation of the iron, while the other half is unexcited. In machines with small airgap the rotor core may pull hard over towards the stator core, under these conditions.

2. Under normal working conditions, a reduction x of one airgap and an increase x of the diametrically opposite airgap causes in multipolar machines locally an unbalanced magnetic pull per square centimeter, given by the formula

$$4 \cdot \left(\frac{B}{5000}\right)^2 \cdot \frac{x \cdot d\,B}{d\,H} \text{ kilograms} \tag{3}$$

This pull has a maximum value for a comparatively *low* state of saturation. For higher saturation the unbalanced pull falls off. If the magnetic characteristic of the machine is represented by an initial straight part of appreciable length, followed by a distinct "knee", the "critical induction" is in the very beginning of the knee. For an approximate determination of the maximum pull take the induction B_m of that point of the magnetization curve in which the gradient is 5/6 of that of the straight line part and consider B_m as the end point of the straight line characteristic. The maximum unbalanced pull in kilograms per square centimeter is

$$4 \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1}$$

g_1 being the "virtual" airgap of the machine.

(Virtual airgap, an airgap which would take for an induction within the straight line characteristic as many ampere turns as the real airgap and iron path of half the magnetic circuit together.)

3. In multipolar machines, the total unbalanced pull, for a given displacement x of rotor and stator centers, is independent of the rotor position. Its maximum value in kilograms is

$$F_x = c \cdot \pi\, D\, L \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1} \tag{6a}$$

c, the "pole factor", being approximately $^2/_3$, both for machines with salient poles and for those with cylindrical fields and distributed windings. In machines with salient poles and constant airgap the maximum unbalanced pull is reached at a low saturation and falls off distinctly at higher saturations.

In machines with distributed field winding and also in salient pole machines with graded airgap, the maximum pull is reached for higher saturations and changes less with changes in saturation.

In machines with cylindrical fields which are not slotted evenly around the periphery, and machines with salient poles and graded airgap, the saturation curve from which B_m and g_1 are taken, is a mean of the component saturation curves.

4. In bipolar machines, the unbalanced pull has different values for displacement in the direction of the field axis and at right angles thereto. The highest value for a cylindrical field with distributed winding is reached for high peak saturation, and can be calculated from formula (**6a**) with a pole factor of approximately $^2/_3$. Only the saturation curve of the *slotted* part should be taken to determine B_m and g_1, in bipolar machines which contain a non slotted middle part of each pole. For salient pole machines with 120 deg. pole arc, the pole factor is about 0.4. The saturation curve, from which B_m and g_1 are taken, should exclude in bipolar machines the m.m.f. required for the bodies of stator and rotor cores.

5. Assuming that under the influence of unbalanced magnetic pull the airgap experiences equal changes through the deflection of the rotor shaft and through the combined deflection of all other machine parts, the magnetic pull caused by a displacement equal to the "gravity deflection" of the shaft must not exceed 1/6 of the rotor weight, to ensure that the displacement, at the moment of sudden excitation, does not exceed twice the value of the original mechanical displacement.

If the other parts of the machine were infinitely stiff, the magnetic pull caused by a displacement equal to the gravity deflection could be allowed to approach ⅓ of the rotor weight.

("Gravity deflection" is the static deflection of the *horizontally situated* shaft under the influence of the rotor weight.)

6. The critical speed is lowered by the unbalanced magnetic pull and varies with changes in excitation. For the greatest permissible value of unbalanced pull, mentioned in the preceding rule, the reduction of critical speed amounts to 18 per cent. In machines, the winding of which has full equalizing qualities, no reduction of the critical speed takes place.

Presented by publication only.

ECONOMIC PROPORTION OF HYDROELECTRIC AND STEAM POWER

BY FRANK G. BAUM

ABSTRACT OF PAPER

This paper describes a new method of determining for any power system, what proportion of generation should be hydroelectric, and what steam, from the standpoint of economics.

A method is outlined for obtaining a curve showing "Total cost per kilowatt-year for hydroelectric and steam power", for any percentage combination of generation.

With system load curve, fixed charges on steam and hydroelectric plants, cost of fuel and other steam energy charges all known quantities, this curve can readily be calculated, and one can see at a glance the limiting economical percentage of steam power for the given conditions.

IT IS, of course, well known that steam power is usually less expensive for low load factors than hydroelectric power, and the latter becomes economical only when the load factor is favorable. To determine the economical division between the two there are usually given curves of cost varying with load factor. Such curves show that at certain load factors the cost of steam power exceeds the cost of water power, but the actual yearly cost of power for any assumed proportion between water power and steam power must be calculated for each case. This becomes laborious.

The results can, however, be shown in a much more illuminating way if presented as shown in Fig. 1. In this figure, abscissae from left to right (from O to O^1) show percentage of total load carried by water power and from right to left (from O^1 to O) the abscissae show the percentage of total load carried by steam power. The sum of the steam power and water power must of course equal 100 per cent for every condition, hence the sum of the two abscissae is 100 per cent at any point.

If now we take the yearly cost per kilowatt of hydroelectric power as $O^1\,h$, taken in the figure at $22 per kilowatt-year, and draw the straight line $O\,h$, this line will represent by any ord nate the yearly charge per kilowatt against the water power for any proportion between steam and water power. For it is clear that if we have one-half water power then the yearly charge per kilowatt against the entire 100 per cent load is $11.

(In comparing the cost of power we must of course include total cost of delivery to center of load.)

Similarly, if we take the yearly fixed cost per kilowatt of steam power as $O\ s$, taken in the figure as $11 per kw-year, and draw the line $O^1\ s$, this will represent by any ordinate the yearly charge per kilowatt against the steam power for any proportion between steam and water power.

The straight line $s\ h$ then represents the total fixed charge against the steam and hydroelectric power for any proportion of steam and water power. ($O\ h$ and $O^1\ s$ being straight lines and $s\ h$ being derived by adding the ordinates, gives another straight line.)

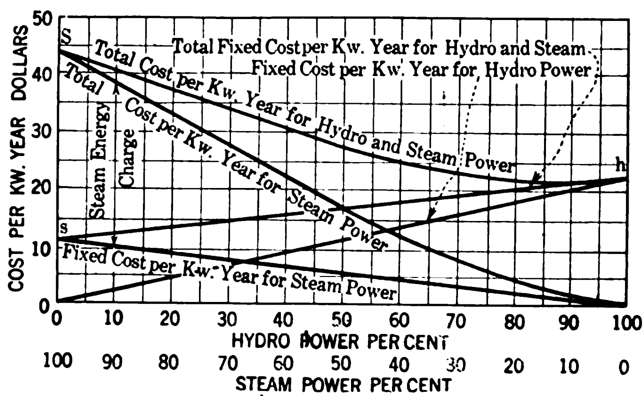


FIG. 1—PROPORTION OF WATER TO STEAM POWER

Water power cost per kw-yr	$22.00
Steam, fixed cost per kw-yr	11.00
Steam energy charge per kw-yr. and 100% load factor	44.00

For example, let total load equal 100,000 kilowatt, divided 70 per cent hydroelectric and 30 per cent steam; then the yearly charge against the water power and steam will be

Water power, fixed charge	70,000 kw. × $22	$1,540,000
Steam power, fixed charge	30,000 kw. × $11	330,000
Total power, fixed charge		1,870,000

or $18.70 per kw-year, as shown by the ordinate of the line $s\ h$ at 70 per cent water power, 30 per cent steam power. All water power fixed charge would cost $2,200,000 and all steam power fixed charge $1,100,000 per year. All steam power costs $4,400,000.

For any other assumption of cost per year of water power or steam power, it is only necessary to determine the yearly fixed charge against steam power ($O\ s$) and water power ($O\ h$) and draw the line $s\ h$, and we have immediately the total fixed charge for any proportion of water power and steam power. This very much simplifies the problem and visualizes the results.

To determine the total charge per kw-year against the combined steam and water power, it is necessary to add the kw-hr.

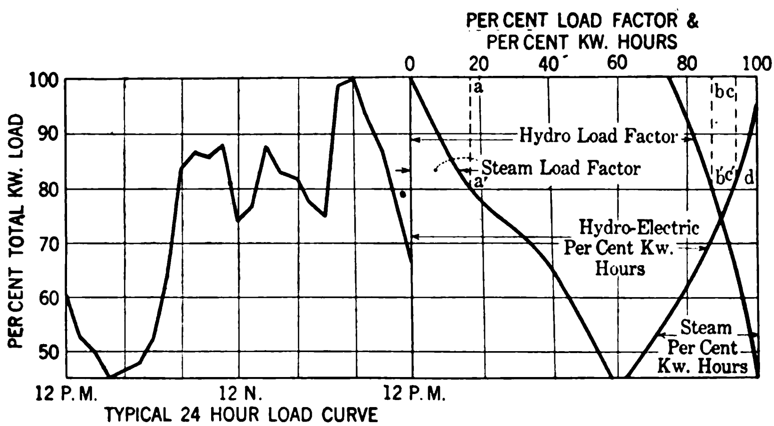


FIG. 2—LOAD FACTORS AND KW-HR. FOR STEAM AND WATER

—EXAMPLE—

For 20% peakload carried by steam and 80% by water the curves show

Steam load factor = 17% = $o\ a$
Water load factor = 87% = $o\ b$
Water kw-hr. = 94% = $o\ c$
Steam kw-hr. = 6% = $c\ d$

NOTE—Steam carries everything above 80% line in example, and the curves will show per cent load factor and per cent kw-hours for any other per cent load taken.

charge against the steam power. (It is of course assumed that all charges against the water power are fixed charges.)

To determine the yearly charge against steam power for any load factor we must start with the load curve of the power system. The load curve assumed is that shown on the left of Fig. 2. The ordinates, it will be noted, are plotted in percentage of the load, the peak load being 100 per cent.

Now we must determine the kw-hours carried by steam for any percentage of the total load carried by steam, it being assumed of course that the steam power takes the load off the

top of the curve. For this purpose we determine the curve of kilowatt-hours to be carried by steam power when 10 per cent, 20 per cent, etc. of the top of the load curve is carried by steam. To do this we take the area of the load curve above 90 per cent for example, and determine what percentage this is of the entire area of the load curve. Similarly for areas above 80 per cent, 70 per cent, etc. The results are shown in the curves to the right of the load curve.

For example, if all load over 80 per cent is carried on steam, we get the steam load factor = 17 per cent, and steam kilowatt-hours 6 per cent of the total as shown by $o\ a$ and $c^1\ d$. Also we get a hydroelectric load factor 87 per cent and kilowatt-hours 94 per cent of the total as shown by $o\ b$ and $o\ c$.

From the curves in Fig. 2 and the cost per kilowatt-hour of fuel and other strictly steam energy charges, we determine for any percentage of load carried by steam power the yearly energy charge per kilowatt-year. Assuming $33 per kilowatt-year for energy charge where all the energy is supplied by steam for the particular load curve under consideration (which corresponds practically to $44 per kilowatt-year, or 0.5 cent per kilowatt-hour for 100 per cent load factor, as the load factor of total load is 75 per cent) and adding the energy charge to the fixed charge for steam power, we obtain the total cost of steam power as shown by the curve "cost steam power" $O^1\ S$ in Fig. 1.

To obtain now the total cost of all power for any proportion of steam power to water power, we add the ordinates of $o\ h$ the "fixed cost of hydroelectric power" to the ordinates $o^1\ S$ the "Total cost of steam power" and obtain the curve $h\ S$ the "total cost per kilowatt-year hydroelectric and steam power." This curve starts at $22 per kilowatt-year if all power is water power. By adding some steam power to take off the peaks, we see there is a slight decrease in the yearly charge until the steam power carries about 15 per cent of load. At 20 per cent of load the cost again comes to about $22 per year, and then a gradual increase in power cost results.

At 30 per cent load carried by steam for this particular load curve there is little difference in the yearly charge per kilowatt and we would not for this case be warranted in installing less than 30 per cent steam; for naturally, unless very material savings result, the decision will always be to install steam power because of the smaller capital cost.

At 50 per cent of total load taken by steam installation and 50 per cent by water power, we have the yearly charge as follows:

50 per cent × \$22..\$11.00..yearly fixed charge water power
50 per cent × \$11.... 5.50..yearly fixed charge steam power
Total............ \$16.50..yearly fixed charge total power

At 50 per cent load factor practically $33\frac{1}{3}$ per cent of the kilowatt-hours are carried by steam and this adds the energy charge of \$33 × $33\frac{1}{3}$ per cent or \$11. Therefore the total yearly charge is \$16.50 + \$11 = \$27.50, as shown by the curve *h S* by the ordinate at 50 per cent. The added charge over all water power costs here is \$5.50 per kw. year, but on a system with 100,000 kw., the yearly excess charge is \$550,000, a very substantial sum.

The curve of total yearly power costs per kilowatt shows graphically what we want to know, and after we have the curve of energy cost of steam power at various load factors, we can very quickly make up total costs per kilowatt-year for any assumption of fixed cost of hydroelectric and steam power. It is believed this method will assist engineers in their work. The actual proportion of steam to hydroelectric power will of course be somewhat influenced by service insurance conditions.

This paper is merely an outline of the general principles and gives the general method to follow. There are, of course, many details and different conditions in different sections of the country. It is hoped that the method presented will appeal to engineers and managers.

Presented by publication only.

APPLICATION OF HARMONIC ANALYSIS TO THE THEORY OF SYNCHRONOUS MACHINES

BY WALDO V. LYON

ABSTRACT OF PAPER

It is shown that the flux distribution in the airgap of a synchronous mac ine consists of a series of component distributions that are simple harmonic wave trains, either stationary or moving at constant velocities. Methods are suggested for determining the effects of slots and the saturation of the magnetic circuit on the magnitudes of these component distributions. Expressions for the voltage generated and the power developed thereby are given. The theory is applied to the operation of a three-phase, synchronous machine under different conditions of load, both qualitatively and quantitatively. A table of comparative calculated field currents is appended.

A COMPLETE theory of the performance of alternating-current machinery must recognize the presence of harmonics in the current and voltage and in the distribution of the airgap flux. The principle of the following analysis is well known but the author believes it has never been so fully developed. It seems best to show how the method may be applied in a variety of cases rather than to carry any one case into all of its manifold refinements. Much of the analysis is qualitative rather than quantitative. It is useful to know, however, what harmonics may be present even when their magnitudes cannot be calculated. Where it is possible to determine certain coefficients with sufficient accuracy the method should prove of considerable practical importance. The fundamental principle upon which the analysis is based is that of the superposition of magnetic fields. For example, if a constant magnetizing force produces at any point a certain magnetic flux density when it alone is acting, and another constant magnetizing force produces at the same point another flux density when it alone is acting, then when both magnetizing forces act together the resultant flux density at the point in question is the vector sum of these two flux densities, provided the permeability at every point in the region concerned is the same in each of the three cases. Thus the component flux densities should be calculated for a condition of the magnetic region which is assumed to be identical with that existing when the components act simultaneously. When the magnetic forces

are variable the theorem does not hold if eddy currents are produced. If the resultant magnetizing force is constant, however, the eddy currents that may be produced by the component magnetizing forces should be neglected. With the laminated magnetic masses that are usually employed when the resultant flux density is variable, the effect of the eddy currents can often be neglected without undue error. Such is the case at 25 or 60 cycles with laminations no thicker than are ordinarily used in transformers. At high frequencies or with laminations of two or three times this thickness, however, the effect of the eddy currents may be comparatively great. This would occur with the practically solid rotors of large turbo-alternators.

The procedure followed in this analysis is to decompose the actual flux distribution in the airgap into a series of simple sinusoidal distributions and then to determine how each of these is effective in the generation of voltage and in the development

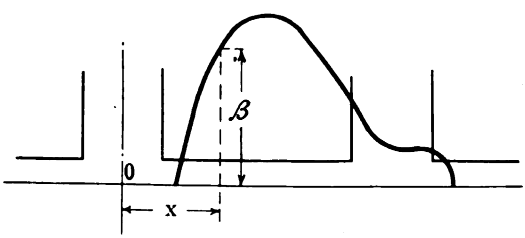


FIG. 1

of power. At any instant the flux distribution, or flux density, in the airgap of an alternating current machine may be represented by a curve such as shown in Fig. 1. The abscissas, x, of this curve are measured in electrical radians from some point fixed of reference, O, in the airgap. In the case of a synchronous machine it is convenient to choose this point as half way between the centers of adjacent field poles. It is also simpler to assume that this point of reference is stationary and that the armature is rotating. This will be done throughout the analysis. The ordinates represent the normal flux density in the airgap at the surface of the armature—or, at the surface of the field poles if this is desired. A distribution of this sort may be of constant magnitude and wave shape, or both may vary periodically, and in either case it may be fixed or moving at a constant or a variable angular velocity. In any case the most complex distribution can be decomposed into a series of simple sinusoidal dis-

tributions of different wave lengths and amplitudes all of which move at a constant but not the same velocity. In fact, some move in one direction through the airgap and some in the opposite direction.

As an illustration of this process of decomposition, consider the case of a salient pole alternator in which but one armature coil per pole is carrying current. At any instant the flux distribution per ampere (see Fig. 1 for example) can always be represented by a Fourier series consisting of both sine and cosine terms. As the coil moves through the airgap the shape of this distribution will vary periodically due to the non-uniformity in the reluctance of the magnetic circuit. If, therefore, the coefficient of each term of the distribution series is plotted against time its value may be represented by another Fourier series. That is, the airgap flux distribution per ampere may be represented by a series of products of sine and cosine terms. Each of these products may be broken up into a sum or difference of sine or cosine terms of constant amplitude, wave length and angular velocity. If instead of there being one ampere in the coil the current in it should vary periodically, each of these sine or cosine terms must be multiplied by still another Fourier series, representing the current. The resulting products of sine and cosine terms would again be broken up into a series of sine and cosine terms which represent simple harmonic wave trains that are either stationary or moving at constant angular velocities.

General Expressions for Voltage and Power

Let the equation for one of these component wave trains be

$$\beta = B_q \cos (q\,x - j\,\omega\,t + \alpha_{qj}) \tag{1}$$

This represents a sinusoidal distribution of maximum value B_q and of wave length $2\,\pi/q$, such as might be produced by $q \cdot P$* poles moving through the airgap at an electrical angular velocity of $j\,\omega/q$ in the same direction as does the armature. See Fig. 2. If the sign before $j\,\omega\,t$ is positive the direction of rotation is opposite to that of the armature. If the coefficient j is zero the distribution is stationary with respect to the field poles. The angle α_{qj} determines the phase of the distribution. When $\omega\,t$ equals zero or any multiple of π the value of the flux

*P represents the actual number of field poles. q is throughout the harmonic order of the flux distribution and is usually 1, 3, 5, 7, etc. j is usually 0, 2, 4, 6, etc.

density at the reference point, O, due to this distribution alone is $\pm B_q \cos \alpha_{qj}$.

The frequency of the voltage generated in the field winding by this distribution is proportional to the equivalent number of poles, $q \cdot P$, and the relative velocity, $j\ \omega/q$. The frequency is $j f$, where f is the fundamental frequency. If the field circuit is closed a current of this frequency will be produced in it and a torque will be developed tending to move the field in the direction in which the distribution is moving. This altermating current in the field winding will also produce a harmonic variation in the field flux which will in turn cause a higher and a lower harmonic in the armature electromotive force.

The rotating flux distribution will also directly produce an electromotive force in the armature winding, the frequency of which is proportional to the equivalent number of poles, $q \cdot P$,

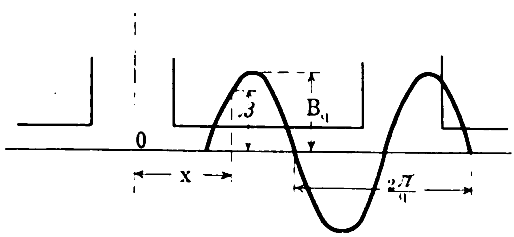


FIG. 2

and the relative angular velocity, $(\omega - j\ \omega/q)$. The frequency is $(q - j) f$. If there exists a current of this frequency in the armature winding power will be developed if the phase relation of the current and the generated voltage is favorable. The equation for the voltage produced in the armature winding by this flux distribution of the q^{th} order is

$$e_{(q-j)} = \sqrt{2}\, E_{q-j} \left[\cos\,(q - j)\ \omega\, t + \alpha_{qj}\right]^* \tag{2}$$

The average power developed in a single-phase winding by

*The effective value of this voltage is

$$E_{q-j} = \frac{K_q}{K_1}\ \frac{B_q}{B_1}\ \frac{q-j}{q}\ E_1$$

where E_1 is the effective voltage due to the fundamental flux distribution, whose maximum value is B_1, and K_q and K_1 are the voltage reduction factors for the qth harmonic and the fundamental flux.

this electromotive force and a harmonic current of the s order whose equation is $i_s = \sqrt{2}\, I_s \sin (s\, \omega\, t + \theta_s)$† is

$$P = \pm E_{q-j} \cdot I_s \sin (\alpha_{qj} \pm \theta_s)\ddagger \qquad (3)$$

The average power is zero, however, unless $(q - j \pm s)$ is zero, that is to say, unless the voltage and current have the same frequency. When the average power is positive electrical power is developed, but when it is negative mechanical power is developed.

In a balanced three-phase alternator, power is developed only when $q \pm s = j = 0, 6, 12$ etc. if there are no even harmonics present or 0, 3, 6 etc. if there even harmonics in the current. The power is

$$P = \pm 3\, E_{q-j} \cdot I_s \sin (\alpha_{qj} \pm \theta_s) \qquad (4)$$

Electrical power is developed when $(\theta_s - \alpha_{qj})$ is an angle of lead, and mechanical power is developed when it is an angle of lag. For this reason it may happen that though power is developed in each of the phases of a three-phase alternator, the net power developed is zero. In this case at least one of the phases would develop electrical power while at least one of the other two would develop mechanical power. That is, the windings of a three-phase alternator may act as a self-contained motor-generator set.

Reduction Factors for Flux. Most frequently the flux in the airgap is due to the action of several magnetizing coils per pole. The resultant flux distribution is the sum of the distributions due to the individual coils acting separately. If these component distributions are decomposed into their elementary sinusoidal parts, as can always be done, those of the same wave length, which are stationary or which travel in the same direction at the same velocity can be combined according to the principles of vector addition. This process of addition is simplified if the components are of the same amplitude and are equally displaced from one another in space-phase position. In this case the method of addition is the same as that for a number of equal voltages of the same frequency differing from each other in time phase by the same amount. With n such distributions of wave length $2\,\pi/q$ differing from each other in position by an angle γ on the fundamental scale, in which the pole pitch equals

†When the conductors of a phase belt are at the zero point midway between the poles, the s harmonic component of the current in that belt has a value $\sqrt{2}\, I_s \sin\ \theta_s$

‡In this equation the signs are either both plus or both minus.

π radians, the sum is what it would be if they were coincident multiplied by

$$K_b = \frac{\sin \frac{q\,n\,\gamma}{2}}{n \sin \frac{q\,\gamma}{2}}$$

This is the belt reduction factor.

If it is further desired to add two such distributions which are equal in magnitude but differ in phase position by an angle λ measured on the fundamental scale, their sum is what it would be if they were coincident multiplied by $K_p = \cos \frac{q\,\lambda}{2}$. The angle λ is some multiple of γ. This is the pitch reduction factor.

The total reduction factor for the qth harmonic flux distribution is

$$K_q = \frac{\sin \frac{q\,n\,\gamma}{2}}{n \sin \frac{q\,\gamma}{2}} \cdot \cos \frac{q\,\lambda}{2}$$

This holds true only when the flux per ampere is independent of the position of the coil, that is, when the airgap is uniform.

Reduction Factors for Voltage. Having calculated the harmonic components of the resultant flux distribution in the airgap, the electromotive force generated in either the stator or rotor winding that links this flux can be found. The belt reduction factor for voltage depends upon the space displacement of the coils and upon the order of the harmonic in the flux distribution. It does not depend upon the velocity of the distribution. The frequency of the voltage produced in the winding, however, depends upon the order of the harmonic in the distribution and upon its velocity with respect to the winding. Thus neither the belt nor the pitch reduction factor for voltage depends upon the frequency of the voltage. If there are n equal coils per pole displaced from one another by an angle γ and having a pitch deficiency of λ, the total reduction factor for voltage for the qth harmonic in the flux distribution is

$$K_q = \frac{\sin \frac{q\,n\,\gamma}{2}}{n \sin \frac{q\,\gamma}{2}} \cos \frac{q\,\lambda}{2}$$

Both γ and λ are measured on fundamental scale.

Flux vs. Voltage Reduction Factor. The angles γ and λ in the voltage reduction factors are always equal to the angles between the coils in which the voltage is produced. It is only with a uniform airgap, however, that the angles γ and λ in the flux reduction factor are equal to the angles between the coils which are producing the flux. If a winding is designed to eliminate a given harmonic from the flux distribution that it produces, a harmonic flux distribution of the same order will generate no voltage in the winding no matter what its relative velocity may be. There may be produced, however, an electromotive force of this order of harmonic frequency by another component in the distribution which is moving at the proper speed with respect to the winding.

Distributing a winding is thus no sure means of reducing or of eliminating a given harmonic in the voltage. For example, a fifth harmonic in the flux distribution moving at fundamental velocity will produce a fifth harmonic in the voltage, as will also a third harmonic in the flux distribution moving at five-thirds of the fundamental velocity. A winding designed to eliminate the harmonic voltage due to the first of these distributions will fail to eliminate that due to the second. All that can be accomplished by distributing the winding is in the reduction or elimination of the harmonics caused by given harmonics in the flux distribution. Another method of reducing the harmonics in the voltage would be to reduce the troublesome harmonics in the flux distribution. If the harmonic flux is in motion with respect to the field, this can be accomplished by means of a short-circuited winding set in the field poles. A winding of this sort is most effective when it is of low resistance and is brought into close proximity to the field winding it protects. The total variable flux that links this amortisseur or damper is just sufficient to account for the resistance drop in it.

In some cases, particularly when there is not an integral number of slots per pole, the reduction factors for flux and voltage must be calculated for each individual case, as no general formula is applicable.

Control of Flux Distribution. The distribution of flux in the airgap of an alternator may be controlled to a certain extent in either of two ways,—by shaping the pole surface if salient poles are used, or by properly distributing the magnetizing winding if the airgap is uniform. The first method will not be discussed.

The first and simplest case assumes that the airgap is uniform,

as it approximately would be in a turbo-alternator. The direct effect of the slots in both the field and armature in producing harmonics is neglected. This effect can be largely eliminated by closing the slots with magnetic wedges and by choosing the proper relative numbers of stator and rotor slots. As is customary the reluctance of the field and armature cores is neglected. An approximate method that corrects for this omission is given later.

A single field coil with a pitch equal to the pole pitch will then produce a rectangular flux distribution in the airgap which may be represented by the Fourier series

$$\beta = \frac{4}{\pi} B_0 \sum \frac{1}{q} \sin q\, x \tag{5}$$

q is the order of the harmonic in the distribution. β is the flux density in the gap at an angular distance x radians from one side

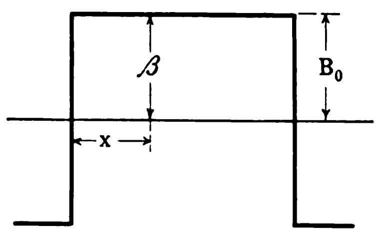


FIG. 3

of the coil, and B_0 is the average flux density in the gap. See Fig. 3. The relative magnitudes of the harmonics are: 1.00: 0.33 : 0.20 : 0.14., etc.

If the field winding is distributed in more than one slot per pole the relative magnitudes of the harmonics are reduced. The amount of this reduction depends upon the method of distributing the field winding. With n coils per pair of poles, each of unity pitch* and having the same number of turns, and equally displaced by an angle γ, the flux density at an angular distance x from the middle of a belt is

$$\beta = \frac{4\, B_0}{\pi} \sum \frac{\sin q \frac{n}{2} \gamma}{n \sin q \frac{\gamma}{2}} \frac{1}{q} \sin q\, x \tag{6}$$

*As a matter of fact the field coils are arranged concentrically but the magnetic effect is the same as if the end connections were made to give unity pitch. See Fig. 4.

If the winding were concentrated the average flux density would be B_0. The relative magnitudes of the harmonics depend upon the number of magnetizing coils, n, and the angle, γ, between them. If, for example, there are six coils per pair of poles displaced by 20 degrees, the relative magnitudes of the harmonics are 0.83 : 0.0 : 0.038 : 0.022. The third harmonic in the distribution vanishes and the phase of the fifth harmonic is reversed. The qth harmonic or any odd multiple of it will vanish if $\frac{q\,n\,\gamma}{2} = \pi$ or any multiple of π. Thus, by properly choosing the breadth of the winding, any harmonic in the flux distribution may be theoretically eliminated, although practical winding conditions will limit this possibility. Since the harmonics are much less prominent than with a concentrated winding the distribution is more nearly sinusoidal. The fundamental flux is now 83 per cent of what it would be if the winding were concentrated. This might be called the efficiency of the winding.

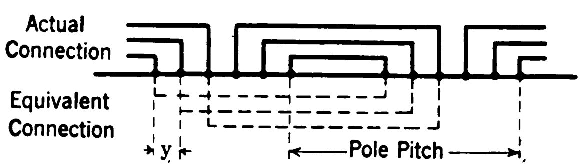


FIG. 4

If the field winding is equivalent to two similar distributed windings displaced by an angle λ as shown in Fig. 5, the relative magnitudes of the harmonics are still further reduced. By properly choosing the pitch deficiency angle, λ, any harmonic in the distribution may be theoretically eliminated. The angle, λ, is always some multiple of the angle, γ, between the slots. The distribution is represented by

$$\beta = \frac{4}{\pi} B_0 \sum \frac{\sin \frac{q\,n\,\gamma}{2}}{n \sin \frac{q\,\gamma}{2}} \cdot \cos \frac{q\,\lambda}{2} \cdot \frac{1}{q} \sin q\,x \qquad (7)$$

The average flux density for a concentrated winding having the same number of turns is B_0. Fig. 5 shows two windings, each like that shown in Fig. 4, which are displaced by two slots or 40 degrees. The relative magnitudes of the harmonics in this case are:

0.78 : 0.0 : 0.0066 : 0.017

The effect of the pitch factor is to reverse both the fifth and seventh harmonics so that the fifth now appears in phase with, and the seventh in opposition to the fundamental. The harmonics are very small. The efficiency of this winding is 78 per cent.

By properly choosing the breadth of one belt or layer, $n\gamma$, and the pitch deficiency, λ, any two harmonics may be eliminated theoretically. There is a practical limitation to this, however, fixed by the minimum allowable angle between the slots. If the angle between the slots is $\pi/15$, and the breadth of one layer of the winding is 10 slots and the pitch deficiency is three slots, the third harmonic will be eliminated by the distribution, and the fifth harmonic by the pitch, of the winding. In this case the seventh harmonic is 1.4 per cent of the fundamental. The efficiency of this winding is 0.79.

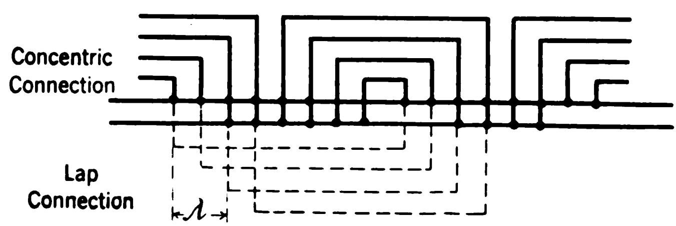


Fig. 5

Correction for Saturation. Thus far the reluctance of the field and armature cores has been neglected. The following method of correction is an approximate one designed to compensate for this omission. Both the field and armature are assumed to be uniformly slotted. The flux distribution in the airgap due to a single magnetizing coil instead of being rectangular, is less at the center than at the conductors, as shown in Fig. 6* The simplest equation that can be given such a curve between $x = 0$ and $x = \pi$ is

$$\beta = B_0 \left(1 + \frac{b_1}{B_0} \cos 2x\right)$$

*Because some parts of the magnetic circuit are more highly saturated than others the component distributions due to the individual coils will not be alike. This effect, however, is disregarded, although it may be of prime importance in determining the magnitudes of the harmonics. Other assumptions as to the shapes of the component distributions might be made which would increase the precision of the results.

B_0 is the average flux density in the gap and $(B_0 + b_1)$ and $(B_0 - b_1)$ are the maximum and minimum flux densities respectively. The ratio of b_1 to B_0 is approximately equal to the ratio of the ampere turns required for the field and armature cores to those required for the airgap and the teeth.

The Fourier series that represents this distribution is

$$\beta = \frac{4\,B_0}{\pi} \sum \left(\frac{1}{q} + \frac{b_1}{B_0}\,\frac{q}{q^2 - 4} \right) \sin q\,x \qquad (8)$$

If the ratio $\frac{b_1}{B_0}$ is 10 per cent, the relative magnitudes of the harmonics produced by a single magnetizing coil are

1.0 : 0.406 : 0.232 : 0.163

instead of

1.0 : 0.333 : 0.200 : 0.143

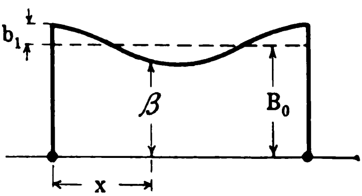


FIG. 6

as they are on the assumption that the reluctance of the field and armature cores is negligible.

The corrected series for a distributed fractional pitch winding is

$$\beta = \frac{4\,B_0}{\pi} \sum \frac{\sin \frac{q\,n\,\gamma}{2}}{n \sin \frac{q\,\gamma}{2}}$$

$$\cdot \cos \frac{q\,\lambda}{2} \left(\frac{1}{q} + \frac{b_1}{B}\,\frac{q}{q^2 - 4} \right) \sin q\,x \qquad (9)$$

At highe flux densities the ratio $\frac{b_1}{B_0}$ is greater and the harmonics become more prominent. The general effect of this is .o make the flux distribution in the airgap flatter than it is at lower saturations.

Correction for Unslotted Portion of Field Surface. Usually the field winding is distributed over only a portion of the polar pitch as shown in Fig. 7. In this case the flux density between the points a and b is greater than it would be if the field were slotted in this region. The full ampere turns of one pole are acting to produce the flux density here and the increase in the density between a and b over what it would be provided the entire field were slotted may be represented roughly by the curve shown in Fig. 8. The length of the unslotted portion is η. The Fourier series that represents this curve is:

$$\beta = \frac{4\,B_0}{\pi} \sum \frac{b_2}{B_0} \sin \frac{q\,\pi}{2} \cdot \sin \frac{q\,\eta}{2} \cdot \frac{1}{q} \sin q\,x \,^* \qquad \textbf{(10)}$$

B_0 is the average flux density with a uniformly slotted field produced by a concentrated field winding having the given number of turns per pole. This series represents the flux density which should be added to that calculated on the assumption that the entire field is slotted.†

Large turbo-alternators are now built with a field winding arranged as shown in Fig. 9. The method of analysis, including

---

*

$$\frac{b_2}{B_0} = \frac{(N\,I)_1 - (N\,I)_2}{(N\,I)_2}$$

where: $(NI)_1$ = ampere turns required to produce B_0 with a uniformly slotted field.

$(NI)_2$ = ampere turns required to produce B_0 with an unslotted field.

†The magnitude of the correction is shown by the following example in which $\frac{b_2}{B_0}$ is taken equal to 0.20 and η equal to $\frac{\pi}{3}$.

$$\beta = \frac{4\,B_0}{\pi}\,(0.10 \sin x - 0.067 \sin 3\,x + 0.020 \sin 5\,x + 0.014 \sin 7\,x + \ldots)$$

The corrected distribution for the winding shown in Fig. 4 if

$\frac{b_1}{B_0} = 0.10$ and $\frac{b_2}{B_0} = 0.20$ is:

$$\beta = \frac{4\,B_0}{\pi}\,(0.91 \sin x - 0.067 \sin 3\,x - 0.023 \sin 5\,x + 0.039 \sin 7\,x + \ldots)$$

The third harmonic which had been cut out by the breadth factor is now 7.4 per cent. of the fundamental.

The corrected efficiency of the winding is 85 per cent.

The uncorrected efficiency as previously calculated was 83 per cent.

the corrections for the effects of saturation and of the unslotted portions of the field, is similar to that already outlined.

The magnitude of any harmonic in the flux distribution depends not only upon the pitch and breadth reduction factors but also upon the degree of saturation and the breadth of the unslotted portion of the field. Harmonics that have been

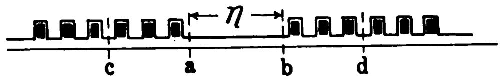


FIG. 7

entirely eliminated by the first two factors will still appear on account of the last. It is only with a uniform airgap that harmonics can be surely eliminated by the distribution of the field winding. Field slots that are closed by magnetic wedges tend to produce this condition. In general the effect of increased saturation is to flatten the flux distribution curve.

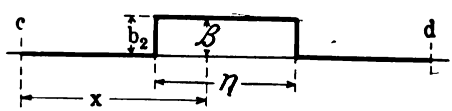


FIG. 8

Flux Distribution with Special Winding. It has been shown that if the equivalent field winding consists of equally spaced coils having the ***same*** number of turns, ***any two*** harmonics or their odd multiples can be eliminated from the flux distribution by properly choosing the breadth of the winding and the pitch of the coils. Thus if a winding is chosen that eliminates the third

FIG. 9

and fifth harmonics, it will also eliminate the ninth, fifteenth, twenty-first, twenty-fifth, etc. On the other hand, if the field winding consists of coils having ***unequal*** numbers of turns more can be accomplished in the elimination of the harmonics. Only the case in which the field is uniformly slotted will be considered. the second correction does not then have to be applied. The illustration here given is for a field winding consisting of four

coils per pole contained in slots displaced by 20 degrees as shown in Fig. 16. See Appendix I. The central slot is not occupied. If the relative numbers of turns in the coils 1 to four are respectively 1.0, 0.88, 0.65 and 0.35, all of the harmonics in the flux distribution will be eliminated up to and including the fifteenth. The first harmonics that do occur, viz. the seventeenth and nineteenth, have the same relative magnitudes as they do with a concentrated winding.*

Analysis of Airgap Flux. The flux distribution in the airgap of a synchronous machine can be modified to a considerable extent by shaping the pole surfaces, or, as has been shown with non-salient poles, by properly distributing the field winding. In either case the value of the flux density in the airgap may be represented by a Fourier series which may be written:

$$\beta = i_f \left(\Sigma A_q \sin q x + \Sigma C_q \cos q x \right) \qquad (11)$$

x is the electrical angle measured from a point midway between the poles, i_f is the value of the exciting current, and the coefficients, A_q and C_q are the maximum values of the various harmonic distributions per ampere of field current at the particular saturation considered.

With a smooth-core armature the magnetic circuit is constructed so that the flux distribution is at all times symmetrical about the center of the field poles. This disregards the effect of the cross magnetizing action of the armature currents which increases the saturation of one-half of the magnetic circuit above that of the other half. With a symmetrical flux distribution the C coefficients are all zero. When the armature is slotted, as it almost invariably is, the magnetic circuit is symmetrical only when a slot or a tooth is exactly at the center of a field pole. The C coefficients are no longer zero but vary periodically with the time.

There are three cases to consider. The first is for a smooth armature and a constant exciting current. The flux distribution is then independent of the position of the armature and i_f and the coefficients A_q are all constant. The second case is for a smooth core armature and a periodically varying field current. i_f is then a periodic function of the time but the coefficients A_q are still assumed to be constant. The third case is for a slotted armature core but with a constant field current. Here i_f is constant but the coefficients A_q and C_q are periodic functions of the time.

*See appendix I.

The armature winding in which the electromotive force is generated is distributed in n slots per pole per phase, displaced by an angle γ and with a pitch deficiency, λ.

I. *Constant Flux*. In the first case, which is the simplest, the electromotive force generated in any armature conductor has the same wave form as the flux distribution. This is not true of the other cases.

If the fundamental flux per pole is ϕ_1 the q^{th} harmonic flux per pole is

$$\phi_q = \frac{1}{q}\frac{A_q}{A_1}\phi_1$$

The voltage produced in the armature winding due to this harmonic flux is

$$E_q = 4.44\, K_q\, N\, q\, f\, \phi_q\, 10^{-8}$$

N is the number of turns in one phase of the armature winding and f is the fundamental frequency. K_q is the total reduction factor for the q^{th} harmonic in the flux distribution.

By properly choosing the distribution and the pitch of the winding it is theoretically possible to eliminate any two harmonics from the electromotive force. A three-phase winding arranged in 15 slots per pole, having a phase belt of 10 slots and a pitch deficiency of three slots will entirely eliminate the third and fifth harmonics. In a well distributed three-phase winding, a pitch of five-sixths reduces the fifth and seventh harmonics to about four or five per cent of their full value. Special windings can be designed to eliminate several harmonics. For example, a single-phase winding such as previously described in connection with the flux distribution will eliminate all of the harmonics up to and including the fifteenth.

Power will be developed in the armature only when there is a harmonic in the current of the q^{th} order and then only if the time phase displacement is not $\pi/2$.

Since the flux is of constant magnitude it produces no voltage in the field winding and therefore no reaction with it.

II *Variable Field Current*. In the second case it is simplest to assume, though incorrectly, that the distribution of the flux in the airgap has a fixed wave form. If the variations in the field current are not large the approximate effect of a periodic field current may be represented by

$$i_f = I_0 + k\,\Sigma\, I_s \sin\,(s\,\omega_1\,t - \theta_s).$$

The frequency of the fundamental variation in this current is $f_1 = \frac{\omega_1}{2\pi}$. It may or may not be equal to the frequency of the fundamental in the armature electromotive force. The constant term I_0 produces the harmonics that were calculated in the first case. Any harmonic in the field current produces a flux distribution in the airgap which is equivalent to two distributions of the same magnitude and wave length, which rotate in opposite directions with respect to the field poles. The frequencies of the voltages produced in the armature by these rotating fluxes are $(q f \pm s f_1)$. Thus in a 60-cycle alternator, a 25-cycle current in the field winding produces voltages having frequencies of 35 cycles and of 85 cycles, due to the variation in the fundamental component of the flux distribution.

If the flux distribution contains a third harmonic, the 25-cycle field current produces voltages in the armature winding with frequencies of 155 cycles and of 205 cycles. This method of resolving a pulsating field into oppositely rotating fields was used by Ferraris in analyzing the operation of the single-phase induction motor.

III *Effect of Slots.* In the third and last case to be considered the field excitation is constant but the *permeance* of the magnetic circuit varies periodically on account of the slots in the surfaces of the field and armature. The wave form of flux distribution in the airgap varies cyclically while the armature is advancing the distance of the armature tooth pitch.* The periodic variation in the shape of this flux distribution produces a corresponding variation in the coefficients of its harmonic components. Each of these coefficients may then be represented by a Fourier series.

If n represents the number of slots per pole the series which represent any of the coefficients such as A_q and C_q are

$$A_q = \Sigma\, {}_p a_q \cos 2\, n\, p\, \omega\, t$$

and

$$C_q = \Sigma\, {}_p c_q \sin 2\, n\, p\, \omega\, t$$

The first is a cosine and the second is a sine series for the same reasons as outlined in the analysis of the armature reaction.

*Time is measured from the moment that an armature slot is directly opposite the zero point, that is, midway between the field poles.

(See Appendix II). The general expression for the flux distribution in the airgap reduces to

$$\beta = \frac{I_0}{2} \{ \Sigma ({}_pa_q + {}_pc_q) \sin (q\, x + 2\, n\, p\, \omega\, t) + \Sigma ({}_pa_q - {}_pc_q) \sin (q\, x - 2\, n\, p\, \omega\, t) \}\dagger \qquad (12)$$

${}_pa_q$ is the coefficient of the p^{th} harmonic term in the Fourier series which represents the variation in the coefficient of the q_{th} harmonic sine term, A_q, in the flux distribution, and ${}_pc_q$ is the coefficient of the p^{th} harmonic term in the Fourier series which represents the variation in the coefficient of the q^{th} harmonic cosine term, C_q, in the flux distribution.

The flux distribution in the airgap is thus resolved into two components one of which (for $p = 0$) is stationary with respect to the field poles and the other of which (for $p = 1, 2, 3$ etc.) is rotating. This second component can be further resolved into two wave trains which rotate in opposite directions through the airgap. The effect of the stationary component has already been analyzed. The harmonics existing in it may be due to the shape of the pole surface or, in the case of non-salient poles, to the distribution of the field, winding or to the field slots. The rotating fluxes are due wholly to the presence of the armature slots although their magnitude is influenced by the shape of the field poles or the arrangement of the field winding. The fundamental slot harmonics, *i. e.*, $p = 1$, are probably the most important.

The periodic variations in the A coefficients indicate corresponding variations in the flux per pole, while the periodic variations in the C coefficients indicate corresponding distortions of the flux distributions. The latter are equivalent to an oscillation of the flux across the pole face.

The coefficients ${}_pa_q$ and ${}_pc_q$ can be determined by analyzing the flux distribution for successive positions of the armature with respect to the field poles. If the values of the coefficients of any particular harmonic are then plotted against time, this curve can be analyzed for the coefficients of its harmonic components, *i. e.*, ${}_0a_q$, ${}_1a_q$, ${}_2a_q$, etc.

The order of the harmonic in the generated voltage depends upon the order of the harmonic in the flux distribution which is affected by the slot pulsations, and the number of slots per pole. It is $(q \pm 2\, n\, p)$. The most important slot harmonics are due to the variation in the fundamental flux and are of the orders $(2\, n \pm 1)$.

$\dagger I_0$ is the field current.

Distributing an armature winding has no effect in eliminating the harmonics caused by the pulsations due to the slots except in so far as it modifies the reduction factor for any particular harmonic in the flux distribution.

Harmonics due to the armature slots may be very prominent in the flux distribution and yet generate little or no voltage in the armature winding. This is because these harmonics move synchronously with the armature and will thus be effective in producing voltage only when their magnitudes vary so that they produce a net variation of flux through the armature coils.

In Fig. 10 assume that the flux distribution in the airgap is represented by the expression

$$\beta = B_0 \sin x \left[1 - a \cos 2\, n\, (x - \omega\, t)\right]$$

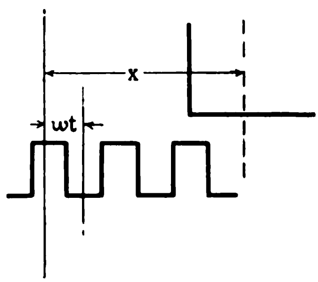


FIG. 10

where n is the number of slots per pole. The value of a determines the magnitude of the slot harmonics. This flux distribution may be thought of as the result of a sinusoidal distribution of magnetomotive force acting on a magnetic circuit whose permeance varies with the time as shown by the bracketed term. The flux linking a coil of unity pitch is

$$\phi = 2\, B_0 \left(1 + \frac{a}{4\, n^2 - 1}\right) \cos \omega\, t\ *$$

Thus, in this case the flux through the coil varies sinusoidally and there are no higher harmonics in the voltage, although the armature slots are the cause of large harmonics—depending upon the value of a—in the air-gap flux distribution. These harmonics are of the orders $(2\, n + 1)$ and $(2\, n - 1)$, but their amplitudes vary with a frequency of $2\, n$ times the fundamental frequency. The net effect of this is to produce a fundamental voltage only. Further, the flux from one pole is

$$\phi_0 = 2\, B_0 \left(1 + \frac{a}{4\, n^2 - 1} \cos 2\, n\, \omega\, t\right) \dagger$$

*This assumes that the center line of the slot marks the division between the flux which enters the tooth at the right and the one at the left. This is of course not strictly true.

$$\dagger\ \phi_0 = \int_0^{\pi} \beta\, d\, x \text{ (assumed)}$$

The flux is greatest when a slot is midway between the poles, and least when a tooth is midway between them. The slots may thus cause a periodic variation in the total flux per pole and yet there may be no corresponding harmonics in the generated voltage.

Armature Reaction. The flux produced by the armature current will be divided into two distinct parts; that which exists in the airgap and that which does not. The magnitude and distribution of the first varies with the excitation and power factor, but the second portion is very nearly independent of each of these factors, and for the sake of simplicity it is assumed to vary only with the armature current.

The effect of the armature current of an alternator in modifying the airgap flux generally influences the operation of the machine more than anything else. The method here used to analyze this effect is to first obtain an expression which shows how the flux density in the airgap, due to a single armature coil of unity pitch, varies when the coil is placed in *any* position relative to the field poles. The resultant effect due to a single-phase winding is the sum of the separate effects due to the individual coils of the winding. An extension of this method of combination will give the resultant reaction for a three-phase alternator.

In Fig. 1, β is the normal component of the flux density in the airgap due to a single coil in which there is a current of i amperes. The equation of this distribution may be written in the form of a Fourier series.

$$\beta = \frac{N_c}{N_f} i \left\{ \Sigma A_q \sin q (x - \omega t) + \Sigma C_q \cos q (x - \omega t) \right\} \qquad (13)$$

N_c and N_f are respectively the turns per armature coil and the turns per field pole. A_q and C_q are respectively the instantaneous maximum values of the q^{th} harmonic sine and cosine flux distributions for an armature current equivalent to one ampere in the field circuit.

As the coil moves through the airgap, *i. e.*, as ωt varies, the coefficients A_q and C_q of the series will vary periodically at double frequency. If the magnetic circuit is assumed to be symmetrical about the center line of the poles the series for A_q will consist of cosine terms only and the series for C_q will consist of sine terms only. The proof of this is given in Appendix II.

The series for A_q and C_q may be written

$$A_q = {}_0a_q + \Sigma\, {}_ha_q \cos 2\, h\, \omega\, t$$
$$C_q = \Sigma\, {}_hc_q \sin 2\, h\, \omega\, t$$

where h may equal 1, 2, 3, 4, etc.

If these values are substituted in equation (**13**) and the products of sine and cosine terms are expanded, the result gives

$$\beta = \frac{N_c}{N_f}\, i \left\{ \sum {}_0a_q \sin (q\, x - q\, \omega\, t) \right.$$
$$+ \frac{1}{2} \sum \sum ({}_pa_q + {}_pc_q) \sin (q\, x - (q - 2\, p)\, \omega\, t)$$
$$\left. + \frac{1}{2} \sum \sum ({}_pa_q - {}_pc_q) \sin (q\, x - (q + 2\, p)\, \omega\, t) \right\} \quad \textbf{(14)}$$

This represents the flux distribution in the airgap due to a single armature coil carrying a current of i amperes. The angular displacement of the coil from the center of the pole is $\omega\, t$. ${}_0a_q$ is the average value of the amplitude of the q^{th} harmonic in the flux distribution. ${}_1a_q$ and ${}_1c_q$ are respectively the fundamental amplitudes of the variation in the maximum values of the q^{th} harmonic sine and cosine distributions.

With a distributed winding the values of $\omega\, t$ are different for the individual coils and the total reduction factors are

$$K_{q \pm 2\, p} = \frac{\sin \dfrac{(q \pm 2\, p)\, n\, \gamma}{2}}{n \sin \dfrac{(q \pm 2\, p)\, \gamma}{2}} \cdot \cos \frac{(q \pm 2\, p)\, \lambda}{2}$$

If the expression for the current is $i = \Sigma\ \sqrt{2}\, I_s \sin (s\, \omega\, t + \theta_s)$ the general expression for the flux distribution, due to the armature current alone, in the airgap of a single-phase alternator reduces to

$$\beta = \frac{1}{\sqrt{2}} \frac{N_P}{N_F} \left\{ \sum \sum K_q \cdot {}_0a_q \right.$$
$$\cdot\, I_s \left[\cos (q\, x - (q + s)\, \omega\, t - \theta_s) \right.$$
$$\left. - \cos (q\, x - (q - s)\, \omega\, t + \theta_s) \right]$$

$$+\sum\sum\sum K_{q-2p}\cdot\frac{({}_pa_q + {}_pc_q)}{2}$$
$$\cdot I_s\,[\cos(q\,x - (q - 2\,p + s)\,\omega\,t - \theta_s) - \cos(q\,x - (q - 2\,p - s)\,\omega\,t + \theta_s)\,]$$

$$+\sum\sum\sum K_{q+2p}\cdot\frac{({}_pa_q - {}_pc_q)}{2}$$
$$\left.\cdot I_s\,[\cos(q\,x - (q + 2\,p + s)\,\omega\,t - \theta_s) - \cos(q\,x - (q + 2\,p - s)\,\omega\,t + \theta_s)\,]\right\}^{*} \tag{15}$$

Three-Phase Armature Reaction. In a three-phase alternator the resultant flux distribution due to the armature current alone is the combination of three such distributions. With a balanced load the currents are equal in magnitude and differ in phase by 120 degrees. The three armature windings also differ in their relative positions by 120 degrees. The sum of the three single-phase distributions will then be zero except when the coefficients of $\omega\,t$ in equation (**15**) are zero, three, or some multiple of three. In the coefficient of $\omega\,t$, q is always odd; $2\,p$ is always even and ordinarily the current contains no even harmonics and s is therefore odd. $(q \pm 2\,p \pm s)$ is thus either zero or even. If, however, the current contains even harmonics, this coefficient may be odd.

The general expression for the armature reaction flux in a three-phase alternator is given in equation (**15**). If the current contains no even harmonics only those terms can appear in which the coefficient of $\omega\,t$ is zero, six, or some multiple of six. If the armature current contains none but even harmonics only those terms can appear in which the coefficient of $\omega\,t$ is zero, three or some odd multiple of three.

In a turbo-alternator with a uniform airgap, the armature flux is independent of the position of the armature coil with respect to the field poles. In this case all of the coefficients except ${}_0a_q$ vanish, *i. e.*, p has no value but zero, and the complete expression for the armature reaction is much simplified, consisting as it does of only the first two terms in equation (**15**).

---

*N_P and N_F are the armature and field turns per pole.

For the method of calculating the coefficients ${}_pA_q$ and ${}_pC_q$ and their values see Appendix III.

The only assumption made in deriving this expression for the armature reaction flux is in regard to the symmetry of the magnetic circuit. But for this it is perfectly general.

Armature Flux, Voltage and Power. In an alternator with a uniform airgap the armature reaction is stationary with respect to the field poles only when the harmonic orders of the component flux distribution and the component armature current are the same. With a salient pole alternator, however, the fundamental current may produce a stationary third harmonic in the armature flux, or a third harmonic component in the armature current may produce a stationary fundamental component in the armature flux distribution. With a salient pole alternator there is likely to be a considerable third harmonic component in the armature reaction which is stationary with respect to the field poles and is due solely to the fundamental part of the armature current.* If the armature is delta-connected, a third harmonic current will exist in the windings which may of itself produce a stationary fundamental component in the flux distribution.† The latter would affect both the fundamental voltage and the power developed.

With a uniform airgap, in which case $p = o$, no average power is developed. That is, no power is developed in an alternator having a uniform airgap if the field excitation is zero. With a salient pole alternator, however, in which case p is an integer, power may be developed due to the armature reaction flux alone, or, in other words, when there is no field excitation. This power is proportional to the sine of twice the lead angle of the fundamental armature current. The power is greatest with a phase angle of 45 degrees. Thus an unexcited synchronous machine may develop considerable power as a generator if the current leads, and considerable power as a motor if the current lags. These are well known facts. Furthermore a current of one harmonic order may produce a flux that reacts with a current of a different harmonic order and produces power.

When a synchronous motor is being brought up to speed by hysteresis and eddy current action, the fundamental current is relatively a fifth harmonic at one-fifth of synchronous speed, a third harmonic at one-third of synchronous speed and a second harmonic at one-half of synchronous speed. If one of these har-

---

*See fourth term, equation (**15**) $q = 3$, $p = 1$, $s = 1$.

†See sixth term equation (**15**) $q = 1$, $p = 1$, $s = 3$.

monics produces a torque greater than that due to hysteresis and eddy currents, the motor will tend to lock in at that speed and not come up to synchronism. Whether it does or does not will depend upon whether the momentum of the rotating part is sufficient to carry it into a phase position that is unfavorable for the development of motor torque due to the reaction of the airgap flux and the current. The hysteresis and eddy current torque is greater at one-fifth speed and the synchronous torque developed by the current is smaller than at either one-third or one-half speed. The tendency of the motor to lock in at one-half speed is greater than at either one-third or one-fifth speed. It depends upon the design and principally upon the ratio of pole arc to pole pitch.

Harmonics in Transient Short-Circuit Condition. The transient short-circuit in an alternator gives a good illustration of the effect of even harmonics in the armature current. We will consider only the case of a three-phase turbo-alternator which has its field and armature windings so distributed that the harmonic components in the flux distributions they produce are negligible compared with the fundamental components.

The general equation for this current is

$$i = \Sigma A\, e^{-\alpha t} + \Sigma B\, e^{-\beta t} \sin s\, \omega\, t + \Sigma C\, e^{-\gamma t} \cos s\, \omega\, t \qquad (16)$$

As far as the writer is aware the constants in this equation have never been evaluated.

Due to the first term in this expression there is a decaying flux distribution which rotates at synchronous speed with respect to the field poles. There will thus be produced a current of fundamental frequency in the field winding. The periodic flux produced by this current will generate a voltage of double frequency in the armature winding, causing a second harmonic current, which will react on the field circuit and produce a third harmonic current in it. This third harmonic variation of the field flux will produce a second and fourth harmonic in the armature winding. The effect of a second harmonic has already been accounted for. The fourth harmonic in the armature current will produce only a third harmonic in the field winding, the effect of which has already been considered. The fifth harmonic which would have been produced in a single-phase alternator is not present, since the component fluxes which would produce it sum up to zero in the case of a three-phase alternator.* There will thus be a fundamental and a third har-

*First term, equation (15) $q = 1$, $s = 4$.

monic in the field current, and a fundamental, a second and a fourth harmonic in the armature current, but none higher. The magnitude of these harmonic components will be influenced to a considerable extent by the eddy currents produced in the practically solid rotor.

Calculation of Field Current. One of the most interesting applications of this theory, is to the problem of calculating the field current of an alternator for a given load condition. A three-phase, Y-connected generator, with neutral free, is chosen so that there can be no troublesome third harmonics in the armature current. There is generally little error in neglecting the effect on the terminal voltage of the fifth or higher harmonics in the armature flux if the magnetic circuit is designed so that there is no prominent fifth harmonic n the flux distribution due to the

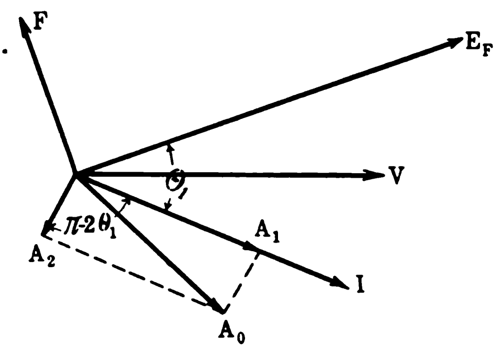


FIG. 11

field. If it be assumed that the armature currents are sinusoidal and balanced, that the harmonics in the armature flux above the third are negligible and that the harmonic variations in these components above the second are also negligible,* the expression for the stationary armature reaction flux distribution reduces to

$$\beta = \frac{1}{\sqrt{2}} \frac{N_P}{N_F} K_1 I_1 \left\{ -{}_0a_1 \cos(x + \theta_1) + \frac{{}_1a_1 + {}_1c_1}{2} \cos(x - \theta_1) - \frac{{}_1a_3 + {}_1c_3}{2} \cos(3x + \theta_1) + \frac{{}_2a_3 + {}_2c_3}{2} \cos(3x - \theta_1) \right\} \quad (17)$$

*In equation (15) $s = 1$, $q = 1$ and 3 and $p = 1$ and 2 only.

The first of these terms is that ordinarily given for the armature reaction and it is the only one present when the air gap is uniform. The other terms are due to the non-uniformity of the magnetic circuit for different positions of the armature coils on account of the spaces between the salient poles. The third and fourth terms are due to the presence of variable third harmonics in the flux distributions produced by the armature coils. They have no direct effect on the fundamental armature voltage but they do alter the saturation of the main magnetic circuit and may be important on this account. If they increase the flux from each pole it requires more field current to give a specified armature electromotive force than is shown by the no-load saturation curve.†

The vector diagram showing the phase relations of the fundamental armature reaction, the armature current and the airgap flux is given in Fig. 11. F and E_F are the impressed field and the voltage it would produce on open circuit. V is the terminal voltage. The armature current, I, lags behind the excitation voltage E_F by an angle θ_1. A_1 and A_2 are the first and second terms of the resultant armature reaction A_0. From equation (**17**) it is seen that

$$A_1 = \frac{1}{\sqrt{2}} \cdot \frac{N_P}{N_F} \cdot K_1 \cdot I_1 \, ({}_0a_1)^*$$

and
$$A_2 = \frac{1}{\sqrt{2}} \cdot \frac{N_P}{N_F} K_1 \cdot I_1 \cdot \frac{{}_1a_1 + {}_1c_1}{2} \quad *$$

A_1 is in phase with the current while A_2 lags behind the current by $(\pi - 2\,\theta_1)$ if $({}_1a_1 + {}_1c_1)$ is positive, as it usually is. Thus the resultant armature reaction lies more nearly in line with the impressed field than if the airgap were uniform. This condition occurs for two reasons; first, because an armature coil produces more flux when opposite a pole than when half way between poles $({}_1a_1 > 0)$, and second because the axis of the armature flux tends to conform with the axis of the field poles $({}_1c_1 > 0)$. At unity power factor, A_1 and A_2 are in opposition, while at zero power factor they are in conjunction. Thus the magnitude of the armature reaction flux is much greater at zero power factor than at unity power factor.

†See *Effect of Third Harmonic on Saturation.*

*Values of the coefficients ${}_0a_1$, ${}_1a_1$ and ${}_1c_1$ etc. are given in appendix III.

Fig. 12 shows the vector diagram which may be used in calculating the regulation of an alternator. The power factor (external) at which the alternator is operating is cos θ.* The sum of the terminal voltage, V, the resistance drop, $I\,r_e$†, and the leakage reactance drop, $I\,x_a$†, gives the voltage E_a, which is produced by the resultant fundamental flux in the airgap. This flux is due to the effect of the field and armature currents acting in conjunction on the magnetic circuit. If the permeability of all parts of the magnetic circuit is now assumed to be constant, the airgap flux which produces E_a may be replaced by the component fluxes which would be produced in the airgap by the independent action of the field and armature currents. With such an assumption the fluxes and the equivalent field currents

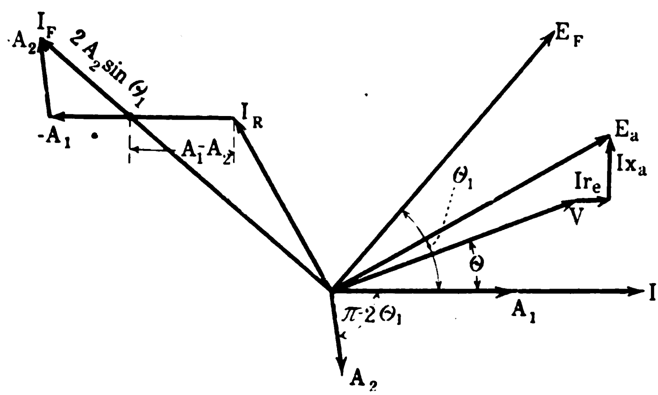


FIG. 12

producing them are proportional. I_R is the field current corresponding to E_a, obtained from the open-circuit saturation curve. A_1 and A_2 are the equivalent field currents corresponding to the two parts of the armature reaction. I_F is the actual field current. It is readily proved by geometry that the vector drawn from the terminus of I_R to the intersection of A_1 and I_F is equal to $(A_1 - A_2)$ and that the distance from this intersection to the terminus of I_F is $2\,A_2 \sin \theta_1$. Thus the phase position of I_F is determined by the armature reaction at unity power factor.

For the purpose of calculation it is more convenient to rotate I_R and I_F through 90 degrees clockwise so that the vector diagram appears as in Fig. 13.

*An angle of lead is positive and an angle of lag is negative.

†r_e and x_a are the equivalent single phase values.

The armature leakage reactance may be determined from the open- and short-circuit characteristics. At short circuit the internal power-factor angle, θ_1, is essentially 90 degrees. Calculate the field current equivalent to the total armature reaction at zero power factor, viz. $A_1 + A_2$. Observe the voltage E_{AO} that would be produced on open circuit by this field current. Referring to Fig. 14 the leakage reactance is shown to be

$$x_a = \frac{E_F - E_{AO}}{I_a}$$

There are two minor corrections which will increase the precission of the results. They are the effect of the third harmonic and the increased pole leakage.

Effect of Third Harmonic on Saturation. The flux in the airgap

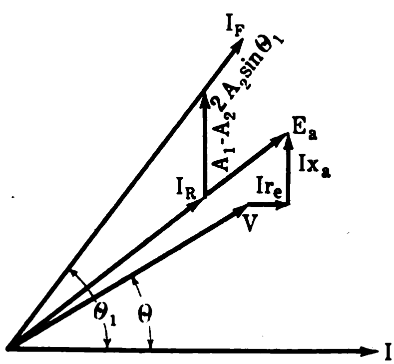


FIG. 13

and in the main magnetic circuit is produced by the magnetizing action of the field current and the armature current. Part of this flux is sinusoidally distributed and produces a measurable voltage across the terminals of the armature, while a small part of it is a third harmonic distribution and produces no voltage across the terminals. The saturation of the magnetic circuit, however, is determined by the entire flux. The armature voltage, E_a, (See Fig. 13) is due to the combined action of the field current and the sinusoidal portion of the armature reaction. The third harmonic portion of the armature reaction increases the flux per pole, when the current is a lagging one, and therefore more net field current is required to produce E_a under load conditions than at no-load. The higher harmonics in the armature flux may also increase the saturation, but their effect is

usually small. The magnetizing action of the third harmonic component, in equivalent field current, is somewhat less than

$$A_{03} = -\frac{1}{3}\,\frac{1}{\sqrt{2}}K_3\,\frac{N_P}{N_F}\,I_1\left(\frac{{}_1a_3 + {}_1c_3}{2} + \frac{{}_2a_3 + {}_2c_3}{2}\right)\sin\theta_1\text{*}$$

In Fig. 15 E_a is the armature voltage which would be produced by I_R on open circuit. Under load, because of the increased saturation due to the third harmonic, it requires I'_R amperes to produce this same voltage. The entire magnetizing action for this condition is equivalent to a field current of $(I'_R + A_{03})$ amperes. On open circuit this would produce a voltage E. Perhaps the simplest method of determining the field current,

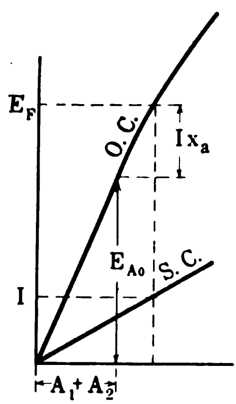


FIG. 14

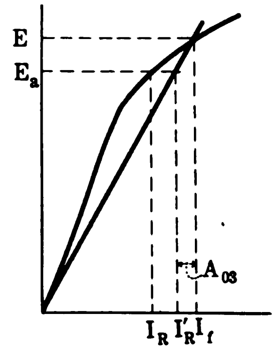


FIG. 15

I'_R, necessary to give the required armature voltage is to assume two values of E somewhat above the actual armature voltage, E_a, to calculate the corresponding values of E_a and then to interpolate between them. The values of I'_R and E_a corresponding to an open circuit voltage E are

$$I'_R = I_f - A_{03} \text{ and } E_a = \frac{I'_R}{I_f}E$$

It is suggested that values of I_f be chosen equal to $I_R + A_{03}$ and $I_R + 3\,A_{03}$. This correction is qu te small unless the saturation is high and the power factor is low. Except in a few cases to which attention is called no account of this effect is taken in the calculations that follow.

*For a lagging current $\sin\theta_1$ is negative.

Increased Field Leakage. On open circuit the field current produces not on y the flux in the airgap but also the leakage flux between the poles. Under load conditions the field current must usually be increased if the flux is to be maintained constant, on account of the demagnetizing effect of the armature reaction. Since the pole leakage is nearly proportional to the field current, it will usually be greater under load conditions than at no-load for the same armature voltage. The increased leakage requires additional field current, which is proportional to the difference between the calculated value of the field current, I_F, and that required to produce the armature voltage, namely I'_R. An approximate value of this added field current is

$$\left(\frac{v-1}{v}\,u\,y\right)\cdot(I_F - I'_R)$$

in which v is the leakage coefficient of the field, u, the saturation factor for the field poles and yoke, and y, the ratio of ampere turns required for the field poles and yoke to those required for the rest of the magnetic circuit. These coefficients should all be calculated for a condition of the magnetic circuit corresponding to the field current I'_R.

Numerical Calculation of Field Current

1080—h p., 3-phase synchronous motor.

Volts	6600
Poles	6
Slots	108
Series conductors per slot	14
Coil pitch	0.83
Turns per field spool	462
Pole arc: pole pitch	0.642
Field Leakage coefficient	1.083
Ratio $\dfrac{\text{amp. turns, poles and yokes}}{\text{amp. turns airgap, teeth and arm. core}}$	0.026
Saturation factor—field poles and yokes ($E_a = 6025$)	5.6
${}_0a_1$	0.803
$\dfrac{{}_1a_1 + {}_1c_1}{2}$	0.269

Calculation of field current when 98.5 amperes is delivered at a power factor of 0.42, leading

$$0.707\,K_1\,\frac{N_P}{N_F}\,I_a = 0.707 \times 0.923 \times \frac{18 \times 7}{462} \times 93.1$$

$$= 16.57 \qquad \text{for } I_a = 93.1$$
$$= 17.52 \qquad \text{for } I_a = 98.1$$

Armature reaction at zero power factor

$$A_0 = 1.072 \times 16.57$$
$$= 17.76 \text{ equivalent field amperes.}$$

Field current on short circuit for armature current of 93.1 amperes is 19.2 amperes. Armature voltage on open circuit for field current of 50 amperes is 2250 volts.

$$I\,x_a = \frac{19.2 - 17.76}{50} \times 2250$$

$= 648$ volts for armature current of 93.1 amperes
$= 686$ " " " " " 98.5 "

$$V = 6600 \cos\theta - j\,6600 \sin\theta \qquad \cos\theta = 0.42$$
$$= 2772 - j\,5990$$

$$I\,z_a = \underline{92 + j\,686} \qquad I\,r = 92.\ \text{volts (between terminals)}$$
$$E_a = 2864 - j\,5304$$
$$= 6025 \text{ volts}$$

Corresponding field current from saturation curve.

$$I_R = 14.2 \text{ amperes}$$
$$= \frac{2864}{6025} \times 14.2 - j\,\frac{5304}{6025} \times 14.2$$
$$= 6.75 - j\,12.49$$

Armature reaction at unity power factor for armature current of 98.5 amperes.

$$A_1 - A_2 = 0.535 \times 17.52$$
$$= 9.38$$
$$I_R + j\,(A_1 - A_2) = 6.75 - j\,3.11$$
$$= 7.43$$

$$2\,A_2 \sin\theta_1 = 0.537 \times 17.52 \times \frac{3.11}{7.43}$$
$$= 3.94$$
$$I_F = 7.43 - 3.94 \qquad \theta_1 \text{ is an angle of lead}$$

$$\frac{v-1}{v}\,u\,y = \frac{8.3}{108.3} \times 0.026 \times 5.6$$
$$= 0.011$$

$$I'_F - I_F = 0.011\,(3.49 - 14.2)$$
$$I'_F = 3.37$$

Measured value of field current = 2.0

It is difficult to predetermine accurately the field current of a synchronous generator for a given load condition, particularly when it is delivering a leading current. If the so-called magnetomotive force method or the method proposed by the A. I. E. E. is taken as the standard of simplicity, then it can be said that there is no simple method of calculating the field current that will give reliable results for every load condition. The effect of the armature current is much too complicated to be calculated so easily. The general method, in which the leakage reactance and armature reaction are determined by the Potier triangle, is perhaps as good a compromise as any between accuracy and simplicity.

TABLE OF COMPARATIVE FIELD CURRENTS.

Output	Voltage	P.F.[1]	FIELD CURRENT				
			Gen. method	M. M. F.	A.I.E.E.	Proposed	Observed
500 kv-a.	240	−0.0	....	121.5	....	134	135.5
1,500 kv-a.	2,300	−0.0	....	120.5	....	135.7	130.
2,000 kv-a.	2,300	−0.0	....	1 9.	....	214.	227.
2,000[2] kv-a.	2,100	−0.0	....	52.5	....	60.5	64.5
2,000[2] kv-a.	2,100	+0.0	−12.4	−5.6	....	−8.7	−10.0
2,500 kv-a.	2,300	−0.0	....	257.	....	310 327[4]	330.
2,500 kv-a.	2,300	+0.0	−12.	10.[5] −42.[6]	67	−6	−4.8
5,000 kv-a.	2,300	−0.0	....	235	....	258	251.
5,000 kv-a.	6,600	−0.0	....	221	....	246[4,7]	250.
5,000 kv-a.	6,600	1.0	173	171	165	170	169.
83.5 amp.[3]	11,150	1.0	40.0	39.5	36	41.1[4,7]	42.8
68.0 amp.[3]	11,550	+0.784	53.	48.1	56	58.2	59.
12,000 kv-a.	6,600	−0.0	....	372	....	406	410.
93.1 amp.	6,600	−0.0	....	35.2	....	36.8	36.9
110 amp.	6,600	+0.72	16.6	16.5	16.7	13.	12.0
98.5 amp.	6,600	+0.42	9.2	9.0	16.1	3.37	2.0

1. + sign indicates a leading current; − sign, a lagging current.
2. Two-phase.
3. As a motor.
4. Corrected for effect of third harmonic.
5. Armature m.m.f. from short circuit data.
6. Armature m.m.f. from zero power factor data at rated voltage.
7. Not corrected for increased field leakage.

The author wishes to aknowledge his indebtedness to the General Electric Company and Westinghouse Electric and Manufacturing Company for the data from which these results were calculated.

APPENDIX I

Refer to Fig. 16. N_1, N_2, N_3 and N_4 indicate the numbers of turns in the coils. Let B_0 be the average flux density per ampere turn when one coil alone is acting. The flux density per ampere at a distance x, measured from the center of a belt of conductors, reduces to

$$\beta = \frac{8 B_0}{\pi} \sum \left(N_1 \cos \frac{q \gamma}{2} + N_2 \cos \frac{3 q \gamma}{2} + N_3 \cos \frac{5 q \gamma}{2} + N_4 \cos \frac{7 q \gamma}{2} \right) \frac{1}{q} \sin q x$$

If any harmonic is to be eliminated its coefficient in this series must be zero. Since there are three independent variables in each coefficient, viz. $\frac{N_2}{N_1}$, $\frac{N_3}{N_1}$, $\frac{N_4}{N_1}$, at least three harmonics and their odd multiples may be eliminated. Choose these

FIG. 16

variables so that the third, fifth and seventh harmonics will vanish. Since there are 9 slots per pole the angle $\gamma/2$ equals 10 degrees. The equations are

$$N_1 \cos 30 + N_2 \cos 90 + N_3 \cos 150 + N_4 \cos 210 = 0$$
$$N_1 \cos 50 + N_2 \cos 150 + N_3 \cos 250 + N_4 \cos 350 = 0$$
$$N_1 \cos 70 + N_2 \cos 210 + N_3 \cos 350 + N_4 \cos 490 = 0$$

The ninth harmonic vanishes for any number of turns in the coils. The equation for the eleventh harmonic coefficient is identical with that for the seventh except that the signs are changed. It therefore vanishes. The same is true for the thirteenth and fifteenth harmonics. Thus the first harmonic that appears is the seventeenth. The other higher harmonics that may be present are the nineteenth, twenty-third, twenty-ninth, etc. The first harmonics that appear, viz. the seventeenth and the ninteenth are of the same order as would be produced by the rotor slots. They could be reduced to small proportions by closing the slots with magnetic wedges. The solution of the above equations for the relative numbers of turns in the field coils gives

$$\frac{N_2}{N_1} = 0.88$$

$$\frac{N_3}{N_1} = 0.65$$

$$\frac{N_4}{N_1} = 0.35$$

The efficiency of the winding is 80 per cent.

The reduction factor for the seventeenth harmonic is the same as that for the fundamental but the harmonic is reversed in phase. The same is true of the nineteenth harmonic. Each of these harmonics thus occurs in the same relative magnitude as it would in the distribution due to a concentrated winding. Making the correction for saturation, neither alters the efficiency

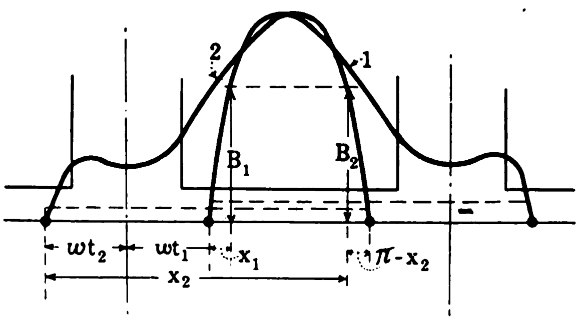


FIG. 17

of the winding nor the reduction factors for the seventeenth and nineteenth harmonics, but does alter their relative magnitudes. If $\frac{b_1}{B_0}$ is 0.10, they are both about thirteen per cent greater than at low saturation.

APPENDIX II

In Fig. 17, $B_1 = B_2$ if $x_1 = \pi - x_2$ and $\omega t_1 = - \omega t_2$ and the magnetic circuit is symmetrical about its center line.

$B_1 = \Sigma A_q' \sin q x_1 + \Sigma C_q' \cos q x_1$

$B_2 = \Sigma A_q'' \sin q x_2 + \Sigma C_q'' \cos q x_2$

But since $x_1 = \pi - x_2$, and q is always odd

$B_2 = \Sigma A_q'' \sin q x_1 - \Sigma C_q'' \cos q x_1$

From this it follows that $A_q' = A_q''$ and $C_q' = - C_q''$.

But A_q and C_q are periodic functions of $2 \omega t$ and since $\omega t_1 = - \omega t_2$, A_q must be a cosine series and C_q a sine series.

If one tip of the field pole is more highly saturated than the other, as it usually would be, the magnetic circuit will not be quite symmetrical about its center line. This distortion is probably never very great since most of the reluctance is in the airgap. Its effect would be to introduce sine terms into the series for A_q and cosine terms into the series for C_q. The final

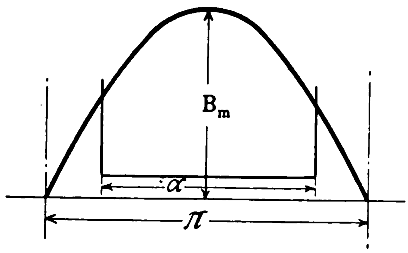


FIG. 18—FLUX DISTRIBUTION DUE TO FIELD CURRENT

result would be to produce small quadrature electromotive forces in the armature winding which would otherwise be absent and which are neglected throughout this analysis.

APPENDIX III

Calculation of Coefficients. The following assumptions in regard to the distribution of flux in the airgap are considered

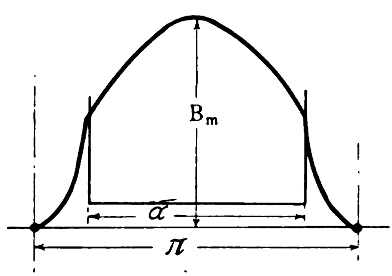


FIG. 19—FLUX DISTRIBUTION DUE TO ARMATURE CURRENT. ZERO DEGREES.

ideal. They are chosen to represent the actual conditions as nearly as possible and still allow relatively simple calculations to be made. With these assumptions, however, it requires about fifteen pages of close calculation to obtain the desired coefficients. In practise, it may happen that the departure of the actual distributions from these ideal forms may be so great as to warrant new calculations that will more nearly fit the

special case. The flux distributions can also be determined experimentally and the coefficients deduced therefrom.

With one ampere-turn acting per field pole the flux distribution in the airgap is assumed to be sinusoidal, with a maximum value of B_m. See Fig. 18. For the same saturation of the main magnetic circuit with one armature turn carrying one ampere, the

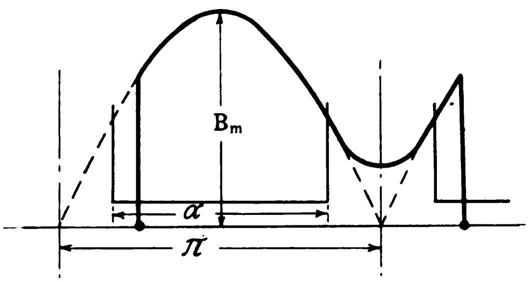


FIG. 20—FLUX DISTRIBUTION DUE TO ARMATURE CURRENT. 45 DEGREES

flux distributions for different positions of the turn are shown in Figs. 19, 20 and 21. At zero degrees, with the armature coil directly opposite a field pole, the distribution is an inverted sine curve between the poles and coincides with the distribution due to a field ampere-turn under the poles. The maximum value is B_m. At 45 and at 90 degrees, the distribution is also an

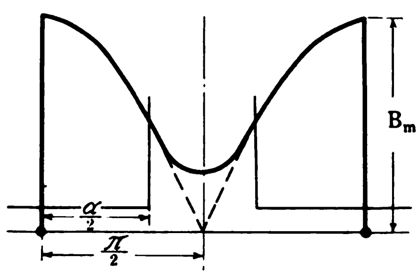


FIG. 21—FLUX DISTRIBUTION DUE TO ARMATURE CURRENT. 90 DEGREES.

inverted sine curve between the poles and coincides with the distribution due to a single field ampere-turn under the poles. Taking the maximum value in each of these four cases as B_m assumes that the reluctance of the armature core, pole and yoke is negligible compared with that of the airgap and teeth. This is nearly so at low saturation. For operating conditions the

saturation is higher and the flux densities due to an armature coil will be relatively greater for those positions that give flux paths that do not include the field poles, yokes and core. In order to make the calculations as simple as possible it is assumed that this applies only to the distribution at 90 degrees. It should also apply to the distribution at 45 degrees. The method of estimating this increase in the flux density, however, assumes that the condition of the magnetic circuit for a coil position of 90 degrees is the same as at low saturation. This assumption neglects the increase in reluctance of the armature core and pole faces and thus makes a compensating error. The method of estimating the increase in the flux density for a coil position of 90 degrees is illustrated in Fig. 22. If the saturation of the main magnetic circuit is determined by the voltage E_a on the open-circuit saturation curve, the value of the maximum flux density should be taken as $\frac{E'}{E_m} B_m$ for the distribution at 90 degrees.

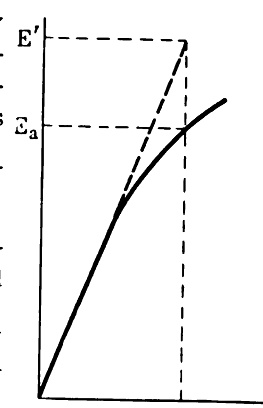


FIG. 22

In the two latter positions of the armature coil the amplitude of the inverted sine curve is taken as $1 - \frac{2}{\sqrt{4 + \frac{b^2}{g^2}}}$ of the density at the pole tip*, where b is the width of the interpolar space and g is the length of the airgap at the pole tip. An average value of this amplitude is about 0.85 and it is taken as such in the calculations. The Fourier analysis of these assumed flux distributions gives formulas for the coefficients of the sine and cosine terms of the fundamental and third harmonic distributions. If the coefficients of the third and higher harmonics in the *variations* of these coefficients are assumed to be negligible, the approximate values of ${}_0a_1$, ${}_1a_1$, ${}_1c_1$, ${}_1a_3$, ${}_2a_3$, ${}_1c_3$ and ${}_2c_3$ can be calculated. At the end of this appendix these coefficients are tabulated for a ratio of pole arc to pole pitch of 0.70 for different saturations. For ratios of pole arc to pole pitch between 0.65 and 0.75 the armature flux is nearly constant.

*This formula is due to Mr. F. W. Carter, who, however, uses it in connection with the problem of determining the flux distribution opposite an armature slot placed in an otherwise uniform airgap.

Different assumptions in regard to the shape of the flux distributions for different positions of the armature coils would give different values for these coefficients. Considerable research may be necessary before reliable representative values can be chosen. Oscillographic measurements made at low saturation on a 7.5-kw. generator give a ratio of third harmonic to fundamental component at unity power factor of 52 per cent, and at zero power factor of 18 per cent. The ratio between the fundamental fluxes for unity power factor and zero power factor is 45 per cent. Referring to the table of coefficients it will be seen that the calculated values of these ratios are respectively 64 per cent, 22 per cent and 50 per cent. The coefficients given in this paper are only roughly indicative of the general results that may be expected, and the following conclusions are liable to some error. Within its usual values the ratio of pole arc to pole pitch has apparently little effect on the armature reaction. At unity power factor the reaction is about 3 per cent greater for a ratio of 0.75 than for one of 0.65. For a ratio of the length of the airgap at the pole tip to the length of the interpolar space of 0.05 the reaction at unity power factor is about 2.5 per cent less than for a ratio of 0.10. Saturation has no effect on the armature reaction at zero power factor but at unity power factor the armature reaction increases with the saturation. At higher saturations the third harmonic in the armature reaction is more prominent than at lower saturations at unity power factor, but at zero power factor the third harmonic component is practically constant.

TABLE OF COEFFICIENTS.*

Saturation...........	1.0	1.11	1.2	1.3
$_0a_1$..................	0.80	0.82	0.83	0.85
$\frac{_1a_1 + {}_1c_1}{2}$............	0.27	0.26	0.24	0.23
$\frac{_1a_3 + {}_1c_3}{2}$............	−0.29	−0.30	−0.32	−0.33
$\frac{_3a_3 + {}_1c_3}{2}$............	0.05	0.07	0.09	0.10

*To obtain the best results the coefficients should be calculated for the proper ratio of pole arc to pole pitch and for airgap to interpolar space.

Presented at a Section Meeting of the American Institute of Electrical Engineers, Pittsburgh, Pa., November 13, 1917.

(Subject to final revision for the Transactions.)

ELECTRIC ARC WELDING

BY A. M. CANDY

ABSTRACT OF PAPER

The exacting demands of the present World War have given a tremendous impetus to the application of electric arc welding methods for making rapid repairs to a great variety of machinery and apparatus required in the successful conduct of the conflict. The British Admiralty has adopted the process extensively for ship construction including minor structural parts, caulking of seams, and for partially eliminating riveted construction. At present the United States Shipping Board Emergency Fleet Corporation has an active Welding Committee working upon the application of electric welding with a view to ultimately producing merchant ships which will be largely built by welding instead of the present riveted construction.

Probably the chief reasons for the apparent apathy of the general industrial world toward arc welding are the haphazard methods employed, using any kind of electrical equipment, any kind of wire for electrode material and any kind of laborer or general handy man as a welder. This situation is being gradually improved, owing to a number of articles appearing recently in the technical press which emphasize the proper methods for the execution of satisfactory welds, general methods of supervising and inspecting welding work, and the selection of proper electrical equipment.

HISTORICAL

THE process of arc welding may be defined as the utilization of the intense concentrated heat, produced by the electric arc, for melting and fusing the metals to be welded. The electric arc produces the hottest flame known to science and therefore it is uniquely adaptable to the welding art.

That the arc welding process is not new is evidenced by the fact that N. V. Benardos of St. Petersburg (Petrograd) in 1887, secured a patent for the now generally known "carbon electrode process." (Fig. 1.) A few years later Slavianoff introduced a process for casting metals into blow-holes of defective castings by producing an arc between an electrode comprising a metallic rod and the casting. The arc produced melted the rod and heated the casting sufficiently so that the melted metals united. This process is now generally known as the "metallic electrode process." Another process was invented by Dr. H.

Zerener, of Leipsig who produced an arc between two carbon electrodes, the resulting flame being deflected on the metals to be welded, by means of an electromagnet.

Fig. 2 indicates the original scheme of Benardos. It will be observed that he used a storage battery for the source of power. It is undoubtedly true that the slow progress made in the arc welding art during the early years following its introduction was due principally to the non existence of generating equipment suitable for the service.

Fig. 3 indicates a special adaptation by De Meritens who was also one of the first workers in the art. He provided for operating the arc in a comparatively small enclosure or case, the top being of glass of a color suitable for the protection of

BENARDOS PROCESS

ARC DRAWN BETWEEN THE METAL TO BE WELDED WHICH FORMS ONE TERMINAL OF AN ELECTRIC CIRCUIT AND A CARBON ELECTRODE WHICH FORMS THE OTHER TERMINAL.

SLAVIANOFF PROCESS

ARC DRAWN BETWEEN THE METAL TO BE WELDED WHICH FORMS ONE TERMINAL OF AN ELECTRIC CIRCUIT AND A METAL ELECTRODE WHICH FORMS THE OTHER TERMINAL.

ZERENER PROCESS

ARC DRAWN BETWEEN TWO CARBON ELECTRODES THE METAL TO BE WELDED BEING PLACED IN CONTACT WITH THE ARC.

FIG. 1

the operator's eyes; the case being ventilated by means of the passage as illustrated. This arrangement must surely have made the operation of manipulating the electrode rather difficult, and furthermore the collection of burnt gases and smoke must have interfered with the vision of the operator. However, welds free from burning and pitting must have been easily produced, due to the arc and molten metal being surrounded by inert gases produced by the welding operation.

Fig. 4 illustrates a form of the apparatus introduced by Zerener. As will be observed the two carbon electrodes are mounted at the bottom of the apparatus, their axes being at an angle so that the arc may be operated close to the work. Immediately above the tips of the carbons is the winding of the electromagnet, the iron core or circuit of which terminates

in two curved poles located at either side of the arc stream. The magnetic field produced by this electromagnet deflects the arc downward. This apparatus is in reality of historical interest, only, because it is not used today in this country and only in a few factories in Germany.

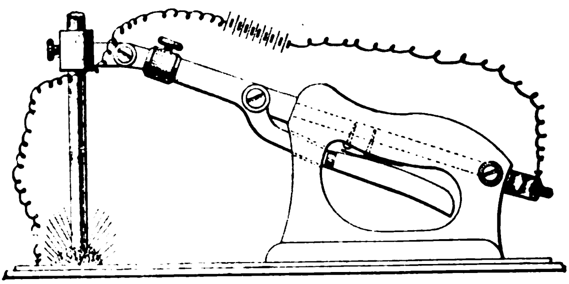


FIG. 2

PRESENT PRACTISE

In present practise the apparatus required may be summarized as follows:

1. A low voltage direct current generator.
2. Switchboard for control of generator and welding circuits.
3. Electrode holders for both carbon and metallic electrodes.
4. Welding materials such as spare metallic and carbon electrodes, and filling materials.

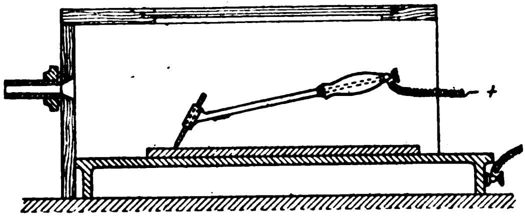


FIG. 3

5. Protective coverings for operators.
6. Moulding blocks, fire clay and preheating furnaces. These are required for special applications only.

Arc welding is inherently a low voltage application of electric power because it is a characteristic of the electric arc that it is most stable when operated at from 20 to 50 volts. The arc, however, is inherently an unstable body having a characteristic which can be best expressed as a negative resistance

coefficient. In other words, as the current increases the voltage drop across the arc decreases, or what is more important, with a fixed voltage across the arc the current increases indefinitely. For this reason therefore, it is necessary to provide a resistance in series with the arc when the source of supply is a constant potential circuit. It is generally customary therefore, to employ a source of supply having a potential of from 60 to 75 volts, d-c. Direct-current sources of 110, 220, 400, 550, and 600 volts can be used with entire satisfaction relative to the stability of the arc, but the power loss in series rheostats in such instances is excessive. As a rule low voltage d-c. sources are not available and therefore, in the majority of instances it is necessary to install suitable generating equipment. Such an equipment will comprise one of the following: (1) a generator only to be driven by an engine or line shaft, (2) a d-c. to d-c. motor-generator set or (3) an a-c. to d-c. motor-generator-set.

Motor-Generator Sets. Fig. 5 illustrates a 1600-ampere, a-c. to d-c. motor-generator set which has been in use daily for over ten years in a large electrical manufacturing plant. Fig. 6 illustrates a 600-ampere, a-c. to d-c. motor-generator set. Fig. 7 illustrates a 600-ampere, d-c. to d-c. motor-generator set.

Generators for welding service are usually compound wound, and should be provided with commutating poles because of the momentary overload produced when the arcs are struck. The capacity of the generating equipment required is determined by the character of welding work to be performed and the number of operators who will work simultaneously. As a rule welding service is of a very intermittent nature, but if the equipment is used for cutting work, such as risers on steel castings, the service may be practically continuous.

Control Equipment. Switchboards and control apparatus for arc welding service have been materially improved within recent years. The first welding plants were one-man outfits arranged so that the welding was performed in an enclosure adjacent to the generator. The successful application of arc welding, however, has increased to such an extent in the industries that it has been found desirable and profitable to expand the one-man outfit into a system permitting a number of operators to work simultaneously at various positions distributed over an extensive area. This development has resulted from a more extensive use of the metallic electrode process,

which has been generally adopted in railway and locomotive shops for many varieties of work.

Fig. 8 illustrates control panels which are typical of present practise. The panel at the right is a combination motor-generator and welding circuit control panel. The panel at the left is for the control of one welding circuit and is generally termed an "Outlet Panel."

The metallic electrode process generally is most satisfactory if the current required is within the limits of 50 to 175 amperes. Some applications however, may require a minimum of 25 amperes and still others a maximum of 225 amperes. For the carbon electrode service satisfactory results cannot be secured with less than 300 amperes. For some applications as much as 500 to 800 amperes may be required, especially if much cutting is to be done at an appreciable speed.

Accessories. Because of the intense heat generated by the arc it is necessary to secure the electrode (carbon or metallic) in a properly designed holder. There is a distinct type of holder designed for each of the two types of electrodes. Such holders are illustrated in Fig. 9 the one above being for a carbon electrode and the one below for a metallic electrode.

The carbon holder comprises an aluminum rod, one end being fitted with a suitable connection for attaching the feeding cable; to the other end a tube is welded, which contains two jaws or clamps for holding the electrode. The portions of the clamps in the tube are tapered and, therefore, the clamps grip the electrode when they are forced into the tube. The cable end of the holder is provided with an insulating, heat-resisting handle and a disk shield to protect the operator not only from electric shock but also to protect his hand from the heat of the arc The electrode is a solid carbon, or more strictly speaking a graphite rod, one inch in diameter and from six to eight inches long. This holder is sufficiently light to be handled easily and is strong mechanically; the current-carrying parts have ample capacity for 500 amperes. For current above 500 amperes, up to and including 800 amperes, a larger holder of similar design is available.

The metallic electrode holder is considerably lighter than the carbon holder and the disk shield is omitted because the heat generated at the arc is much less intense. The electrode is clamped in the holder by a cam device which is designed so as to accommodate electrodes of various sizes.

The intense brightness of the light rays radiated by the electric arc necessitates thcrough protection of all parts of the human body. For this reason if at all practicable it is advisable to perform all welding within an enclosure especially if other persons such as workmen are located within the vinicity of the welding equipment. Furthermore it is essential that the head and hands of each operator and helper be effectively protected. Simply shielding the eyes with colored glasses is not sufficient because the rays emanating from the arc will produce a burning quite similar to and equally as painful as sun burn. The effect will not be noticed at the time of exposure but will manifest itself within a period of ten to twelve hours thereafter. For the protection of the body, ordinary clothing is usually sufficient, but the wrists and hands should be protected by heavy gauntlet gloves. Under no circumstances should the arms be exposed by rolling up sleeves. For the protection of the head and eyes, nothing is superior to the helmet indicated at the left of Fig. 9. This helmet is a cylinder of micarta or aluminum normally held in place by leather straps, and suitably shaped at the lower end so as to rest easily on the shoulders. At the front of the helmet is an offset or recess at the end of which is a frame containing glasses through which the operator observes the work when welding. This frame is held by springs in the position indicated and is designed to be turned up so that the glasses are out of range when the operator desires to inspect welding work performed, thus obviating the necessity of removing the hood so frequently. The glasses are mounted an appreciable distance from the operator's eyes to minimize the heat radiated from the glass to the eyes of the operator.

The glass window of the helmet is of primary importance because it is absolutely essential to protect the eyes against the ultra-violet rays which are very abundant in an arc drawn between carbon and iron or iron and iron. Scientific investigations have revealed the fact that ultra-violet rays will penetrate human tissue to a depth of approximately one inch. These same rays have been successfully employed for the treatment of cancer and similar malignant growths. Recent investigations, however, have indicated that continued exposure of the eyes to the ultra-violet rays will lead to the formation of cataracts resulting ultimately in total blindness. The helmet glasses must, therefore, successfully absorb these harmful rays. The best possible arrangement is that of three layers of glass.

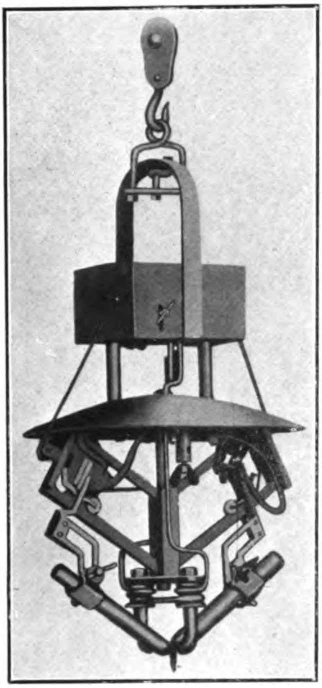


Fig. 4

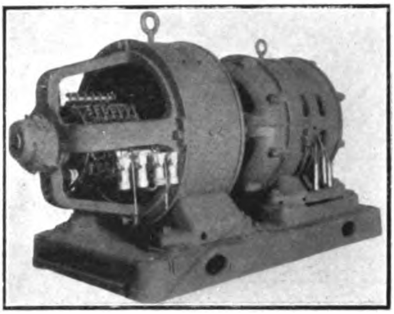


Fig. 6

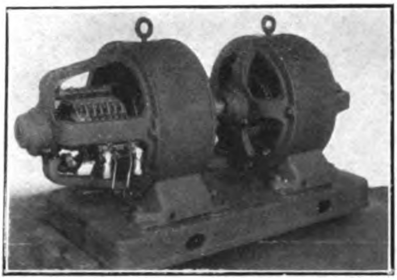


Fig. 7

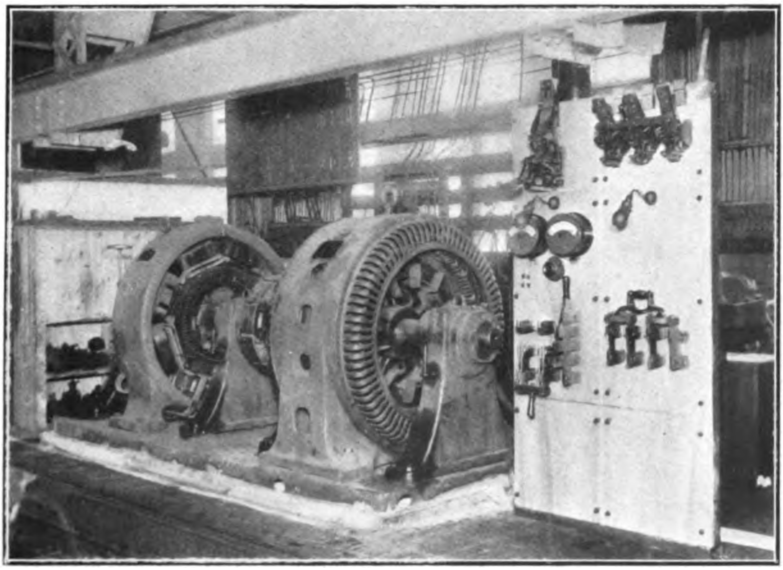


Fig. 5

[CANDY]

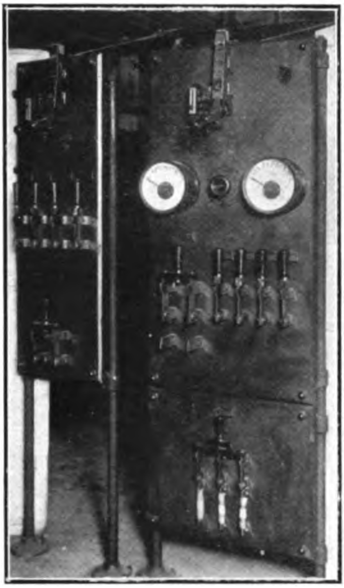


FIG. 8

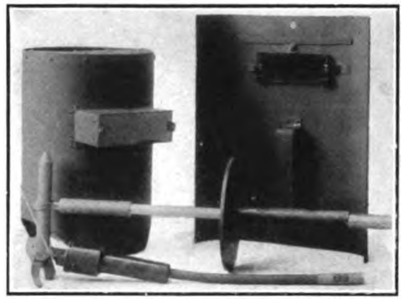
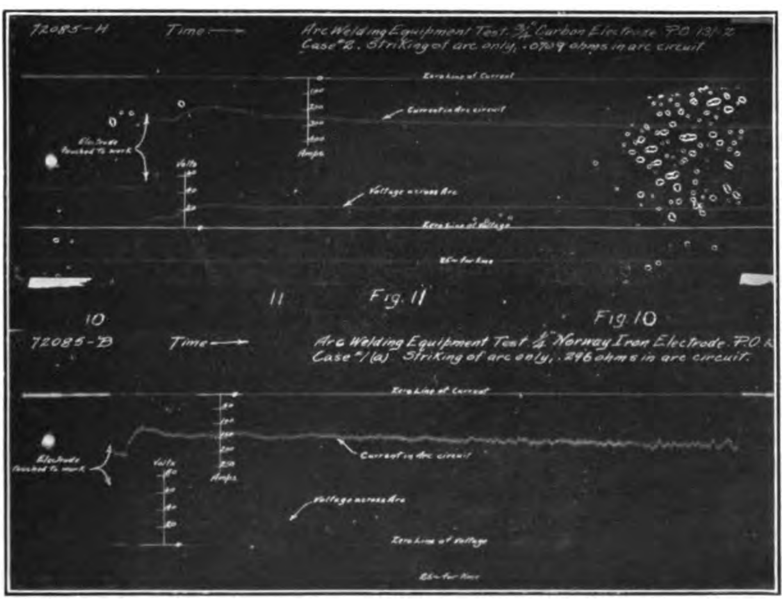


FIG. 9

FIG. 12



FIGS. 10 AND 11 [CANDY]

The outside layer (toward the arc) to be ordinary clear glass. The second and third layers should be of a greenish or amber tint and must be capable of absorbing the ultra-violet rays, as will "Noviweld" glass which has proven very satisfactory.

In many applications of the metallic electrode process the actual periods of welding are short and furthermore the operator has one hand free. It has been found, therefore, that a shield is much more desirable than a hood or mask. Such a shield is similar to the mask. In other words if a mask were flattened out into a sheet and a handle provided it would be practically the same shape as a shield.

Filler Material, etc. The principal materials used for arc welding comprise metallic electrodes, carbon electrodes, and filler material. The most satisfactory metal for general metallic electrode work probably is welding wire which can be readily procured in rods of $^3/_{32}$ to $^1/_4$ inch in diameter and about 14 inches long. Carbon electrodes are usually a special graphite material about $^3/_4$ to one inch in diameter and about 12 inches long. These electrodes are consumed slowly but are subject to breakage and therefore may require replacement frequently. Filler material may be classified the same as metallic electrodes as it serves the same ultimate purpose, namely that of supplying the bond for the piece or pieces welded. As a rule no flux is necessary; however, authorities disagree on this point and, therefore, a flux is used by some operators. The theory of the functioning of the flux is that, if carbon is introduced in the weld from the carbon electrode, it will unite with the oxygen of the flux, thus leaving pure iron in the weld, the carbon having disappeared in a gaseous form, namely that of carbon dioxide. In practise, it produces additional slag which must be removed from the weld.

Execution of Arc Welding

Welding by the electric arc process is an art, which like all arts is not subject to any definitely fixed rules of execution. There are, however, certain fundamental principles which must be observed. The degree of success attained will depend almost wholly upon the skill and intelligence of the operator. Some workmen become experts in a comparatively short time while others experience difficulty and are never able to produce consistently satisfactory results.

The process of welding may for convenience be considered as comprising three distinct operations, namely; (1) prepara-

tion of the work, (2) striking the arc, (3) manipulation of electrode, filling material, and the article or articles being welded.

Preparation of the work. One of the most important factors in producing good welds is the absolute freedom from dirt, grease, scale, etc. of the surfaces to be welded so as to produce good fusion and eliminate slag. A stiff wire brush is very useful for such purposes, although in many instances chipping must be done. A sand blast if available will be found very desirable, especially if much area must be cleaned. Frequently it is permissible or even advisable to clean the work with the carbon electrode arc, no attempt being made to weld until the impurities have been burned off. If necessary, the article to be welded may be tipped so that the molten impurities will flow off readily.

The next step is to properly shape the surfaces on which the weld is to be made. If a crack in a plate or a bar is to be filled in, it should be chipped so that a V-shaped 90 deg. groove is formed. If a casting is to be repaired, temporary supports for the broken member may be required. If a projection such as a lifting bail or lug must be replaced, it will be necessary to use a mould of fire clay, or moulding blocks of carbon or iron. In some instances it may be necessary to preheat the article before welding in order to prevent unequal cooling strains in the metal. For such work a preheating furnace is required. If the piece to be repaired is large it may be necessary to construct a temporary furnace, provided with removable portions so that when the metal is at a cherry red heat the welding electrode may be readily inserted for performing the welding operation.

Striking the Arc. After the piece to be welded is properly prepared it should be connected to one terminal of the welding circuit. If the piece be small the most convenient arrangement is to place it on a metal table or support to which the terminal of the welding circuit is connected. If the piece be large the terminal may be secured to it directly either by means of clamps, or by forcing the terminal into some hole or pocket located in the article to be welded. The circuit breaker and line switch mounted on the panel must then be closed in the order named. The operator then slips the mask into place, and touches the electrode to the work withdrawing it instantly, thus striking the arc. The carbon electrode if possible should be withdrawn at least two inches. Practically no difficulty will be found in striking and maintaining an arc with the carbon electrode. With the metallic electrode, however, considerable difficulty

may be experienced especially at first because the electrode will have a tendency to stick (freeze) to the work. Furthermore, it is not possible to successfully maintain the arc much in excess of $^3/_{16}$ of an inch long. As the electrode melts and forms the weld it is necessary for the operator to feed the electrode toward the work at practically the same speed as it melts in order to maintain the arc and prevent its rupture. The novice or beginner will do well to master the handling of the carbon electrode first.

When an arc is struck there is no arc voltage and the contact voltage drop between the electrode and the work is comparatively small. The flow of current, therefore, is limited only by the resistance and reactance of the circuit including the welding or ballast resistance. Typical fluctuations of the voltage and current are accurately depicted by the curve indicated by Fig. 10. When the metallic electrode was touched to the work the current increased from 0 to about 220 amperes in about $^1/_{60}$ of a second at which value it remained for about $^1/_{20}$ of a second. The current then decreased to about 125 amperes in $^1/_{20}$ of a second after which the average value is somewhere near 140 amperes. Therefore, the entire period of time between striking the arc and obtaining the welding current is about $^3/_{25}$ of a second. A portion of the 25-cycle timing wave is indicated at the lower right corner of the slide. However, a similar curve Fig. 11 illustrating the current and voltage fluctuations incident to striking an arc with a carbon electrode, indicates that the initial current is but little in excess of the normal welding current. This is no doubt due to the high contact resistance between the carbon electrode and the metal.

Manipulation of Arc and Weld

The Carbon Electrode. After striking the arc it should be played over the surface of the piece to be welded by a rotary motion of the hand so as to heat a considerable surface uniformly. When the metal becomes molten, the flux (if any is used) and the filler material should be fed a little at a time. Sufficient filler should be melted and deposited at the proper location to form a satisfactory weld when the metal cools and solidifies. As the metal begins to cool it should if possible be hammered thoroughly to prevent sponginess and produce a finer grain in the finished weld.

The Metallic Electrode. The metallic electrode is at once the

heating electrode and filler material, therefore the arc must be played at the exact spot where the welding is to be done. Otherwise, the manipulation of the weld is the same as specified for the carbon electrode, except that a short 20 volt arc should be maintained.

Polarity

It is essential that the electrode be of the proper polarity, namely negative, for several reasons. First, for carbon electrode work, if the arc is maintained comparatively long, hard welds which are difficult to be machined will not be formed. This is because the direction of the flow of current is from the work to the electrode and therefore the possibility of injecting carbon from the electrode into the weld is minimized. Second, a greater proportion of the heat developed by the arc is generated at the positive terminal, and therefore is generated where it is most desirable, namely at the surface of the metal to be welded.

Carbon Versus Metallic Electrodes

If the weld to be executed is of considerable size and if strength is not of paramount importance, the carbon electrode process will produce satisfactory results in less time than required for the metallic electrode. The carbon electrode is always used for cutting.

If the ultimate strength of the finished weld is an important factor the metallic electrode should be used. It should also be used if it is essential to have the heating of the work localized and restricted to a small area, furthermore, small welds can be better executed with the metallic electrode.

Application for Arc Welding

The field for the application of electric arc welding is a very broad one. Practically every industry making use of iron and steel can utilize the arc welding process to advantage. For instance, in the shops of a large electrical manufacturing plant, both the metallic and carbon electrode processes are used principally for the repair of broken and defective steel castings. In one instance, however, it is used extensively in the regular manufacture of modern motor frames. The frame of such a motor is a slab of open hearth steel, rolled to the form of a ring by a special forging machine. This ring after being tested for size and diameter is allowed to cool and is then sent to the welders who weld together the two ends of the slab, thus forming a continuous ring.

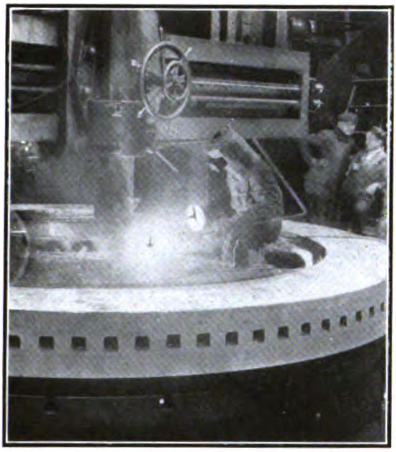


FIG. 13

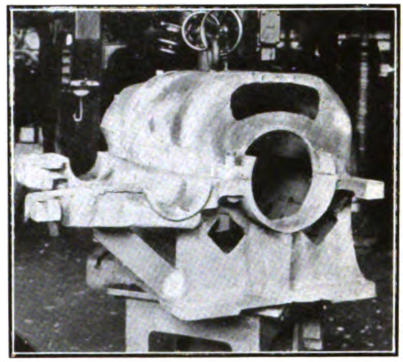


FIG. 14

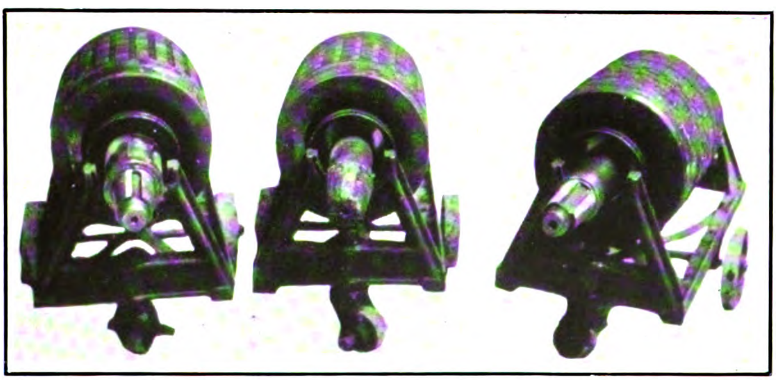


FIG. 15

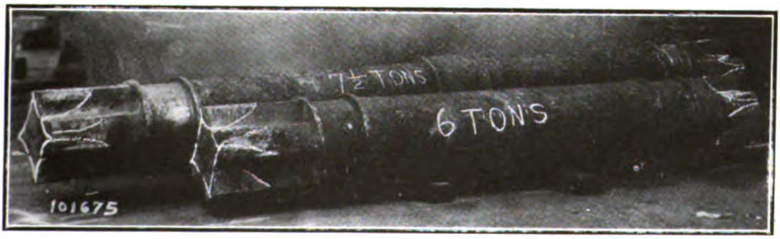


FIG. 16

[CANDY]

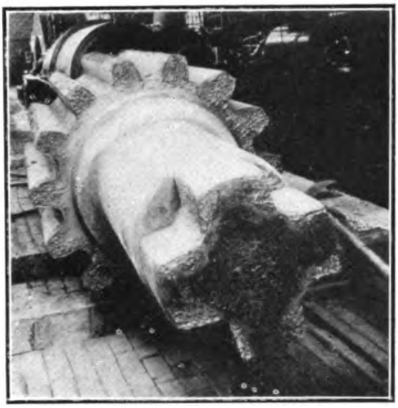


FIG. 17

FIG. 19

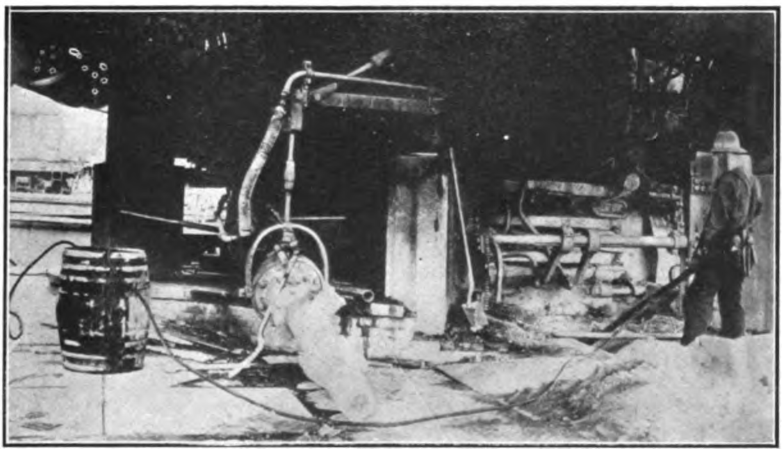


FIG. 18

[CANDY]

In foundries and machine shops, many applications for arc welding equipments may be found, the principal one of which is probably that of cutting risers off castings. The cross sectional area of such risers is frequently quite large but by using a current of 600 to 800 amperes they can be removed readily. Fig. 12 illustrates holes cut with the electric arc. Occasionally, during the operation of machining a casting, blow-holes will be revealed which may seriously impair the strength and appearance of the casting. Without removing the piece from the machine such defects can be readily repaired by arc welding, as is illustrated by Fig. 13. In this instance a large blow-hole was discovered in the hub of the cast steel flywheel. The defect was satisfactorily repaired in 15 minutes. Fig. 14 illustrates a most interesting instance of reclaiming a defective casting through the medium of arc welding. Due to an oversight in a foundry six cast steel frames for large mill motors were cast without the supporting members for the axle brackets. The loss involved was quite an item and therefore it was decided to endeavor to repair the castings by the arc welding process. Supporting pieces of steel were prepared, the ends being welded to the frame and axle brackets as indicated. A very successful job resulted which has been entirely satisfactory.

Street railway systems have many applications for arc welding, such as, repairing broken motor frames, brake rigging, worn rails, and in fact all steel parts subject to wear. Fig. 15 indicates a very common example, namely, that of armature shafts which have been worn excessively. It is practicable to build new material on these shafts by the arc welding process, after which the shafts may be machined to size in the usual manner. This process is considerably less expensive than pressing out the old shaft, which must be scrapped, and pressing in a new shaft. If the armature punchings are built directly on the shaft then the saving is very marked because in many such instances the removal of the shaft means practically rebuilding the armature. Another interesting application is that of building material on worn track frogs and crossings which are subject to severe and rapid local wear at the points, which are hammered by the wheels of passing cars. It is frequently difficult and expensive to replace such pieces of special track work in busy streets of large cities and, therefore, the arc welding process is very commendable as the worn parts can be built up and ground to shape without being removed. There are also

authorities who recommend welding the fish plates or rail joints to the rails instead of the bolted or riveted construction used heretofore.

In steel mills electric arc welding can be used to advantage in many applications. For example, Fig. 16 illustrates two steel mill wobblers, the ends of which have been repaired by the arc welding process; the white lines indicate the new metal deposited. It is interesting to observe that these wobblers are worth approximately $1000 a piece, whereas the cost of repairing by the arc welding process is but $85 each. These wobblers could not be saved in any other manner. Fig. 17 indicates a large pinion which was reclaimed in a similar manner.

Blast furnaces, which are an adjunct of steel mills, present a very interesting application, namely that of burning out the furnace top holes. By using the carbon arc it is possible to burn the cold metal plugging the top hole so the molten metal will flow (See Fig. 18). If the hole becomes clogged with cinder or other non-conductor, it is only necessary to drive a steel bar into the hole until it makes a contact with the molten metal, after which the steel bar may be melted by the carbon arc thus releasing the molten metal. For this service special appliances are necessary. First, a carbon rod several inches in diameter and about four feet long must be inserted in a suitable piece of wrought iron pipe to which a long wooden handle is secured. Second, arrangements must be made for supporting this electrode so that the operator need not work in a position too close to the furnace for safety.

Probably the most extensive field for arc welding, however, is that of steam railway repair shops. All steam locomotives are subject to rapid deterioration, many parts requiring frequent repair or replacement. Not infrequently the frame of a locomotive will crack in service, the usual method of repair being to entirely dismantle the locomotive, the frame being sent to the blacksmith shop for welding. This, of course, is an expensive operation and furthermore the withdrawal of the locomotive from service increases the expense. With the arc welding process such breaks may be repaired without dismantling as is illustrated by Fig. 19.

A locomotive boiler is probably subject to more repairs than any other parts. Due to the continual strain and vibration when in service, the joints and flues are sure to become loose and leak. Fig. 20 indicates two operators in the interior of a locomo-

FIG. 20

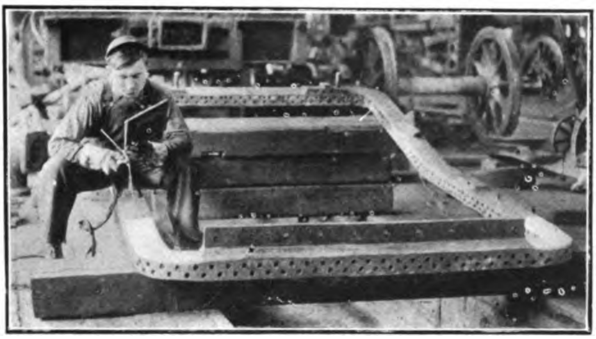


FIG. 21 [CANDY]

tive fire box; the one at the left uses carbon electrode to prepair a seam and the one at the right uses the metallic electrode to weld a crack in the fire box. Neither of the arcs were in operation, however, when the picture was being secured, for obvious reasons. There are other applications also, such as plugging holes in driving box collars, welding spokes of driving wheels, repairing cracks in sides and door sheets of fire boxes, welding bridge in flue sheets, welding guide yokes, repairing mud rings and other parts too numerous to mention. Fig. 21 illustrates an operator who is building new metal on worn and corroded surfaces of a mud ring. The welding of flues in locomotive boilers is probably the most profitable application in steam railway shops. Prior to the introduction of the arc welding process the flues were inserted in the rear fire sheet and the end of the tube allowed to extend beyond the sheet a sufficient distance so that it could be flanged or turned; the tube was also expanded slightly just inside of the sheet. Flues mounted in this manner will hold without severe leaking for quite a time but will gradually become sufficiently loose to necessitate repairs. Experience has demonstrated that the tubes can be welded to the flue sheet by the electric arc process. A number of the large railroads have adopted this process and are obtaining very satisfactory results.

Presented at a Section Meeting of the American Institute of Electrical Engineers, Toronto, Ont., November 16, 1917.

(Subject to final revision for the Transactions.)

CURRENT TRANSFORMER RATIO AND PHASE ERROR BY TEST RING METHOD

BY H. S. BAKER

ABSTRACT OF PAPER

This method of testing may be indicated briefly as follows:

A special current transformer or test ring is connected with its primary and secondary respectively in series with the primary and secondary of the current transformer under test. The number of turns in the secondary coil of the special current transformer is varied until the primary and secondary ampere turns of the special current transformer or test ring are equal to each other and show a balance.

Then the ratio of turns in the special current transformer is the current ratio of the transformer under test.

Of course, the primary and secondary ampere turns can never actually balance, because they are not exactly in step or opposition, but a point can be found where the vector difference between them is at right angles to one of them and this point is taken as the balance point.

This actual minimum value of vector difference gives a measure of the phase difference between primary and reversed secondary amperes.

IN giving a detailed description of the apparatus, the case will be taken of a current transformer in place, operating under normal running conditions, and feeding its actual secondary circuits of ammeters, wattmeters, or whatever it may be.

In Fig. 1 a loop in series with the primary of the current transformers under test is shown, marked $L\ P$, and feeds a heavy current, well-insulated winding on the test ring. The test ring, consists of a laminated iron ring carrying three windings: First, the heavy current winding of one, two, or four turns according to whether series or parallel connections are used. Second, a winding of No. 8 wire in which any number of turns can be used from one to 230. Third, a search-coil winding which feeds the moving coil of a special wattmeter.

The No. 8 wire winding is connected as shown in Fig. 1. in secondary loop $L\ S$, and the direction of flow of current in this winding is such as to oppose the direction of flow in the heavy current winding.

The special wattmeter shown has its field energized from voltage $A\ B$ Fig. 1 or from voltage $C\ B$ as required for the different readings.

Having partly described the apparatus we can now follow the procedure of the test in the case cited. The transformer under test was marked 120 to 1, hence a one-turn connection of heavy current coil, and a No. 8 wire winding of 119 to 121 turns were chosen.

With the No. 8 winding set at 119 turns the field of the wattmeter was energized from voltage $A\ B$, and a reading was taken of plus 295 on the wattmeter. Then with the wattmeter field on $C\ B$ we got a reading of plus 175.

These two readings determined the value and phase of the

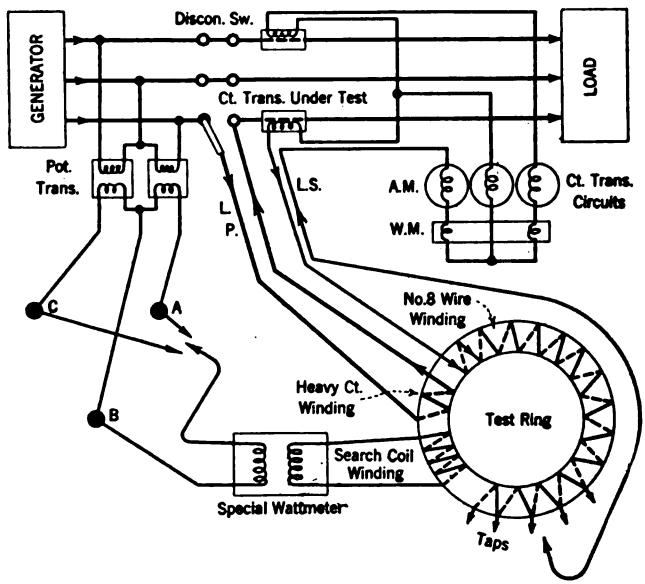


FIG. 1

amperes flowing in the search coil winding, since the wattmeter indicated the components of these search coil amperes along two directions. This search coil current (or ampere turns) give a measure of the vector difference between the ampere turns in the heavy winding and in the No. 8 wire winding.

In Fig. 2, the angle $A\ O\ C$ was set off equal to 60 degrees and the first reading of plus 295 was measured along $O\ A$ to a point R_1 and the perpendicular R_1, P_1 was drawn. Then the second reading (+ 175) was measured along the line $O\ C$ to a point L_1, and the perpendicular L_1, P_1 drawn determining the point P_1. Now it should be evident that the ampere vector $O\ P_1$ represents

the amperes in the search coil winding because this is the only vector whose components along $O\,A$ and $O\,C$ respectively give the wattmeter readings actually read. $O\,P_1$ then is a measure of the vector difference between ampere turns in the heavy current and the No. 8 wire windings when 119 turns of No. 8 wire were used.

We then increased the number of turns of No. 8 wire to 120 thus increasing the ampere turns in the No. 8 wire coil in the ratio of 120 to 119 but without changing the phase direction of these ampere turns.

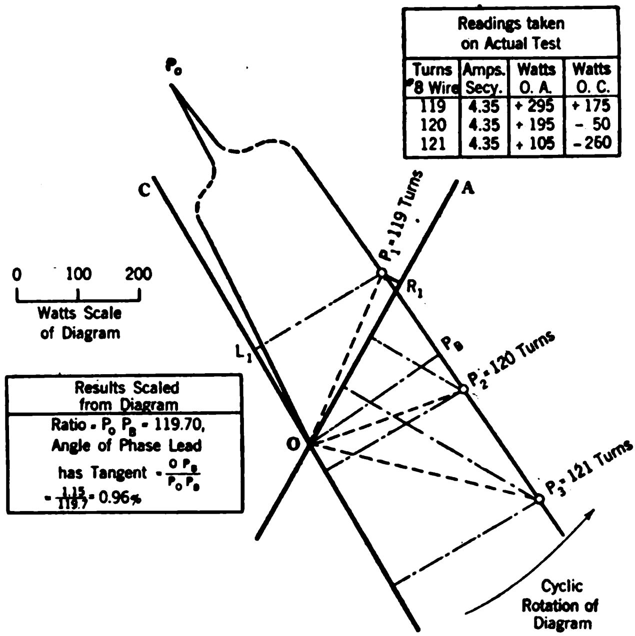


FIG. 2

Then a second pair of readings (+ 195 and − 50) determined the point P_2 for 120 turns of No. 8 wire, and we see the direction of the vector of secondary amperes is along P_1, P_2, because its increment is P_1, P_2. A third pair of readings determine the point P_3 for 121 turns of No. 8 wire. At least three points are generally taken as it adds very little to the time of test and if the points plot out in nearly a straight line the readings are considered consistent.

If we were to extend the line P_3, P_2, P_1 by 119 spaces, each equal to the distance P_2, P_1 we would come to the point P_0 and

the vector $O\,P_0$ would represent the vector difference between the primary ampere turns and the No. 8 wire ampere turns when the No. 8 wire turns were zero.

Hence $O\,P_0$ is the vector of primary ampere turns and $P_0\,P_1$ is the vector of No. 8 wire ampere turns when 119 turns were used. Now to get the point where the heavy current ampere turns and the No. 8 wire ampere turns were equal we might measure a distance $P_0\,P_b$ equal to $P_0\,O$ or for practical purposes drop a perpendicular from the point O upon the line P_1, P_2, P_3. The number of turns represented by the line $P_0\,P_b$ (which scales to be 119.70) divided by the number of turns used in the heavy coil is the ratio of the test ring turns when the ampere turns are equal. Thus we know that test ring ratio of turns = 119.70 at the time when test ring ampere turns are equal.

Hence ampere ratio = 119.70 which is the current ratio of the transformer under test.

The angle $O\,P_0\,P_b$ is the angle between the vector of primary and secondary ampere turns and is the angle of phase error of the current transformer under test. Its tangent is the ratio of $O\,P_b$ to $P_b\,P_0$, and $O\,P_b$ scales to be 1.15 times $P_1\,P_2$.

Hence $\dfrac{O\,P_b}{P_b\,P_0} = \dfrac{1.15}{119.70} = 0.96$ per cent = tangent of angle of secondary lead of transformer under test. All the parts of the diagram Fig. 2 have standard rotation because the reference lines $O\,A$ and $O\,C$ are plotted in keeping with voltage $B\,A$ and $B\,C$ Fig. 1.

The readings for the determination of the above ratio and phase error were taken in about two minutes and the plotting and calculation required about five minutes. Of course the diagram shown Fig. 2 is more complete and complicated than is necessary in actual work.

If the points P_1, P_2, P_3, had not plotted to a straight line the cause would probably have been a change of amperage in the current transformer while the six readings were being taken and the readings could have been repeated. If the amperes can be held for two minutes within a five per cent variation, good results are obtained. Even ten per cent variation permits of results of sufficient accuracy for most commercial requirements. Even on a varying load fed by a number of generators in parallel, a steady load can be delivered by one generator running at a fixed gate opening.

The plotting of the readings and scaling off of results are quickly done by use of a transparent celluloid scale shown in Fig. 3. For instance in plotting the first pair of readings taken, the scaled centre line $x\,y$ of the transparent scale is laid along $O\,A$ (see Fig. 2) and moved till the reading plus 295 falls on O, then a line is ruled along the edge $L\,M$ and this line is the perpendicular $R_1\,P_1$ referred to. Similarly $L_1\,P_1$ is drawn etc. The divided triangle D, E, F is used for scaling off the decimal fraction of a turn represented by the line P_1, P_b, Fig. 2, and for measuring the length of $O\,P_b$ in terms of P_1, P_3. The angle marked 60 degrees is used for laying off the angle $A\ O\ C$ Fig. 2.

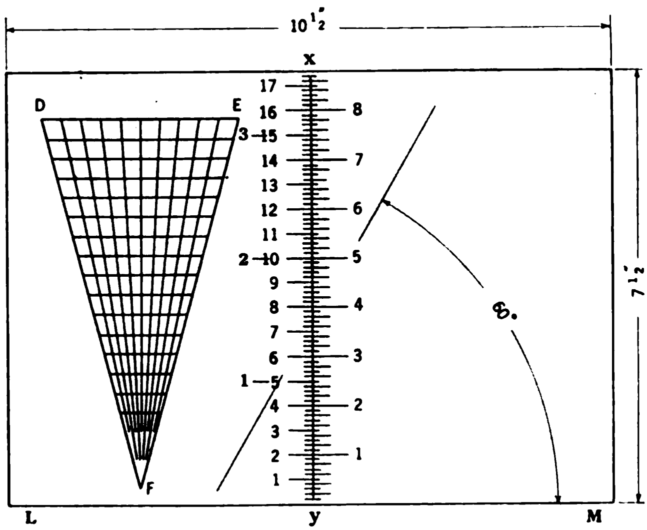


FIG. 3

Current transformers are generally available for test in the laboratory before being installed but the difficulty in this case is in the duplicating in the laboratory of the secondary circuit conditions under which they will operate in service. A very common instance of this difficulty is illustrated in the diagram Fig. 1. Here we have two current transformers feeding three ammeters, and having one common return wire. This center ammeter has a very different effect upon the ratio of one transformer than it has upon the other because the drop across the ammeter terminals is at a very different phase angle from the amperes in the two transformers respectively.

The case of secondaries connected in delta also is difficult to reliably duplicate in the laboratory. However, after making a good "guess" at the amount of resistance and inductance to use in the laborary in the secondary circuit of the transformer under test we can get ratios sufficiently accurate for practical requirements by the following variation upon the method described above.

There are usually two transformers available and they are connected as per Fig. 4. First one is used for a generating transformer to generate the heavy current for test while testing

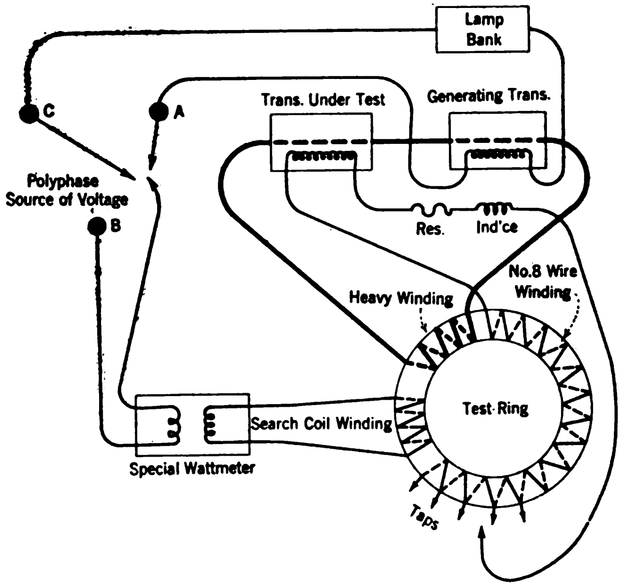


FIG. 4

the other, then the other is used to generate current to test the first one.

The diagram Fig. 4 should explain itself and the procedure of test is the same as described above. Care should be taken to avoid having the two transformers too close together or having a heavy current lead too close to the transformer under test as stray fields affect the ratio of the transformer.

Care should also be taken in cases where a greater accuracy than two-tenths of one per cent is required, to remove the residual magnetism from the transformer under test. At moderate amperages this residual will remain and affect the ratio and

phase errors. This residual can be removed by inserting a resistance in series with the secondary circuit and smoothly cutting it down to zero. If enough resistance is used to raise the secondary volts to say 50 the residual will be effectively removed.

This residual can be caused by the occurrence of even a small pin spark in the secondary circuit which gives momentarily the effect of high secondary resistance and allows the iron of the transformer to become strongly magnetized. On the test ring a switch is provided to avoid this spark on changing taps on No. 8 wire winding.

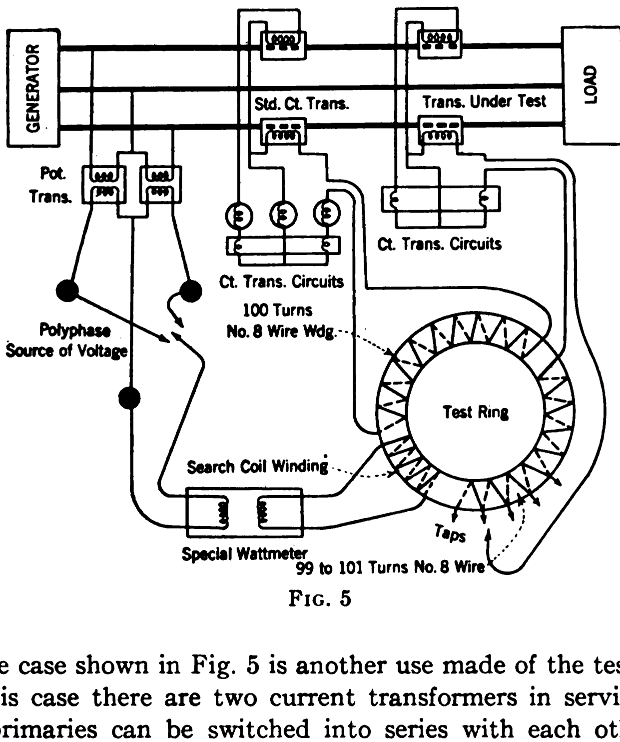


FIG. 5

The case shown in Fig. 5 is another use made of the test ring. In this case there are two current transformers in service and the primaries can be switched into series with each other by some combination of station switching. The ratio and phase error of one is known and the other is tested by comparing the secondary currents in the test ring.

One secondary is fed through 100 turns of the No. 8 wire coil, and the other secondary is fed through a separate section of from 99 to 101 turns. The results obtained will be the difference of ratio and the difference of phase errors between the two current transformers.

The polyphase sources of voltage shown in Figs. 1, 4 and 5 are taken as three-phase. Any other polyphase source may be used provided the angle *A O C* in the plotting diagram Fig. 2 is set off to suit the angle between the two phases of voltage used to feed the field of the special wattmeter. Ordinary unbalance of voltage does not seriously harm the accuracy of results obtained.

The special wattmeter used should have its moving coil wound with a size wire that will not give a resistance many times that of the search coil winding, because it is very desirable to keep the total resistance of the search coil circuit low enough to insure the flux density in the iron of the test ring being exceedingly low. In the test ring described the flux density figured from search coil amperes and turns and resistance figures to less than one hundred lines per square inch for as much as ten ampere turns difference between primary and secondary ampere turns on the ring.

The special wattmeter must have zero mutual inductance between its field and moving coil, hence it must be a zero reading instrument in which the moving and stationary coils have a definite relative position when the reading is being taken. The low resistance required in the moving coil together with the sensitivity required, necessitate a strong field. It is the strong field that necessitates the zero mutual inductance condition. Without zero mutual inductance the wattmeter will act as a transformer sufficiently to effect results. The wattmeter should be sufficiently independent of stray fields to permit of its use in the neighborhood of heavy current conductors.

Returning to the low flux requirement in the test ring, the one hundred lines per square inch referred to figures to a very small voltage (0.04 volt) in say 100 turns of the No. 8 wire winding, and hence the extra resistance added to secondary circuits due to No. 8 wire winding is practically ohmic and is generally a negligible quantity.

The heavy current highly insulated winding on the test ring consists of four U bolts of No. 0000 copper extending up through a fibre face plate and set evenly around the test ring. Special strap connectors are provided to get any series or parallel combination required for one, two or four turns.

The No. 8 wire winding consists of a number of separate windings, each of which is distributed evenly around the ring. There are four coils of 40 turns each, two coils of 20 turns, one coil of ten turns and one coil of 20 turns distributed twice around the ring and having taps at every turn.

The search coil winding is 196 turns of No. 16 copper but for future designs a larger number of turns of finer wire would be used so that the special wattmeter could be more easily designed to fill requirements, and to decrease the percentage effect of stray fields upon the leads between the search coil winding and the moving coil of the special wattmeter, should these not be carefully twisted together.

The test ring iron is laminated, eight inches inside diameter and two by two inches in section. The present ring happens to be made of armature iron which has low iron loss rather than low magnetizing current. In future designs care would be taken to use more suitable iron.

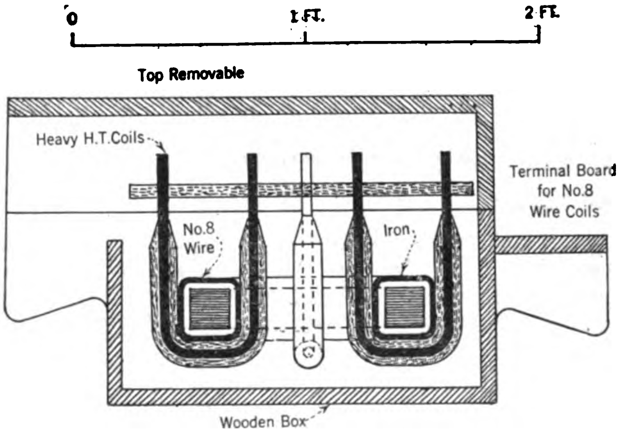


FIG. 6

Fig. 6. shows a section of test ring and its containing box and terminal boards.

In the heavy coil it is not necessary that the heavy current split evenly when parallel circuits are used. A test was made using only one of the four heavy coils and the resultant error was less than two-tenths of one per cent. However, the link connectors used on the heavy coil are proportioned to give practically even splitting of current.

The current in the search coil winding in the present test ring is not nearly in step with the vector difference between the heavy and No. 8 wire ampere turns but it is not necessary that it should be. It can be shown that if there is a constant angular displacement and constant numerical relation to the above vector difference then no error results. The angle and numerical

relation in the present ring are sufficiently constant over the range of one test, as shown by a special test and as shown also by the fact that the points P_1, P_2 and P_3, plot nicely to a straight line.

The following tables show ratio and phase error reports on two current transformers by the Washington Bureau of Standards and by the test ring method. It will be noted that the greatest percentage difference between reports for similar points is seven-hundredths of one per cent for points at two amperes or over. The greatest difference in phase error reports is four minutes of angle over the same range.

Ratios of current transformer No. 50259, 500/5.						
Secondary amperes	5	4	3	2	1	0.5
Washington	99.07	99.15	99.23	99.32	99.45	99.55
Test ring	99.14	99.20	99.27	99.32	99.70	99.50
Ratios of current transformer No. 47113, 500/5						
Secondary amperes	5	4	3	2	1	0.5
Washington	98.87	98.91	98.95	98.95	99.00	99.05
Test ring	98.90	98.92	98.92	98.96	98.98	99.10
Phase errors in minutes (leading) No. 50259, 500/5						
Secondary amperes	5	4	3	2	1	0.5
Washington	32	38	46	57	82	106
Test ring	32	34	43	56	98	125
Phase errors in minutes (leading) No. 47113, 500/5						
Secondary amperes	5	4	3	2	1	0.5
Washington	25	29	35	43	57	73
Test ring	25	27	33	44	65	83

This method of testing the ratio of current transformers has been in commercial use for seven years by the Ontario Power Company and was described in the *Electrical World* of January 26, 1911. At that time a wattmeter was used which did not have specially low resistance in the coil fed from the search coil winding. The consequence was that the iron in the test ring did not run at an extremely low magnetic density and the test ring itself formed a part of the secondary circuit of the transformer under test which could not be neglected and had to be

allowed for in duplicating running conditions in the laboratory. The article referred to describes also a method of determining secondary volts of the transformer under test, but this is not found necessary when the amount of resistance and inductance fed by the transformer under test is known to be substantially correct.

When testing the ratio of current transformers in service as in Fig. 1, if the current varies so badly that the points in Fig. 2 do not plot to a straight line, then it may be necessary to watch an ammeter and take the readings of the special wattmeter only when the amperes are of the value desired. If an ammeter has to be inserted into the secondary circuit especially for this purpose it may be desirable to make a small correction to the ratio and phase errors obtained to allow for the extra ammeter being fed by the transformer under test. A slight correction can also be made, where the greatest accuracy is required, for the ohmic resistance of the coil of No. 8 wire on the test ring itself. This resistance is of the order of 0.06 ohm and the inductive effect of the No. 8 wire is negligible as shown above.

If the polyphase source of voltage used to feed the field of the special wattmeter is seriously unbalanced, the readings of this wattmeter can be corrected before plotting. If the angle between the voltages used is other than 60 deg. then the angle *A O C* Fig. 2 can be plotted to suit.

Presented at a Section Meeting of the American Institute of Electrical Engineers, Philadelphia, Pa., September 16, 1918.

ELECTRIC WELDING—A NEW INDUSTRY

BY H. A. HORNOR

ABSTRACT OF PAPER

This paper covers a brief review of the uses of electric spot and arc welding in this country prior to the formation of the Electric Welding Committee of the Emergency Fleet Corporation. It compares the status of the art at the present time and emphasizes the developments that have been made in apparatus in the last six months. It treats of the activities of the Welding Committee in applying electric welding processes to the shipbuilding industry and points the way to the general applicability of this method to other industries. It shows that the results obtained by investigation and physical tests prove that the applications of this process to heavy work are satisfactory.

INTRODUCTION

ABOUT a year ago the Chairman of the Standards Committee of the Institute was requested to investigate and standardize spot welders and the apparatus connected with them. It occurred to the members of this committee that electric welding could perform an important function in increasing the progress of steel ship construction. The work which was started by the Standards Committee was then transferred to the General Engineering Committee of the Council of National Defense. Last Winter the Council of National Defense abolished all advisory committees but at this time the Emergency Fleet Corporation of the U. S. Shipping Board had become so much interested in the subject that they decided to adopt the Committee. The Committee is composed of representatives covering broadly the whole field of welding activities in this country and, although electric welding has been the subject of all the investigations up to the present time, it is now proposed to include gas welding with representatives from all the gas welding associations and companies connected with this industry.

The two main processes of electric welding, namely, arc welding and spot welding, were found by this committee applied in the first case to repairs and in the second case to certain factory quantity production jobs. The work done was in the case of spot welding only on light material, and in neither case very extensive. The processes to be successful in their application

to the construction of merchant vessels would have to show reliability in the joining of steel plates from a half-inch to one inch in thickness. To this and kindred problems the committee immediately turned its attention.

The work had all been done in the field where it had been applied by practical men. It was first necessary to formulate the proper nomenclature and symbols. This was thoroughly investigated and a very comprehensive set of symbols has been approved by the committee and is in daily use by those now actively engaged in this new application. The approved nomenclature introduces the subject to the designing and calculating engineer and gives him the instrument by means of which he is able to place his thoughts rapidly and conveniently on drawings.

The manufacturers of apparatus joined the practical man in the study of the problems of electric welding. Apparatus and so-called processes introduced various types of machines suitable for the conversion of electrical supply to the proper values of current and voltage needed at the arc or at the spot. The manufacturer in his eagerness to meet the problem naturally encountered many difficulties. These difficulties increased until a point was reached as referred to above where he demanded some standards upon which his apparatus could clearly be rated. Therefore, the manufacturer was only too pleased to co-operate with the Welding Committee and is today conscientiously aiding in straightening out the difficulties in which he was involved prior to last year.

Arc welding in this country has largely been done in the railroad repair shops. It was discovered that the process was much cheaper and could be performed more rapidly than by any of the gas welding methods. It also could be applied without preheating and in many cases without the expense of disassembling complicated pieces of machinery. Spot welding besides being used in many different industries was sought for by the railroad man and there has been built a gondola car which has seen some seven or eight years of service. It is interesting to note here the difference in practise between Great Britain and the United States. The former knowing little or nothing about spot welding had the practise and application of arc welding very well under way; the latter exactly the reverse.

Apparently the attempts to train operators were rather crude and it was early observed that the reliability of the electric weld depended substantially upon the skill of the welder. The manu-

facturers of apparatus and the superintendents in railway shops had struggled with the problem of training operators but intensive study had not been given the subject so that there existed in this respect a great deal of groping in the dark.

PRESENT STATUS OF ELECTRIC WELDING

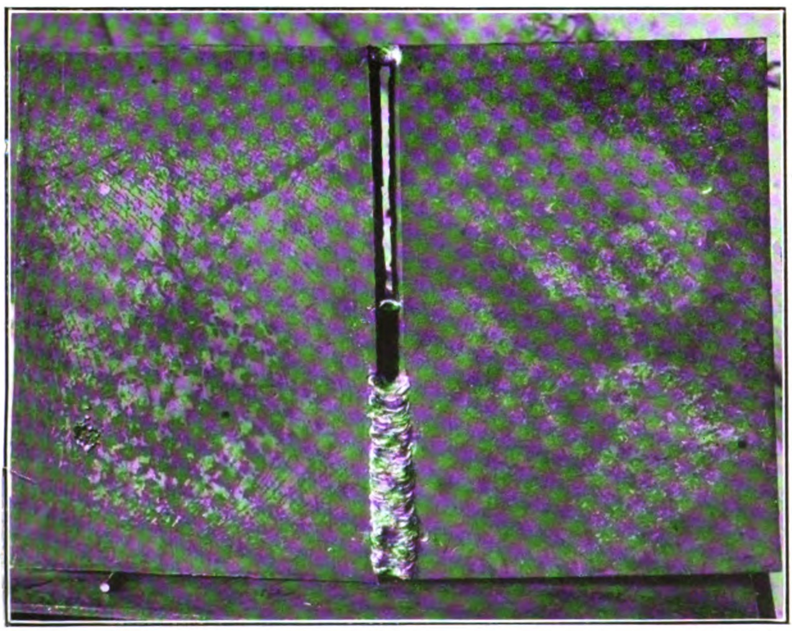
Investigations were immediately undertaken to answer the question whether spot welding could be successfully accomplished using one-inch thick steel plates. An experimental apparatus of large size was erected and put into operation, the results showing that no difficulty was encountered with half-inch and three-quarter inch plates. The same remark applies to one-inch steel plates. In fact, this experimental machine was successful in welding three thicknesses of one-inch plate, a condition which far exceeds the requirements of merchant ship construction. This operation has its historical significance in that this was the first time that any spot welding of this magnitude had been performed. The successful outcome of these experiments has led to the design and construction of large spot welders to be used in the fabrication of ship sections. The practical application of a large five-foot gap spot welder will be made at a demonstration of a forty-foot section of a standard 9600-ton ship to be built at the plant of the Federal Shipbuilding Company, Kearney, New Jersey. This is the largest portable spot welder ever built. It will prove two points in ship construction by the electric method, namely, the clamping of the ship's structural parts for assembly thereby reducing the time in working the material as well as for the erection of the ship material; and secondly, by the speed of spot welding it will prove the decrease in time for joining the material together. The consensus of opinion is that the large stationery spot welder of five- or six-foot gap will undoubtedly play an important part in increasing the speed of fabricating sections of standard steel vessels. Further investigations are being made and designs are being worked out for special spot welders for use in the construction of bulkheads. The designs proposed are chiefly for shop processes, but it can be asserted that such apparatus will be of undoubted value in the saving of time and man power.

Arc welding had been tried in a great variety of work but there was no conclusive evidence that it could be developed to the stage of joining ship plates with the certainty of full strength. The first stage of this investigational work is now

almost completed. Sample welds of half inch ship structural steel were taken by a special sub-committee to fourteen or fifteen different places where electric welding was being performed. This sub-committee saw the welding done, noted the conditions of current, voltage, electrode, operator, etc., and then prepared the welded samples for tests. The samples were forwarded to the Bureau of Standards in Washington so that the tests should be conducted by parties absolutely disinterested and without knowledge of how the samples were obtained. The results of these tests showed a remarkable similarity especially when it is realized that they were made by several firms with different electrode materials and under varying conditions of the electrical circuit. Practically all of the welds pulled at over 50,000 pounds per square inch and several over 60,000 pounds the average being about 58,000. On the bending test one of the samples was bent to an angle of 78 degrees before a crack started and final failure reached 80 degrees. In another case the sample was bent to 65 degrees before the crack started and final failure did not occur until 86 degrees. The point of importance here is that all the welds showed a reliability and satisfactoriness which makes conclusive the opinion that electric arc welding is applicable for the joining of steel where the structure is submitted to live loads, bending strains, static pressure, or the like. The Sub-committee on Research is pursuing this subject and practical samples are being prepared for similar tests using three-quarter and one-inch stock material. The results of these tests will be available as soon as the reports are presented and approved by the Welding Committee. The Research Committee is also preparing various types of joints in heavy plating. These will be submitted to all the regulation tests and in addition to shock and fatigue tests and tests to destruction.

To give a further indication of the large size practical tests which are being carried on at the present time it may be stated that three 12-foot cube electrically welded tanks are now being constructed. These tanks are built in such a way that from twelve to fifteen different designs of joints are used in their construction. After these tanks are built they will be subjected to a static strain and the deflection of the seams will be directly measured. Afterwards they will be tested by external shock and crushed to destruction. Portions of the joints will be cut, sent to the Bureau of Standards, and again tested for the sake

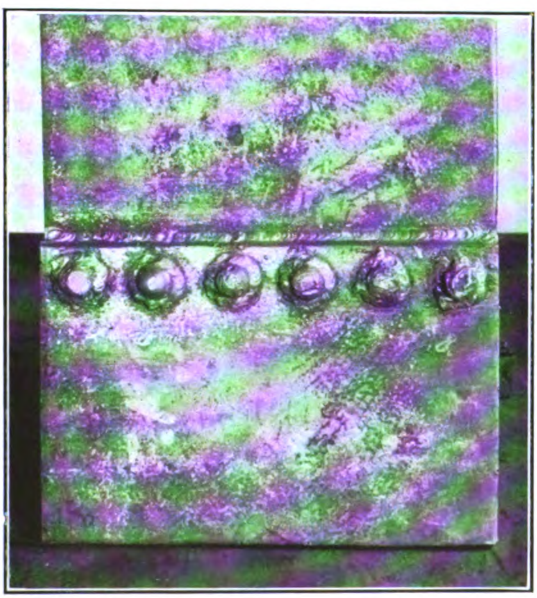
Sample Showing Indifferent and Good Built Up Welds



[HORNOR]

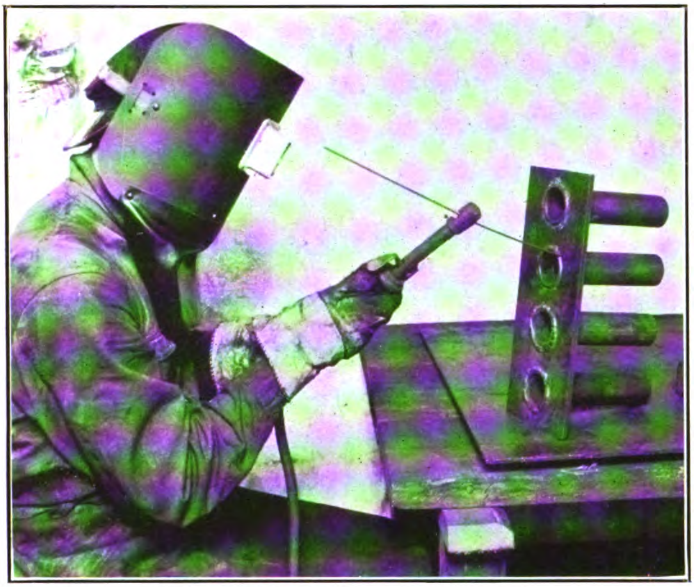
Partially Completed Arc Welded Seam—Notice Tack Welds to Hold Plates Together

ARC WELDING IN THREE DIFFERENT POSITIONS—FLAT—VERTICAL AND OVERHEAD



[HORNOR]

ARC WELDING AROUND RIVET HEAD TO PREVENT LEAKAGE

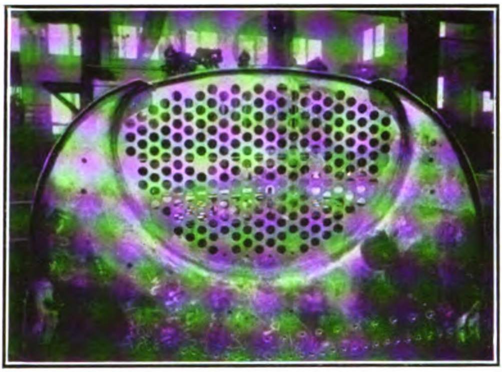


Arc Welding Boiler Flues

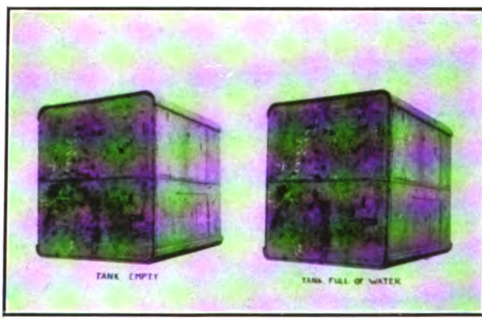
Saving Piston by Building Up with Electric Arc Weld

[HORNOR]

Section of a Smoke Stack—Seams Welded by Electric Arc Welding Process—Metal 1/4 in. Boiler Plate



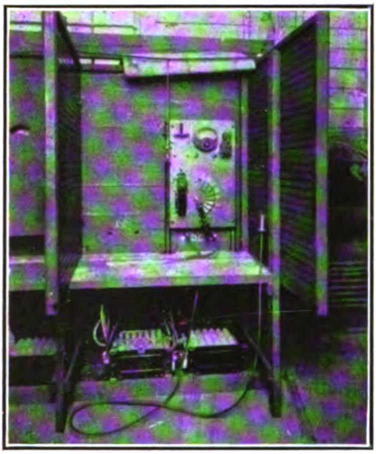
Back Tube Sheet Welded to Head



Twelve Foot Cube Tank all Seams Electrically Welded for Experimental Purposes—Built at Pittsfield, Mass.

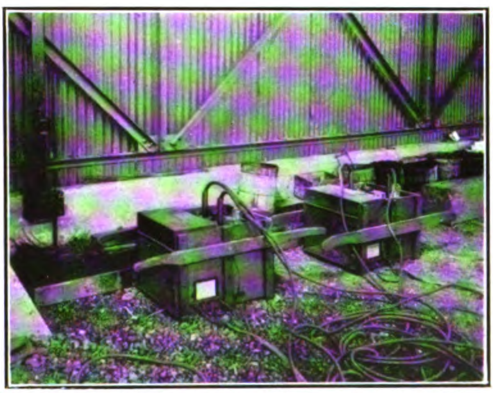
[HORNOR]

School for Electric Arc Welders Established by the Emergency Fleet Corporation at the Plant of the General Electric Company

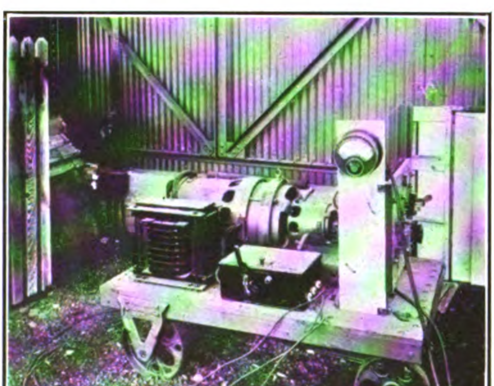


FRONT VIEW WELDING BOOTH—G. E. WELDING SCHOOL

THIRTEEN FOOT MAST ELECTRICALLY WELDED



ONE-MAN A. C. TRANSFORMER OUTFIT FOR ARC WELDING



ONE-MAN PORTABLE D-C. ARC WELDING OUTFIT [HORNOR]

of accumulating precise data. In this connection there is being built at the Norfolk Navy Yard a battle-towing target. The keel of the target 110 ft. long will be entirely electrically welded and the results of this practical demonstration will be carefully recorded after it has been put in regular service.

It is to be expected that the manufacturers of apparatus being keenly observant of the increased interest in electric welding as well as in the future, which is probably now unquestioned, would be active in their desire not only to improve their present facilities and their design of apparatus but also to proceed themselves to follow the trend of the investigations made by the Welding Committee. The consequence of this has been a large increase in output of apparatus and it may be unhesitatingly stated that there are no difficulties in the way of obtaining all the electrical welding apparatus that is needed. One interesting point is that certain manufacturers who were decidedly of the opinion that direct current was the only proper current to use for arc welding have within a very recent period changed their point of view and are willing to admit that alternating current may have certain advantages in the development of this art.

The electric arc requires a reduced voltage and this is difficult to attain with direct current without relatively expensive machines or a useless expenditure of energy. The practise in this country in manufacturing establishments of any size has been toward an increase in the supply voltage so that very few large manufacturing plants use less than 220 volts direct current. With this voltage the only economical method of transformation is in the use of a motor-generator set. The efficiency in this case is in the neighborhood of 50 to 60 per cent. It is possible to use a supply voltage of 110 volts with a variable resistance which cuts down the voltage to the arc volts. This gives a very poor efficiency. In the case of alternating current the supply voltage can be reduced by a transformer which will supply as in the case of direct current a sufficient voltage for striking the arc and a satisfactory reduction when the arc has been struck. On the other hand, if a low voltage alternating current is provided a simple reactance may be introduced which has some of the same wasteful characteristics of the resistance used with the direct current. The average apparatus will permit of electric arc welding consuming about six to eight kilowatts per welder but if low voltage is provided there are

certain outfits which will reduce the consumption as low as three and one-half kilowatts per welder, or even less.

Without entering into an elaborate analysis of the relative costs of electric welding, it may be broadly stated that there is hardly any question that the electric process is cheaper than any other. The same may be said as regards speed and also reduction of man power. In a recent discussion of this subject President Adams stated that at one of the Eastern shipyards the total number of parts on the welding program of the standard riveted ships now building at that yard amounted to 225,000. The labor cost for riveting these pieces is about 245,000 dollars and for welding about 99,000 dollars making a saving of 146,000 dollars. But this is only a drop in the bucket when compared to what might be profitably done in this line. He stated further that in certain particular instances the saving is as great as 90 per cent.

One of the interesting questions discussed with some fervor by the members of the Welding Committee is the advantages of the bare and covered electrode. Regarding this discussion no definite facts can be stated. In England the practise has been to use the covered electrode which protects the welding arc from contact with the air thus guarding against too great a formation of oxide. The practise in the United States up to the present time has been largely bare wire. Recently, American investigators have discovered the important fact that there are advantages in the covered electrode and many experiments are now being made, some with results. It is important to observe that in the above mentioned tests of welds, the best one of these samples was made with a coated (not an asbestos covered) electrode using alternating current. The point in this case seems to rest upon the question of the ductility of the weld and it would seem that the bare electrode does not make as ductile a weld or at least one as easily bent as the coated or covered electrode. The question of the ductility of the weld is one of much importance in the application to ship construction and will doubtless be of importance in other allied industries. It is, therefore, a question of serious importance and constitutes an important part of the work of the Sub-committee on Research.

No matter what the type of electrode is nor its composition, no matter what kind of shank material is to be welded, no matter what kind of apparatus is employed, the reliability of

the weld rests mainly upon the man who makes it. This man if he has been properly trained and is skilled in the art knows instantly whether he is making a weld or not. He becomes after much practise able to judge fairly well upon looking on a finished weld whether it is a good weld or not. The work of training electric welding operators early became a part of the functions of the Education and Training Section of the Emergency Fleet Corporation. The men connected with this work are members of the Welding Committee. Schools for the training of operators as well as for the conversion of operators into instructors, are established in many parts of the country. The objects held in view by the training department are first to give the man intensive practise work so that he becomes a good craftsman. The methods are simple to start with, as the exercise of the right arm muscles must become flexible enough to permit the operator to give the required movement to the electrode. By a graduated series of exercises this is accomplished in about eight weeks. The man is allowed to do production jobs in the shop which gives him confidence through responsibility. It becomes desirable at this time to give the man some outside work on ships and where this is practicable it is done. The man is then turned over to an instructor who gives him an intensive course in pedagogics lasting from five to six weeks. At first sight it would not seem necessary to so instruct a man but it is not generally understood that teaching after all is itself a trade. The experience with the men in this respect is most interesting. In nearly every case the man has resented this course at the start but at the end has turned completely around and in many cases has desired an even more extensive training. What is really accomplished is to give the man the necessary confidence to impart the knowledge that he has gained to another green man. The men under training are taken from the various industries especially the shipbuilding industry and after they have finished their instructor training course are returned to their employer to carry on the instruction, in their own plant. The men who go through this training as provided by the Emergency Fleet Corporation are certificated when they have shown themselves to be entirely proficient. It is not possible nor expedient for the Emergency Fleet Corporation to require the certification of all electric welders. It is the consensus of opinion that all industries doing serious work with the electric arc should use men who are certified as to their ability in the art

of electric welding. The main reason for this opinion is that the operator must be a conscientious workman or the weld will not be of perfect quality.

This brings forward another problem upon which a great deal of experimental work has been and probably will continue to be done, namely, a practical and scientific method of testing a welded joint after it has been made. There have been a number of suggestions made for the solution of this problem. They are briefly, as follows:

(a) Mechanical. By hammering the weld or by chipping at frequent intervals.

(b) Electric. By means of resistance or voltage drop.

(c) Magnetic. By means of the permeometer or the change of conditions of the magnetic circuit.

(d) X-ray. By means of an exposure on an X-ray plate.

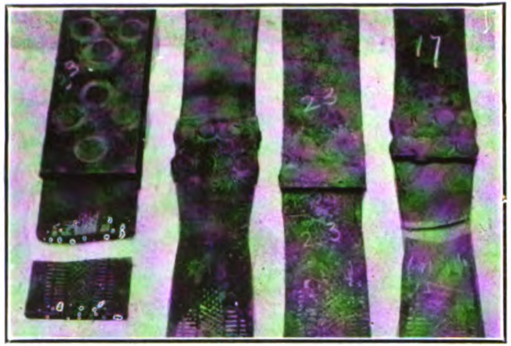
At the present time none of these suggested methods have been productive of conclusive results and recourse must be had to the purely mechanical methods of striking heavy blows on, or adjacent to, the weld or by using a chipping hammer and making intermittent examinations. It would seem by far the best procedure to make the inspector proficient in the art so that he may closely observe the welders while at work. This may be accomplished by a two or three weeks attendance of nspectors at any one of the electric welding training centers.

Methods of Electric Welding

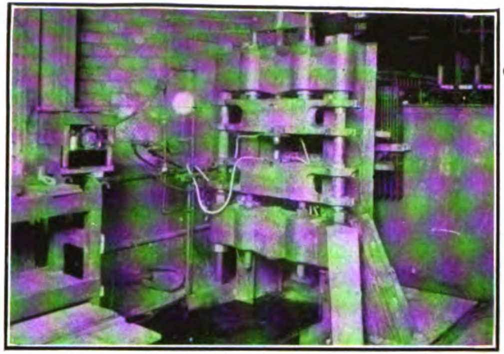
There are many methods and processes of electric welding but the two main ones that interest the committee at the present time and alone have been mentioned so far are the spot welding and arc welding. It may be a surprise to some of the old time welders to consider electric welding as a new industry. In substantiation of this statement it may be well to describe briefly what is meant by electric welding as it is practised today.

Spot welding is not much different in the methods of procedure or in design of apparatus as when it was first introduced. Copper electrodes, water-cooled in the heaviest machines, are placed on opposite sides of the material to be welded together. The joint is a lap joint. Machines are now so designed that two spot welds may be made at one time. The routine of the operation is as follows:

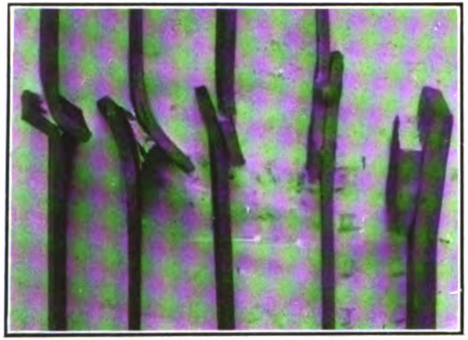
The electrodes are brought into contact with the materials to be joined, current is supplied sufficient to give the required heat, pressure



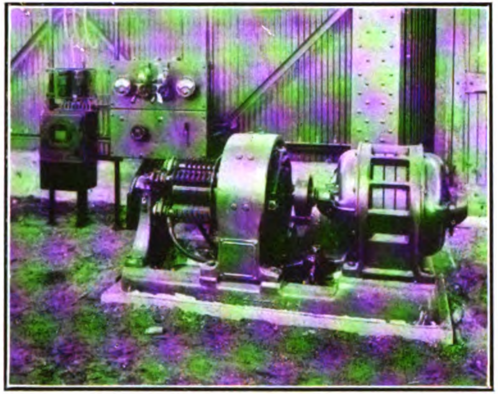
ELCTRICALLY WELDED SAMPLES—FAILURES IN TENSION OCCURRING AWAY FROM WELD—STEEL PLATES 3/4 IN. BY 7 IN. AND 1/2 IN. BY 6 IN.



MACHINE FOR EXPERIMENTAL WELDING FOR HEAVY SYEEL BY ELECTRICITY—CURRENT CAPACITY 100,000 AMPERES— PRESSURE CAPACITY 36 TONS

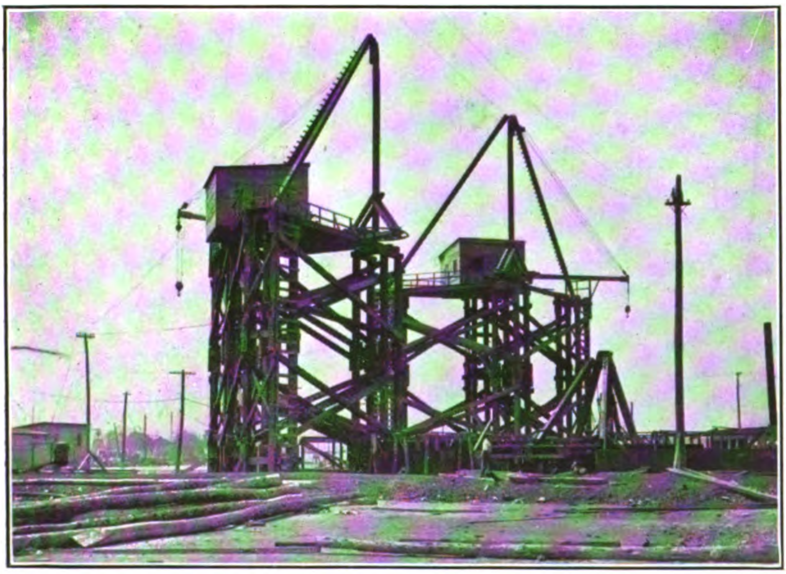


ELECTRICALLY WELDED STEEL PLATES 1/2 INCH AND 1 INCH THICK AFTER BEING TESTED IN TENSION—SHOWING EFFECT OF COMPOUND STRESSES DUE TO EFFECT IN LINE OF TENSION CAUSED BY THE LAPPING OF THE POLES



DIRECT-CURRENT MOTOR GENERATOR SET FOR SUPPLYING 8 ARC WELDERS

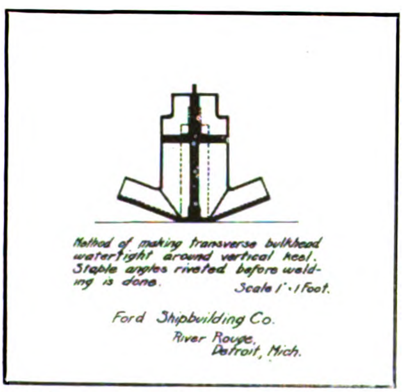
STRUCTURAL DRAWINGS OF EXPERIMENTAL SECTION OF STANDARD SHIP BEING BUILT AT KEARNEY, NEW JERSEY, UNDER THE DIRECTION OF MR. A. J. MASON



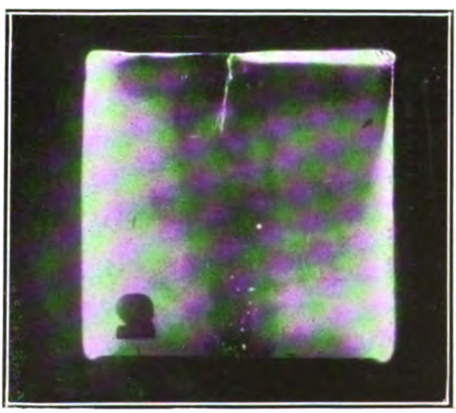
[HORNOR]

ARRANGEMENTS AT KEARNEY, NEW JERSEY, FOR THE BUILDING OF EXPERIMENTAL SECTION OF STANDARD SHIP

ARRANGEMENTS AT KEARNEY, N. J., FOR THE BUILDING OF EXPERIMENTAL SECTION OF STANDARD SHIP—NEAR VIEW SHOWING KEEL BLOCKS

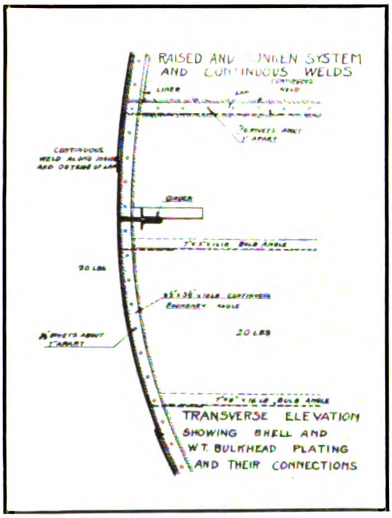


METHOD OF MAKING TRANSVERSE BULKHEAD WATERTIGHT AROUND VERTICAL KEEL—STAPLE ANGLES RIVETED BEFORE WELDING IS DONE

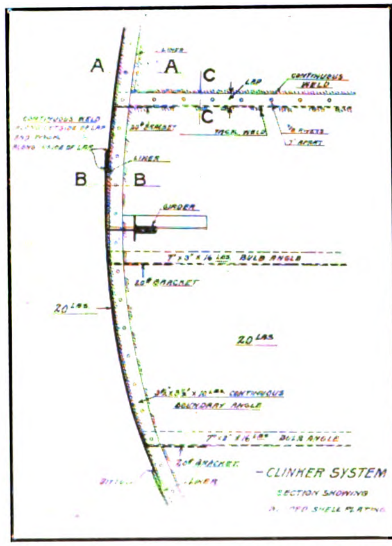


[HORNOR]

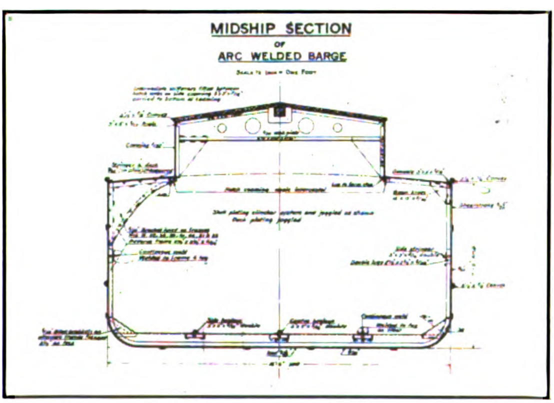
X-RAY PHOTOGRAPH OF WELD



Raised and Sunken System and Continuous Welds



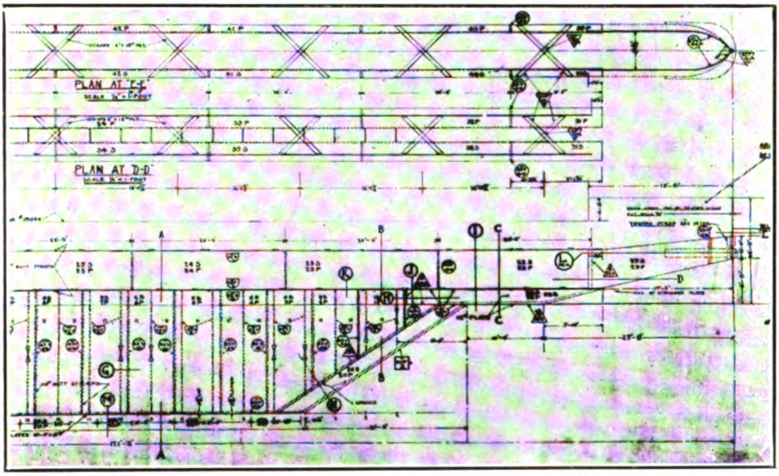
Clinker System—Section Showing Welded Shell Plating



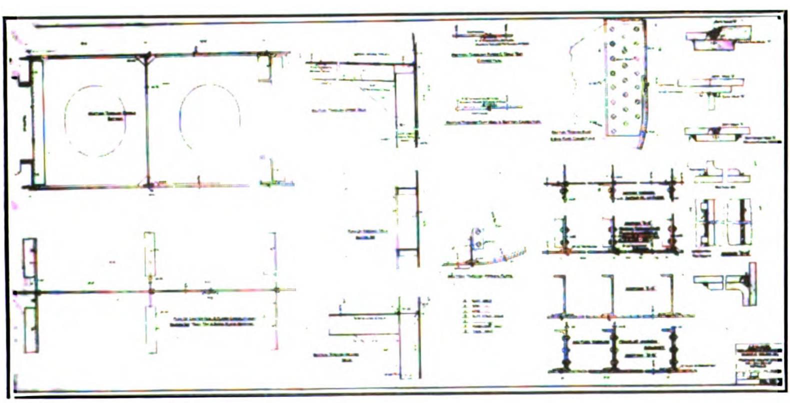
Midship Section of Arc Welded Barge

[Hornor]

TOTALLY ELECTRICALLY WELDED BARGE BUILT IN ENGLAND



PORTION OF DRAWING FOR KEEL OF BATTLE TOWING TARGET SHOWING THE USE OF APPROVED SYMBOLS AND THE SIMPLICITY OF THE DRAWING



[HORNOR]

DETAILED DESIGN FOR PROPOSED ELECTRICALLY WELDED SHIP

is then applied, the current is removed, and the pressure is removed, the weld is then complete.

The operator has a perfect indication of making a good spot weld by the use of a button placed under the electrode observing which he knows exactly the proper timing of the operation. There is therefore no question as to a good, bad, indifferent, spot weld. Automatic spot welders have been designed and built but it is the general opinion that they add complication to a process which in itself is very simple.

The process of arc welding is as follows:

One side of the electric circuit is connected to the material to be welded, the shank material is usually prepared by beveling the edge of the pieces to be welded together. The other side of the electric circuit is connected to the electrode. The operator is provided with a holder which carries the electrode. By touching the electrode to the shank material the arc is drawn. The skilled operator now moves the electrode from side to side of the groove giving a semi-circular motion while at the same time moving the electrode along the groove.

It is important that the arc "bite" into the shank metal creating a perfect fusion along the edges and the movement of the electrode is necessary for the removal of any mechanical impurities that may be deposited. In the coated electrode it is further necessary that the slag which forms for the protection of the pure metal be worked up to the surface and it is extremely important in the event of a second or third layer that the slag or impurities be carefully scraped away before the virgin metal is again laid on.

The operator in arc welding is protected with either a hand screen covering his face with special glass through which to observe his work. The electric arc emits dangerous invisible rays in both the upper and lower spectrum scale and it is quite evident that both the infra-red and ultra-violet are dangerous in their effect, the former is pathological the latter actinic. The operator further uses gloves for his hands and for the very difficult work of overhead welding it is necessary for him to use a helmet which partly covers his breast.

Developments

The tendency of developments in spot welding has already been slightly touched upon. In their nature as applicable to shipbuilding the advancement will naturally have to proceed toward means for accomplishing spot welding in very cramped locations. This makes an exceedingly difficult problem as the

power requirements are such as to preclude any very small device. In riveting one-half of the apparatus is on one side of the work and the other half on the opposite side and it is difficult to conceive of any method of spot welding that will admit of such an arrangement. In shipbuilding it is quite probable that designs may be made that will permit of a large or at least increased amount of spot welding in the actual construction of the vessel. Certainly, present designs of riveted ships will not allow of this to any great extent. As already stated, spot welding can now take its place in the fabricating shops and it is to be expected that within a few months spot welding will begin to supplant riveting in this field. The only drawback to this will be the sufficient production of spot welding apparatus.

The tendency of development in arc welding is toward the automatic machine to obviate the responsibility that has to be placed upon the skilled operator. Intensive work has been done within the last few months in the line of automatic arc welding machines and at the present time sample tests of welds made by such apparatus have been sent to the Bureau of Standards. These machines will occupy a very important position in repetition work. They will not immediately supersede the skilled operator in repair work, or in special jobs but it may be expected that the development of such machines will bring about apparatus which can be man-handled and will eventually take the place of most of the hand work as it is now known.

Of the scientific advancement in the art of electric welding there is so much to be treated that only a general outline can be considered at this time. The research work has only just begun. Practise has preceded the scientific investigation. The field, therefore, is full of most interesting problems. Those who have been following the development of the past six months are deeply interested to know the fundamental reasons. The investigational questions may be grouped into three main divisions:

1. Metallurgical.
2. Physical.
3. Electrical.

The metallurgist has yet to tell us what the conditions of the metals are after the electrode material has fused with the parent metal, and to determine what the proper conditions must be to produce a good weld. This problem has in it a great many variables. The physicist must explain the atomic or

electronic conditions which permit of the combinations at the high temperatures involved and must explain the phenomenon of overhead welding. The electrical investigator must determine all the various phenomena connected with the preferences between and the advantages of the use of different forms of electrical energy and the varying characteristics of the electric circuit in producing different types of welds.

CONCLUSION

From the preceding remarks it must be conceded that the Welding Committee of the Emergency Fleet Corporation has already crystalized the problems connected with this art. The working functions of this Committee have been laid down upon the broadest possible lines. Liberal opportunity has been given every one to state in detail his opinion and to express the reasons for his preference on every point connected with this subject. The Committee goes even further than this. It furnishes those interested with every new idea that is brought to bear upon the subject after sifting from the suggestions any question of doubt or misstatement of fact. All suggestions of improvement or problems of special application are gladly taken in hand, thoroughly investigated, and reports made. It will welcome any comments that those connected with the industries may desire to lay before it. The personnel is at the present time such that it can devote not one but many minds to the solution of any specific problem that is laid before it.

The Committee early discovered that the literature of electric welding was very much clouded by misstatement of fact or half-baked theory and much of the time of the Committee has been taken up in disproving such statements. In order to spread the results of this work to all quarters a handbook is now being prepared which will contain only definite facts and results of investigations as are approved by the whole Committee. This handbook will be made available to all those who desire to acquaint themselves with the proper means of accomplishing good and reliable electric welding.

To be presented at the 342nd meeting of the American Institute of Electrical Engineers, Philadelphia, Pa., October 11, 1918.

THE USE OF ELECTRIC POWER IN THE MINING OF ANTHRACITE COAL

BY J. B. CRANE

ABSTRACT OF PAPER

This paper gives figures as to the power cost and current consumption of anthracite mines and the reasons for these being in excess of the requirements bituminous mines.

Estimates are also given as to the additional coal that will be released by the electrification of the anthracite mines.

Illustrations are included showing representative installations of electric drive.

THE production of anthracite coal in the United States for the past five years was as follows:

Years	Total Gross Tons
1913	81,809,782
1914	81,580,479
1915	79,803,374
1916	78,406,387
1917	89,720,982

The undeveloped beds are to such a large extent owned or controlled by the large producers that no extensive opening of new mines has taken place in recent years nor is to be expected in the near future.

In the bituminous fields there are many independent holdings, and when the coal business was flourishing many new companies were formed and opened up new mines, using electric drive, mainly on account of lower first cost for installation of machinery. The resulting economies were such as to force the former steam-operated mines to electrify even where it was necessary to throw away expensive steam equipment.

Coal producers are conservative and not having the many examples of successfully operated electrified mines before them, the anthracite operators have been slower in seeing the benefits to be derived and the economies to be effected by throwing out their steam-operated and replacing it with electrically-operated

Manuscript of this paper was received August 12, 1918.

equipment. The margin of profit, due to the better organization of the producers, was also higher and the necessity of saving in the cost of production had not been brought home to them with the same force. For these reasons the possibilities of making a greater progress, in the decrease in amount of coal burned for power, in increased production of coal, in decrease in amount of labor and in the general living conditions for the employees, exists to a greater extent in the anthracite than in the bituminous coal regions.

The anthracite operators, are at the present time keenly alive to this fact and are making every effort to take advantage of the improvements to be made by electrification of their mines, but are handicapped by the difficulty of securing electrical equipment and by the impossibility of securing sufficient power from the central stations supplying power to the anthracite regions.

The mines that have been electrified show some surprising results. The following figures are taken from a paper read before the A. I. M. E. (Economy of Electricity over steam for Power Purposes in and about Mines, by R. E. Hobart, Feb. 18th, 1918.)

	Steam Operation April 1914 to April 1915	Electrical Operation Nov. 1916 to Nov. 1917
Cost of Power	$46,992	$21,590
Cost of Heating	Included in above	8,700
Total	$46,992	$30,290
Tons of coal mined	343,665	435,073
Cost per ton	$0.137	$0.0696

When it is remembered that the value of the coal burned with steam operation has more than doubled and that the output of coal per man per year employed has increased from 540 tons to 647 tons it will be seen the figures above, large as they are, represent only part of the saving to be effected.

Some other figures obtained are interesting as showing costs of electrification, power used, etc.

Item	Mine No. 1	Mine No. 2
Gross tons-yearly	641,533	670,000
Kw-hr. consumption	2,312,195	3,477,876
Kw. demand	870	1,135
Annual L. F.	30.4	35

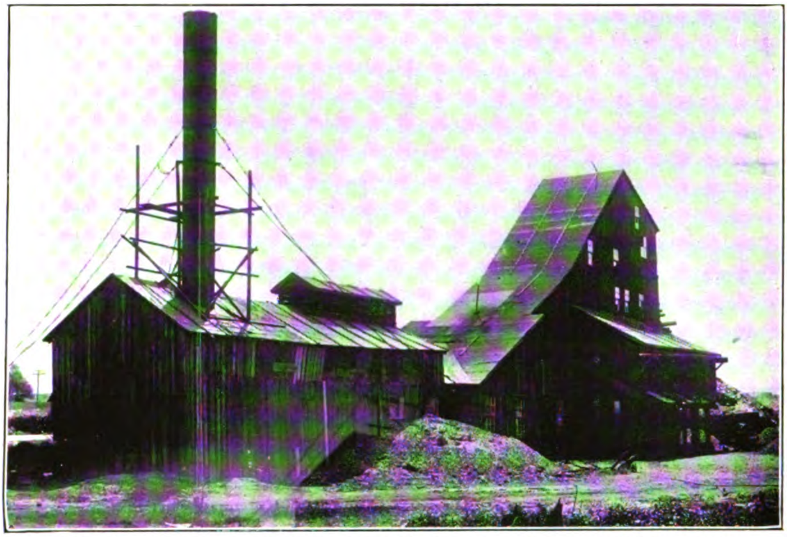


FIG. 1—AN EARLY ANTHRACITE BREAKER—STILL IN OPERATION

[CRANE]

FIG. 2—A MODERN ANTHRACITE BREAKER—ELECTRICALLY EQUIPPED

Kw-hr. per ton mined....	3.6	5.2
Kv-a. trans. capacity....	2,000	3,500
H. P. connected.........	2,400	2,535
Cost of electrification....	$134,500	$325,000
Cost per h. p. connected.	56.04	128.20
Per cent kw-hr. per pumping..................		50
Average depth of mine...	650 ft.(198.1m.)	800 ft.(243.8m.)

Mine No. 2 had a boiler plant of 800 h. p., used 900 tons of coal per month and 11 men have been put to other work about the mine. The output will be largely increased this year as the above investment provides for additional equipment, not yet in operation.

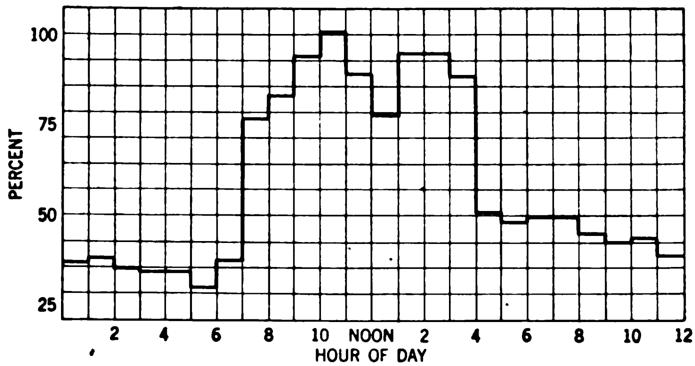


FIG. 3—TYPICAL ANTHRACITE MINE LOAD

Mine No. 1 has no pumping, hence the low kw-hr. per ton mined.

The best figures I have yet been able to obtain indicate that 12 kw-hr. per gross ton of coal mined is the average figure for an anthracite mine. This is of course subject to wide variations in individual cases on account of depth of mine, amount of air required for ventilation, quantity of water to be pumped as well as the head pumped against, and amount of work necessary to prepare coal for market. The writer, three years ago secured figures from over fifty bituminous mines and it is interesting to note the average kw-hr. per gross ton mined was 3.57.

The less power required for mining bituminous coal is due to various causes, among which may be mentioned:

Depth of Mine. Bituminous mines in many cases are situated above tipple, so that coal is brought to the surface on level tracks,

and from this point is loaded into the cars by gravity, while the anthracite mines are from 300 ft. (91.4 m.) to 1000 ft. (304.8 m.) in depth and all of the coal has to be raised to the surface and from there to the breaker.

Pumping of Water. The anthracite mines have more water to be taken care of and this water has to be elevated from the lowest level.

Ventilation. In spite of the fact that there is less volatile in the anthracite coal there is generally more gas to be taken from the mine and a larger amount of ventilation has to be provided for and the additional space used for shafts, etc. has to be ventilated.

Preparation. The breaker of the anthracite mine requires

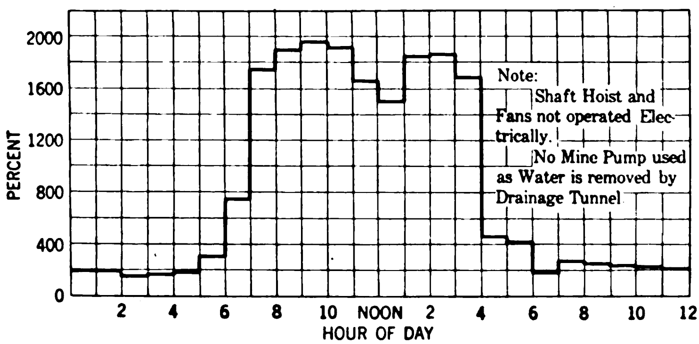


FIG. 4—TYPICAL BREAKER LOAD AND MINE HAULAGE

more power as the coal is sized to a greater extent than is bituminous coal.

One anthracite mine had meters installed for the different services for one year and the following average figures were obtained.

Operation	Kw-hr. per gross ton of coal produced
Haulage	1.73
Ventilation	1.62
Drainage	1.30
Lighting (inc. charging station)	0.12
Hoisting	1.02
Air compressor	2.11
Breaker	4.75
	12.65

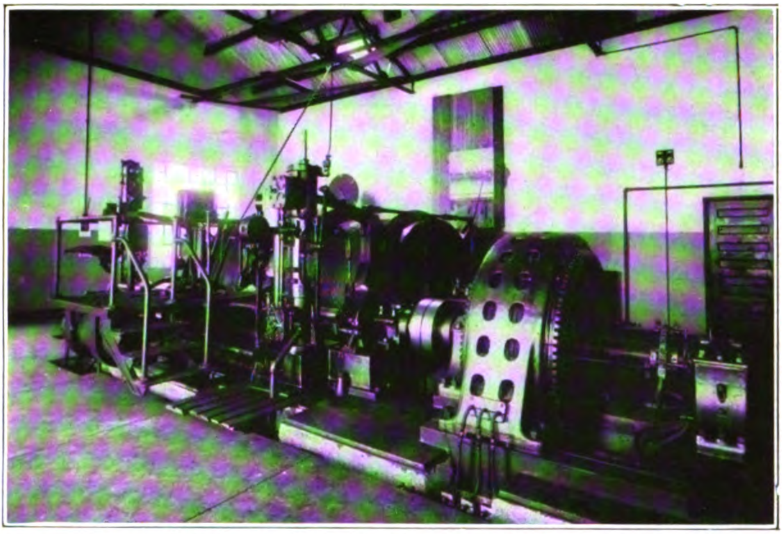


FIG. 5—AN ELECTRICALLY DRIVEN HOIST

60-Cycle—2200-volt—675 h. p.—450 rev. per min.—wound-rotor motor with liquid rheostat—double drum hoist running 1500 feet per minute with two 3-ton cars—slope 1100 feet long—95 cars per hour.

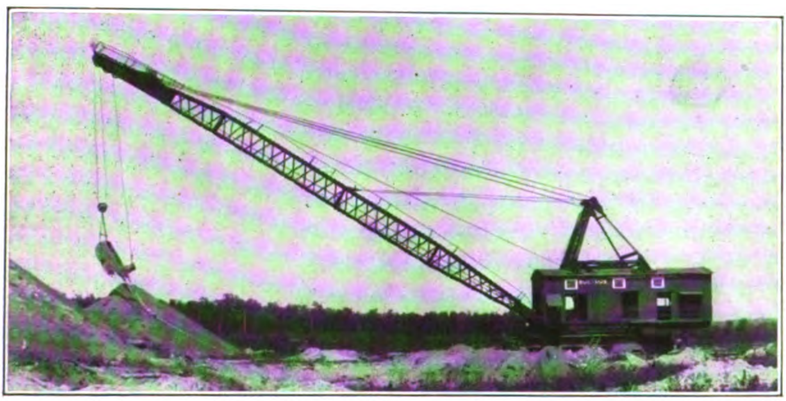


FIG. 6—AN ELECTRIC SHOVEL [CRANE]

Current supplied through a bank of three 100-kw-4000/400 volt transformers.

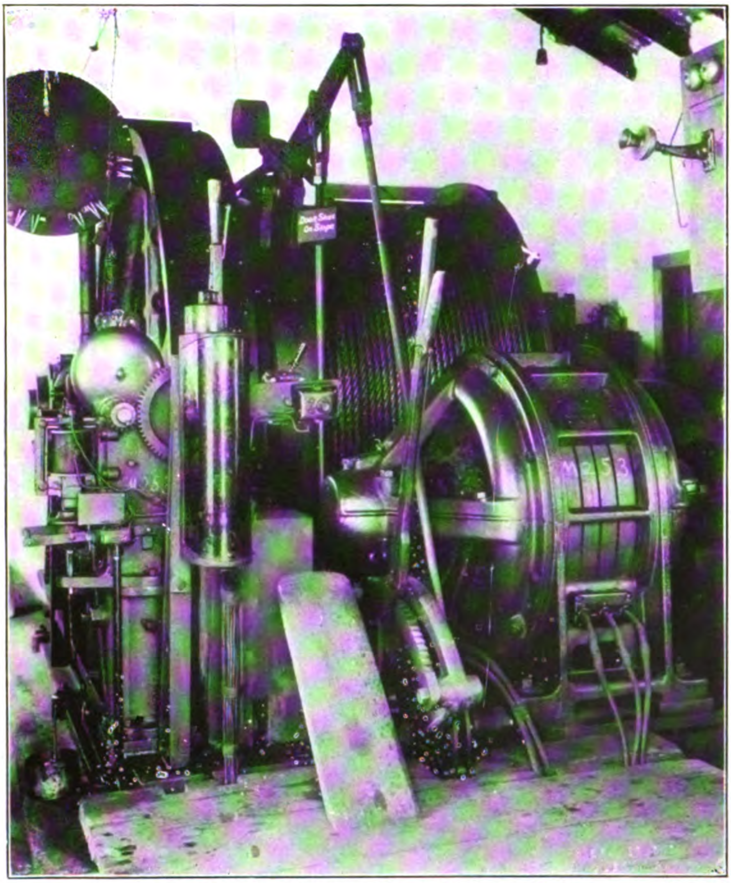


FIG. 7—AN ELECTRICALLY DRIVEN HOIST [CRANE]

60-cycle—440-volt—125-h. p.—500-rev per min.—wound-rotor motor—single drum hoist 900 feet per minute—one 3-ton car—slope 1000 feet long 52 degree pitch—18 cars per hour.

The above mine produced 558,394 tons for the year.

If we assume 12 kw-hr. per ton of coal produced there would have been required in 1917 if all the coal had been produced electrically.......................... 1,076,652,000 kw-hr.

There was actually used in 1917 in electrically operated anthracite mines from central station and mine plants..........	215,000,000
Total kw-hr. necessary to produce remainder of coal electrically..............	861,652,000

At the present time 10 per cent of the coal produced is used to provide power to mine the remainder. There are some very bad cases, at one colliery 400 tons of coal is burned in the boiler plant for every 1000 tons of coal shipped. The above 10 per cent does not include the coal used at central stations so that to produce the 89,720,982 tons in 1917 there was burned under boilers.................................. 8,972,000 tons

In large stations we can produce 1 kw-hr. for 2.5 lb. of small anthracite coal.......... (2.5 × 861,650 divided by 2240)	965,000
There would be released for sale............	8,007,000 tons
At one colliery 2.5 per cent of the men were released by electrification. There are employed in the anthracite region 150,000 men. If 2.5 per cent are released by electrification this makes a total of 3,750 men additional, which would be put to mining coal. Each man produces 550 tons per year so we should get additional (550 × 3750)....................	2,062,500
Additional coal produced by total electrification..................................	10,069,500
The cost of providing for this is estimated as follows:	
Additional kw-hr. required................	861,652,000
At 40 per cent load factor this represents a station capacity of......................	394,000 kw.
Cost of plant at $75 per kw................	$29,550,000
Transmission and distribution lines 250 miles at $5,000..................................	1,250,000
	$30,700,000

Substation and mine installations 900,000 h. p. at $40 per h. p.		$36,000,000
The savings to be effected would be 10,069,500 tons of coal at $2.00 per ton		20,139,000
Reduced mining cost, 700,000,000 tons at 7c. per ton		4,900,000
		$25,039,000
870,000,000 kw-hr. (861,652,000 plus losses) at 11 mills for current delivered at mine	$9,570,000	
Depreciation 5 per cent on $36,000,000	1,800,000	11,370,000
		$13,669,000

This represents 38 per cent on an investment of $36,000,000.

At the present time the mining companies are financially able and are installing as rapidly as possible electrically driven machinery for increasing production and cutting down the requirements for labor. They cannot at this time secure the material for power houses and the central stations are prevented from supplying this power by the impossibility of securing the necessary capital to add to their facilities. It is suggested that means should be found for supplying the amount required to finance additions to these plants so that the vital needs of this industry for power can be met.

To be presented at the 342nd meeting of the American Institute of Electrical Engineers, Philadelphia, Pa., October 11, 1918.

DRUM SHAPES AS AFFECTING THE MINE HOIST DUTY CYCLE AND MOTOR RATING

BY F. L. STONE

ABSTRACT OF PAPER

The standardization of mine hoists, from an engineering standpoint is considered impossible on account of the wide variation in the conditions and methods under which anthracite coal is mined.

The problem of drum shape consists in varying the diameter of different parts of the winding drum so that the load may be accelerated and retarded at the beginning and end of its travel with the minimum consumption of power.

Numerical examples of the performance of various drum shapes under assumed conditions are given.

IN order to make the subject clear I propose to describe in a very general way why there is a problem connected with drum shape.

Coal in this country lies at a depth of from 40 ft. (12.1 m.) to 1800 ft. (548.6 m.) or more below the surface. There may be only one vein or there may be a number of veins lying over each other. In some instances the parent vein will split and form two or more veins. The pitch of these veins in any given direction may vary from horizontal to vertical.

In nearly all cases the veins outcrop on the surface somewhere though they may be so thin and pinched at the outcrop that it would be unprofitable to attempt mining operations at that point.

Possibly the most violent upheavals in coal measures have occurred in the anthracite mines in the Panther Creek Valley section. An approximate cross section of this locality is shown in Fig. 1. From this you will see how the measures twist and turn and what a different mining problem is found as compared with mining flat coal.

Fig. 2 shows a hard coal mine in which all conditions of mining are encountered. This picture tells in plain language the various methods involved. This model is 9 ft. (2.7m.) by 16 ft. (4.8m.). The front portion about two thirds the area of the

Manuscript of this paper was received August 15, 1918.

model shows the workings of an 8-ft. (2.4m.) vein with all the pitches from flat to 80 deg. Back of this the overlying measures are shown in section with a shaft and a skip hoist or gun boat by means of which the coal is taken to two breakers. A conveyor is shown taking the coal from the shaft to a breaker. A stripping operation is also shown with two steam shovels in operation. The back ground is formed by a painting of characteristic coal region scenery.

The coal is mined in the various rooms and brought to the foot of the shaft or slope in mine cars which hold from one and one-half to five tons. It is usually gathered with small locomotives and made up into trips and taken to the foot by a large haulage locomotive. This locomotive returns with the empties.

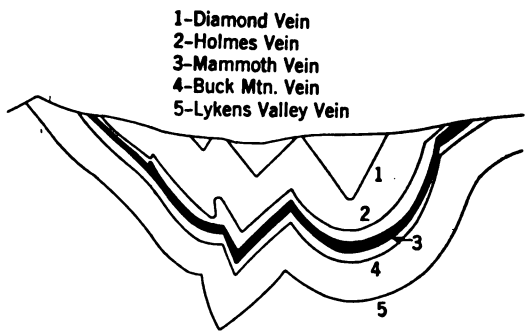
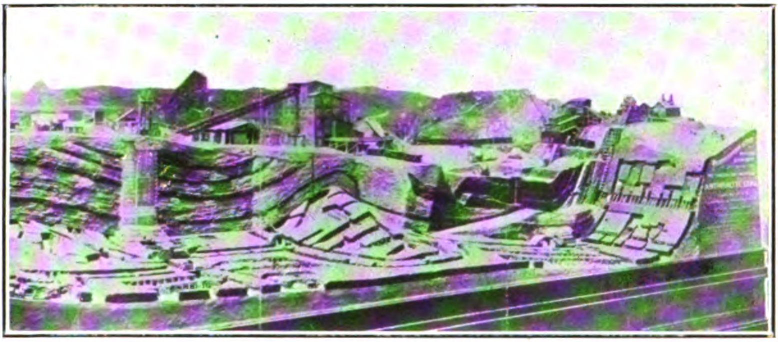


FIG. 1—SECTION OF COAL MEASURES IN PANTHER CREEK VALLEY

The coal is brought to the surface in a number of different ways. First the loaded car is put on the cage and hoisted to the landing while an empty is being lowered. At the landing the car is taken off the cage and sent to the breaker or tipple over the surface. The second method is that of using a dumping cage, that is to say the loaded car is taken to the surface and then the cage tilted and the car door opened and the contents dumped into a chute, the car never leaving the cage.

The third method is that of dumping the coal directly into a skip, the skip holding one or two mine car loads, the car never leaving the mine. This method involves an increase in the breakage of coal but not necessarily to any great extent. The fourth method is somewhat similar to the third except that the coal is dumped from the cars into a hopper and then loaded into the skip.



[STONE]

FIG. 2—MODEL SHOWING METHODS OF MINING ANTHRACITE COAL UNDER THE SEVERAL CONDITIONS OF ITS OCCURRENCE, ALSO TYPES OF MINING PLANTS

This method has many features to recommend it. Heavy loads can be lifted per trip at a comparatively low speed—for example if a given tonnage is required and it takes three trips per minute to get this output by the first or second method, a motor or engine of a h. p. X would be required. If the output could be obtained by the fourth method carrying heavy loads at low speed, the capacity of the drive would be reduced to $X/2$ or more. The mechanical parts, would, of course have to be stiffened up to take care of the increased rope stress.

From the foregoing remarks you will realize that standardization of mine hoists from the engineers standpoint is out of the question, since there are so many factors entering into the calculations for determining the proper size of motor and a change of any one may change the result very materially.

The following are the important factors which enter into the calculations:

Tonnage per hour
Weight of material per trip.
Total lift.
Rest period between trips.
Amount of material to be accelerated and retarded per trip.

You can readily see how at least one of these values will vary in every problem.

The problem is usually stated as follows:

Mine hoist is required to hoist—
Tons per day of eight hours *a*
Total lift..................*b*
Weight of material per trip *c*
Weight of cages and cars...*d*
Rest between trips.... ....*e*

If the average rope speed is comparatively high, say 20 ft. (6m.) per sec. or greater and the time of a trip comparatively short, say 20 sec. including rest period, then one may be fairly sure that the horse power required to accelerate the mass will be a large factor in determining the motor rating.

The ordinary horse power—time curve of a hoist using a cylindrical drum follows the general shape shown in Fig. 3. Motor starting from rest accelerates the total mass to running speed. This is held until the cage approaches a landing; then the

speed is reduced to rest. The cars are then changed and the cycle repeated.

The heating of the motor is, of course, proportional to the square of the current and the current is proportional to the torque, and therefore the r. m. s. value of the area from start to

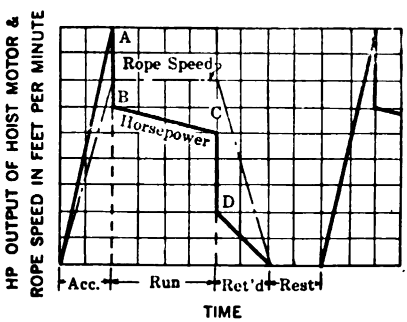


FIG. 3—TYPICAL MINE HOIST DUTY CYCLE

start (making proper allowances for the acceleration, retardation and rest periods) will be the rating of the motor.

You can appreciate the important part played by the accelerating area in the motor rating. It is this peak which, by the shape of the drum, we are trying to reduce.

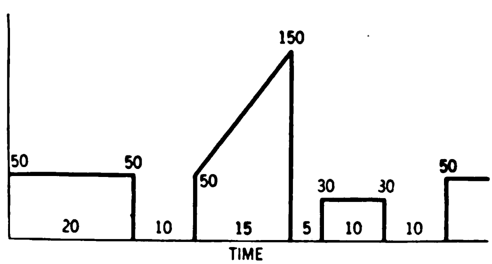


FIG. 4

It is obviously impossible to reduce the $\frac{m v^2}{2}$ but we can reduce the point B to which the h. p. of acceleration is added.

The depths I have selected are 300 (91.4m.) and 500 ft. (152.4m.). I have assumed the same tonnage output in each case.

In none of the examples chosen has the armature acceleration or retardation been included as this would only complicate

matters and would not change the relative result to any great extent.

The method of arriving at this motor rating is the usually accepted one of r. m. s. By this is meant the square root of the average squared ordinate of the entire area under the duty cycle. This is not strictly correct but sufficiently accurate for the normal hoist cycle.

I will illustrate the method by the following simple examples:

1st. A motor running at constant speed and the load varying rapidly. Fig. 4.

$$50^2 \times 20 = 50{,}000$$

$$\frac{(50^2 + 150^2 + 50 \times 150)}{3} 15 = 162{,}500$$

$$30^2 \times 10 = 9{,}000$$

$$221{,}500 = \text{Total squared area.}$$

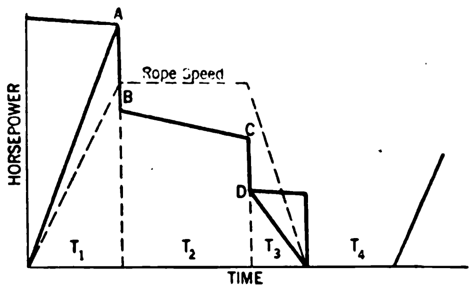


FIG. 5

$$\frac{221{,}500}{70} = 3164 = \text{mean squared ordinate.}$$

$$\sqrt{3164} = 56.2 \text{ r. m. s.}$$

If the values are amperes then the same heating would be obtained in the motor as if it were running continuously at 56.2 amperes. The same is equally true if the values are in horse power or kw. direct current, or kv-a. alternating current.

Now take the case of an intermittent running motor such as a hoist drive. Assume a cycle similar to Fig. 5.

$$\sqrt{\frac{(A^2 \times T_1) + \left(\dfrac{B^2 + C^2 + B\,C}{3}\right) T_2 + D^2 \times T_3}{\dfrac{T_1}{K} + T_2 + \dfrac{T_3}{K} + \dfrac{T_4}{L}}} = \text{r. m. s.}$$

Since the torque must be held constant or nearly so during acceleration the current corresponding to A horse power is flowing in the motor during the time T_1, but since the speed is O at start the horse power must be O also. For the slightly cumbersome expression $\left(\frac{B^2 + C^2 + B\,C}{3}\right) T_2$ may be substituted the expression $\left(\frac{B + C}{2}\right)^2 \times T_2$ if the difference between B and C is not greater than 10 or 15 per cent. The line joining B^2 and C^2 is in reality a curve and the area under this curve is represented actually by the expression above. The same remarks regarding A apply to D, the torque must be held constant from the time retardation begins until the hoist comes to rest. In getting the average square ordinate it would be incorrect to use the full value of T_1, T_3 or T_4 as in the first two cases the motor starts from rest and only reaches full speed at end of T_1 and is at rest at end of T_3. Therefore the ventilation is cut down quite materially. Experience has shown that for a-c. motors the value 2 may be used for K, and for d-c. machines which are more exposed the value 1.33 may be used, while in the case of L_1 for a-c. machines 4 is used and for d-c. 2.

In the calculation of all the cases, the moment method has been used. By that I mean that the moment of all the forces that :equire power such as the up-cage car and coal, up-rope, acceleration and friction are working against the motor and considered as positive moments while the down-cage and car, down-rope, and retardation are assisting the motor and therefore considered negative. The total negative values are subtracted turn for turn from the total positive values and the net moment found. From the shape of the drum and the depth, the total turns may be readily found.

From the total turns may be calculated the r. p. s. as follows:

$$\text{R. P. S.} = \frac{\text{Turns}}{T - \dfrac{t_a + t_r}{2}} = V$$

Where T = total running time, t_a = time of acceleration, t_r = time of retardation.

In regard to the friction value which has been assumed as 25 per cent of the useful work done in all cases. This

value covers the mechanical losses in hoist from motor coupling to the actual work done in the shaft, including gear and bearing losses, rope bending, guide friction and cage windage. There is, of course, no absolute method of predetermining this value and an intelligent guess is about the best method for practical cases. Many formulas have been evolved for the calculation of this value but they are based on assumptions which may or may not be correct.

Having the total moment including friction the horse power can be found by substituting in the following simple formula:

$$\text{H. P.} = \frac{2\,\pi\;r\,M}{550}\quad (r = \text{r. p. s.},\; M = \text{Moment.})$$

This formula is derived as follows:

at any point $M = W \times$ radius

Distance traveled in one second $= 2\,\pi \times$ radius $\times$ r. p. s. $= V.$

$$V \times W = \text{ft. lb. per sec.}$$

$$\frac{V \times W}{550} = \text{h. p.} = \frac{2\,\pi \times \text{Radius} \times W \times \text{r. p. s.}}{550}$$

$$= \frac{2 \times M \times \text{r. p. s.}}{550}$$

We now come to the consideration of the cases selected. Only in the case of the cylindro-conical drum (Case 4) are the actual figures of the calculation shown. This case will serve to illustrate the general method used.

Weight of coal hoisted	5,000	lb.
Total lift	300	ft.
Weight of cage	11,000	lb.
Weight of car	4,000	lb.
Rope	1.25	in.
Number of trips per min....................	3	
Rest between trips.........................	4	sec.

Case 1.—Cylindrical drum.

Case 2.—Conical drum.

Case 3.—Cylindro-conical with long cone.

Case 4.—Cylindro-conical with short cone.

One of the most important points to be determined in the calculation of these cycles is the weight and radius of gyration of the rotating parts of the hoist. The figures used have been

secured from various hoist builders at various times and are believed to be fairly accurate.

Calculation of Case 4:—

Acceleration in 5 sec., run 6 sec., retard 5 sec., rest 4 sec.

To find number of turns on small drum to take up acceleration.

x = r. p. s.

$\frac{5\,x}{2}$ = Turns during acceleration.

$\frac{5\,x}{2} \times \pi \times 6$ = length of rope wound on during acceleration. (1)

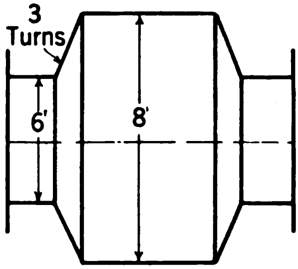


Fig. 6

$\frac{5\,x}{2} \times \pi \times 8$ = length of rope wound on during retardation. (2)

$3 \times 7\,\pi$ = length of ropewound on cone. (3)

$6\,x$ = turns during constant running.

$6\,x - 3$ = turns on large diameter during constant run.

$(6\,x - 3)\,8\,\pi$ = Rope wound on. (4)

$$\frac{5\,x}{2} \times 6\,\pi + \frac{5\,x}{2} \times 8\,\pi + 21\,\pi + (6\,x - 3)\,8\,\pi = 300$$

$$30\,\pi\,x + 40\,\pi\,x + 42\,\pi + 96\,\pi\,x - 48\,\pi = 600$$

$$166\,\pi\,x - 6\,\pi = 600$$

$$166\,\pi\,x = 618.9$$

$$x = \frac{618.9}{166.\pi} = 1.187 \text{ r. p. s.}$$

Turns acceleration $\frac{1.187 \times 5}{2}$	2.97
Turns on cone..................................	3
Turns on large diam. $6 \times 1.187 - 3$..............	4.11
Turns Retard large diam. $\frac{1.187 \times 5}{2}$	2.97
Total Turns..........	13.05

Distance passed over.

Acceleration.............	$2.97 \times 6\,\pi$.............	56 ft.
Up cone.................	$3 \times 7\,\pi$.............	66 ft.
Large diam. constant speed	$4.11 \times 8\,\pi$.............	103.3 ft.
Large diam. retard........	$2.97 \times 8\,\pi$.............	74.7 ft.
Total Lift..................................		300.0

Moments.

Up load

11,000 cage
5,000 coal
4,000 car

20,000 total.

Turns	Weight		Radius		Moment.
0	20,000	×	3	=	60,000
2.97	20,000	×	3	=	60,000
5.97	20,000	×	4	=	80,000
10.08	20,000	×	4	=	80,000
13.05	20,000	×	4	=	80,000

Down Load

11,000 cage
4,000 car

15,000 total.

Turns	Weight		Radius		Moment.
0	15,000	×	4	=	60,000
2.97	15,000	×	4	=	60,000
7.08	15,000	×	4	=	60,000
10.08	15,000	×	3	=	45,000
13.05	15,000	×	3	=	45,000

Up-rope (1.25 in. diam. weight 2.45 lb. per foot)

Turns	Weight		Radius		Moment.
0	300 × 2.45	×	3	=	2,205
2.97	244 × 2.45	×	3	=	1,795
5.97	178 × 2.45	×	4	=	1,743
10.08	74.7 × 2.45	×.	4	=	732
13.05	0				0

Down-rope

Turns							
0	0						0
2.97	74.7	×	2.45	×	4	=	732
7.08	178.	×	2.45	×	4	=	1,743
10.08	244.	×	2.45	×	3	=	1,795
13.05	300	×	2.45	×	3	=	2,205

Plot all curves on coordinate paper—Moments against turns.

Sum of Moments.

			UP			
Turns......	0	2.97	5.97	7.08	10.08	13.05
Up-load....	60,000	60,000	80,000	80,000	80,000	80,000
Rope......	2,200	1,790	1,740	1,500	730	0
Total up...	62,200	61,790	81.740	81,500	80,730	80,000
			DOWN			
Down-load.	60,000	60,000	60,000	60,000	45,000	45,000
Rope......	0	730	1,500	1,740	1,790	2,200
Total......	60,000	60,730	61,500	61,740	46,790	47,200
Total up...	62,200	61,790	81,740	81,500	80,730	80,000
Total down.	60,000	60,730	61,500	61,740	46,790	47,200
Net.......	2,200	1,060	20,240	19,760	33,940	32,800
Friction....	4,600	4,600	4,600	4,600	4,600	4,600
Gross M...	6,800	5,660	24,840	24,360	38,540	37,400

Friction Determination.

$$\frac{2200 + 1060}{2} \times 2.97 = 4,830$$

$$\frac{1060 + 20,240}{2} \times 3 = 31,950$$

$$\frac{20,240 + 19,760}{2} \times 1.11 = 22,200$$

$$\frac{19,760 + 33,940}{4} \times 3 = 80,550$$

$$\frac{33,940 + 32,800}{2} \times 2.97 = \underline{99,000}$$

Area in moment turns 238,530

Average ordinate

$$\frac{238,530}{13.050} = 18,300$$

$$\frac{18,300}{0.80} = 22,900$$

Friction ordinate

$$22,900 - 18,300 = 4600$$

Acceleration and retardation of loads in shaft due to cone.

$$a = \frac{v_1 - v_0}{T}$$

Acceleration up-load $v_0 = 6\,\pi \times 1.187 = 22.4$

$$v_1 = 8\,\pi \times 1.187 = 29.85$$

$$T = \frac{3}{1.187} = 2.53 \text{ sec.}$$

$$a = \frac{7.45}{2.53} = 2.93$$

Allowing 750 lb. for rope in shaft.

$$F \text{ of acceleration } = \frac{20{,}750 \times 2.93}{32.2} = 1887.$$

Moment at 2.97 turn = 1,887 × 3 = 5661.
Moment at 5.97 turn = 1,887 × 4 = 7548.

Retard down-load.

$$F \text{ of retardation } = \frac{15{,}750 \times 2.93}{32.2} = 1435$$

Moment at 7.08 turn = 1435 × 4 = 5740
Moment at 10.08 turn = 1435 × 3 = 4305

Acceleration of up-load from start to full speed of drum.

$$v = 6\,\pi \times 1.187 = 22.4 \qquad a = \frac{22.4}{5} = 4.48.$$

$$w = 20{,}000 + \text{Rope } 750.$$

$$F = \frac{20{,}750 \times 4.48}{32.2} = 2870 \qquad M = 2870 \times 3 = 8600.$$

Acceleration down-load.

$$v = 8\,\pi \times 1.187 = 29.85 \qquad a = 5.97$$

$$w = 15{,}000$$

$$F = \frac{15{,}000 \times 5.97}{32.2} = 2780$$

$$M = 2780 \times 4 = 11{,}120$$

Drum and 1/2 rope.

$WR^2 = 350{,}000$, W at 3 ft. $= 38{,}900$

$$V = 22.4 \qquad a = 4.48$$

$$F = \frac{38{,}900 \times 4.48}{32.2} = 5410$$

$$M = 5410 \times 3 = 16{,}200$$

Total acceleration moment

Up-cage, car, coal and rope.......... 8,600
Down-cage and car 11,120
Drum.......................... 16,200

35,920

• Retardation

Up-load $V = 29.85 \qquad a = 5.97$

$$M = \frac{20{,}000 \times 5.97}{32.2} \times 4 = 14{,}900$$

Down-load and rope $V = 22.4 \qquad a = 4.48$

$$M = \frac{15{,}750 \times 4.48}{32.2} \times 3 = 6560$$

Drums as before 16,200

Total retard moment up............ 14,900
Down 6,560
Drums 16,200

37,660

SUM OF MOMENTS—FINAL

Turns	0	2.97	2.97	5.97	5.97	7.08	7.08	10.08	10.08	13.05
Gross	6,800	5,660	5,660	24,840	24,840	24,360	24,360	38,540	38,540	37,400
Acc. ...	35,920	35,920	...	...	...	...	...	...	...	...
Acc. on cone ...	...	...	5,661	7,548	...	...	...	...	...	...
Retd. on cone ...	...	...	...	...	...	...	−5,740	−4,305	...	...
Retd.	...	...	...	...	...	...	...	...	−37,660	−37,660
Total	42,720	41,580	11,321	32,388	24,840	24,360	18,620	34,235	880	−260
Time.....	0	5	5	7.53	7.53	8.47	8.47	11	11	16
H. P.	580	564	154	438	338	330	253	466	11.9	3.5

R. M. S.

$$\left(\frac{580 + 564}{2}\right)^2 \times 5 = 1{,}640{,}000$$

$$\left(\frac{154^2 + 438^2 + 154 \times 438}{3}\right) \times 2.53 = 239{,}000$$

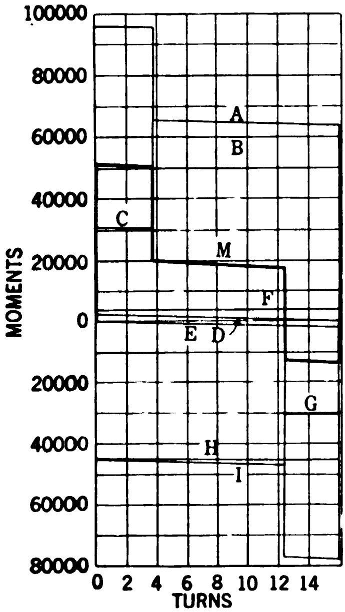
$$\left(\frac{338 + 330}{2}\right)^2 \times .94 = 110{,}500$$

$$\left(\frac{253^2 + 466^2 + 253 \times 466}{3}\right) \times 2.53 = 338{,}000$$

$$\left(\frac{11.9 - 3.5}{2}\right)^2 \times 5 = 88$$

2,327,588

$T_1 = 5$	$K = 2$	Equiv. Time =	2.5
$T_2 = 6$		" "	6.
$T_3 = 5$	$K = 2$	" "	2.5
$T_4 = 4$	$L = 4$	" "	1.
Total Equiv. Time...........			12.0



FGI. 7—MOMENT DIAGRAM—CASE I—SINGLE CYLINDRICAL DRUM—$W R^2$ ROTATING PARTS 200,000

A = Total monent up load
B = Moment up cage car and ore
C = Acceleration moment
D = Up rope moment
E = Down rope moment
F = Priction moment
G = Retardation moment
H = Down cage and car moment
I = Total down moment
M = Total net moment

$$\frac{2{,}327{,}588}{12} = 194{,}000$$

$$\sqrt{194{,}000} = 440.$$

Looking at Case I, Figs. 7 and 8, you will note the acceleration peak goes to 825 h. p. and the retardation goes below the zero line which indicate the motor must be plugged or the mechanical brakes applied to stop the hoist. This in the case of an induction motor drive is not conducive to good operation as the operator will invariably plug, which is expensive from a power standpoint, or he will take longer to retard which reduces the output. The r. m. s. value of this cycle is 600 h. p.

Case II is that of a conical drum 6 to 8 ft. in diameter. It will be noted the acceleration peak is considerably less in this than in Case I. This is brought about by the fact that the point *B* in the master diagram, has been reduced to approximately 200 h.p. Here, again during retardation the value becomes negative and as in Case I the motor must be plugged or stopped by means of mechanical brakes, though not to any such great extent as in Case I. The r. m. s. value of this cycle is 482 h. p. This is shown in Figs. 9 and 10.

Case III is that of a cyclindro-conical drum in which the

cylindrical parts are just large enough to take the turns during acceleration and retardation. The turns on the large diameter are common to both ropes, the drum being symmetrical around the center line. Here we have reduced the accelerating h. p. to approximately 600, and the retarding h. p. above the line which indicates that power must be kept on to a very small extent during retardation. The r. m. s. value in this instance is 452 h. p. Shown in Figs. 11 and 12.

Case IV, the calculation of which has been given in the previous pages is that of a cylindro-conical drum with a very steep pitch or spiral cone. This gives a good many of the characteristics found in Case III and a r.m.s. value of 440, so that

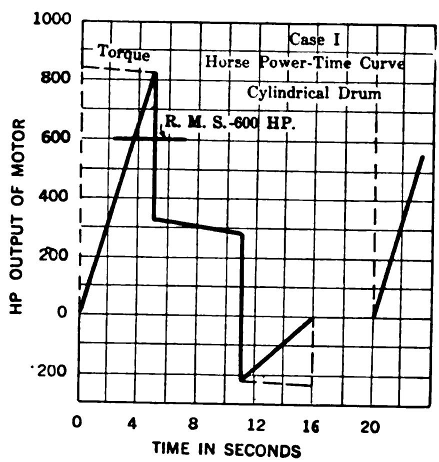


FIG. 8

apparently little is gained in the use of the steep pitch cone over the medium pitch cone of Case III and there is always some question in the minds of some operators, whether or not the pitch cone is a very desirable design. Personally I am not experienced enough with their operation and life to express an opinion, but feel that if the same result can be obtained with a moderate pitched cone, I personally would prefer the latter. Figs. 13 and 14 show results clearly.

In Cases VI, VII, VIII and IX the same tonnage output has been used, but the depth of the shaft increased to 500 ft. This, of course, means a very much more rapid speed of hoisting.

Case VI is that of a cylindrical drum which shows a very undesirable cycle from an induction motor standpoint. It has

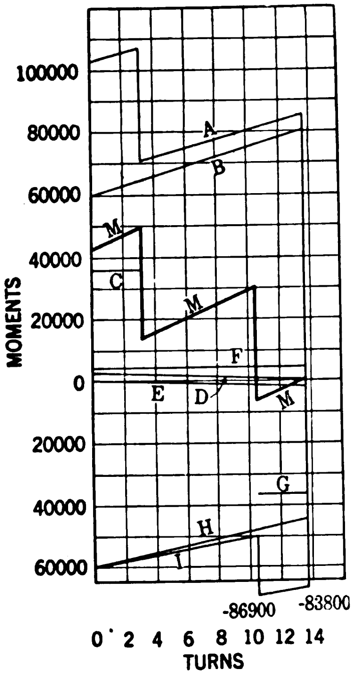


FIG. 9—MOMENT DIAGRAM—CASE II—CONICAL DRUM 6 FT. TO 8 FT.—WR^2 ROTATING PARTS 300,000

A = Moment up load, total
B = Moment up car, cage and ore
C = moment acceleration
D = Moment up rope
E = Moment down rope
F = Moment friction
G = Moment retardation
H = Moment down car and cage
I = Moment total down side
M = Moment total net

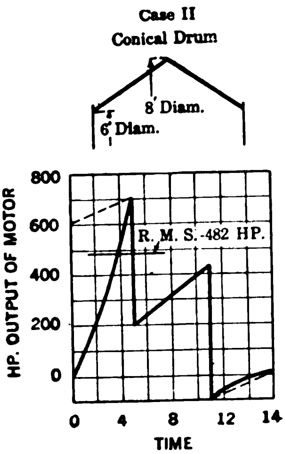


FIG. 10

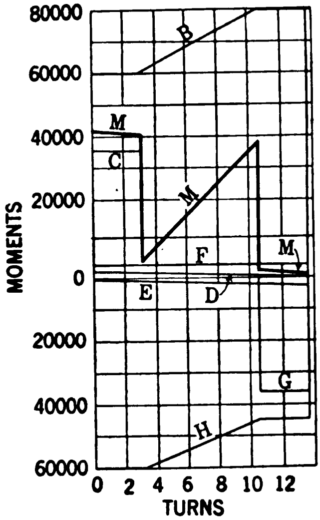


FIG. 11—MOMENT DIAGRAM—CASE III — CYLINDRO-CONICAL DRUMS 6 FT. TO 8 FT.—WR^2 ROTATING PARTS 300,000

B = Moment up car, cage and ore
C = Moment acceleration
D = Moment up rope
E = Moment down rope
F = Moment friction
G = Moment retardation
H = Moment down cage and car
M = Total net moment

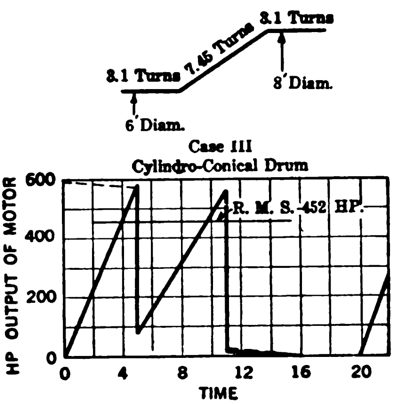


FIG. 12

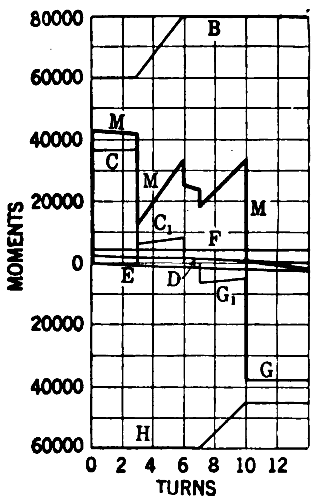


FIG. 13—MOMENT DIAGRAM—CASE IV—CYLINDRICAL CONICAL DRUM—3 TURNS ON CONE—WR^2 ROTATING PARTS 350,000

B = Moment up cage, car and ore
C = acceleration
C_1 = Acceleration up load on cone
D = Moment up rope
E = Moment down rope
F = Moment friction
G = Moment retardation
G_1 = Retardation down load on cone
H = Moment down cage and car
M = Moment total net

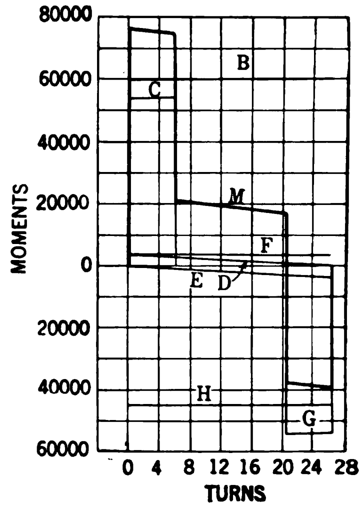


FIG. 15—CASE VI—MOMENT DIAGRAM—CYLINDRICAL DRUM—WR^2 = 250,000—DEPTH 500 FT.

B = Moment up car cage and load
C = Moment acceleration
D = Moment up rope
E = Moment down rope
F = Moment mechanical friction
G = Moment retardation
H = Moment down cage and car
M = Total net moments

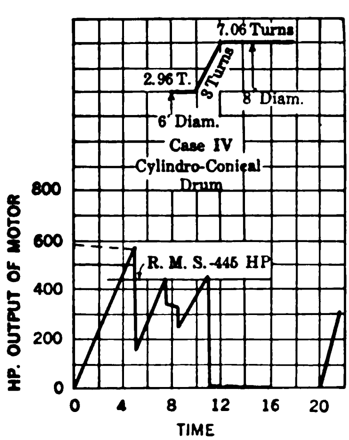


FIG. 14

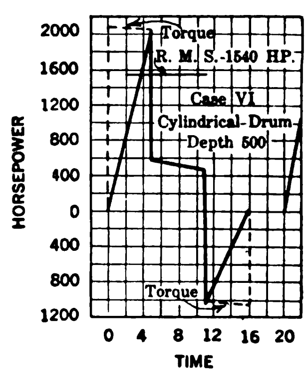


FIG. 16

an acceleration peak of over 2000 h.p. and r.m.s. value of approximately 1540 h. p. with a large quantity of braking to be done to bring the hoist to rest. Figs. 15 and 16.

Case VII is that of a conical drum 6 to 9 feet. It shows a r. m. s. value of 1210 h. p. and a considerable amount of area below the line and an acceleration of approximately 1800 h. p. Figs. 17 and 18.

Case VIII, which is a cylindro-conical drum 6 to 9 ft. with only sufficient cylindrical portions to take care of the active turns during acceleration and retardation. In this the peak is reduced still further and is the case of the conical drum, namely to approximately 1460 h. p. The r. m. s. value of 1128 h. p. and incidently the negative area has been considerably reduced. Figs. 19 and 20.

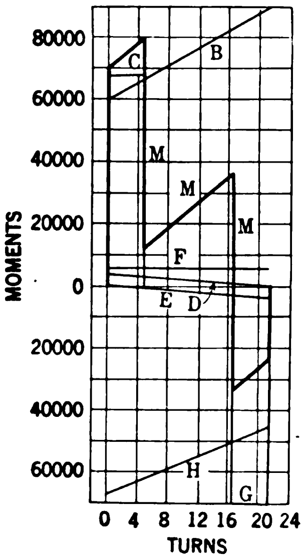


FIG. 17—CASE VII—CONICAL DRUM 6 FT. TO 9 FT.—DEPTH 500 FT.—$W R^2$ = 400,000

B = Moment up cage car and load
C = Moment acceleration
D = Moment up rope
E = Moment down rope
F = Moment mechanical friction
G = Moment retardation
H = Moment down cage and car
M = Total net moment

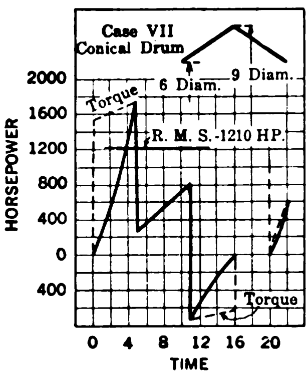


FIG. 18

Case IX is a cylindro-conical drum with steep pitched cone. This shows approximately the same peak as the drum with the long cone, in Case VIII. but has a slightly less r. m. s. value of 1070 h. p. Figs. 21 and 22.

None of the latter cases, I believe, are really suitable cycles for induction motor operation and should be driven by Ward-Leonard equipments.

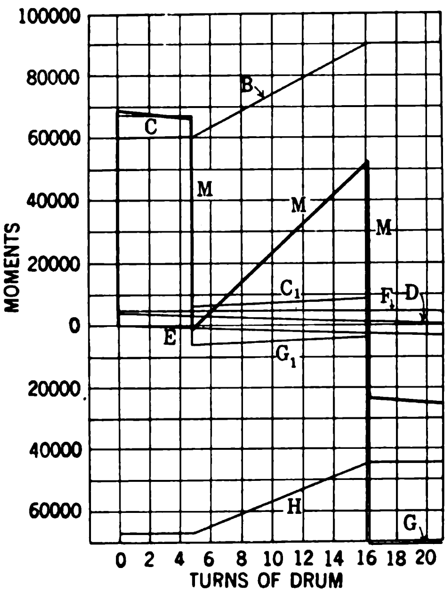


FIG. 19—MOMENT DIAGRAM—CASE VIII — CYLINDRICAL-CONICAL DRUM — $W R^2$ 400,000

B = Up cage car and load	F = Mechanical friction
C = Total acceleration	G = Retardation
C_1 = Acceleration up cone	G_1 = Retardation down cone
D = Up rope	H = Down cage and acr
E = Down rope	M = Total net moments

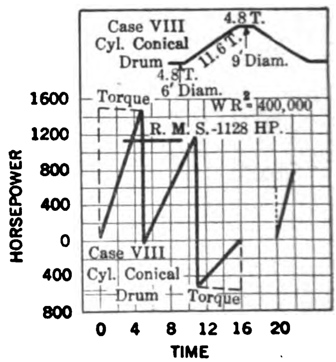


FIG. 20

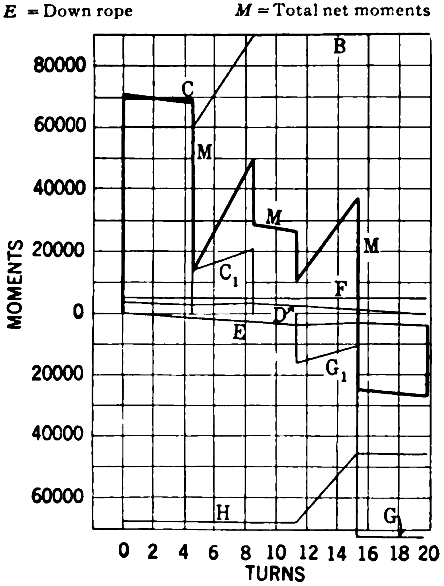


FIG. 21—MOMENT DIAGRAM—CASE IX—CYLINDRICAL-CONICAL DRUM

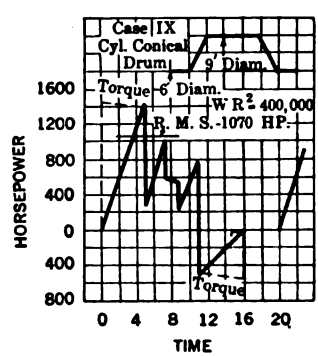


FIG. 22

I realize, of course, that in this short paper I have only touched the high spots in the problem of drum shapes, but I have, I hope selected sufficient examples to show that there is in the proper selection of drum an opportunity for some real engineering.

I have kept away from the problem of drum shapes connected with hoisting from great depths as in this class of problem the rope weight is usually the reason for any departure from the cylindrical drum. The reduction of the acceleration peak in deep hoisting is, of course, an advantageous point which comes automatically with the coning of the drums. The weight of the hoisting cable in some of the deep shafts exceeds by a considerable amount the weight of the active material hoisted. As before stated this is a problem by itself.

THE NATIONAL ENGINEERING SOCIETIES AND THE NATIONAL RESEARCH COUNCIL

BY GEORGE ELLERY HALE

IN an address delivered in the Engineering Societies Building on May 28, at the kind invitation of the Engineering Foundation, I briefly sketched "the War Activities of the National Research Council."* The wide scope of my subject, calling for some reference to the work of the Council in the various branches of the physical and biological sciences, as well as in agriculture, medicine, and other arts, forced me to touch very lightly upon engineering. I therefore beg permission to return to this phase of the subject in the present paper.

As shown in the address just cited, the charter membership of the National Academy of Science, constituted in the midst of the Civil War, comprised a notable group of engineers. Indeed, engineering was the only one of the arts represented in the Academy, which based its elections, then as now, upon creative work and original contributions to knowledge. The war was the immediate stimulus that led to the establishment of the Academy, but the published opinions of well-known visitors from abroad indicate that there was urgent need for such a body in this country.

De Tocqueville, in a chapter entitled "How the example of the Americans fails to prove that a democratic people cannot possess aptitude and taste for science, literature and art", wrote in 1840 as follows: "It must be admitted that among the civilized peoples of our time, there are few in which the higher sciences have made less progress than in the United States."† This he attributed to our Puritan origin, our pursuit of the wealth which is so easily acquired in a new country, and our dependence upon England for intellectual things. "I consider the people of the United States as that portion of the English people which is charged with the exploitation of the forests of the new world, while the rest of the nation, enjoying more leisure

*See "A Comprehensive Organization for the Engineering Profession in America".

†De la démocratie en Amérique, 17 ed., vol. 3, p. 58.

and less preoccupied with the material needs of life, may devote itself to thought and to the development of the human mind in every field."*

But although he regarded the United States as exceptional, he fancied that he recognized in all democracies conditions of disturbance and unrest which leave little opportunity for the quiet and repose essential to the cultivation of pure science. These he carefully distinguished, however, from great upheavals of the body politic. "When a violent revolution occurs among a highly civilized people, it cannot fail to give a sudden impulse to feeling and imagination."† Thus the French achieved their highest development in science soon after the revolution of 1789.

In 1963, when the National Academy was incorporated, De Tocqueville would probably have considered our intellectual dependence upon England to be materially less than at the time of his visit to the United States, thirty years earlier. Doubtless he would have attributed the improved condition of American science to the effect of the Civil War, and the considerable increase in wealth and leisure. In 1873, if we may judge from Tyndall's remarks in the concluding lecture of his American series, European opinion saw hope for the future of science in the United States, but recognized few important accomplishments. "If great scientific results are not achieved in America, it is not to the small agitations of society that I should be disposed to ascribe the defect, but to the fact that the men among you who possess the endowments necessary for profound scientific inquiry, are laden with duties of administration, so heavy as to be utterly incompatible with the continuous and tranquil meditation which original investigation demands."‡ At this time Henry was Secretary of the Smithsonian Institution, Barnard was President of Columbia College, and Rogers was President of the Massachusetts Institute of Technology. There was thus some justification for Tyndall's remark, though the amount of scientific research in progress was much larger than one would infer from his statement of the case. Moreover, though deprived by other duties of the privilege of personal work in the laboratory, these very men, charter members of the National Academy, were nevertheless laying the foundation of science in America.

*Op. cit. p. 60.

†Op. cit. p. 70.

‡Six Lectures on Light, 2nd. ed., p. 226.

By uniting in one national body the representatives of research in both science and engineering, they set an example which their successors should keep steadily in view.

The half century which elapsed before the United States was again stirred to its depths by another great war was a time of specialization, both at home and abroad. Once fairly launched, both science and the arts made rapid progress, but they inevitably grew apart. Indeed, the tendency toward specialization which divided the arts from the sciences also separated the sciences into many distinct groups and split the arts widely asunder. Thus in engineering, many societies were organized, first those comprising the major fields of civil, mechanical, mining, and electrical engineering, subsequently those dealing with the special problems of naval architecture, illumination, refrigeration, and still more narrowly limited branches. At the same time numerous major and minor societies were formed in the general field of medicine; others marked out special territories in the name of agriculture, forestry, and fisheries; and the process of subdivision and separation still goes on.

It is plain that these effects of specialization, while natural and essential elements in the development of science and the arts, involve certain consequences which are far from advantageous. The underlying motive of the investigator, to advance knowledge and to improve practise through the utilization of new ideas, is common to all fields of action. His point of view is much the same, whether his problems be those of the biologist or the engineer. Moreover,—and this is a matter of prime importance—the principles and methods of research developed in one field may be equally applicable in another. Thus there is an essential solidarity of research, which should bring into active cooperation the men engaged in all of its various branches. Recent experience, both in peace and war, has shown how effectively the physicist and chemist can join forces with the engineer: in fact, how men drawn from the most diverse fields can utilize their varied experience to common advantage.

The remarkable development of engineering in the United States is indicated by the success of the four great National Societies, which aggregate more than thirty thousand members. Nine-tenths of the work of the engineer is organization and construction rather than research. While the chief interests of the National Societies thus lie in other fields, the importance of research is such as to demand a large measure of support from

each of them. Moreover, great benefit will result from a joint effort, involving the cooperation of the National Engineering Societies with the National Academy of Sciences in a new and powerful movement to promote research in every branch of science and the arts. The establishment of the National Research Council, and the duties laid upon it by the President in his recent Executive Order, plainly indicate the steps to be taken.

It is natural that the first effective contact between the National Academy of Sciences and the National Engineering Societies should have been established through the Engineering Foundation, endowed by Mr. Ambrose Swasey, a mechanical engineer who has contributed greatly to the progress of astronomy through the perfection of the powerful telescopes built by the firm of Warner and Swasey. It is equally natural that the engineers who, with Mr. Swasey, took leading parts in the movement toward a consolidation of interests were also men fitted by experience to appreciate both sides of the question. The National Academy owes a special debt of gratitude to Mr. Gano Dunn, who immediately grasped the purpose in view, and has worked unceasingly toward its accomplishment. Though prevented by his heavy responsibilities as a construction engineer from conducting research in a professional way, Mr. Dunn's private activities as an investigator are well-known to his friends who therefore understand how whole-heartedly he has devoted himself to the task of breaking down the artificial barriers between the engineer and the man of science. Others who were most active in the initiation of the movement, including particularly Colonel Carty and Dr. Pupin, also combine experience in research with exceptional capacity as engineers. With their effective aid, and with the active support of the officers of the Engineering Foundation and those of the National Societies, the difficulties of the initial steps were soon removed, and the way was prepared for the intimate cooperation subsequently realized.

The National Academy, probably because of the general tendency toward separate development of the arts and sciences already mentioned, failed to maintain on its rolls the same percentage of engineers with which it originally set out. At the annual meeting in April, 1916, however, the following resolution, presented by the Council, was adopted by the Academy:

> That the Council express to the Academy the opinion that it is desirable that a section of engineering be developed which shall include men who

have made original contributions to the science or art of engineering; that to this end the council suggests to the Academy that the present section of physics and engineering be designated the section of physics, and that the Council, under the authority granted by section 4, article 4, of the constitution, nominate to the Academy, after inviting suggestions from the members of the Academy, two or three engineers each year until such time as it shall seem advisable to establish a separate section of engineering, any engineers elected as the result of such nominations being in the meantime assigned to that one of the existing sections to which their work is most closely related.

Since that time six eminent engineers have been elected to membership in the Academy, and the Section of Engineering will soon be established.

Another means of connection between the Academy and the engineering profession was initiated at the same meeting. Our relations with Germany, after repeated submarine attacks on merchant ships, were in a state of high tension, and the need of some preparation for coming war was plainly evident. The Academy's offer of service to the President was at once accepted, and the National Research Council was formed, at the President's request, for the purpose of federating the research activities of the country.*

The first duty laid upon the National Research Council by President Wilson in his Executive Order of May 12, 1918, reads as follows:

1. In general, to stimulate research in the mathematical, physical and biological sciences, and in the application of these sciences to engineering, agriculture, medicine and other useful arts, with the object of increasing knowledge, of strengthening the national defense, and of contributing in other ways to the public welfare.

This definition of the scope of the Council indicates its purpose to give equal attention to research in all branches of science and the arts. The Council fully recognizes the solidarity of research to which reference has already been made, and its efforts will be directed to promoting the closest cooperation between investigators in every field. It should be clearly understood that the National Research Council was not organized as an independent body, but as a means of federating existing research agencies.

War Duties

It is a matter of prime importance that in all researches bearing on the war the scientific and technical societies of the entire

*See "War Activities of the National Research Council". Proceedings A. I. E. E., July, 1918.

country should work in close cooperation, both to avoid unnecessary duplication and to insure the utilization of all ideas and facilities available for the solution of the most difficult problems. The National Research Council affords the necessary means of bringing representatives of these bodies together and into contact with the various technical bureaus of the Army and Navy and other departments of the Government. The advantage afforded by the Research Information Service, and the other facilities for international cooperation provided by the Council, are described below. Here we may observe how some of the work in engineering is conducted.

The appointment of Mr. Gano Dunn as Chairman and Dr. W. F. Durand as Vice-Chairman of the Council's first Engineering Committee insured that its work would be ably directed. Mr. Dunn's engineering duties made it necessary for him to retain his headquarters in New York, but his close contact with the Engineering Foundation and the national societies proved very advantageous. His activities, in fact, led directly to the Council's first step in securing general cooperation in the organization of researches bearing on the submarine problem. In the initiation and development of many other undertakings he played an equally important part. Dr. Durand's joint duties in Washington, as Vice-Chairman of the Engineering Committee and as Chairman of the National Advisory Committee for Aeronautics, gave him opportunity for valuable work in the organization and conduct of many investigations of an engineering nature. When the Research Information Committee was established, Dr. Durand's qualifications for the position of Scientific Attaché and representative of the Research Council in Paris were so exceptional that he was transferred to this important post.

As a typical illustration of the work of the Engineering Committee we may describe the organization and activities of the special sub-committee on Protective Body Armor, which includes in its membership the curator of arms and armor of the Metropolitan Museum of Art, representatives of the Ordnance Department of the Army, well-known metallurgists, and several able engineers experienced in different fields. The close cooperation of this sub-committee with the Ordnance Department enabled it to carry on its work very effectively and to make all necessary tests of the special types of helmets and body armor that were devised. The form of the helmet was materially influenced by

the extensive knowledge of ancient armor possessed by Doctor (now Major) Bashford Dean, who also went to France to familiarize himself with conditions of trench warfare. The value of this experience has been abundantly proved by the tests to which the helmets have recently been subjected. The metallurgical experiments were carried out in Dr. Howe's own laboratory. The results of the sub-committee's work promise to be of great practical importance in the protection of our troops. Another illustration of the work of the Engineering Committee, which unfortunately cannot be given in detail because of the confidential nature of the problem, is the development of a special form of gun for the Ordnance Department of the Army. This involved the cooperation of several engineers, machine designers drawn from universities and other organizations, ordnance experts, and manufacturing establishments.

Without going into further details of many other research problems studied by the Engineering Committee, we may now turn to the work of the recently organized Engineering Division, which the natural development of the work of the Research Council has brought into existence. The constantly increasing demands upon Mr. Gano Dunn's time resulting from the large war contracts upon which his firm is engaged, and the departure of Dr. Durand for France, made it necessary to select new officers to carry on the engineering work in Washington. Dr. Henry M. Howe was accordingly made chairman, and Mr. W. J. Lester vice-chairman of the Engineering Division, the purpose of which is described in the following excerpt from the remarks of Dr. Howe at the first meeting of the Advisory Committee of the Division.

After referring to the establishment of the National Research Council, and speaking of its general purposes, Dr. Howe went on to describe the object of the meeting:

"It is to consider how we may best carry out this general purpose of 'co-ordinating the scientific resources of the entire country', as regards engineering and how we may best 'secure the cooperation of all engineering agencies in which research facilities are available' that you have been called together. We are asked to do something wholly new, and, by the intentional breadth of our charter we are in effect told to devise ways of doing it. We have a free hand.

"Let me tell you what plans we have already made in this early and formative stage of our growth: Our most pressing

duty is to help the existing governmental agencies in every possible way to win the war, taking the attitude that, however perfect their several organizations, after all they are finite, that is, limited, whereas the demands which the most rapid possible development of our military strength makes on them are unlimited. We therefore seek and welcome ways of helping them. In general our natural function here has been to develop ideas, often initially nebulous, far enough to make their usefulness clear to the military authorities, using this term broadly to include the land, sea, and air forces, and then to leave the active production to them. In many cases our work is confined strictly to perfecting the design, in other cases models have been made. In this way the Division of Physics has developed a great number of very important instruments and devices relating to submarine, subterranean, aircraft, and other matters, and the Division of Medicine and Related Sciences, besides organizing many researches in medicine, has developed a system of psychological tests which have been adopted in the Army for both officers and privates.

"Our own division has already formed five sections,—on mechanical engineering under Mr. W. J. Lester, prime movers under Prof. Lionel Marks, metallurgy under Dr. Bradley Stoughton, electrical engineering under Dr. Stoughton and Prof. C. A. Adams, and military "tanks"; and we ask your advice today about forming others on ordnance, clearing house, and the fatigue of metals. The National Advisory Committee for Aeronautics acts as our section on aircraft. Our section on Metallurgy has two important committees, on helmets and on body armor under Major Bashford Dean, and on Smelting Ores of Manganese under Mr. J. E. Johnson, Jr.

"How we may 'secure the cooperation of engineering agencies' as President Wilson wishes, is illustrated first by our working in close cooperation with the Bureaus of Mines and Standards, the latter of which has placed a laboratory at my disposal, and second by our research on the saving of manganese in steel-making by replacing it in part with other deoxidizing agents.

"Here the deoxidizing agents used must bear such a ratio to each other that the sum of the resultant oxides will be fusible at the steel-making temperature, and hence will coalesce and rise to the surface by gravity instead of remaining entangled in the steel to its great harm. But before we can do this we must

learn what the fusible combinations of the oxides of available deoxidizing agents are. To this end we have secured the cooperation of the Geophysical Laboratory, whose Dr. R. B. Sosman is one of the first, if not the first, authority on this subject, to select the most promising field, and we are now seeking the cooperation of a large number of laboratories, industrial, educational, and governmental, in determining the actual melting points of large numbers of these combinations of oxides. We thus seek a truly scientific solution of the problem instead of one by trial and error. Here we may have as many as twenty separate institutions collaborating on this one problem, with corresponding saving of time.

"It is to be hoped that our present cooperation with the Bureaus of Standards and of Mines may be matched by like cooperation with the Naval Consulting Board, whose important work of sifting out the promising inventions from the great mass submitted to it seems to be well complemented by our natural work of developing promising ideas."

Dr. Howe then discussed the question of the men needed and the expenses involved in the proposed work. The Engineering Division of the Research Council already has thirty thousand dollars available for its office and organization expenses during the current year, and additional funds will probably become available in the near future.

Since that meeting the work of the Sections on Mechanical Engineering and on Metallurgy has developed rapidly. The former has taken over the laboratories and machine shop of the Carnegie Institution at Pittsburg so as to control the construction of the devices which it is perfecting. Through its Committee on Fatigue, under the Chairmanship of Prof. H. E. Moore of the University of Illinois, it has begun the systematic study of fatigue phenomena, having especially in view the requirements of aircraft crankshafts and welded ship plates. It has brought the development of two special types of guns so far that one is now ready for firing, while the other will probably be fired before this paper is in print. Beyond this it is actively developing ten devices, a special gun for use in aircraft, a special mechanism for controlling it, a new control for aircraft, aircraft fuel, tanks of various types, mechanism for controlling trucks, a new type of tractor, special telescopes, special balloons, parachutes, and a new type of aircraft engine.

The work of the Section on Metallurgy promises to develop

chiefly through the creation and direction of committees which shall mobilize the latent skill and patriotism in the metallurgical works themselves and in their laboratories, metallurgical, chemical and mechanical, and in the laboratories of our institutions of learning. Thus in addition to the committees mentioned by Dr. Howe, this Section has organized, under the Chairmanship of Col. W. P. Barba of the Ordnance Department, a committee containing the metallurgists of the great ordnance works, Bethlehem, Midvale, Standard and the United States Steel Corporation, to formulate detailed directions for the procedure in making and treating steel ingots for objects needing the very best quality, such as cannon, shells, armor and crankshafts. Under the chairmanship of Dr. George K. Burgess, of the Bureau of Standards, it is now organizing a committee to develop a pyrometer for determining the temperature of the molten steel in the open-hearth and electric steel processes. Other committees with aims of this general class are projected.

Research Information Service

The organization and work of the Research Information Committee, which now has offices in Washington, London, Paris, and Rome, were described in the address previously cited. The subsequent action of the Secretary of War in issuing the following general order to all scientific and technical bureaus of the War Department has led to an important expansion of the work of the Committee.

1. The Secretary of War directs that you be informed as follows:

2. The Research Information Committee was formed to establish machinery by means of which the general staff of the Army, the various bureaus of the Army and Navy, the Scientific Organization in the United States, who are working on problems connected with war production and invention, and the various committees of the Council of National Defense charged with work of this nature, may be put in touch with the developments and experimental work being carried on, not only in this country, but in Europe, and kept mutually informed of the state of development of work of this nature.

3. In pursuance of the order of the Secretary of War, establishing this Committee and in order effectively to do this work, it is vitally necessary that the utmost of cordial cooperation be shown by each of the Bureaus and Committees in question with the Research Information Committee. To secure this the following is directed:

(a) All Military Bureaus requiring scientific and technical information are given official status on the Research Information Committee in Washington, D. C.

(b) Representatives of Military Bureaus or of research committees collecting information abroad will be instructed, by their chiefs, to put themselves into direct relationship with the joint committees of the Research Information Committee sitting in Paris or London, or later in Rome, in order that information be at once dispatched to the Research Information Committee at Washington, D. C. All communications of scientific investigations or research shall be routed through these channels, even though other channels are employed at the same time.

(c) Official means of intercommunication, such as memorandums, bulletins and the like, between Bureaus of the Army and Committees for research shall be developed to such a degree of efficiency by the Research Information Committee that the distribution of information shall be practically automatic.

(d) Before sending officers or civilians abroad for investigation work, all Army Bureaus or civilian research committees shall get in touch with the Research Information Committee at Washington, D. C. for information and guidance.

(e) The present method of routing information memoranda for file and distribution through the Military Intelligence Branch will not be discontinued.

(f) You will immediately notify this office and the Research Information Committee of the name of the officer who shall represent your Bureau before the Research Information Committee.

By order of the Secretary of War:
(Sgd.) Paul Giddings, Adjutant General.

In accordance with the principles embodied in this order of the Secretary of War, the Organization of the Research Information Service has been expanded to include official representatives of all the Military and Naval Bureaus, together with the more important Government civilian bureaus and committees. The present organization of the Washington and foreign offices is given below. A meeting of the Washington representatives was held on August 29, when plans for perfecting the operation of the service were developed.

PRESENT ORGANIZATION OF RESEARCH INFORMATION SERVICE

COMMITTEE IN CHARGE:

The Chief of the Military Intelligence Branch, Brig.-Gen. Marlborough Churchill.

The Director of Naval Intelligence, Rear-Admiral Roger Welles.

The Chairman of the National Research Council, who acts as general executive officer of the Information Service.

WASHINGTON BRANCH:

Officers:

Executive Secretary, Dr. Graham Edgar.

Representative of Physics and Engineering.

Representative of Chemistry and Chemical Technology, Dr. Graham Edgar.
Representative of Medicine and Related Sciences, Dr. R. M. Pearce.

Representatives of Military Bureaus:
Division of Military Aeronautics, Capt. A. Ames.
Military Intelligence, Capt. P. M. Buck.
Bureau of Ordnance, Major C. J. Brown.
Quartermaster General, Major W. F. Dodd.
Office of the Signal Corps, Capt. G. F. Gray.
Chemical Warfare Service, Major S. P. Mulliken.
Tank Corps, Capt. Phil D. Poston.
Engineer Corps. Capt. L. D. Rowell.
Office of Surgeon-General, Col. F. F. Russell.
Bureau of Aircraft Production.

Representatives of Naval Bureaus:
Office of Naval Intelligence, Lieut.-Com. H. H. Whittlesey.
Bureau of Steam Engineering, Lieut. M. Pendleton.
Bureau of Construction and Repair, Capt. W. G. Du Bose.
Operations-Aviation, Ensign A. F. Lippmann.
Bureau of Ordnance, Ensign C. L. McCrea.

Representatives of Civilian Bureaus:
War Industries Board.
Bureau of Standards, F. J. Schlink.
Bureau of Mines, Dr. F. G. Cottrell.
Bureau of Chemistry, Dr. H. D. Gibbs.
Explosives Investigations Committee, Dr. C. E. Munroe.
Nitrate Investigations Committee, Dr. John Johnston.
National Advisory Committee for Aeronautics, Dr. W. G. Sabine.

Representatives of Foreign Missions:
British Embassy and War Missions.
French Embassy and War Missions.
Italian Embassy and War Missions.
Japanese Embassy and War Missions.
Canadian War Mission.

LONDON BRANCH:
The Military Attaché.
The Naval Attaché.
Scientific Attaché, Dr. H. A. Bumstead.
Engineering Associate, Dr. S. J. Farnsworth.
Chemical Associate.

PARIS BRANCH:
The Military Attaché.
The Naval Attaché.
Scientific Attaché, Dr. W. F. Durand.
Physics Associate, Dr. K. T. Compton.
Chemical Associate.
Medical Associate, Dr. R. G. Perkins.

ROME BRANCH:
The Military Attaché.
The Naval Attaché.

Scientific Attaché, Mr. S. L. G. Knox.
Physics Associate, Dr. Edgar Buckingham.
Chemical Associate, Dr. H. S. Washington.

In this field there will necessarily be close cooperation between the National Research Council and the National Engineering Societies, already well begun through the acceptance of the offer of the American Society of Mechanical Engineers referred to in the address so frequently cited. The policy of the Information Service will be to render available to accredited persons all sources of information relating to research, both at home and abroad. Its chief function at present will relate to the war; but this naturally includes extensive duties of an industrial nature, in addition to more strictly military and naval work. Through the scientific attachés at the various embassies, the Army and Navy Intelligence Services, and the officers of the scientific and technical bureaus of the Government, and through various other agencies with which the National Research Council is in touch, a large collection of valuable information will be brought together and collated for easy reference.

International Cooperation in Research

The work of the Research Information Service, which has already led to the establishment of the position of Scientific Attaché by the State Department, is part of an extensive plan for international cooperation in research which is being developed by the National Academy of Sciences and the National Research Council. A detailed plan for cooperation among the Allies in all researches bearing on the war has been prepared by the Council of the National Academy, for submission at a meeting soon to be held in London, at which the United States will be represented by a delegation appointed by the National Academy.

It is evident that each of the National Engineering Societies, in addition to its special reasons for securing effective international cooperation in its particular field, has broader interests that necessarily involve joint action with the representatives of other branches of science and the arts. The plan prepared by the National Academy provides a means, through the Section of Foreign Relations of the National Research Council, by which such joint action can be arranged for. While the time is not yet ripe to enter into the details of the scheme, it is worthy of mention here because of its bearing upon the subject of this paper.

Industrial Research

I may conclude this paper with a brief reference to the common interests of the National Engineering Societies and the National Research Council in the promotion and organization of industrial research, already mentioned in my New York address. The members of the Advisory and Active Committees of the Industrial Relations Section of the National Research Council dined together in New York on May 29. Among the speakers who strongly emphasized the importance of promoting industrial research were Hon. Elihu Root, Mr. Theodore N. Vail, Col. J. J. Carty, Mr. Ambrose Swasey, Dr. Henry S. Pritchett, Mr. Pierre S. duPont, Mr. George Eastman, Mr. Arthur H. Fleming, Dr. L. H. Baekeland, President Richard C. Maclaurin, Dr. M. I. Pupin, and Dr. Willis R. Whitney. Mr. Theodore N. Vail was elected Chairman of the Advisory Committee, and it was decided to organize the work of the Section and to begin the publication of a series of bulletins on the value of research and the advantages resulting from the establishment of research laboratories. The Active Committee, of which Dr. John Johnston is Chairman, has stimulated the organization of several successful conferences on research in the industries, and the outcome of its work is very promising.

Here is a field where the Engineering Societies and the Research Council can cooperate to special advantage through the Engineering Foundation, which is already taking an active part. The possibilities of developing this work, through the establishment of special laboratories and by other means, are obvious, and advantage will be taken of the present exceptional opportunity to influence favorably the industries which have hitherto failed to appreciate the value of research.

To be presented at the 343d Meeting of the American Institute of Electrical Engineers, New York, November 8, 1918.

ELECTRIC MOTORS IN THE CEMENT INDUSTRY

BY R. B. WILLIAMSON

ABSTRACT OF PAPER

This paper has been compiled by the Committee on Industrial and Domestic Power to give general information regarding the different classes of machinery used in the cement industry and the sizes and types of motor best adapted for the work. The paper gives, first, a brief description of the process. This is followed by an outline of the various kinds of machinery used together with data as to power requirements. The types of motor best suited to each application together with starting characteristics, overload capacity, torque and other features are indicated.

PRACTICALLY all modern mills manufacturing Portland cement are driven by electric motors, and since the process is mainly one of crushing and grinding, a large amount of power is required.

Before considering the various motor applications, it may be advisable to give a brief outline of the process of manufacturing Portland cement, which constitutes about 99 per cent of the cement of all kinds manufactured in the United States.

GENERAL DESCRIPTION OF PROCESS

Portland cement is generally formed by an artificial combination of calcareous and argillaceous materials. These materials are mixed either before grinding or during the process of grinding. After being ground so that from 80 to 90 per cent of the material will pass through a screen having 200 meshes per lineal inch, it is burned to incipient fusion in a rotary kiln, after which the resulting clinker is again ground so that at least 78 per cent will pass the 200-mesh screen.

The Portland cement clinker, after it leaves the rotary kiln, requires no further treatment than grinding, excepting the addition of about 2 per cent of gypsum to retard the setting. In some parts of the United States natural cement rocks are found which contain nearly the proper proportions of materials to produce Portland cement; but, even in these localities it is generally necessary to add either limestone or shale in order

to get the proper mixture. Certain deposits of cement rock so nearly approach the proper proportions of materials, that at one time limestone must be added and at another time shale.

In the United States, limestone furnishes the largest supply of calcareous material for Portland cement. Other sources of supply for this class of material, in addition to cement rock mentioned above, are marl, chalk and alkali waste. Shale and clay are most commonly used for obtaining the argillaceous material for Portland cement. Cement rock mentioned above, as well as blast furnace slag, are also used as a source of argilla-

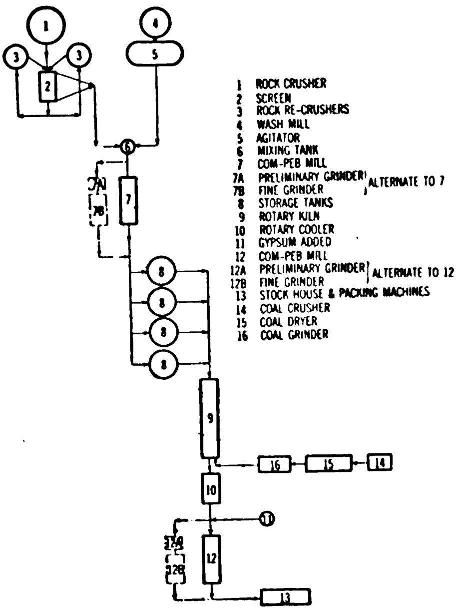


FIG. 1

ceous material. Blast furnace slag, like cement rock, contains a considerable proportion of lime and has the further advantage that this lime is found as calcium oxide instead of the carbonate; therefore, the heat necessary to dissociate the carbon dioxide gas from this part of the lime is not required.

The preliminary crushing of limestone, cement rocks and shale is ordinarily done in the crushers of the gyratory, jaw or roll type. Materials may be ground either in the wet or dry state. Fig. 1 shows a typical flow sheet for a wet mill and Fig. 2 for a dry grinding mill. In a dry grinding plant, the two classes

of materials are preferably dried separately and then weighed, after which they are ground. In a wet grinding plant, the materials are generally measured or weighed in their natural state as excavated, and then ground. Where shale or cement rock is used as the argillaceous material, it must be crushed in rock crushers. In the past, it has been the practise in certain wet grinding cement plants to dry the clay in order better to store and handle it, and also to eleiminate gravel and other foreign material. The better practise, however, is to dump the clay in its natural state into a wash mill where sufficient water is

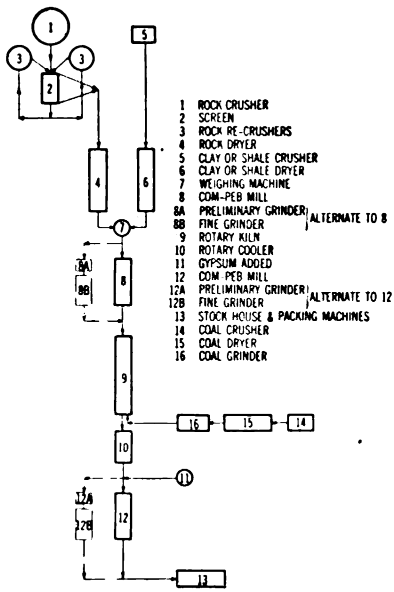


FIG. 2

mixed with it to allow the heavy pieces of gravel to settle out, and also to form a slurry which can easily be stored in tanks. The flow sheet, Fig. 1, for a wet grinding cement plant shows a wash mill installed for handling the argillaceous material. Wash mills are used only for clays, marls and similar materials, which, by mixing with water, can be kept in suspension. If shale were used as argillaceous material, a crusher of some type would have to be provided instead of a wash mill.

The same general type of grinding machinery can be used for either wet or dry grinding. In the past, it has been customary to use a preliminary and a finishing grinder; but, the latest

development in cement practise is to use a combination mill which combines the preliminary and finishing grinder in one machine, eliminating considerable expense in cost of installation and reducing materially the cost of operation. The combination mill has its greatest advantage in eliminating one set of storage bins and a great deal of elevating and conveying machinery. Combination mills consisting of a preliminary compartment charged with steel grinding balls and a finishing compartment charged with flint pebbles were tried many years ago, but mills of this type have never been successful until the introduction of the iron grinding body in the finishing compartment. The introduction of iron grinding bodies in the finishing compartment has also increased the capacity of mills of a given size, resulting in greatly simplifying the design of a cement plant, also reducing the cost of installation and operation.

While most mills have in the past been arranged for dry grinding, there is a strong trend towards wet grinding in new installations. The principal advantages of the wet process are, cleanliness, less tendency for different materials to segregate, and better opportunity to correct the mixture to obtain the exact proportions desired.

In the early days of the manufacture of Portland cement, the ground raw material was made into briquettes and then charged into a vertical kiln and burned. This process of burning was more economical in fuel consumption than rotary kilns, except of the very latest type; but had the disadvantage that a great deal of the material was wasted on account of being improperly burned and the labor cost was very high. In the United States, fuel was comparatively cheap and the cost of labor high, and, as might be expected, the rotary kiln is an American invention. The improvements made in the rotary kiln have resulted in such saving in the cost of burning cement that it is now generally used throughout the world.

The clinker leaves the rotary kiln at a high temperature, and before grinding it must first be cooled. Most modern cement plants use a rotary cooler after the kiln. The cooler is preferably placed below the kiln in order to conserve the sensible heat of the clinker, the air for cooling the clinker passing up into the kiln to support combustion. In addition, however, to cooling the clinker, it is desirable to age it before grinding, as aging not only improves the quality of the cement, but also increases the "grindability" of the clinker. The aging of cement

can either take place before grinding or after grinding; but, the cement can be more cheaply stored in the condition of unground clinker than after it is ground.

The grinding of the clinker into finished cement requires more power than any other step in the process, as the clinker ordinarily is much harder to grind than the raw materials. It is common practise in American cement plants to grind the raw material and the clinker to about the same fineness. There is at the present time, a tendency to demand more finely ground cement, specifications for Portland cement in the United States having been changed in 1916 from 75 per cent through 200 mesh to 78 per cent through 200 mesh. There is a greater tendency, however, to grind the raw material extremely fine. There are three advantages in the extremely fine grinding of raw material; fuel is saved, a better cement is produced, and the clinker from finely ground raw material is more easily ground than clinker from the same material not so finely ground.

Natural gas and fuel oil are used in a few cement plants, but the most commonly used fuel for burning Portland cement is finely pulverized bituminous coal. Bituminous coal is crushed in a preliminary crusher, or the screenings from the coarse coal are used, after which it is passed through a rotary dryer and then ground so that about 85 per cent passes through a 200 mesh sieve. Finely pulverized coal is blown into the rotary kiln by an air blast; the pressure used varies greatly but is generally from two to six ounces.

General Conditions Affecting Motors

The same reasons that hold for the preference of a-c. to d-c. motors in general industrial work make the a-c. motor more desirable for cement mill operation. There are two main reasons for this. First, the relatively higher alternating voltage that can be used, *i. e.* 440 or 550 volts or even 2200 volts for some of the motors in large mills; and second, the greater mechanical simplicity of the a-c. motor, particularly that of the squirrel cage type. The great majority of cement plants use alternating current, though there are a number of mills equipped with d-c. motors which have given excellent service. There is nothing inherently injurious about the action of cement dust on the commutators of d-c. machines, and they are successfully employed in some plants, but, all in all, the a-c. motor is preferred and used. The following features should be borne in mind in selecting motors for a cement mill.

Service. Continuous. Shut down for repairs only.

Starting Conditions. Severe on many applications and making phase-wound-rotor motors preferable or even essential.

Rating. May be affected by heavy accumulations of dust on the motor windings, thus interfering with ventilation. Machines should be frequently blown out and cleaned off.

Bearings. Should also be made dust proof by the use of felt and steel washers, as cement has a highly abrasive action and greatly increases bearing wear when present in the bearings.

Outboard Bearings. It is good practise to supply outboard bearings for belted motors of 75 h. p. and larger. Pulleys larger than usual in both diameter and length are often specified since the cement dust causes more than normal belt slippage and makes necessary tighter belts and increased strain on bearings.

Drive. Motors should not be direct geared to crushing machinery on account of excessive and severe vibration. Low speed motors direct-connected through flexible couplings, or belted, are the proper application. So far as possible, motors and belts should be placed in a separate room for protection from dust and to prevent their agitating the dust in the room where the mills are located.

Type of Motors. Alternating-current preferred; squirrel-cage where starting conditions are fairly easy, and wound-rotor motors where starting is severe. Low- or moderate-speed motors should be used. On account of the unfavorable belt conditions large speed reduction ratios should be avoided. Moreover, low- or moderate-speed motors are more substantial mechanically than high-speed and give less bearing trouble.

Collector rings are not injured by cement dust, which, when dry, does not show the abrasive action that it does in the bearings. It is advisable, however, to protect collectors so that dust cannot settle on them in large quantities.

While the induction motor has been used almost exclusively for cement plant operation, there are indications that synchronous motors will be adapted to this work more in the future. This is specially so in the case of low-speed motors for geared tube mills as mentioned later.

Frequency. Frequencies of 60 cycles and 25 cycles are in common use. Most of the large plants that are operated in connection with steel mills have 25-cycle equipment since they usually get their power from the same source as the steel mill, and 25 cycles has been generally used for steel mill operation.

A few plants are in operation at frequencies other than 25 or 60 cycles but these are on the whole exceptional.

Voltage. 440 volts preferred to 220 on account of copper distribution; 550 volts is satisfactory in a dry process plant. In some large mills 2200 volts has been used for the large motors but this practise is not common.

Load Factor. May be as high as 95 per cent since the machines run at all times under constant full load. The load factor of the average complete cement plant runs from 65 to 75 per cent taken through the entire year and is probably the highest of any industry.

The various kinds of machines that may be employed for the different processes in a cement mill may be grouped as follows:

1. Elevator and conveying machinery for raw material.
2. Crushers.
   Gyratory   Jaw   Roll.
3. Intermediate crushers
   Roll   Hammer mills.
4. Rotary Dryers.
5. Preliminary Grinding Machinery.
   Ball mills   Rolls
   Ring-roll mills   Huntington mills
   Hammer mills   Griffin mills.
   Fuller mills
   Raymond mills.
6. Finish Grinding Machinery.
   Tube mills
   Combination mills.
7. Conveying Machinery for Finished Product.
8. Rotary kilns.   Coal feeding machinery.
9. Cement Grinding Machinery.
   (a) Preliminary grinding
       Ball mills
       Rolls
       Griffin mills
       Ring roll mills.
   (b) Finish grinding
       Tube mills
       Ball peb mills
       Combination mills.
10. Coal Grinding and Drying Machinery. Same class of machinery as used for raw material.

ELEVATORS AND CONVEYERS

Every cement plant requires a large number of elevators. and conveyers for handling the material between the various

steps in the process. One modern 3000-barrel plant, as an example, has 14 elevators, 2 belt conveyers and 8 screw conveyers in its equipment. It is very important that these auxiliaries be kept operating, as an accident to a conveyer or elevator may shut down an entire department of the plant until repairs can be made.

Various methods of driving this class of machinery have been installed in different mills. The chief difficulties are that the driving shafts run at extremely low speeds and that elevators must be driven from the top. The first necessitates a number of countershafts and belts or gears to give the necessary speed reduction and the second means that the driving motor, if individual drive is used, must often be located in an inaccessible place and in many cases be subjected to considerable vibration due to weak foundations. A very good method of drive which is sometimes applicable, especially in the case of crushers or group drive of grinding machinery, is to belt the elevators and conveyers from a pulley on some part of the mill shafting or from a small pulley on the front end of the motor.

Another method adopted in some plants is to group several elevators and conveyers on one motor, driving through countershafts and belts or chains.

The third possibility is to put an individual motor on each conveyer and elevator. In applying individual motors to this class of work, however, there are certain points which should be borne in mind if trouble is to be avoided. The machinery is rather roughly built, especially bucket elevators and bucket and drag type conveyers. These usually consist of two parallel chains driven by rough cast or forged sprockets, and carrying the buckets between them. It sometimes happens that the chains will climb up on the teeth of the sprockets and throw heavy overload on the driving motor which may be enough to stall it. Screw conveyers occasionally wear their bearings down so far that the screw may rub against the trough for most of its length with consequent overload. These possibilities make it advisable, in the case of individual drive, always to install motors from 25 to 50 per cent larger than necessary to meet the average power demand. This is not true when group drive is used as it is very unlikely that more than one unit of the group would be subjected to overload at the same time and this would probably not affect the operation of the motor to any extent. At the same time, group drive has the

disadvantage or requiring more belts and countershafts and of the probability of crippling an entire department if an accident happens to any unit or any part of the transmission.

The lengths and types of elevators and conveyers differ so much with different plants that it is impossible to give a table which will be of much assistance in estimating the power required, but the following test results will probably be of some service in such work.

Screw conveyer on dry clay.	
Diameter	16 in.
Pitch	16 in.
Length	120 ft.
Speed of screw	65 rev. per min.
Horse power	5.5
Screw conveyer on pulverized coal.	
Diameter	16 in.
Pitch	16 in.
Length	162 ft.
Speed of screw	55 rev. per min.
Horse power	3.7
Belt conveyer on finished cement.	
Width	20 in.
Length ... between centers	230 ft.
Height material is lifted	40 ft.
Speed	200 ft. per min.
Horse power	10–14
Belt conveyer on crushed stone.	
Width	20 in.
Length ... between centers	135 ft.
Height material is lifted	22 ft.
Speed	211 ft. per min.
Horse power about	5
Elevator on crushed stone.	
Buckets	18 x 10 x 16 in.
Height material is elevated	52 ft.
Speed	83 ft. per min.
Horse power	3 to 5.5

Elevator same as preceding except elevating to 60 feet instead of 52 feet. Horse power 2 to 3 (probably due to better lubrication.)

Elevator on crushed stone.	
Buckets	48 x 8 x 12 in.
Height material is raised	62 ft.
Speed	95 ft. per min.
Horse power	8 to 10 (occasional peaks above 15)

The starting torque of elevators and conveyers is likely to be above normal running torque, but standard squirrel-cage

motors, if large enough to take care of the peak loads as mentioned above, should have no difficulty in starting them.

CRUSHERS

All dry process cement mills require extensive crushing plants for their limestone cement rock. Three types of crusher are in common use for this work. Gyratory crushers are used more than any other, although those of the jaw type, or crushing rolls, are preferred in some cases.

Gyratory Crusher. The gyratory crusher has a vertical shaft with a bearing at the top and bottom. The shaft is driven by a bevel gear and pinion at the lower end. The shaft is eccentric with respect to the bottom gear and journal so that the shaft gyrates as the gear revolves, the amount of side movement being a maximum at the bottom and a minimum at the top. Attached to this shaft is a conical crushing head of chilled iron or special steel, sometimes smooth and sometimes corrugated. The shell or mouth of the crusher surrounds the crushing head and is lined with special iron or steel plates. The shell is conical but inverted so that the distance between its surface and the crushing head is a maximum at the top and a minimum at the bottom. As the shaft rotates, its gyratory motion causes the head to alternately approach and recede from the shell so that the large pieces of rock which are fed in at the top are gradually broken up and fall lower and lower in the shell until the pieces are small enough to fall out through the space between the bottom of the head and shell on to a sloping diaphragm between the bottom of the shell and the driving gear, which directs the crushed pieces into the discharge spout.

In all cement plant crushing, the problem is to reduce the rock from the size as it comes from the quarry down to a size small enough to pass an inch or a ¾ inch ring. It has been found that it is better to divide this range into two steps instead of making the entire reduction in one machine. Therefore, in most crushing plants, the rock first passes through a large crusher which reduces it to three- or four-inch cubes, after which it is reduced to the required size (usually ¾ to 1 inch) in smaller crushers. Occasionally, revolving screens are installed to grade the material as it comes from the crusher, all that which will not pass through the screen being returned for further reduction.

There is a considerable difference of opinion regarding the

size of the first crusher. The rock as it comes from the quarry is made up of pieces of various sizes ranging from small chips up to pieces three or four feet in diameter, the only limit being what can be handled by a steam shovel. The great majority of the pieces, however, are under 20 inches. Some cement manufacturers install for the first crusher a machine large enough to take the largest pieces, while others use a crusher large enough for the average size and break up the extremely large pieces with dynamite before taking them to the crusher house. The advantage of the first method is that it eliminates this dynamiting, thus saving labor and expense in the quarry. Its disadvantage is that a crusher large enough to take care of the few exceptionally large pieces will probably have a much greater capacity than necessary so that it will be idle a great deal of the time, and will represent considerably more investment than a smaller machine, as well as a greater consumption of power.

The size of the crushers is designated by an arbitrary number. Practically the same system is used by the different manufacturers, so that a No. 8 crusher, for instance, is about the same size capacity, etc., whether one make or another; Table 1 is therefore approximately correct for all makes. As a No. 3 crusher is the smallest which is likely to be used in any cement plant, data are omitted on number 0, 1 and 2.

TABLE 1.
Power Requirements of Gyratory Crushers.

Makers No.	Size of opening on each side of top bearing arms	Size cube which can be crushed	Horse power
3	7 × 28	7	10– 20
4	8 × 34	8	15– 25
5	10 × 40	10	20– 35
6	12 × 44	12	25– 40
7-½	15 × 55	15	50– 70
8	18 × 68	18	65–100
9	21 × 76	21	100–140
10	24 × 99	24	125–175
18		36	150–200
21		42	150–200
24		48	175–225
27		54	200–250

The horse power figures given in Table 1 must be used with discretion because there are a number of factors which affect the power required by crushers. Chief among these are the hardness of the material to be crushed and the method of feeding. On soft rock or shale the power will be lower than when crushing very hard stone, the difference being about as indicated by the

two limiting powers in the above table. When crushers are fed intermittently as for example the very large crushers which take the material when it first comes from the quarry and which are of such great capacity that they can run a small carload of rock through in less than a minute, after which they may run light for several minutes, the driving motor can be somewhat smaller than for crushers of the same size fed from bins where the power is more nearly continuous. It is the usual experience that crushers are "over motored", and the horse power data given in the above table are more likely to be high than low.

Belt drive is used almost exclusively, an incidental advantage being that the belts afford some protection against damage in

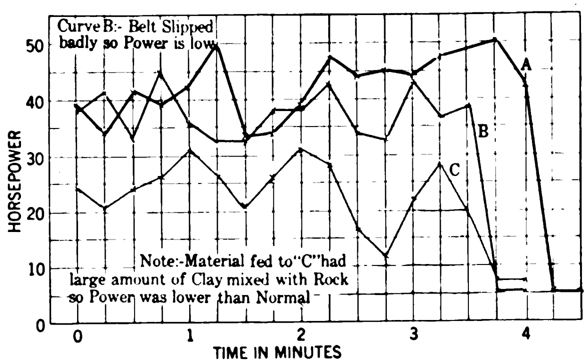


FIG. 3—TESTS ON THREE NO. 5 CRUSHERS—EACH DRIVEN BY 25 HORSEPOWER 600 REV. PER MIN.
A, CRUSHER WITH CORRUGATED HEAD—1½ IN. PRODUCT
B, " " " " —1¼ IN. "
C, " " SMOOTH " —1 IN. "

case pieces of iron etc., get into the rock accidentally. Installations seem to be about evenly divided between those using individual motors and those in which the entire crushing plant including possibly three or four crushers with their elevators and conveyers are driven in a group from one large motor.

The starting torque required by a gyratory crusher is small unless it is accidentally stopped with material in it, in which case, it is usually necessary to dig it out before it can be started.

Typical load curves on crushers are shown in Figs. 3 and 4. The former illustrates the difference that may exist between the power required by crushers of the same size due to the quality of the material.

JAW CRUSHERS

In jaw crushers of the Blake type the crushing takes place between the two jaws set at an acute vertical angle, one fixed and one movable; the feed opening at the top being greater than the discharge at the bottom, gravity brings the material into the crushing zone and discharges it when crushed. The movable jaw is suspended from the top of the crusher frame and is given a reciprocating motion by means of an eccentrically operated vertical pitman and a pair of toggle plates.

The most favorable condition for jaw crushers is obtained when the ratio of size of feed to product does not exceed 6 to 1. When this ratio is exceeded, the efficiency of the crusher is greatly decreased, and an increase in the power requirement will be noted. With a ratio of 6 to 1, the power requirement remains practically constant whether the reduction is from 18 in. to 3 in.,

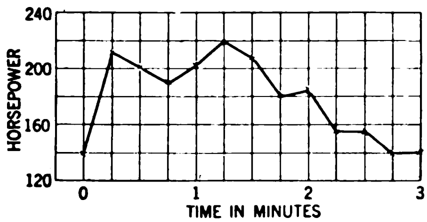


FIG. 4—POWER REQUIRED TO DRIVE MAMMOTH CRUSHERS WHILE CRUSHING ONE CARLOAD OF ROCK—FEED AVERAGE SIZE FORM QUARRY —REDUCES TO 4 IN.-5 IN. SIZE

or from 6 in. to 1 in. The amount of material crushed, however, will be in proportion to the width of the discharge opening, *i. e.*, when reducing from 18 in. to 3 in., the crusher will handle approximately three times as much material as when reducing from 6 in. to 1 in., all other things such as size of feed openings, speed, etc. remaining constant. Any increase or decrease within certain limits in the speed of the crusher will increase or decrease the volume of material handled and the power required.

The power required to operate any given size of jaw crusher, depends on the power required to operate the crusher when running idle and the power required to do the work of crushing the material.

Consider these in the order as given above: The power required to operate a crusher when it is running idle is due principally to friction, and this will vary considerably for different

sizes, speeds and makes of machines. When figuring the power requirements of a jaw crusher, it would be preferable to obtain the friction of the crusher direct from the manufacturer.

For the Blake crusher, there is one theoretically correct speed based on the law of gravity. The time period of the stroke of the movable jaw must not exceed the rate of movement of the material due to gravity. Therefore, there is one theoretical speed for all sizes of crushers, but it is obvious that there is a mechanical limitation when a certain size of crusher is reached that will necessitate the use of a lower speed.

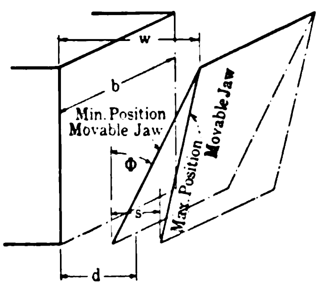


FIG. 5

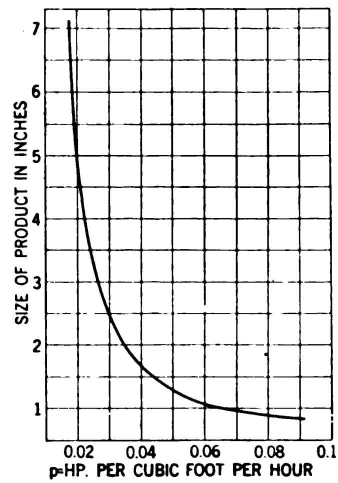


FIG. 6

This curve is drawn on the assumption that the size of the product will never be less than 1 inch and that the reduction will never be more than 6 to 1.

An approximation of the power required to operate the crusher when it is doing no work is

$$p_m = 0.016 \times b \times w$$

where

p_m = horse power
b = breadth of opening of jaws in inches. See Fig. 5
w = width of opening of jaws in inches. See Fig. 5

Power required to crush the material.

The amount of material M passed by the crusher is*

$$M = \frac{sb\,(2d + s)\,R}{57.6 \tan. \phi}$$

Where M = cubic feet of material per hour.
s = stroke in inches measured at the throat of crusher. See Fig. 5

*Wiard, "Theory and Practise of Ore Dressing."

d = average distance in inches between the fixed and movable jaws and the minimum and maximum position of the swing jaw as measured at the throat of the crusher. See Fig. 5.

b = breadth in inches at opening of jaws. See Fig. 5

R = revolutions per minute.

The angle ϕ varies for different makes of crushers from 20 deg. to 26 deg.

The above formula gives the theoretical capacity and is the one used in figuring the power requirements. In order to obtain the actual capacity, the theoretical capacity should be divided by 1.6. This is on account of irregularity in feed, hardness of material, size of material, etc.

In the smaller sizes of crushers, where the breadth b is small and the stroke s is large, comparatively, it is found that the theoretical capacity M is as much as 3¼ times the actual capacity. This is due to the fact that these smaller sizes of crushers are very inefficient when compared to the larger sizes; this is found for example to be the case of the 10 in. x 7 in. machine as shown in Table 2. Therefore, the empirical formulas as herein given will not apply to these small crushers.

Power required to crush the material (dry limestone)

$$p_c = p\,M$$

where p_c = horse power.

p = take from curve—Fig. 6.

It is assumed that the size of the product will never be less than one inch and that the reduction will never be more than 6 to 1.

The total power required to operate the crusher when doing work

$$P = p_m + p_c$$

Where P = horse power.

The above formulas have reference only to the crushing of dry limestone. The weight of one cubic foot of broken dry limestone is approximately 100 lb.

As the starting conditions are usually severe, the slip ring type of induction motor is invariably recommended to drive jaw crushers.

Fairmount Crusher

Many of the more recent crushing plants for cement works use the Fairmount type crusher. This machine consists of a single roll working against an anvil plate. The roll is provided

TABLE 2.

Power Requirements of Blake Crushers.

Dry Limestone.

Size or jaw opening b x w	Capacities in tons per hour											Approx. horse power req'd
	Size of product											
	¾ in.	1 in.	1½ in.	2 in.	2½ in.	3 in.	4 in.	5 in.	ϕ	s	R	
10 × 7 in.	1.5	2.5	4	5					26°	$5/8$ in.	275	7
15 × 9 "		6	8	10	12				26°	$7/16$ "	275	10
20 × 10 "			10	15	17.5	20.			26°	$1/2$ "	275	15
24 × 12 "				20	25	30.	35		26°	$9/16$ "	275	25
30 × 18 "					20	37.5	45	50	26°	$9/16$ "	275	40

with knobs from two in. to three in. in height for crushing against the anvil plate and with sledging knobs from three in. to four in. in height for breaking the large pieces of stone on top of the roll, which are too large to be directly gripped between the roll and the anvil plate. These machines have a very large capacity when operating continuously, but are not objectionable for small capacity on account of their very low friction load when running idle.

The Fairmount type crusher is built in sizes 24 × 36 in., 24 × 48 in. and 24 × 60 in., also 36 × 60 in. and 60 × 84 in., in each case the size being referred to diameter and face of the roll. The size which has so far been used in cement plants is 36 in. diameter by 60 in. face. This size of machine ordinarily requires a 150-h. p. motor to drive it and it has a capacity of from 200 to 500 tons per hour, depending upon the character of the material to be crushed. One of the particular advantages of the Fairmount type crusher is that it makes a very large reduction in one operation so that the number of secondary crushers is reduced and the product from the primary crusher being smaller can be much more easily handled into and out of storage bins.

The starting torque required for the Fairmount crusher when idle, is very low so that squirrel-cage motors are often used. While it is not possible to start the crusher full of stone even with slip ring type induction motor, the latter will give sufficient torque to start up the crusher without entirely cleaning it out, if it should happen to stop under load. Slip ring type motors are therefore, recommended for this class of work.

Crushing Rolls

The most favorable condition for crushing rolls is obtained when the ratio of size of feed to product does not exceed 4 to 1. When this ratio is exceeded, the efficiency of the roll is greatly decreased and an increase in the power required will be noted. With the ratio of 4 to 1, the power requirement remains practically constant whether the reduction is from 4 in. to 1 in. or from 1 in. to ¼ in. The amount of material crushed, however, will be in proportion to the size, *i. e.*, when reducing from 4 in. to 1 in. the rolls will handle four times as much material as when reducing from 1 in. to ¼ in; all other things such as size of rolls, speed, etc., remaining constant. Any increase or decrease in the speed of the rolls will increase or decrease the quantity handled and the power required.

Due to irregularity of feed, variation in size and hardness of material, etc., it has been found that the power required is not steady but fluctuates as shown by Fig. 7 which is a power diagram of a 42 × 16-in. roll operating at 60 rev. per min. and reducing dry iron ore from a maximum of two inches to an average of one-half inch, the average power being 13.18 h. p.

Fig. 8 shows the relation existing between horse power per cubic foot per hour and different ratios of reduction assuming maximum size of feed material and average size of product, all friction of the rolls being neglected. This curve has been plotted as an average of points obtained from actual tests. It is to be noted, however, that while dry material has been rolled, most of the points indicate power readings taken while crushing iron ores, and that only one point refers to the rolling of dry

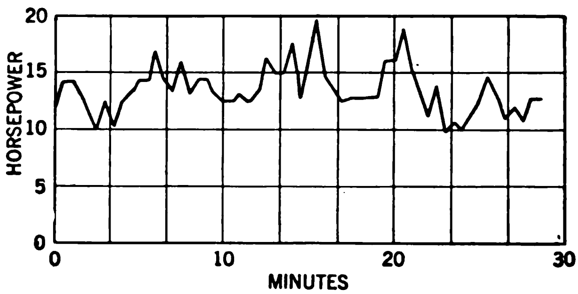


Fig. 7

limestone such as is used in the cement industry. As the iron ore is a harder substance than the limestone, it is reasonable to assume that the curve as drawn shows values slightly higher than would be obtained had more data been available on dry limestone.

The friction of crushing rolls, that is, the power required to drive the rolls at their normal speed and without load, varies considerably for different sizes, speeds and makes of machines, but an approximate average is obtained from the following:

$$P_1 = 0.0835\ (D + W)$$

where p_1 = horse power

D = diameter of rolls in inches

W = face of rolls in inches.

When figuring the power requirements of crushing rolls it is preferable to obtain the friction of the rolls direct from the manufacturers.

The amount of material passed by the rolls is

$$M = \frac{d \times W \times S}{144} \times 60$$

where M = cubic feet of material per hour
d = distance in inches from edge to edge of roll (See Fig. 9)
W = face of rolls in inches
S = peripheral speed of rolls in feet per minute.

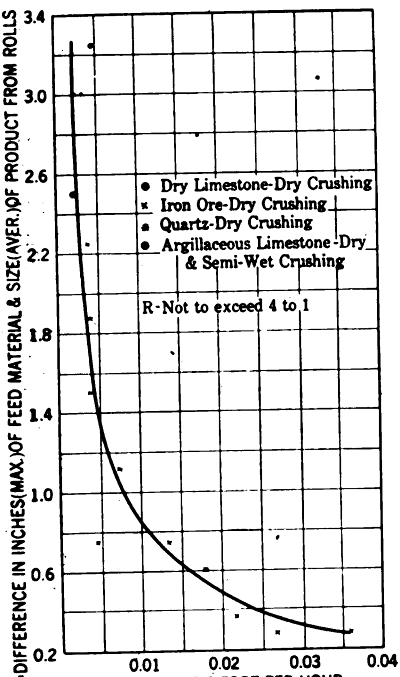


FIG. 8

The above formula gives the theoretical capacity and is the one used in figuring the power requirements. In order, however, to obtain the actual capacity, the theoretical capacity should be divided by 4. This is on account of irregularity in feed, etc.

The total power required to drive rolls for any different rolling condition is

$$P = (p \times M) + p_1$$

where P = total horse power
p = horse power per cubic ft. per hr. (Fig. 8)

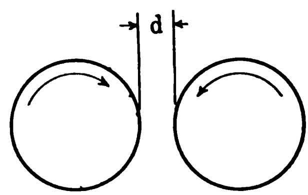


FIG. 9

As the starting conditions are not severe, the squirrel-cage induction motor makes a suitable machine for driving the rolls, provided the conditions are such that this type of motor is connected to a source of power where line disturbances are not objectionable. If line disturbances are objectionable, the slip ring type of induction motor should be used. When shutting down the mill, the usual procedure is to first cut off the feed to the rolls, allowing same to empty themselves, before closing down the motor. No difficulty, however, is likely to be exper-

ienced with the squirrel cage type of motor, if called upon to start the rolls when full of material and with the feed open, as even under these conditions the starting requirement is not severe.

Intermediate Crushers

Hammer Mills. These machines are made in several styles but all make use of the same principle, namely, the use of hammers revolving at high speed and abrading the material against a slotted iron casing. Unlike the crushing rolls, where a reduction not exceeding 4 to 1 is most favorable, the hammer mill is used for reducing material of from 2½ to 1 in. to 20 mesh or less, the maximum reduction in this case being 76 to 1. As the power required to drive this type of mill varies considerably, due to irregularity in feed, it is most important that this remain steady and therefore a feed regulator is highly desirable. In addition to this, any variation in the size and hardness of material, provided the feed remains fairly constant, will also cause power fluctuations which will however be within reasonable limits.

There is very little general power data available on hammer mills other than for a specific manufacturer's type, trade size and specific speed or speeds. The latter are supposed to be the most efficient speed at which the particular mill should operate. A general rule gained by experience indicates that in order to crush one ton of dry limestone in one hour, reducing from a maximum size of 2½ in. to 20 mesh, requires six to seven horse power. This would be the output of a motor and would include the friction load, in addition to the crushing load, of the mill. The actual capacity of any mill is proportional to the size of the product and to some extent to the character of the feed so that for different sizes of product on any given mill, the power requirements will be different.

Sturtevant Hammer Mills are of two types known respectively as hinged hammer and hammer bar mills. The hinged hammer mills are of the general type noted above under this class. The bar hammer mill differs in that there is only one set of hammers instead of six or more and the hammers themselves have an axial length the full width of the rotating spider which carries them. Both mills take material in pieces from 5 to 6 in. wide. The hinged hammer mill reduces from one inch to 20 mesh and finer. It is built in three sizes and operates at a speed of 1000 to 1500 rev. per min. The hammer bar mill is built in two

sizes and reduces from ½ in. to 20 mesh. A belted wound-rotor motor is the preferred application. Squirrel-cage motors have, however, proved satisfactory, particularly if used when line disturbances at starting are not objectionable. In any case, if a mill is shut down it will have to be clear of material before started again.

Hammer bar mills are made in two sizes, No. 0 and No. 1. The No. 0 mill will handle from 1½ to 4 tons per hour, depending on material, and requires from 8 to 10 h. p. The No. 1 handles from 4 to 12 tons per hour and takes 20 to 25 h. p.

Hinged hammer mills are made in three sizes 30 by 12 in., 30 by 18 in. and 30 x 24 in. The power varies widely with the material. The 30 x 12-in. machine, for example, might take a maximum of 30 h. p. or a minimum of 10 h. p. The wider machines take increased power in proportion to the width.

TABLE 3

Power Requirements of Hammer Mills.
(Allis-Chalmers)

Trade size	Rev. per min.	Hammer circle	Face	H. p.
No. 1	1800	24 in.	9 in.	15
" 2	1200	30 "	12 "	30 to 40
" 4	1200	42 "	24 "	75 to 90

The above requirements are based on crushing dry limestone, reducing from a maximum size of 2½ in. to 20 mesh with a constant feed of material best suited to the particular mill.

Williams' Jumbo Crusher. The machine of this make used for intermediate crushing is usually the No. 6 size. It takes stone three inches in size and under and reduces it to ¼ inch and finer. The capacity is 60 to 65 tons per hour, and the motor requires 150 h. p. The mill operates at from 720 to 750 rev. per min. It prepares material for some one of the various finishing mills.

Vulcanite Crushers. (Williams). The Vulcanite crushers are also swing hammer mills. They are built in eight sizes having capacities from two tons per hour at ¼-inch size up to 50 tons per hour at ½-inch size. The standard unit generally employed for cement work is No. 3. This takes material up to three inches and reduces it to ¼-inch for feeding ring roll mills. It handles 15 to 18 tons per hour, operates at a speed of 1000 rev. per min. and requires 50 h. p.

Universal Grinder (Williams). This is termed a tube mill feeder as it will take limestone 2½ inches and under after passing through the dryers and reduce it to a product, 95 per cent of which will pass a 20 mesh sieve. These are built in ten sizes, the two most commonly used in cement plants being the No. 3 and the No. 9. The No. 3 handles from 8 to 10 tons per hour under the above conditions, requires from 50 to 60 h. p., and should operate at a speed of 1100 rev. per min. The No. 9 size, used in plants requiring very large units, handles from 25 to 30 tons per hour, requires 175 to 200 h. p., and operates at a speed of 720 rev. per min. Wound rotor induction motors are the best application on all three mills. The preferred method of drive is by motor direct connected through a flexible coupling.

Sturtevant Rotary Crushers. This type of rotary crusher operates on the principle of the old fashioned coffee mill. It

TABLE 4.
Power Requirements of Rotary Crushers.

Machine size	Approx. capacity in tons per hour	Rev. per min.	Approx. h. p.
00	1 to 1½	300	1 to 2
0	½ to 2	250	3 to 4
1	1 to 6	300	6 to 10
1½	4 to 10	200	15
2	8 to 15	250	15 to 20

is shaped like an hour-glass or double cone, one cone mounted and reversed on the other. The coarse material is introduced at the top and is acted upon by a rotating cylindrical member which exerts a continual nipping action on the rock since the diameter of the stationary cone decreases as the material falls toward the center. Below the center the material is ground or shredded by the usual mill stone action. The lower portion of the rotating member is cone-shaped and so arranged as to leave a tapering annular space decreasing in size toward the bottom. As the material proceeds by gravity toward the bottom, it is ground and reduced in the annular space by the action of the rotating cone against the stationary one. Both these members are faced with hardened steel burrs or grinding members. These crushers are intended for moderately hard or soft rocks but are not used for hard cutting substances. They crush large rocks

to approximately ⅛ inch without screens in the smallest sizes and as coarse as 1 inch in other sizes. Their sizes, outputs and power requirements are as shown in Table 4.

A wound-rotor induction motor is the preferred application for these crushers but a squirrel-cage motor can handle them if designed with considerable margin in starting torque.

Rotary Dryers

Dryers are used to dry the crushed limestone which contains a considerable amount of moisture that must be driven out before the grinding operation can be performed. They are also used for the same reason, for drying the clay and coal. The dryer consists of a large revolving cylinder which is sufficiently inclined to roll the contents through by gravity. It is heated either by oil, natural gas, or by the waste gases taken from the kilns.

Auxiliary machinery necessary for the dryer, are fans and conveyers, and usually this auxiliary machinery is considered as integral parts of the dryer, and is driven by the same motor. For a fixed rate of feed the load is constant and as a rule the dryers are operated at a given predetermined speed; therefore, squirrel-cage motors are satisfactory for this kind of service. Squirrel-cage motors with 125 per cent to 175 per cent starting torque, are suitable.

Approximate Horse Power requirements are as follows:

Size of dryer	Horse Power	Recommended speed of motor.
6 ft. x 60 ft.	12	700 to 1200
6½ ft. x 60 ft.	15	700 to 1200

On some particular installations, the following was found. A dryer 6 ft. in diameter 45 ft. length, 9 rev. per min., connected to a 24-in. x 50-ft. horizontal pan conveyer and to a 14 in. x 35-ft. bucket elevator, required a 30 h. p. motor. A 7-ft. x 60-ft. dryer for rock crusher, connected to a 20 in. x 45-ft. inclined conveyer, and 16 in. x 40-ft. elevator, required 40 h. p.

Ordinarily, dryers of 5-ft. diameter and 50- to 60-ft. length, revolving at 3 to 5 rev. per min., were found to require between 10 and 20 h. p.

PRELIMINARY GRINDING MACHINERY

Ball Mills. The ball mill consists principally of a revolving horizontal cylinder lined with overlapping iron plates and containing a charge of steel balls. For the first charge, balls of

3½, 4 and 5 inches diameter are supplied, but under running conditions only the largest size are added from time to time to maintain the charge. These mills will take material up to 6 inches in size. The material as ground falls through perforations in the iron plates to the cylindrical screen or hopper which surrounds the mill, and the oversize rock falls back through the same perforations to the grinding plates. The fine product is removed by a conveyer below the machine and is generally of such fineness that all will pass a 20 to 30 mesh. When pulverizing to pass all through a 20 mesh, from 30 to 40 per cent will pass a 100 mesh sieve.

Ball mills are low-speed machines generally running from 20 to 25 rev. per min. The charge of balls revolving at this speed causes very severe vibration in the gearing, and motors geared

TABLE 5.
Power Requirements of Ball Mills.

Make	Weight of charge lb.	Capacity on cement clinker bbl. per hr.	H. p. req'd.
Gates No. 7	3,000	12–16	30– 40
" " 8	4,500	18–24	40– 50
Smidth (Kominuter) No. 53-½	3,000	.....	30
" " " 66	6,600	.....	75
" " " 88	....	.....	100
Krupp No. 7	3,520	12–16	30– 35
" " 8	4,400	16–22	35– 45
Chalmers & Williams	10,000	60–70	100–125

to their countershaft have proved unsatisfactory. Belted motors or low-speed motors for direct connection to the countershaft through flexible couplings give entire satisfaction. When starting, the charge of balls and stone must be rotated through nearly 90 deg. before it begins to roll, and the motor torque required at starting varies from 1¾ to 2 times full-load torque. Squirrel-cage a-c. motors specially designed for this high starting torque have been successfully applied but they have a lower efficiency than wound-rotor motors and make a large current demand at starting. The same starting conditions are found in tube mills and as the size of these has steadily increased of late years, and as the power required has also increased with the use of metallic grinding bodies in the mills, there has been a gradual change to the use of wound-rotor motors in place of the

high-torque squirrel-cage motor. Many of the latest mills are using wound-rotor motors for these grinders of large capacity and the saving in power due to increased efficiency, together with better performance at starting, warrants their use in preference to the squirrel-cage type. Table 5 indicates the power requirement for a few typical ball mills.

Hammer Mills. These mills which have been described above are used for preliminary grinding as well as for crushing.

Ring-Roll Mills

There are a number of different mills that may be classed generally as ring-roll. In these, the material is crushed by means of a ring or large balls rolling inside and against the ring or die. In some types this ring rolls also. There is enough difference in mechanical construction to require a brief explanation of each mill. Under this general type are classified the Kent mill, Griffin mill, Fuller mill, Bonnot mill, Raymond mill, Sturtevant Ring-Roll mill, Huntington mill and others.

These different types of mills are used in many combinations depending on the material and the locality in which they are employed. An analysis of about fifteen mills shows that ball mills are widely used as intermediate grinders taking material from the crushers on the raw side and the kilns on the clinker side and reducing it to a suitable fineness for feeding tube mills. In another combination the ball mills are replaced by ring-roll mills. Another combination uses hammer-mills to reduce the material to ¼ inch and smaller, after which it is crushed in ring-roll mills. In other plants the entire operation of grinding the clinker in the finishing mill is performed by ring-roll mills.

Fuller Mill. Three sizes of this mill are manufactured—33-inch, 42-inch and 57-inch. The smallest mill is best adapted for use on coal or similar materials while the two larger machines are adapted either to coal or harder material such as limestone, rock or cement clinker. This mill accomplishes both intermediate and fine grinding in one operation. It takes material up to ¾ inch in size and reduces it to a fineness of 85 per cent through a 200 mesh sieve. The mill is vertical in construction and the grinding is done by four chilled unattached balls rolling against a circular horizontal die. The balls are propelled by four equi-distant horizontal arms or pushers radiating from the vertical shaft. The shaft is driven at such a speed as to cause the balls to exert a pressure against the die of approximately

1600 pounds. The material is fed in at the top of the mill by a screw and enters the pulverizing zone. By means of fans and screws the material is removed as soon as it has reached the proper degree of fineness. The driving pulley is mounted at the bottom. Squirrel-cage induction motors are used very satisfactorily to drive these mills through belts running in a vertical plane. Special vertical motors have been developed for this application.

The output of these mills and the power required to operate them depends upon the weight and hardness of the material being pulverized and the fineness of finished product desired. On the 33-in. mill the output will vary from two tons per hour when grinding hard material, to four tons per hour when grinding soft material. The power will approximate 45 h. p. for hard material and 25 h. p. for soft material. The above figures are based on grinding to 100 mesh. On the 42-inch mill the output will vary from 4 to 10 tons per hour from hard to soft material, and the horse power will vary from 75 to 45 when grinding to 100 mesh.

The 57-inch mill when grinding raw cement material consisting of limestone and shale or clay or cement rock has a capacity of from 9 to 12 tons per hour and requires from 110 to 125 h. p. The same mill when pulverizing cement clinkers has a capacity of from 500 to 600 barrels per day and requires from 135 to 150 h. p. The performance of the 57-inch mill is based on grinding to 85 per cent through a 200 mesh sieve.

It should be borne in mind that the power required for the different machines as given in this paper represent average performances. It is possible under certain conditions to crowd all these machines thereby increasing their output and also increasing the power consumed so that specific instances are no doubt known to all engineers interested where the figures are somewhat higher than here given. These are matters that take care of themselves in practise.

Kent Mill. The Kent mill consists principally of a revolving ring and three rolls pressing against its inner face. The rolls are convex and the ring is concave and tracks on the rolls. Springs support the rolls yieldingly and the rolls support the ring so that the four crushing parts are free to move. The material falls from the inlets onto the inner face of the ring, and centrifugal force holds it there in a layer an inch deep as it passes under the rolls. The latter are pressed by the springs

outwardly against the rock or the ring with a pressure adjustable to 20,000 pounds, the adjustment being accomplished by means of the screws against the springs. As the rolls pass over the rock they crush it against the ring, while the crushed rock falls off each side of the ring into the casing and then into the discharge. It is claimed that 90 per cent of the rock is abraded on itself in crushing so that the wear on the parts of the machine is materially decreased.

The torque required to start this mill does not exceed full-load torque, since the moving parts are comparatively light and there is only a small amount of rock in the machine at a time. The pull-out torque of the motor supplied should be comparatively high as large pieces of rock passing between the rolls and ring will cause severe momentary overloads.

In order to obtain a fine and uniform product, the ground material is usually elevated from the discharge to a screen or separator above the mill and the oversize is returned to the feed. The elevator and separator will add from 7½ to 10 h. p. to the figures given below for the mill alone. The feed should preferably pass through a one-inch ring but anything up to two inches can be handled. The finished product will all pass a 20 mesh.

These mills are made in three sizes known as the Kent mill, the "Maxecon" and the "Big Maxecon" mill. Their capacity depends on the hardness of the material and the fineness to which it is ground. Round approximations might be 25 barrels per hour for the Kent, 35 for Maxecon and 50 for Big Maxecon. These mills operate at 200 rev. per min. and the usual sizes of motors are 30, 40 and 50 h. p. for the Kent, Maxecon and Big Maxecon respectively. When grinding clinker one h. p. per barrel is the average power consumption to pass 20 mesh; also the product will approximate 50 per cent through 100 mesh and 35 per cent through 200 mesh.

Raw rock varies so in hardness that capacities range from 7 to 16 tons per hour, and power estimates from five h. p. per ton for the hardest to three h. p. for the softest.

Sturtevant Ring-Roll Mill. This ring-roll pulverizer consists of a steel anvil ring or die secured in a head supported and revolved by a horizontal shaft. Against the inner face of this ring the three hammer rolls are equally and elastically pressed with great force and revolved by the ring. Substances to be ground are fed to the anvil face of the rotating ring and held thereto by centrifugal force and are crushed by being drawn

under the rolls, as the ring revolves. The face of the anvil ring is concave and the rolls convex, thereby crowding the material as it is finished to the outside edges of the ring. These mills may be used with elevator and separator, thus securing very uniform product. They take material up to 1½-inch size and reduce it to a fineness varying from ⅛ inch to 100 mesh or finer. They are usually employed as a preliminary pulverizer rather than a finishing mill for cement work. As is the case in mills of this type, the operation of crushing is largely performed by abrading the rock on itself so that the wear on the mill parts is reduced. These mills are made in five sizes; three sizes of single mills and two of duplex mills which are a double unit. Table 6 gives the sizes, horse power and speed. Regarding the output, the manufacturers furnish this on request, since it varies widely with the nature and conditions of the material handled. As an illustration the following figures are cited for the No. 2 duplex mill. Grinding cement clinker to 20 mesh, 80 to 100 barrels per hour, and to 80 mesh 30 to 40 barrels per hour. Grinding limestone to 20 mesh 16 to 20 tons per hour and to 80 mesh 8 to 14 tons per hour.

TABLE 6.
Power Requirements—Sturtevant Ring-roll Mills.

Size	Pulley speed rev. per min.	Ring speed rev. per min.	Approx. h. p.
0	125	125	8 to 12
1	320	80	15 to 20
2	300	63	35 to 45
No. 1 Duplex	375	80	30 to 40
No. 2 Duplex	325	64	70 to 90

The elevator and separator if used add about 8 h. p. per single mill. A squirrel-cage induction motor is the preferred type for these mills.

Huntington Mill. This ring-roll mill is a vertical shaft machine with a horizontal spider from the rim of which are suspended four spindles. These spindles carry at their lower end a heavy horizontal crushing roll. The rotation of the main shaft causes these crushing spindles to swing out by centrifugal action and roll against a ring die. The material is fed against this die and crushed by the rolls.

These mills are made in four sizes, but the largest size is suited only for wet grinding. The other three sizes are 3½ ft.,

5 ft., and 6 ft. and require 5, 8, and 10 h. p. respectively. This mill is a preliminary pulverizer and not a finishing mill. A belted squirrel-cage motor is a satisfactory application.

Bonnot Mill. This ring-roll type of mill employs air separation of the material. The driving shaft is horizontal and the grinding ring or die is held in a vertical plane in the main frame. The rolls are carried by a head or driver mounted on the main shaft and are free to be thrown outward by centrifugal force, thus furnishing the crushing pressure. The material thrown upward into the screening chamber by the motion of the grinding parts is deflected against the screen by the blades of a fan. The coarse material strikes a baffle and falls fack into the grinding chamber while the fine material passes through a screen and drops through passages into the conveyer. The 36-inch size is used for cement; it operates at 180 rev. per min. and requires 75 h. p. It takes pieces up to two-inch size and when working on clinker will handle 13 to 16 barrels per hour of which 96 per cent will pass 100 mesh and 77 per cent 200 mesh. It will be seen from this that it is a finishing mill as well as a preliminary grinder. When grinding raw material and fed through ⅝-in grate it will handle 4 to 5 tons per hour, 97 per cent through 100 mesh. A squirrel-cage a-c. motor is the preferred application.

Griffin Mill. This mill employs in its construction the principle of a roll running against a ring or die. Power is received by a pulley running horizontally, from which, by means of a universal joint, a shaft is suspended in pendulum fashion. To the lower extremity of this shaft is rigidly secured the crushing roll which is thus free to swing in any direction within the case. The base or pan contains the ring against which the roll works and upon the inner surface of which the pulverizing is done. On the bottom of the roll are shoes for stirring up the material and throwing it against the ring, and above the roll are attached blowers for blowing the fine material through the screen which surrounds the base. When at rest the roll hangs vertically beneath the driving shaft, but when revolved it flies out to the die ring and travels around in a direction opposite to that of the driving pulley. There is a pressure of approximately 6000 lb. due to centrifugal force brought to bear on the material being pulverized between the roll and the die. These mills are manufactured in three forms known as the "Bradley Hercules," the "Bradley 3-Roll" and the "Giant Griffin" mill. The Bradley Hercules takes material two inches

and under and reduces it to a fineness of 53 per cent through a 100 mesh screen at the rate of from 40 to 50 tons per hour, using about 250 h. p. in the operation. When used as a preliminary grinder of cement clinker it has an output of from 150 to 175 barrels per hour and requires 250 h. p. to drive.

The Giant Griffin mill has an output when grinding clinker of from 25 to 30 barrels per hour to a fineness of 60 per cent through a 100 mesh screen. It takes material ¾ inch and under for feed and when operating to full capacity requires about 80 h. p. This mill is a very efficient preliminary machine on account of the high percentage of fineness and has sometimes been used as a finishing mill.

The 3-Roll mill in theory is similar to the Griffin, but employs three rolls instead of one. A squirrel-cage a-c. motor is the preferred application for these mills except the Hercules which is better equipped with a phase-wound-rotor motor on account of the starting conditions on so large a unit.

Raymond Mill. This mill performs the operations of preliminary grinding and finishing in one process. The grinding ring is mounted in a horizontal plane and the grinding is performed on its inner surface by means of rolls suspended from a rotating driving head or spider. There are two, three, four, or five of these rollers, depending on the size of the mill. A prominent feature of this mill is the system of air separation employed. When handling raw material the mill will handle about one ton per hour per roll, *e. g.*, the 4-roll and 5-roll mills will handle four and five tons per hour respectively from a size of ¼ inch to 1 ½ inches to a fineness of 95 per cent to 98 per cent through a 100 mesh sieve. There are six sizes in all called two-roller mill, three-roller mill, four-roller mill (low side or high side), and five-roller mill (low side or high side). The power consumption is given as follows:

2 rolls, 45 to 50 h. p.	3 rolls, 55 to 60 h. p.
4 „ 75 to 80 h. p.	5 „ 85 to 90 h. p.

The driving motor is belted. A squirrel-cage induction motor with good starting characteristics is the preferred application.

Emerick Ball Pulverizer. This is a vertical mill and the crushing is done by four or five 12-inch or 14-inch steel balls which are unattached and roll in and against a grinding race. These balls are driven by a head or spider which keeps them properly separated. The centrifugal force set up in the balls by the rotation of the driving head causes them to exert great pressure

against the grinding race, thereby crushing and abrading the material in the mill. The mill is arranged with five 12-inch, four 14-inch, or five 14-inch balls. This mill may be used either as a preliminary grinder or as a finishing mill. When used as a finishing mill it is equipped with an air separator and will reduce from ½ inch and under to 95 per cent to 98 per cent through 100 mesh and 80 per cent to 90 per cent through 200 mesh. On raw material or coal it will handle six to eight tons per hour and clinker 12 to 20 barrels of finished cement per hour. The mill operates at 90 to 130 rev. per min. and requires 35 to 60 h. p. A belted squirrel-cage motor is the proper application either vertical or with a quarter turn belt.

Symons Disk Crusher. This crusher is of a special type in which the crushing is done by two dish shaped disks mounted on horizontal shafts which are separate but one of which gyrates slightly with respect to the other. The two disks are set with their concave sides facing each other and are rotated in the same direction at the same speed. This results in the hollow between the two disks having a wider opening between their edges at one part of a revolution than on another. This progressive pinching action admits a piece of coarse material at one side and as the disks rotate they approach one another thus crushing the material which is then thrown out through the opening between the edges of the disks if it is reduced to the proper fineness; if not, the process is repeated.

A wide range of adjustment is provided by changing the distance between the crushing disks. These crushers are made in four sizes as given in Table 7. A squirrel-cage motor with a maximum starting torque of 1 ½ to 1 ¾ full load torque is satisfactory on this application.

TABLE 7

Power Requirements of Disc Crushers.

Size of crusher	18 inch	24 inch	36 inch	48 inch
Size of material fed	1½ "	2½ "	3½ "	6½ "
Size to which reduced	⅜ "	½ "	¼ "	1 "
Tons per hour	5 to 8	12 to 15	25 to 30	45 to 60
H. p. required	12 to 18	18 to 25	30 to 40	50 to 65

Such material is too coarse to finish in tube mills, but could be handled by some of the types of ring roll finishing mills described in this paper.

FINISHING GRINDING MACHINERY

Tube Mills

For the fine grinding of limestone or clinker, the tube mill is used extensively, both wet and dry. The usual form is that of a rotating cylinder and the grinding is done by attrition or by the rubbing together of the grinding medium and the material to be ground. The mill is generally filled to about the center line, and the grinding medium, or charge, as it is usually called, consists of either flint pebbles, cast iron balls, or forged steel balls, etc., depending on the design of the mill and the compara-

TABLE 8

PLAIN TUBE MILLS WITH FLINT PEBBLES.

Operating in Portland Cement Plants with 20 Mesh Feed and Grinding, to 95% through 100 mesh and 80% through 200 mesh.

Size of Mill	Capacity on raw material		Capacity on clinker	Horse power		R.P.M. mill	Initial charge pebbles
	2000-lb. tons per 24 hr.	666 lb. bbl. per 24 hr.	380-lb. bbl. per 24 hr.	to start	to run		
3 ft. 6 in. × 20 ft.	50	150	120 to 144	60	35 to 40	40	9,800
4 ft. 0 in. × 20 ft.	64	192	192	70	45 to 48	35	11,600
4 ft. 6 in. × 22 ft.	80	240	240	85	55 to 60	31	14,500
5 ft. 0 in × 22 ft.	117	350	350	120	75 to 80	28	20,000
5 ft. 6 in. × 22 ft.	157	470	480	150	95 to 100	26	24,000
6 ft. 0 in × 22 ft.	192	575	575	185	115 to 125	24	29,000
7 ft. 0 in. × 22 ft.	280	840	815	250	160 to 175	20	39,000

The above figures are based on the charge and material level being carried at the center line of the mill.

tive costs of the various grinding mediums. There are several types of tube mills in commercial use, designated as follows:

1. The plain tube mill using flint pebbles as a grinding medium.

2. The plain tube mill with ball-peb compartment. This type of mill has a partition located near the discharge end. The large compartment on the feed end is charged with flint pebbles, while the small compartment on the discharge end is charged with cast iron or forged steel balls.

3. The ball-peb mill using cast iron or forged steel balls as a grinding medium.

4. The combination or compeb mill with a partition located near the feed end. The small compartment of the feed end is charged with large cast iron or forged steel balls while the large compartment on the discharge end is charged with small cast iron or forged steel balls.

The power requirements of these different types of mills vary not only with the size of the mill, but also with the nature and amount of the charge used. Once the mill is started, the load is practically constant and only slight pulsations in power can be noticed. The power required to drive these mills has been determined from actual tests and refers to dry grinding conditions only. For wet grinding the power required is approximately 30 per cent less than for dry grinding, other conditions remaining the same. The power requirements for the four types of mills, as noted above, are as follows:

1. The plain tube mill using flint pebbles—see Table 8.
2. The plain tube mill with ball-peb compartment; add 20 per cent to the power requirements of the plain tube mill using flint pebbles.
3. The ball-peb mill; see Table 9.
4. The combination or compeb mill; see Table 9.

The most economical speed for any mill should be such that the bulk of the charge is carried to the highest possible point in the mill before falling and this without any of the charge or material being carried around by centrifugal force. This is approximately

$$s = \frac{195}{\sqrt{d}}$$

where s = revolutions per minute.
d = diameter of mill in inches, including the lining.

On this basis of speed and for the dry grinding of limestone and clinker, the power required to drive any of these various types of mills may be approximated by the following:

$$P = \left\{ [0.0034\ (W + W_m)]\ \frac{R}{r} + 0.00015\, w \right\} \times \frac{S}{s}$$

where P = Horse power.
W = Weight of charge of pebbles or balls in pounds.
W_m = Weight of the material being ground in mill.
w = Weight of revolving elements of the mill in pounds.

TABLE 9.

BALL-PEB AND COMPEB MILLS.

Operating in Portland Cement Plants with 20 mesh feed and grinding to 95 per per cent through 100 mesh and 80 per cent through 200 mesh.

Dia.	r. p. m.	Periph speed	Length of Mill in feet																	
			5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
			Horse power to run mill																	
	24	362	44	50	58	66	74	82	90	98	106	115	123	131	139	147	155	164	172	180
	25	378	46	52	60	68	77	85	94	102	110	119	127	136	144	153	161	170	178	187
5 ft.	26	392	48	54	63	71	80	89	98	106	115	124	133	142	151	160	168	177	186	195
	27	407	50	56	65	74	83	92	101	110	119	128	137	147	156	165	174	183	192	202
	28	422	52	59	67	77	86	96	105	115	124	134	143	153	162	172	181	191	200	210
	22	368	54	61	71	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220
	23	386	56	64	74	84	94	105	115	125	135	146	156	167	177	187	198	209	219	230
5½ ft	24	402	58	67	77	88	98	109	120	131	142	153	164	175	185	196	207	218	229	240
	25	418	60	69	80	91	102	114	125	136	147	159	170	182	193	204	215	227	238	250
	26	436	62	72	83	95	107	118	130	142	153	165	177	189	201	213	224	236	248	260
	20	367	63	73	84	96	108	119	132	144	155	167	179	191	203	215	227	239	251	263
	21	385	66	76	88	101	113	125	138	150	163	176	188	201	213	226	238	251	263	276
6 ft.	22	403	69	79	92	105	118	131	145	157	170	184	197	210	223	236	249	262	275	289
	23	422	72	83	96	110	124	138	152	165	178	192	206	220	234	248	261	275	289	303
	24	440	75	86	100	115	129	143	158	172	186	201	215	230	244	258	272	287	301	316
	17	366	81	97	113	129	145	161	178	193	209	226	242	258	274	290	306	323	339	355
	18	387	86	102	119	132	154	170	188	204	221	238	250	263	285	307	324	341	358	375
7 ft.	19	408	91	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396
	20	428	96	114	133	152	171	190	208	227	246	265	284	303	322	341	360	379	398	417
	21	452	101	119	139	159	179	199	218	238	258	278	298	318	338	358	378	399	418	438
	15	369	103	129	150	172	193	205	236	257	278	300	321	343	364	386	407	429	450	472
	16	394	115	137	160	183	206	228	252	274	297	320	343	366	389	412	434	457	480	503
8 ft.	17	418	122	146	170	195	219	243	268	292	316	340	364	389	413	438	462	487	511	535
	18	443	130	155	180	206	232	257	284	309	334	360	386	412	437	463	489	515	541	567

The above figures are based on the following:

5 ft. 0 in. dia. mill—charge and material level within 6 in. of center line of mill—1800 lb. initial charge per ft. length of mill
5 ft. 6 in. " " " " " " " 7 in " " " " " 2200 " " " " " " " "
6 0 " " " " " " " " 8 " " " " " " 2600 " " " " " " " "
7 0 " " " " " " " " 10 " " " " " " 3600 " " " " " " " "
8 0 " " " " " " " " 11 " " " " " " 4700 " " " " " " " "

s = Theoretically correct speed in rev. per min.

S = Actual speed of mill in rev. per min.

R = Distance in feet from center of mill to center of gravity of mass $W + W_m$.

r = Distance in feet from center of mill to center of gravity of mass $W + W_m$. Assuming mass comes to center line of mill, $r = 0.0177\, d$.

When a metallic grinding medium is used $W_m = 0.304\, W$.

This is based on one cubic foot of charge and material weighing 314 pounds and consisting of 280 pounds of charge and 34 pounds of material. When a flint pebble medium is used $W_m = 0.121\, W$. This is based on one cubic foot of charge and material weighing 150 pounds and consisting of 115 pounds of charge and 35 pounds of material. In the above equation S should not vary more than 15 per cent above or below s.

For operating ball mills both squirrel-cage and wound-rotor type motors are used. The tendency is towards the larger sizes of mills with metallic grinding bodies thus requiring much larger motors than was common a few years ago. These mills are rather difficult to start since in order to get them in motion the charge has to be lifted, and in the case of large squirrel-cage motors this causes a heavy current demand at low power factor even when special high-torque motors are provided. Also in the larger motors the higher efficiency of the wound-rotor type is an item to be considered. Hence for the large mills wound rotor motors are to be preferred.

The tendency is also towards lower speed motors geared to the mills in place of the higher speed belt driven type previously used. In case of geared drive a flexible coupling should be provided on the motor as there is always considerable vibration present. The low-speed motor has the disadvantage of higher cost and lower power factor, but the saving in space, belting, etc. offsets these disadvantages to a large extent.

There is a trend in some of the latest installations towards the use of synchronous motors for these mills especially where low-speed motors are used. In such applications it is necessary to provide a clutch so that the motor can be started light, but the well known advantages of the synchronous motor are such that it is probable their use for grinding in cement plants will become much more common in the future than it has been heretofore.

Rotary Kilns

For supplying rotary kilns powdered fuel is required in most cases and the crushing plant for this uses the same class of crushing and grinding machinery as has already been described for handling limestone and cement.

The kiln itself is a long cylinder lined with fire brick, revolving on an inclined axis and fired from the lower end. The material is introduced at the upper end and works its way through the kiln in from one to two hours. A speed variation is necessary on the kiln. Therefore either a squirrel-cage motor with speed change gearing or a wound-rotor motor with resistance controller is required; the latter is preferred. The controller should be such as to permit 50 per cent speed reduction on full load torque. Since the kiln radiates a large amount of heat it is preferable to place the motor below the floor or to install it in such manner as to protect it as far as possible from the heat radiation of the kiln. For the same reason motors selected for this work must be liberally rated and have low temperature rise.

TABLE 10

Power Requirements of Rotary Kilns.

Size of kiln	Rev. per min.	H. p.
6 × 60 ft.	1 to 3	5 to 7
7 × 80 "	1 to 2	8 to 12
7½ × 100 "	¾ to 1-½	12 to 18
8 × 120 "	½ to 1	15 to 20
9 × 150 "	3/8 to ¾	22 to 30
10 × 170 "	¼ to ½	30 to 40

Kilns vary in size from 6 ft. 6 in. diameter by 100 ft. length to 10 ft. 6 in. by 200 ft. and revolve at speeds ranging from 3 to ½ rev. per min. They require from 10 to 30 h. p. The load is very steady and the horse power requirements are nearly proportional to the speed of the kiln, to the square of the diameter, and to the length. Approximately 125 to 175 per cent starting torque is required.

Table 10 gives approximate power requirements of rotary kilns. The power requirement varies with the speed as shown in the last two columns and the rating of the motor, on account of the severe operating conditions, should be twice the lower value there given. In selecting the resistance for controlling

the speed, care must be taken to have it proportioned so as to be capable of reducing the speed to one half when the load is that actually required to drive the kiln at half speed.

After the material has passed through the kilns it is ground in the finishing mills. For this work the same class of grinding and conveying machinery is employed as that previously described.

Acknowledgments

This paper has been prepared with the assistance of Mr. A. M. Dudley, Mr. C. A. Kelsey and Mr. H. Weichsel, members of the Committee on Industrial and Domestic Power. Valuable assistance has also been given by Mr. R. C. Newhouse and Mr. Fraser Jeffrey, to whom acknowledgments are due.

To be presented at the 344th meeting of the American Institute of Electrical Engineers, Toronto, Canada, November 22, 1918.

110,000-VOLT TRANSMISSION LINE OVER THE ST. LAWRENCE RIVER

BY S. SVENNINGSON

ABSTRACT OF PAPER

The paper deals with some remarkable construction recently completed by the Shawinigan Water & Power Company near Three Rivers, Quebec. The St. Lawrence river is crossed by transmission line wires on a span of 4800 feet, being the longest span in the world. Due to necessities of navigation clearance, the towers are 350 feet high.

The preliminary investigation leading to the adoption of this construction is outlined, and a general description is given of the design and construction of the towers, insulators and cables. The provisions for protection from ice and the method of sag calculations are also given.

THE Shawinigan Water & Power Company has for a number of years been transmitting power from the generating plants at Shawinigan Falls located north of the St. Lawrence River about 20 miles (32 km.) from Three Rivers, to the towns on the south side of the St. Lawrence River, one branch running to Sherbrooke and supplying various towns and industries between, the other branch feeding the asbestos mines and other industries in the Thetford district. The current is transmitted at 50,000 volts from Shawinigan Falls to the St. Lawrence, where the voltage is stepped down to 25,000 for transmission across the river over the submarine cable, then stepped up to 50,000 volts and transmitted at this voltage to Thetford and Sherbrooke.

At the time the submarine cables were installed, the alternative of putting in an overhead crossing was considered, but the amount of power to be transmitted at that time was so small that it was decided that the expense of an overhead crossing was not warranted. However, the demand for power on the south shore steadily increased, until by the beginning of 1916, five submarine cables were in operation, two three-phase and three single-phase, and the capacity of the transformer house, 10,000 kw., had been reached.

Submarine cables have always been a weak point in this part

of the system and a source of more or less trouble and expense. The current in the river carries them down stream and is sometimes strong enough to pull them apart. In the winter the ice has often put them out of commission, and it has been found necessary at times to erect temporary pole lines across the ice to maintain the service to the south shore. When, therefore, in the Fall of 1916, the demand came for more power for the south shore, partly for war work, and it became a question of putting in an additional submarine or an overhead crossing, the Company decided in favor of the latter.

The construction of an additional submarine crossing would have involved an expenditure of about $150,000 for the purchase and installation of the cables, additional transformers, about 4000 kw. capacity, together with their switches, lightning arresters, etc. and the necessary extension of the transformer houses. Besides this, the weak point in the line would not have been improved.

The overhead crossing was estimated to cost $200,000 the difference between the two being offset, in the opinion of the company, by the elimination of the weak link, in obtaining greater security from interruptions to the service, and a gain of from 2 per cent to 3 per cent in regulation by cutting out the transformers together with the elimination of a considerable amount of operating and maintenance expense. The transformers and other equipment were needed and could be used to advantage in other parts of the system.

Preliminary Investigation

The two shores of the St. Lawrence River upstream, as well as downstream of the cable houses were carefully surveyed in order to find the most advantageous point of crossing. As a result of this preliminary survey it was finally decided to investigate in detail two alternatives:

a. A three-span crossing at Point-du-Lac, each span approximately 2200 ft. (670 m.) long.

b. A single-span crossing between the cable houses 4800 ft. (1463 m.) long.

From a construction point of view the site at Point-du-Lac, about six miles (9.6 km.) up the river from the cable crossing appeared at first to be very favorable for an overhead crossing. The St. Lawrence at this point is about 7000 ft. (2133 m.) wide, but as the water is very shallow, except for a distance of 2000 ft.

(609 m.) in the center, a crossing could have been built using three spans of approximately 2200 ft. (670 m.) each. The towers on either side of the main channel would have been about 205 ft. (62.4 m.) high, while the other two towers would have been about 110 ft. (33.5 m.) high. Although this alternative probably would have been somewhat cheaper, *i. e.* the cost of the crossing itself, it would have necessitated the building of about 15 miles (24 km.) of double-circuit high-tension pole lines in order to connect up with the main transmission lines. This additional cost would have brought the total cost approximately up to that of the single-span scheme. A fairly strong point against the three-span crossing was the inaccessibility of the towers during certain periods in the spring and fall when the river is full of floating ice. The single-span scheme was finally decided on as being the most advantageous, although it was fully realized that there were many difficult problems to solve in connection with the design and construction.

General Description

The crossing as completed consists of a central span 4801 ft. (1463 m.) long and two anchor spans, the north shore span 571 ft. (174 m.) long and the south shore span 951 ft. (289.8 m.) long.

There are two towers 350 ft. (106.6 m.) high and 60 ft. (18.2 m.) square at the base, the upstream and downstream faces tapering to a width of 14 ft. (4.2 m.) at the top. A cross-arm at the top, 14 ft. (4.2 m.) wide by 100 ft. (30.4 m.) long, carries three double-groove sheaves 8 ft. (2.4 m.) in diameter and 50 ft. (15.2 m.) apart, over which the anchor cables pass. The tower foundation is made up of four circular reinforced concrete piers 11 ft. (3.3 m.) in diameter placed on the corners of a 60-ft. (18.2-m.) square. These piers are connected by heavily reinforced concrete beams 4 ft. (1.2 m.) wide by 8 ft. (2.4 m.) deep.

Three lines of cable 50 ft. (15.2 m.) apart span the river between the two towers. The cables are 1 3/8 in. (34.9 mm.) in diameter made of galvanized plough steel. They are composed of six strands of 19 wires each and a stranded core of 30 wires. To each end of the center span cables is yoked two anchor span cables. These are carried over the tower on the 8-ft. (2.4-m.) diameter sheaves and then down to a point about 20 ft. (6 m.) from the anchors. At this point equalizing beams are cut in the lines and the load is transmitted from this point to the

anchor piers by means of short straps of 1 ¾-in. (44.4-mm.) diameter cable. The cables are gripped at the end by means of heavy steel bridge sockets in accordance with the usual practise for suspension bridge cables and other structures of this type.

It was originally intended to use the main cables as conductors and to insulate them from the tower by specially designed insulators. Unfortunately these insulators were not completed in time for erection, and for the present the main cables are used as messengers from which No. 1/0 stranded copper conductors are suspended. These suspended lines are supported every 250 ft. (76.2 m.) by suspension insulators of eight units to a string.

The anchor piers are large mass concrete "dead men," each anchor being designed to take the full overturning moment when submerged.

Foundations

During February 1917 a number of borings were taken about the site of the towers to determine the nature of the river bottom. These borings penetrated to a depth of 100 ft. (30.4 m.), and we found that the foundation on which we would have to build our towers consisted for the full depth of these borings of very fine white sand with occasional strata in which a little clay was mixed with the sand. The difficulty of obtaining a secure pile foundation in this kind of soil and the uncertainty as well as the cost of placing a mat foundation in the dry, led us to adopt the form of pier foundation which we used.

The piers were constructed in the form of hollow cylinders of reinforced concrete with an outside diameter of 11 ft. (3.3 m.) and an inside diameter of 7 ft. (2.1 m.). These cylinders or caissons were poured in 6-ft. (1.8-m.) lifts, the first lift tapering on the inside towards the bottom to a diameter of 10 ft. (3 m.) and being shod with a 6 by 6-in. (15.2-cm.) angle cutting edge. This lift was poured on the working platform and lowered into the water by means of four two-in. (5-cm.) screws. The second lift was then poured and after the concrete had set, the bottom was excavated by means of an orange peel bucket rigged up on a derrick. As the caissons gradually settled successive lifts were poured until they had penetrated the bottom to a depth of about 40 ft. (12.1 m.).

Little trouble was experienced on the north side, but on the south side we encountered large numbers of boulders, some of which were so large that they could not be picked up by the

bucket, so that we had to drill and shoot them. In order to do this the caissons had to be unwatered, a tedious process which delayed the work considerably. When a caisson had reached its penetration of 40 ft. (12.1 m.), a plug of rich concrete was poured in the conical section at the bottom and the inside was then filled with mass concrete. The four piers forming one foundation were finally connected by reinforced concrete beams.

This work was begun early in the year and we expected to have it finished by mid summer, but high water, high winds, rain and labor troubles delayed us so much that it was not completed until about the middle of September.

Cables

The cables are 1 3/8 in. (34.9 mm.) in diameter, of galvanized plough steel, made up of six strands of 19 wires, each and a stranded steel core of 30 wires. Tests made at McGill University showed that the wires had an average yield point of 221,000 lb. (100.243 kg.) per square inch, and an average breaking load of 258,000 lb. (117,026 kg.) per square inch. (6.45 sq. cm.).

The completed cable was tested, the yield point being found to be 158,500 lb. (71,903 kg.) and the ultimate strength 186,400 lb. (84,449 kg.) or 193,000 lb. (87,543 kg.) per square inch, (6.45 sq. cm.) and 227,000 lb. (102,965 kg.) per square inch respectively.

The test of the completed cable, indicated a modulus of elasticity of 7,250,000 lb. (3,288,545 kg.) or 8,800,000 lb. (3,991,613 kg.) per square inch. We were in doubt as to the correctness of our test in this regard on account of the fact that the usually accepted value for the modulus for stranded steel cables is about 21,000,000 lb. (9,525,441 kg.) per square inch. However, the behavior of the cable during erection bore out the results of the test.

The bridge sockets used for connecting the cables were machined out of solid blocks of steel so as to allow a grip of nine in. (22.8 cm.) on the cable. The cable was passed through a tapered hole in the center of the bridge socket and broomed out on the end for a length of 15 to 18 in. (38.1 to 45.7 cm.). The wires were then cleaned with gasoline and held in place by means of a templet made of 1/8 in. (3.1 mm.) steel plate which fitted over the back of the bridge socket. The bridge socket was suspended bottom up and heated by gasoline torches for about half an hour, when spelter was poured into the conical

hole through a one in. (25.4 mm.) diameter hole in the center of the templet. After being allowed to cool, the ends of the wires projecting from the templet were cut off and the templet was removed.

Before adopting this form of connection, tests were run under our direction at McGill University to determine the depth of socket required. We found that if the spelter was heated to just the right temperature, *i. e.*, just hot enough to ignite a sliver of wood thrust into it, that the full breaking strength of the wire was, in the majority of cases, developed in a length of six in. (15.2 cm.).

Shortly after the bridge sockets were poured it was found necessary to shorten two of the cables and the speltered end was cut off. We had one of these cones of spelter cut in the machine shop and found that the spelter adhered so firmly to the wires that the section could be machined without lifting the wires out.

Insulators

The insulators which we propose using eventually in the steel line were devised by our engineering department in conjunction with that of the Canadian Porcelain Company. They consist of a large ring-girder and two spiders.

The ring-girder is eight ft. (2.4 m.) in diameter and made up of two nine in. (22.8 cm.) channels 12 in. (30 cm.) apart, with 3/8 in. (9.5-mm.) cover plates. The upper spider is connected to the ring-girder by means of three 2 1/2 in. (6.2 cm.) bolts 10 ft. (3 m.) long, one at the end of each spider arm. The center spider is supported on the ring-girder by six porcelain insulators of eight skirts each, two insulators at the end of each spider arm. The clear distance between the spiders is about 36 in. (91 cm.).

The porcelain insulators used are special compression insulators having a tested breaking strength of 60 tons each, this is about four times the estimated maximum load. Electrical tests showed a dry flashover of 302,000 volts and a wet flashover of 262,000 volts.

The completed insulator has a net weight of about six tons.

Erecting Cables

Owing to a constant succession of delays that occurred in the construction of the foundations and the erection of the towers we had to abandon our original plan of stringing the cables in

the Fall of 1917 before the ice formed in the river and so decided to do this part of the work after the ice had become thick enough to support the weight of the heavy reels of cable.

Throughout the heavy snows of January and February we managed by constant rolling and scraping to keep a road open between the two towers. Early in March the center span cables were laid out along this road. The anchor cables were then laid out, measured, and cut and their bridge sockets attached.

The three lines were erected one at a time, the middle line first and then the downstream and the upstream lines in succession. The ends of the anchor cables were hoisted over the towers, the south shore cables made fast to the center span cable, drawn over the tower until the bridge sockets touched the main sheave, tied to the top of the tower and attached to the anchor pier. The north shore cables were next attached to the center cable, the suspension insulators and copper line fastened to this and the cable hoisted into place.

The hoisting was done by a steam hoist braced against the center anchor pier. Two $^5/_8$-in. (15.8-mm.) steel hoisting lines reeved through two pairs of three-sheave blocks were used to draw the end of the cable up to within 40 ft. (12.1 m.) of the anchor pier, the final 40 ft. being taken up by means of two $^3/_4$-in. (19-mm.) steel cables reeved through two pairs of six-sheave blocks.

The copper conductor in each line is supported by seventeen suspension insulators spaced about 250 ft. (76.2 m.) apart, the end insulators being about 400 ft. (121.9 m.) from the towers. The copper lines drop from the end insulators to strain insulators on the tower at the 150-ft. (45.7-m.) level, pass through the tower to the back where they are connected to another set of strain insulators. On the north side, the lines pass direct from the main tower to a transmission line tower on the shore, a distance of about 600 ft. (182.8 m.). On the south side a light structural steel truss, 50 ft. (15.2 m.) long, hung from two sets of the anchor cables, provides an intermediate point of suspension, forming two spans of 500 ft. (152.4 m.) each. Access to the insulators attached to the truss is provided by a walkway running up from the anchor pier and suspended from the anchor cables.

After the cables were erected we noticed an almost constant vibration in them, varying in intensity and somewhat similar

to that in a violin string, with definite nodes 12 to 15 ft. (3.6 to 4.5 m.) apart as nearly as could be judged. About a month after the line was put into service this vibration managed to shake loose the bolts connecting two of the suspension insulators to the cable and they dropped and hung suspended on the copper line. Two of the riggers volunteered to go out on the steel cable, fish up the insulators and attach them again. A trolley was rigged up and they had little difficulty in getting out to the point from which the insulators had fallen, about 1000 ft. (304.8 m.) out from the tower. By means of a small tackle line they hauled the insulators back into place and started back towards the tower only to discover that the grade in the cable was so great that they could not pull themselves up. They solved the difficulty by looping the tackle line that they had with them over the steel cable and sliding down the 250 feet to a boat waiting below. A short time later an insulator on one of the other lines broke loose and it was similarly reconnected. This time, however, we profited by our former experience and provided a tail line by means of which the riggers were pulled back to the tower. Since then we have experienced no trouble from this source.

The cables as originally strung allowed the following clearances between the copper conductors and the average water level during the season of navigation:

Downstream	172.5 ft. (52.57 m.)
Center	178.8 ft. (54.5 m.)
Upstream	180.6 ft. (55.0 m.)

The temperature at time of erection was about 20 deg. fahr. As there is a change in sag of approximately one ft. (30 cm.) for each 10 deg. change in temperature the above would correspond roughly to clearances at 110 deg. fahr. of 163.5, 169.8, and 171.6 ft. respectively. (49.83, 51.75 and 52.3 m.).

At the time these cables were erected we naturally expected the sag to increase as the cables stretched under the load until the strands were drawn tightly together. There was no data available with regard to the amount of stretch to expect so that it was impossible to allow for this in sagging the cables. The hoist, therefore, was left in position so that we could pull up the cables when the sag became too great.

In May of this year we found that the sag in the cables had increased by from 24 to 27 ½ ft. (7.3 to 8.37 m.) and that in

order to obtain the necessary clearance over the channel we would have to take up 24 ft. (7.3 m.) in the sag of the downstream cable and 13 and 14 ft. (3.9 and 4.2 m.) in that of the center and upstream cables respectively. The amount by which a cable is to be stretched in order to take up a given amount in the sag varies inversely as the modulus of elasticity of the cable.

Owing to the low modulus which we worked out for the cable from results of the tests made at McGill University we were in doubt as to the amount of take-up required. We found that in order to take up 24 ft.(7.3 m.) in the sag we would need to pull the downstream cable in between 7.2 and 10.4 ft. (2.19 and 3.12 m.) depending on the value of this modulus. This cable was taken in about 8 ft. (2.4 m.) with a consequent reduction in the sag of about 25 ft. (7.6 m.). This corresponds to the result that would be obtained if the modulus of the cable were 17,000,000 lb. (7,711,071 kg.). In other words it would appear that from the time of the original sagging of the cable to the time the cable was resagged, the modulus of elasticity had increased from 7,250,000 lb. (3,188,245 kg.) to 17,000,000 lb. (7,711,071 kg.). This change in modulus is no doubt due to the gradual stretching of the cable causing the wires and strands to draw more closely together under the constantly applied tension of the span.

Ice Protection

Ice conditions in the St. Lawrence River at this point are at times very troublesome, and we considered it advisable to construct some kind of guard piers outside the towers, to obviate the possibility of damage from this source. During the winter we deposited about 3000 tons of field stone on the river bed on each side about 75 ft. (22.8 m.) from the up-stream and river faces of the towers, carrying the rock to an elevation about three ft. (0.91 m.) above the surface of the ice. The ice usually goes out about this level, but last year conditions were exceptional, and before the ice moved it had risen above the tops of our ice breakers, and passed clear over them, piling up around the tower foundations to a height of 25 or 30 ft. (7.6 or 9.1 m.). Fortunately no damage was done. We are at present completing the guard piers, by means of reinforced concrete cribs filled with rock, and carried to about the level of the maximum recorded high water.

Sag Calculations

In our calculations for sags, tension, length of cable etc., under various conditions, we used the parabolic formulas in preference to the hyperbolic formulas for the catenary on account of the greater simplicity of the former. Comparison was made however, between the two sets of formulas and we found, as we had expected, that at working tensions the difference was negligible. The formulas for the parabola gave us about six in. (15.2 cm.) more sag, and about one ft. (30.4 cm.) less length of cable than the catenary formulas for the same conditions of tension and temperature.

The maximum load on the cable, we assumed to be $\frac{3}{4}$ in. (19 mm.) of ice all round, and ten lb. (4.5 kg.) of wind per square foot (0.09 sq. m.) of projected area, for both the steel and copper lines, at a temperature of zero deg. fahr. Under these conditions the calculated tension in the cable is about 106,000 lb. (48,080 kg.), with a sag of 228 ft. (69.4 m.). The normal tension at summer temperatures is about 61,000 lb. (27,669 kg.) with a sag of 185 ft. (56.38 m.).

Conclusion

In our design, we always kept in view the accessibility of various insulators and other working parts that are subject to break down. Automatic hoists have been provided in the towers, as well as ladders which run from top to bottom and provide access to the suspension insulators at various levels.

The crossing has been in uninterrupted service now for about nine months, it has not yet weathered a winter with its low temperatures, gales and sleet storms, so that we still have something to learn about its action under these conditions, but as the allowable stresses have been kept within reasonable limits, we hardly expect serious trouble from this source.

Presented at the 344th Meeting of the American Institute of Electrical Engineers, Toronto, Can., November 22, 1918.

(Subject to final revision for the Transactions.)

ELECTRIFICATION OF THE MONTREAL TUNNEL ZONE

BY WILLIAM G. GORDON

ABSTRACT OF PAPER

The author describes the electrification of the tunnel through Mt. Royal at Montreal which was built to give the Canadian Northern Railway entrance into the heart of the city. The tunnel emerges from the mountain several feet above the level of the city and it is proposed to extend an elevated line at the same uniform grade to connect with the proposed viaduct on the lines of the Harbor Commission. The tunnel is 3.1 miles long and the method of construction is described in detail.

The power is purchased and delivered to a substation near the west portal of the tunnel. The equipment of the substation is described. At present there are six electric locomotives in operation having a one-hour rating of 1280 h.p., and it is proposed to add multiple unit motor cars for handling the local traffic. Details of the equipment and dimensions of both the locomotives and motor cars are given.

The catenary system, which is described in detail, has a number of unusual features due to special local conditions and the extremely low temperatures which sometimes prevail in Montreal.

THE City of Montreal is divided into two principal levels; the commercial and financial quarter being on a plane only a few feet above high water, and the residential and shopping districts being at a height of about 75 ft. above the river. As the space between Mount Royal and the St. Lawrence River is limited, this district has become very congested. Business has largely forced the residence section up and down the River and around the mountain. The tunnel under Mount Royal was built with the idea of giving the Canadian Northern Railway, which property now belongs to the Dominion Government, an entrance into the heart of the City, and to render available a large area for residential purposes, only a few minutes by train from the main terminal.

The location of the present terminal is about midway between the two levels and it is proposed to extend an elevated line, at the same uniform grade, which will connect up with the proposed viaduct on the lines of the Harbor Commission, thus giving

Manuscript of this paper was received November 22, 1918.

direct access to Trans-Atlantic steamers, and all the harbor facilities.

The tunnel as built is 3.1 miles long and is the shortest line that can be devised to take advantage of the geological formation. The tunnel has a uniform grade of 0.6 per cent toward the city to insure proper drainage.

In order to meet the various physical conditions, different cross sections were used; where hard sound rock, unsound rock and soft ground were encountered respectively.

The twin section type of tunnel was adopted for

1. Economy in construction.
2. Ease and economy in ventilation.
3. Protection and safety in case of derailment or accident.

In addition to working from both ends of the tunnel a shaft was sunk one mile from the West Portal at Maplewood Avenue.

It is of interest to note that when the heading from the West Portal met that being driven from the Maplewood Avenue shaft, the lines checked within 1/16 in. on the alignment, and ¼ in. in grade, and that where the headings from the Maplewood Avenue shaft and that from Dorchester Street met under the highest point of Mount Royal, the error was ¾ in. in alignment, and ¼ in. in grade.

The method employed in driving the tunnel was to drive a bottom center heading about 8 ft. high by 12 ft. wide, as this heading could be driven ahead rapidly without much regard to the character of the ground, and from which the full size excavation could be developed at as many places, simultaneously, as desired.

Four drills were used in each heading, supported on a horizontal bar; the drills being operated by compressed air at a pressure of about 100 lb. per sq. in. The break-ups, where the upper part of the tunnel section was excavated to its full width and height, were opened at intervals of from 500 ft. to 800 ft. along the center bottom heading, the practise being to open up as many of these as necessary to keep up with the heading progress.

The compressed air used for operating the drills and other pneumatic machinery was obtained from two plants, one at each end of the tunnel, with an aggregate capacity of 11,000 cu.ft. of free air per minute, compressed to 110 lb. per sq. in.

The muck from the tunnel was handled by two 10-ton and one 8-ton trolley locomotives, and six 5-ton storage battery locomotives.

The load curve was worked up from the following data:

TRAIN WEIGHTS AND SPEEDS ASSUMED FOR MONTREAL TERMINAL ELECTRIFICATION

Class.	Trailing tons.	Speed level.	0.6 per cent up-grade.	Schedule.
Transcontinental	1130	37.0	26.5	21.2
Express and local	550	37.5	27.1	21.6
One motor coach	60	50.0	41.5	22.2
Three motor coaches	180	50.0	41.5	22.2
Three motor coaches and two trail coaches	260	47.8	34.8	21.8
Freight	1000	32.5	23.5	....

The substation is a handsome building, and will harmonize with the buildings which will be erected in the neighborhood.

Power is purchased from the Montreal, Light, Heat & Power Company at 63 cycles, 11,000 volts, three-phase. It is delivered to the substation by a lead-covered, three-conductor cable carried in a duct through the tunnel and also by an overhead line to insure continuity of service. The general arrangement and capacity of the switching equipment provides for the later addition of a steam auxiliary plant at the Back River near the Cartierville Yards for extension of the electrification of the main line to Ottawa.

There are two motor-generator sets with provision for a third, later. Each of these sets consists of a synchronous motor direct coupled to and on a common bedplate with two 750-kw., 1200-volt, d-c. generators, the set running at 600 rev. per min. The generators are connected in series giving 1500 kw. at 2400 volts per unit.

The sets have an overload capacity of 200 per cent for five minutes. The heavy overload capacity of these machines is obtained by the use of a pole-face winding. This winding of tubes and rods through holes near the pole faces is so connected as to directly oppose the armature reaction, thus insuring satisfactory operation up to the heavy overload mentioned. The pole-face windings and the series and commutating field windings are all connected on the ground side of these machines.

The shunt fields of the d-c. generators and the synchronous motor fields are arranged for 125-volt excitation. Each of the synchronous motors is started by a three-phase, 11,000-volt

compensator. This auto-transformer has one coil per phase with suitable starting taps brought out.

The three exciter sets each consist of a 50-kw., 125-volt d-c. generator driven by an induction motor. The generators are commutating pole type, flat compounded for the specified voltage and are especially adapted for exciter work and voltage regulator control. A bank of six 100-kw. single-phase transformers supply the induction motors of the exciter sets and miscellaneous station requirements.

All oil switches on the 11,000-volt circuits, except the synchronous motor magnetizing and starting switches, are enclosed in masonry cells and have two breaks per pole, each break in a separate tank. These switches have a rupturing capacity of 16,000 arc amperes at 11,000 volts. They are motor operated and will open automatically on overload, the incoming line switches excepted, either instantaneously or with a time-limit action, as desired. The incoming line switches operate automatically on the reversal of power only.

The synchronous motor starting switches are remote-control, solenoid-operated, mounted in cells, and have a rupturing capacity of 2000 arc amperes at 11,000 volts.

The main switchboard is of three-section panels of natural black slate, 90 in. high. The 2400-volt direct-current circuit breakers and lever switches are mounted on a panel back of and above the main switchboard. They are operated by insulated handles on the front of the main board so as to eliminate any possibility of the operator coming in contact with the 2400-volt circuit.

The circuit breakers are mounted between fireproof barriers and are equipped with powerful magnetic blowouts. The field switches are mounted on a base back of the panels with the operating handles on the front of the main board.

There are six locomotives in operation. Each locomotive has four axels with all the weight of the locomotive upon the eight driving wheels. The running gear consists of two four-wheel trucks, articulated with a heavy hinge. The equalization of the trucks is accomplished by a semi-elliptic leaf spring over each journal box, connected through spring hangers to the frame and to the equalizer bars. The equivalent of a three-point suspension is thus obtained through the side equalization of one of the trucks and both side and cross equalization of the other truck.

The friction draft gear is mounted in the end frame casting of

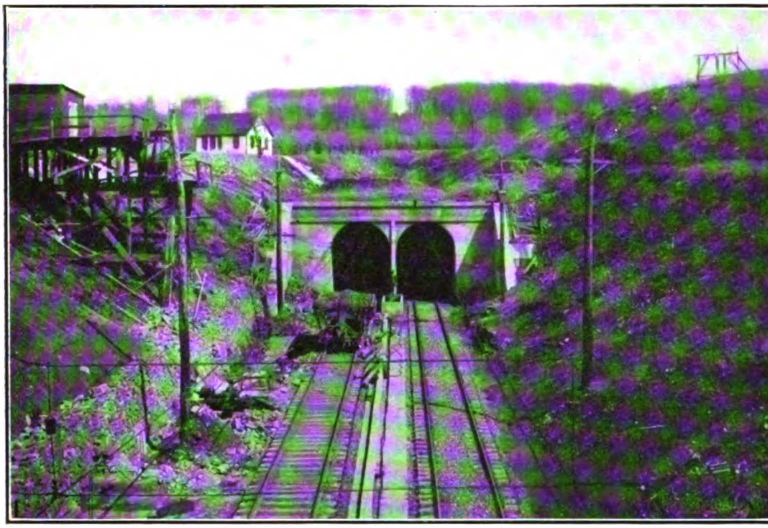


FIG. 1—APPROACH TO WEST PORTAL OF MOUNT ROYAL TUNNEL

FIG. 2—TWO-DEGREE CURVE IN THE TUNNEL [GORDON]

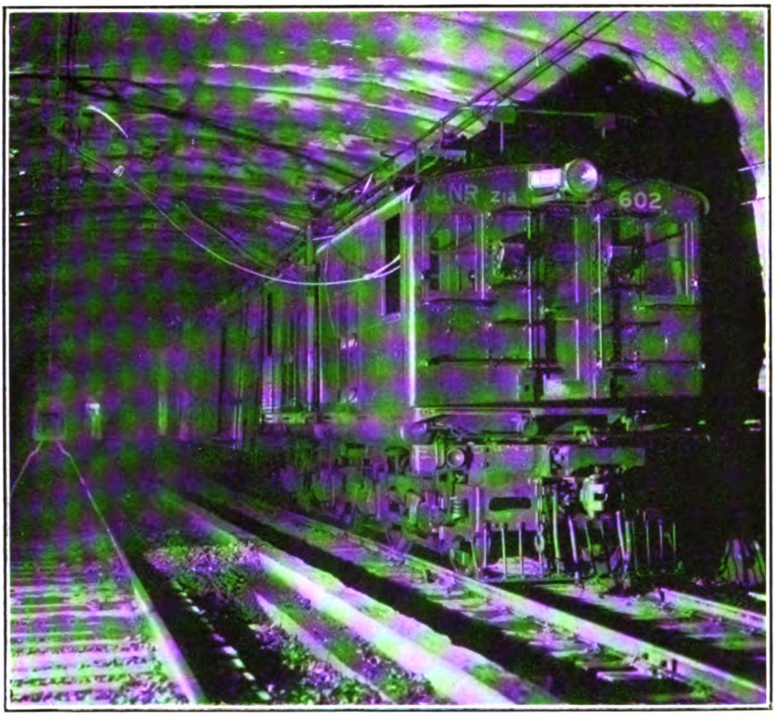


FIG. 3—LOCOMOTIVE PULLING IN MESSENGER AND TAKING CURRENT FROM OPPOSITE TRACK

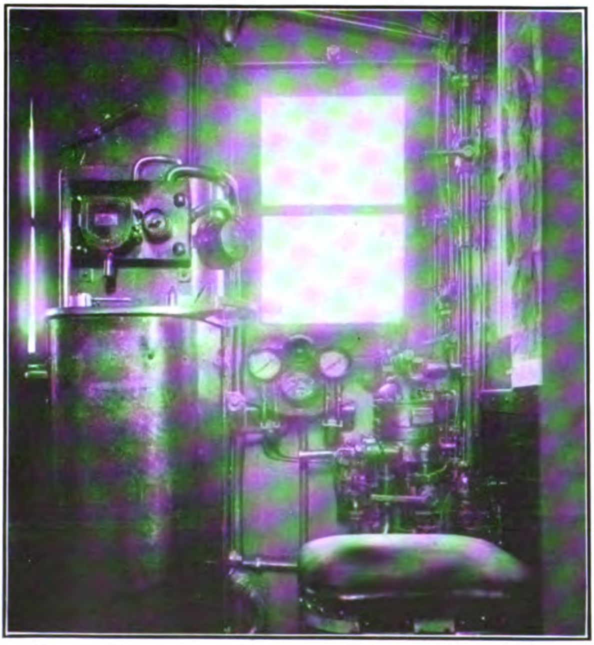


FIG. 4—CONTROL APPARATUS IN OPERATOR'S CAB [GORDON]

the truck. This type of construction restricts the hauling and buffing stresses to the truck side frames and articulated joint, thus relieving the cab and apparatus from the effects of severe shocks. The cab, which is of the box type, is divided into three compartments, the center compartment for the apparatus, and the two end compartments for the operator. Each operator's compartment is supplied with controller, control switches, ammeter, air brake and pantograph control, air gages, 2400-volt cab heater, bell rope, and control for the whistle and sanders, thus providing the locomotive with complete double end control. The motors are nose-supported in the usual way and geared to the axle by means of twin gears, each of four-inch face.

The motor equipment consists of four commutating pole motors wound for 1200 volts and insulated for 2400 volts, two of these motors being permanently connected in series for operating on the 2400-volt trolley circuit. The one-hour rating of each motor is 320 h.p. at 1200 volts. The motors are designed for forced ventilation which is obtained by means of a blower in the locomotive cab. Either pair of motors may be cut out by a special handle on the change-over switch. The locomotives are geared for a free running speed on tangent level track of approximately 45 mi. per hr. with ten points in series and nine points in series-parallel. The master controller used is of the non-automatic type and has two handles, one regulating the applied voltage at the motors and the other controlling the direction of rotation of the motors. The rheostats which form the external motor resistance are placed near the roof of the cab and provided with ample natural ventilation.

The master controller and contactor energizing circuits are designed for 125 volts. Each contactor is easily accessible without any disturbance to adjacent contactors. A special electropneumatic change-over switch is used for making the transition between series and series parallel connection of the pair of motors.

The 125-volt current for operating the contactors and for lighting the cab and headlights is obtained from a motor-generator set, the motor of which has two 1200-volt windings and two 1200-volt commutators in series for operation on 2400 volts. This set is mounted in the center cab and also drives the blower for providing forced ventilation to the main motors.

Fuses of the copper ribbon type placed in the fuse boxes provide protection for each individual circuit as well as the main

circuit from the trolley. These fuse boxes are all arranged to blow into a common chamber designed to take care of the arc. In addition to the fuse on the main circuit, a main switch is also provided. This is of the knife-blade type, being opened and closed by a handle in a position for easy operation in case of emergency, or when it might be necessary to open the circuit while carrying current. This main switch blows into the chamber provided for the fuses and has a powerful magnetic blowout.

The trolleys are of the slider pantograph type, pneumatically operated and mounted on insulated bases. Two pantographs are used per locomotive.

A speedometer, similar to the type largely used on automobiles, but especially designed for locomotives, is located in each operating cab. These are connected to the driving wheels of the locomotive by means of flexible shaft and gearing.

A combined straight and automatic airbrake equipment is provided on each locomotive. This equipment includes a 2400-volt, motor-driven air compressor, the set consisting of two 1200-volt motors operating in series on 2400 volts and direct-connected to an air compressor having a displacement of 100 cu. ft. of free air per minute. The approximate total weight of each locomotive is 83 tons. Some of its principal dimensions and characteristics are given in the following table:

Length inside knuckles	37 ft.	4 in.
Length over cab	31 "	0 "
Overall height, pantograph down	15 "	6 "
Height over cab	12 "	10 "
Overall width	10 "	0 "
Total wheelbase	26 "	0 "
Rigid wheelbase	8 "	8 "
Total weight, all on drivers	83 tons	
Wheel diameter	46 in.	
Tractive effort at 30 per cent tractive coefficient	49,800 lb.	
Tractive effort at one-hour rating	20,300 lb.	
Tractive effort at continuous rating	16,200 lb.	
Speed at rated amperes, one-hour rating	23.4 mi. per hr.	
Total horse power, one-hour rating	1280 h.p.	
Speed at rated amperes, continuous rating	24.6 mi. per hr.	
Total horse power, continuous rating	1090 h.p.	
Gearing 80–25	Reduction 3.2	

The multiple unit motor cars for handling local traffic are not yet in operation. The principal dimensions of these cars are given in the following table:

Length over buffers	67 ft.	5¾ in.
Length over body corner posts	57 "	6¼ "
Truck centers	42 "	9 "
Width over side sill angles	9 "	10½ "
Width over eaves	10 "	2¾ "
Height top of rail over roof	13 "	0 "
Height top of rail to underside of side sill	3 "	7½ "
Center to center of body side bearings	4 "	10 "
Center to center deck sills	5 "	6 "

The approximate weight of the car loaded and equipped is 160,000 lb. The electric hot air system of car heating is used. One complete heater is placed underneath each car and receives its energy direct from the 2400-volt supply. The heater has a capacity of approximately 25 kw. and is constructed for two heat combinations so as to provide for the changes in temperature conveniently and economically.

The complete heating equipment consists of the heating unit, blower and regulating mechanism, the controlling switch and thermostat of the regulating mechanism being arranged for operation from the 600-volt supply. Air is forced over the heating unit by means of the blower and distributed to the car through the air ducts along the sides of the car. The blower used for the circulation of the air is operated by a motor which is connected in series with the heating unit on the ground side. The capacity of the blower is approximately 1000 cu ft. of air per minute.

The motor equipment consists of four fully ventilated, 125 h.p. 1200-volt, commutating-pole motors insulated for 2400 volts. Two of these motors are permanently connected in series for 2400-volts operation. Ventilation of the motor is accomplished by drawing air into the armature at the pinion end by means of the fan on the armature shaft. The air passes longitudinally through the whole interior of the motor and is expelled through an opening in the frame at the commutator end, protected by wire mesh.

The control is of the non-automatic type for multiple-unit operation. The equipment includes a motor-generator set for furnishing 600-volt current for the control circuits, the air compressor and lighting circuits. This set consists of two 1200-volt motors, operating in series on 2400 volts, direct connected to a 600-volt generator. The master controller, contactors, switches, reversers and pantograph are essentially the same construction and appearance as those already described for

the locomotives. The controller has five steps in series and four steps in parallel. It differs from the locomotive controller in having the usual motorman's operating handle instead of a lever. This handle is provided with the so called "dead man's" feature for cutting off power and applying the air brakes in case the motorman removes his hand.

Copper ribbon fuses similar to those on the locomotive are used, and an aluminum-cell lightning arrester is installed on each car.

Special local conditions and extremely low temperatures introduced features making the design of the catenary system for this electrification somewhat out of the ordinary. The electrified track at present is about 10 miles long, and in this distance there is a passenger terminal station and coach yard in the city, a double track tunnel, double tracks in a cut with low clearances under highway bridges, a long stretch of single track, both tangent and curve, and a large freight yard with repair shops and storage tracks. The temperature in the coldest winter weather reaches 35 deg. below zero, while in the hottest summer weather it will go as high as 110 in the sun. In the early spring severe sleet storms sometimes occur.

The poles are of Eastern white cedar. The specifications for these poles and also for the creosote oil used as a preservative, were based upon those of the National Electric Light Association. Steel poles are used in the terminal yard in the city on account of their more sightly appearance. The wood poles are set seven feet in the ground and are all back-guyed. They are long enough to carry two cross-arms for feeders, signal circuit and a three-phase transmission line for supplying the shops in the Cartierville yard with electric power. On top of the poles there is a No. 000 copper ground wire which serves both as a protection against lightning for the circuits on the poles and also as a preventive against any trouble that might be caused by breakage of the rail bonds, which latter are of the welded U type.

The poles throughout the single track construction are spaced 150 ft. on tangents and 20 ft. on the two-deg. curve. On the double-track portion, where the overhead clearance is limited, the spacing is reduced to 105 ft. on tangents.

The messenger for the electrification outside the tunnel consists of a ½ in. seven-strand Siemens Martin steel cable with an ultimate strength of 11,000 lb. and an elastic limit of 6600 lb.

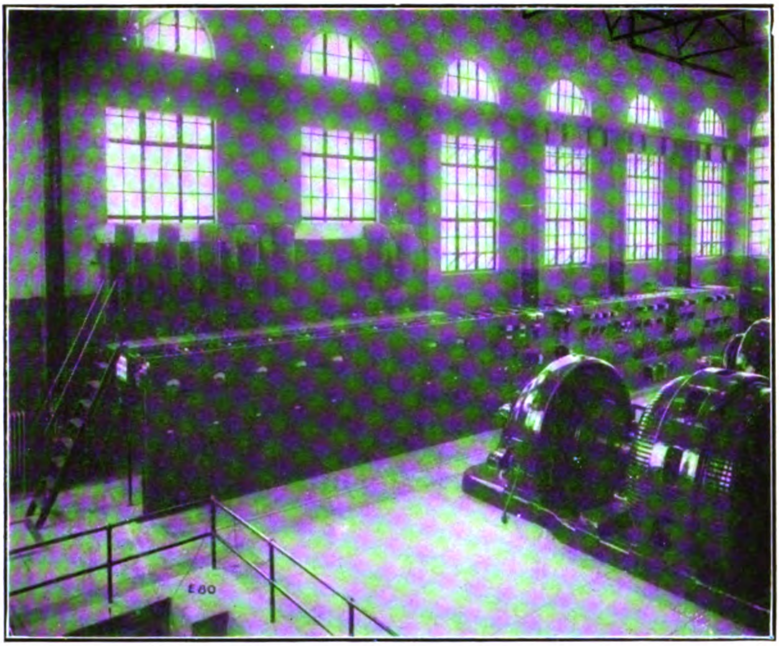


Fig. 5—View of Switchboard in Substation

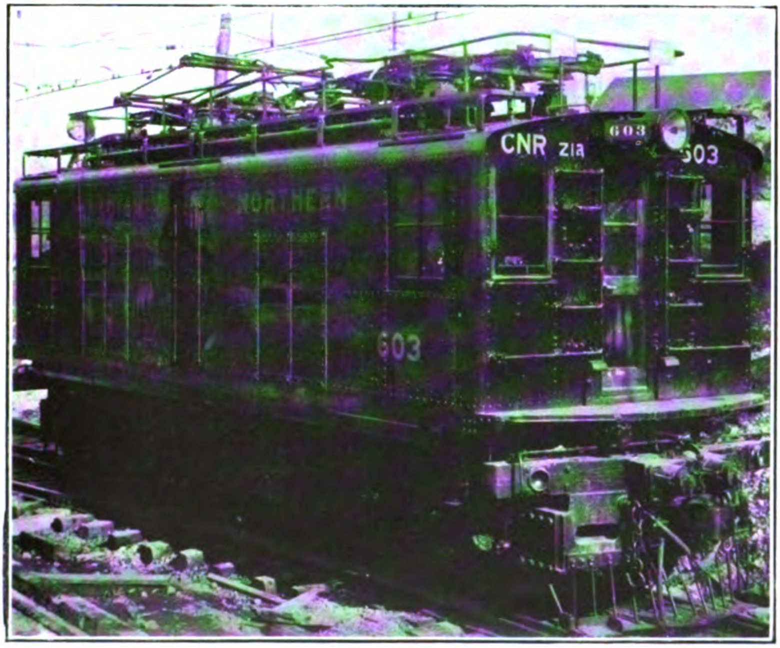


Fig. 6—Locomotive with Low Catenary Construction [GORDON]

FIG. 7—CATENARY CONSTRUCTION ON TANGENTS [GORDON]

Two No. 0000 copper feeders are installed, on the full length of the electrification outside the tunnel and the other for about one mile west of the substation.

The messenger is anchored every half mile. This is accomplished by running the end of one half mile length past the end of the next for a distance of one span. It is then made fast to an anchor eye on the bracket through an insulator and turnbuckle, and the same point of the bracket guyed back to the next pole, which in turn is guyed against this strain. The two messengers where they pass each other, are kept from eight to ten inches apart. By anchoring the trolley wire on the same bracket the anchorage becomes a section insulation, the air space between the messenger and trolley wire forming the insulation. Where a section insulator is not required a copper jumper is placed between the messenger and trolley wires.

For the double-track portion of the line cross-span construction, is used, the cross-span being a 3/8-in. seven-strand Siemens Martin steel cable. The messenger is fastened to this by means of a small malleable clamp. This cross-span is made up with a turnbuckle, strain insulator, and wedge grip in each end, and fastened to the poles by means of eyebolts.

In yard work spanning more than two tracks the construction is similar, but with the addition of a cross messenger of 1/2-in. cable above the 3/8-in. cable. This cross messenger is made fast to the poles directly, without insulators or turn-buckles, and carries the weight of the spans below through lengths of 1/4-in. steel cable. These fasten to eyes in the tops of the messenger hangers and to the cross messenger by means of Crosby clips, There is a strain insulator in each of these lengths.

Pull-offs are used on curves for holding the contact wire and messenger in the correct position over the track and at intervals on long tangents for steadying the contact wire. The pull-offs are made of sherardized steel tubing bent to avoid fouling the pantograph. Each pull-off is fitted with a clamp ear at one end and an eye at the other. Adjustable links are sometimes required with the pull-offs to keep the trolley wires the right distance apart at certain points, such as where the trolley wire for a turnout approaches the main trolley wire at an angle. Each link is composed of two malleable iron brackets, with clamp ears, connected by means of a 1/2-in. pipe, the length of which is adjusted between the brackets and held by set screws.

Porcelain strain insulators are used in two sizes. The larger,

used with ½-in. and ⅜-inch. steel cable, withstands a wet flash-over test of 14,000 volts and has a breaking strength of 22,000 lb. The smaller, used with ⅜-in. and ¼-in. steel cable has a breaking stength of 12,000 lb.

The insulator used on the bracket construction is of the ordinary glazed porcelain, double petticoat, pin type, 4½-in. in diameter. It has a wet flash-over test of 20,000 volts. The messenger rests in the groove in the top of this insulator and is not tied except on curves.

The contact wire is of special bronze composition, size No. 0000, with a breaking stength of 65,000 lb. per sq. in. and an elastic limit of 39,000 lb. per square inch. Its section is the standard of the American Electric Railway Association for No. 0000 grooved trolley wire. The use of this wire instead of hard-drawn copper was thought advisable both because of its longer life when subjected to the wear caused by sliding pantographs, and also because it could be pulled up tighter than copper on account of its greater strength. This latter reason was considered of special importance because of the wide variation in temperature in Montreal, with the consequent great variation in the sag of ordinary copper trolley wire between winter and summer.

The trolley wire is hung straight over the center of the track as the natural side sway of the pantograph is sufficient to prevent wearing grooves in the contact strips.

The height of the trolley wire above the top of rail is ordinarily 23 ft. except along the double track construction and in the tunnel, where it is 16 ft. In this section two wires are used over each track. They hang side by side, supported from the same messenger, the hangers of one wire being staggered with those of the other. These double wires do not raise the hanger loops as high as would a single wire when a pantograph passes along, which is an obvious advantage where the head room is limited. Sparking and consequent wear both of the contact shoes and contact wires is reduced to a minimum, as there is always good contact between the slider strips and one of the contact wires.

The hangers are all of the long-loop type, having a malleable iron, single-bolt clamp ear, and a strap varying in length to suit its position in the span. All parts are sherardized. In spans of all lengths from 150 ft. to 90 ft. the hangers are spaced 15 ft. apart.

Lightning arresters of the magnetic blowout type are installed

at half-mile intervals. The arrester is placed near the top of the pole and the ground wire runs down the pole to a ¾-in. iron pipe driven about ten feet into the ground. Before driving this pipe, a 2-in. pipe was driven down about 5 ft. then withdrawn and the hole filled with rock salt. The ¾-in. pipe was driven down through the salt.

In addition to these arresters on the poles, aluminum cell arresters are installed in the substation on the positive busbars and on each feeder.

In order to string the messenger cable with the proper tension a dynamometer was used. It was therefore necessary for the foreman of the line gang to know what the tension should be at different atmospheric temperatures. The right sag at any given temperature was also of importance as a check on the tension. This information was supplied in the form of tables to which the line gang worked, the sags and tensions being given at 5 deg. intervals.

In the tunnel the overhead clearance was so limited that the catenary had to be very flat. This meant pulling the messenger up very tight for spans of reasonable length. A cable of phosphor bronze was decided upon, composed of nineteen wires, and having an overall diameter of 0.888 in. This cable has an ultimate breaking strength of 22,000 lb. and an elastic limit of 18,600 lb.

This messenger is supported every 90 ft. from the roof of the tunnel by a combination of iron yokes held in the concrete by four one-in. bolts. The cross yoke carries the messenger insulator and is supported on two insulators carried on the two end yokes, so that there are two insulators between the messenger and the ground. The insulators are of glazed porcelain and have a wet flashover test of 20,000 volts. All clamps and small parts of the messenger supports are of malleable iron sherardized. The yokes are of 2-in. by ⅝-in. and 1½-in. by ⅞-in. mild steel, painted with an asphaltum compound as a protection against rust. Two No. 0000 phosphor-bronze contact wires hang side by side from the messenger. The hangers for each contact wire are spaced 15 ft., or 7 ft. 6 in. between adjacent hangers. The hanger lengths vary from 6 in. to 13¾ in. with a 90-ft. span. The two hangers nearest the messenger support, namely those 11¼ and 13¾ in. in length, are made with two loops one sliding inside the other, where the clearance to the roof is small. The remaining hangers are similar to those used outside the tunnel, except that the loop is wider in order to take the

larger messenger. It was found that the two messenger cables and the four contact wires over the two tracks in the tunnel would give ample conductivity, so that no feeders through the tunnel were required. Both the messenger and contact wires are anchored every half mile. Two "bridles" of ½-in. steel cable are fastened to the messenger by means of six ⅞-in. Crosby clips, and the ends of these bridles fastened each way, through two cemented-type strain insulators in series, a turnbuckle and wedge grip to roof plates. The contact wire is anchored by lapping the ends for a distance of one span and then carrying each end up and slightly to one side of the center, making fast to a roof plate through two insulators a turnbuckle and a wedge grip.

At the only curve in the tunnel, one of two deg. two pull-offs are place in each span, over each track, one for each of the contact wires. The pull-offs are fastened to the tunnel arch through two strain insulators in series by means of an expansion bolt. The two pull-offs are placed 7 ft. 6 in. apart and this arrangement prevents hard spots and at the same time keeps the two contact wires close enough together for satisfactory operation.

Presented at a Section Meeting of the American Institute of Electrical Engineers, San Francisco, Cal., February 28, 1918.

INTERCONNECTION OF POWER SYSTEMS

The following symposium presented at a meeting of the San Francisco Section deals exclusively with Pacific Coast conditions, but the problems presented are very important ones and are worthy of study by anyone interested in hydroelectric power development. The utilization of water powers and the interconnection of power systems are closely associated with the present necessity for fuel conservation.

TECHNICAL FEATURES OF THE INTERCONNECTIONS OF ELECTRIC POWER SYSTEMS OF CALIFORNIA

BY P. M. DOWNING

THE subject that is to be discussed this evening is of the utmost importance not only to the people of this State, but to the entire nation because it has to do with the conservation of one of our most valuable resources, viz., fuel oil.

Although it may have been intended to limit the discussion this evening to the technical features involved in the matter of interconnecting the different systems operating in the central part of California, I am going to devote the short time allotted to me to a review of the conditions and circumstances that have made such an arrangement desirable and necessary, leaving the discussion of the technical features to some of the speakers who will follow me.

It is a matter of common knowledge that there is a very serious power shortage in this State, particularly in the central part. The business of the different Power Companies has made a wonderful growth in the last few years, but the construction of new facilities to meet the increased demands for service has not kept pace with the growth of the business. Some idea of this increase may be had when I say that in 1900 the installed hydroelectric capacity in the territory North of Merced, that is, in the territory North of that supplied by the San Joaquin Light and Power Company was, approximately, 15,000, h. p. By 1917 this had increased to 375,000 h.p., an increase in the 17 years of 360,000 h.p. In addition to this 375,000 h.p. of

hydroelectric capacity, there is now installed 187,500 h.p. in steam plants.

Athough there is an abundance of undeveloped hydroelectric power well within distances that it could be economically transmitted to the Bay Counties, it has been allowed to lie idle for no other reason than that capital could not be interested in its development. These projects are unattractive not because of physical or engineering reasons, but solely because their development and utilization are unattractive financially; in other words, they are not looked upon as good investments. Capital can be otherwise invested where it will be more secure, and where it will yield a better return. Before an investor is asked to put his money into any enterprise, he must first be assured that the investment is a reasonably safe one. He must know that there are equities back of the project that will secure him against loss. He must also know that the enterprise is one with an earning capacity that will insure him a fair return on his investment, and at the same time leave a reasonable surplus to properly maintain the property. Public utility securities, unfortunately, have not always afforded all of these inducements, with the result that money has not always been available when it became necessary to provide new facilities for carrying increased loads.

This condition has been brought about, first by the refusal of rate regulating bodies to establish rates that would yield the utilities a return on their investments comparable with the returns on capital invested in other unregulated enterprises, and secondly by the attitude of the Federal Government in the matter of refusing to grant to power companies anything more than temporary permits for the occupancy of public lands. The radical ideas and rulings of the conservationists have for years been a stumbling block in the way of hydroelectric development. Under existing regulations of the Department of the Interior, no one can enter upon any public land within the Forest Reserve for the purpose of constructing any part of a hydroelectric development, except by signing a revocable permit terminable at the discretion of the Secretary of the Interior. Not only are these permits subject to this and other objectionable conditions that might be imposed when they are granted, but they are also subject to such other conditions as the Department might subsequently impose, even after the investment has been made. From the standpoint of the inves-

tor, a very objectionable feature of these permits is that the applicant must agree to turn over the entire property to any municipal, state or federal body at any time upon receiving payment therefor a price to be finally determined by the Secretary of the Interior; in other words, the investor at the discretion of the Secretary of the Interior must give up his property at any time without having any voice in determining the price at which it is to be taken.

It is immaterial how much, or how little of the completed project may lie within the public domain, there is absolutely no way to acquire a fee to the property occupied, and the only possible way to secure even the temporary easement is by signing the permit which would involve the entire project.

As a very large part of the undeveloped water power resources in this State are so situated that at least some portion of almost every development would fall within the Forest Reserve, it will at once be apparent how far reaching a requirement of this kind would be.

A recent report made by a Special Committee of the Chamber of Commerce of the United States appointed for the purpose of reporting on water development as effected by the policy of the Federal Government contains, among other statements, the following:

One of the first things to be clearly perceived, for an understanding of this subject, is that water-power developments are not exceedingly profitable undertakings earnestly sought by capital as a means of securing large returns on a small investment; but that, on the contrary, steam power is the superior of water power in almost all respects. In order to procure the adequate development of water power, inducements must be offered.

There is a very general, but entirely erroneous, belief that any water power running to waste can be utilized at small expense. The initial cost of a steam plant is in general but one-half to one-fifth that of a water-power plant of equal capacity. Moreover, because a steam plant can be more easily enlarged from time to time, the initial development of a water-power plant must, in general, be a much larger proportion of the ultimate contemplated development than in the case of a steam plant. It follows that the investor in a water-power plant is burdened from the very start with a heavy fixed charge, the failure to meet which may mean bankruptcy. The risks to capital in water-power developments are, therefore, much greater than the risks in the case of steam power. The advantage of water power lies in its smaller expense of maintenance, due to the smaller amount of labor and the absence of any cost for fuel.

Water powers will not be developed unless the conditions are made

comparatively favorable. Present demand for the development of such power comes, not from capitalists, but from communities which, on account of the high price and scarcity of fuel, are desirous, in their own interest, of inducing capital to make such developments.

In order to secure the adequate development of water power, it is essential that the subject should be approached with an attitude of mind which recognizes the necessity of making such developments attractive to capital, rather than with that attitude which assumes that such enterprises should be surrounded with as many restrictions as possible.

This short-sighted policy on the part of the Federal Government has retarded hydroelectric developments and forced the companies to take care of their increased load with steam-generated energy.

With oil as cheap as it was prior to 1916, the modern steam turbine, with its high efficiency, its low first cost and the guaranty of the investment, gave promise of being a real competitor of the hydroelectric installations. However, during the past two years, the production of oil has decreased and the consumption increase, thus necessitating a very heavy draft from storage. As a result the approximately 35,000,000 barrels of oil in storage in 1916 is being depleted at the rate of, approximately, 1,000,000 barrels per month. If the present ratio of production to consumption continues, the entire storage will be exhausted by 1920.

In view of the Government activities in building up a Merchant Marine, and the general industrial activity throughout the entire country, there is but little doubt but that even with the increased activity in developing new wells, this shortage will continue. The prevailing high price of oil will be an incentive for greater production by the development of new properties.

The active producing period during which the oil wells of this State can be economically operated, is probably not to exceed 20 to 25 years. Records of the decline in production of wells in the various fields show that during the first five years of the wells' life, the production drops off, approximately, 75 per cent. As the number of wells in the different districts increase, it is reasonable to say that this rate of decline will also increase. With these facts before us it would appear that the added supply to meet the increased demands would have to come almost entirely from new wells rather than from those now being operated.

The accompanying chart giving the sources of the World's

oil supply amounting in 1915 to 427,695,347 barrels shows that a very large proportion of the total is produced in the United States. During that year the production in this country amounted to 281,104,104 barrels, or 65.72 per cent of the total. Russia came next with 68,548,062 barrels, or 16.03 per cent of the total. Of the total United States production amounting to 281,104,104 barrels, the California fields produce 89,566,779 barrels, or, approximately, 32 per cent of the total. Only one other field, viz., the Kansas and Oklahoma produced more than this State. Here the output was 121,920,000 barrels, or 43 per cent of the total.

The average monthly consumption of California oil for the period January to May, 1917, was approximately, 8,370,000 barrels, 7,000,000 barrels of which was used for fuel, the balance going to refineries.

The largest individual users of fuel oil are the railroads, their consumption being, approximately, 40 per cent of the total. Steamships come next with, approximately, 13 per cent of the total, and public utilities third with 10.8 per cent.

The success attending the operation of railroads that have been electrified during the past few years demonstrate beyond a doubt that the problem of changing over from steam to electric operation is one of economics rather than engineering. With oil as the only fuel that is economically available in this State, and the price of this commodity continually increasing, there is every reason to expect that before many years many roads now operating by steam will be changed over to operate electrically, receiving their power from hydroelectric sources.

With steamships, the situation is an entirely different one. Here the motive power must be had from fuel used in some form or other. Oil occupies less space, is more easily handled, and is a better fuel in every respect than coal. With a larger merchant marine, the demand for oil for shipping purposes will be materially increased, and in view of the encouragement and support now being given by the Federal Government to this industry, it is safe to say that every possible effort will be made to encourage the industry by the conservation of oil for this purpose.

Public utilities use oil for two purposes, the first and larger usage being for the manufacture of gas, the other for the operation of steam-driven electric stations. For the manufacture of gas there is no economical substitute for oil. Coal could

be used were it available in sufficient quantities, and at the right price, but at the prices that have always prevailed in this State, gas could not be produced and sold at a rate that would permit of it being used as generally as it is today. Steam-generated electric power can be supplemented by hydroelectric power. It was to accomplish this purpose and to conserve all oil by the utilization of all of the available hydroelectric energy of the different power companies in so far as it was possible to do so, that the present power interchange arrangement was entered into between the companies operating in this vicinity.

Due to abnormal climatic conditions no large quantity of oil has thus far been saved by the interconnection. With an unusually dry season lasting from April 1917 to Feb. 1918, the stream-flows throughout the entire State are much below normal, and none of the water power plants has been able to operate at anything like its normal capacity. The precipitation to date for the winter of 1917-18 is not more than 50 per cent of normal, and unless we have unusually late storms during the coming spring, a very serious shortage of hydroelectric power may be expected before the flood water season of 1918-19. This shortage of water power will have to be made up from the steam plants. Not only will these steam stations have to carry the added load thrown upon them by an unprecedented water situation, but will also be called upon to carry the normal increase in load amounting to between 10 and 11 per cent per year.

No new water power plants are under construction at this time to relieve the situation and reduce the oil consumption. Under present financial conditions there is but little probability of anything being done in the way of making developments, but even if money, material and labor were all available immediately, it would require two years to complete any installation large enough to be much of a factor in accomplishing the desired results.

The combined loads, actual and estimated, with sources of energy supply, oil required, combined peak loads, and peak capacities of the five larger systems operating in the North Central part of the State, viz., Pacific Gas and Electric Co., Great Western Power Co., Sierra and San Francisco Power Co., Northern Calif. Power Co., Cons., and Western States Gas & Electric Co., are given in Table 1. The figures showing the estimated water power available for 1918 to 1921 inclusive, do not contemplate any additional energy from new plants, but do contem-

TABLE I—ACTUAL PAST LOADS AND ESTIMATED FUTURE LOADS OF THE FOLLOWING COMPANIES

Pacific Gas and Electric Company, Great Western Power Company, Sierra and San Francisco Power Company, Northern California Power Company, Cons., Western States Gas & Electric Co., Stockton System.

Expressed in Millions of Kilowatt Hours

Year	Total lead	Water power used	Estimated water power available	Steam used	Estimated steam	Per cent of steam to total	Oil required for steam in bbl.	Peak in kw.	Present peak capacity of steam and water power during draft on storage, in kw.	Deficit of peak capacity in kw.
1915	1,223	915		218		17.8	1,090,000	216,000		
1916	1,357	1,151		206		15.2	1,030,000	239,000		
1917	1,507	1,268		239		15.8	1,195,000	265,000		
1918	1,665		1,380		285	17.1	1,425,000	294,000	338,000	
1919	1,850		1,380		470	25.4	2,350,000	327,000	338,000	
1920	2,050		1,380		670	32.6	3,350,000	362,000	338,000	24,000
1921	2,280		1,380		900	39.5	4,500,000	403,000	338,000	65,000

Note: The above figure assumes operation of all plants at 100% of their plant capacity.

plate normal water conditions with a very decided shortage. Thus far during this year there is every reason to expect the water power output to fall short a considerable amount and the steam production to increase correspondingly. However, a more serious situation will obtain in 1920 unless additional generating facilities are provided. By this time the combined peak load will have increased to 362,000 kw., with an available peak capacity of 338,000 kw., or a shortage of peak capacity of 24,000 kw.

With the situation such as the foregoing figures indicate it to be, it will at once be apparent that something should be done immediately in the way of developing additional hydroelectric facilities not only to conserve fuel oil, but more especially to meet the increased demands of service.

ELECTRIC POWER IN NORTHERN AND CENTRAL CALIFORNIA

BY GASKELL S. JACOBS

THIS review covers generally the situation with reference to the present and future production and transmission of electrical energy by the four largest companies of Northern and Central California.

The electric utilities of this section are confronted with a combination of conditions that will tax both their physical and financial resources to the utmost, if they are to continue to meet the growth of their territory and maintain the high standards of service that have been established in the past. Among the factors contributing to the present emergency are:

1. The rapid growth of load, due not only to the normal increase in the demands for power, but also to the developments in shipbuilding and other industries, and to the stimulation of food production involving the use of electrical power for the irrigation of lands.

2. The scarcity of fuel oil, used exclusively in this section as a source of auxiliary steam-generated energy during the peak hours of the day and during seasons of reduced hydroelectric supply.

3. The cost of fuel oil for steam power plant operations has increased from 70 per cent per barrel in September 1916, to $1.50 per barrel, the prevailing market price in San Francisco since September 1917. Not only has this doubled the cost of pro-

duction of steam-generated energy to the utilities themselves, but it has resulted in the substitution of electric power for steam power by many industries that were unable to bear the increased cost of oil fuel.

In addition, practically all of the gas sold in California is produced from fuel oil so that the increase in price has even more seriously affected the gas utilities than the electric systems. There are today less than 30,000,000 barrels of oil in storage as compared with nearly double this amount in June 1915, and the oil stocks on hand are being drawn down at the rate of 1,000,000 barrels per month to offset the production deficit. The oil situation in California was the subject of an exhaustive investigation, and a comprehensive report on the many phases of this problem was made in July 1917 by the Petroleum Committee of the State Council of Defense.

Conservation of oil resources in California is the watchword today and much is being accomplished through the efforts of the United States Fuel Administration. The future shows every indication of the necessity for increased hydroelectric development and for the electrification of steam railways which are now operated entirely with oil fuel.

4. The curtailment of the expansion of generating and transmission facilities, due partly to the unsettled condition of the money market and of utility credits in war times, and to the absence of agreement between the utility interests and the Federal Government, which controls the available water power on public lands.

5. A large proportion of the skilled labor essential to the operation of the electric utilities is either in the service of the nation or has been attracted to more remunerative forms of activity.

6. The lack of rainfall and snowfall in the mountains and the depletion of stored water resources. The present season to date has been one of the driest experienced in over 20 years, and has resulted in a curtailment of hydroelectric power, not only during the flood water season, but also points to a greatly reduced output of water power during the summer and autumn, during which period the melting snow and the reservoirs filled in the winter and early spring, are generally sufficient to insure the continued operation of the hydroelectric plants during the dry season.

The four largest utilities operating in Northern and Central California, considered in this review, are,

1. Pacific Gas and Electric Company,
2. Great Western Power and Electric Company,
3. Sierra and San Francisco Power Company,
4. Northern California Power Company, Cons.

There are, of course, many other smaller electric utilities operating in this section, some of which generate their own power, but most of them are dependent for their power supply on the larger generating companies. The operations of these four companies constitute 90 per cent of the power business in this section. They are interconnected to the extent that the Pacific Gas and Electric Company purchases upward of 25,000 kw. from the Northern California Power Company and the Great Western Power Company.

The extent of the territory covered by the operations of these companies and the location of generating stations and main transmission lines and present points of interconnection are shown in Fig.1. Attention is directed not only to the extent to which this territory is covered, but particularly to the short gaps to be constructed in order to make a unified system. There are no differences in the operating voltages or frequencies of these companies that would prevent unification.

The total installed generating capacity is in excess of 400,000 kw., of which 260,000 kw. are installed in 26 hydroelectric plants and 140,000 kw. in seven auxiliary steam power plants. The hydroelectric plants are located principally on the western slopes of the Sierra Nevada Mountains, their average distance from San Francisco being about 130 miles. The steam stations, with one exception, are located either in San Francisco or Oakland on tide water. The combined transmission networks aggregate 2600 miles of line, of which 400 miles are operated at 100 kv. or over, 1800 miles at 60 kv., and 400 miles at voltages less than 60 kw. During the year 1917 nearly 1,600,000,000 kw-hr. of energy were produced in the power plants of this group, 16 per cent of which was from auxiliary steam plants. The simultaneous peak load of the year was in excess of 260,000 kw. The week-day load factor varies from 68 per cent to 77 per cent, while the minimum load on these systems rarely drops below 100,000 kw. The diversified char-

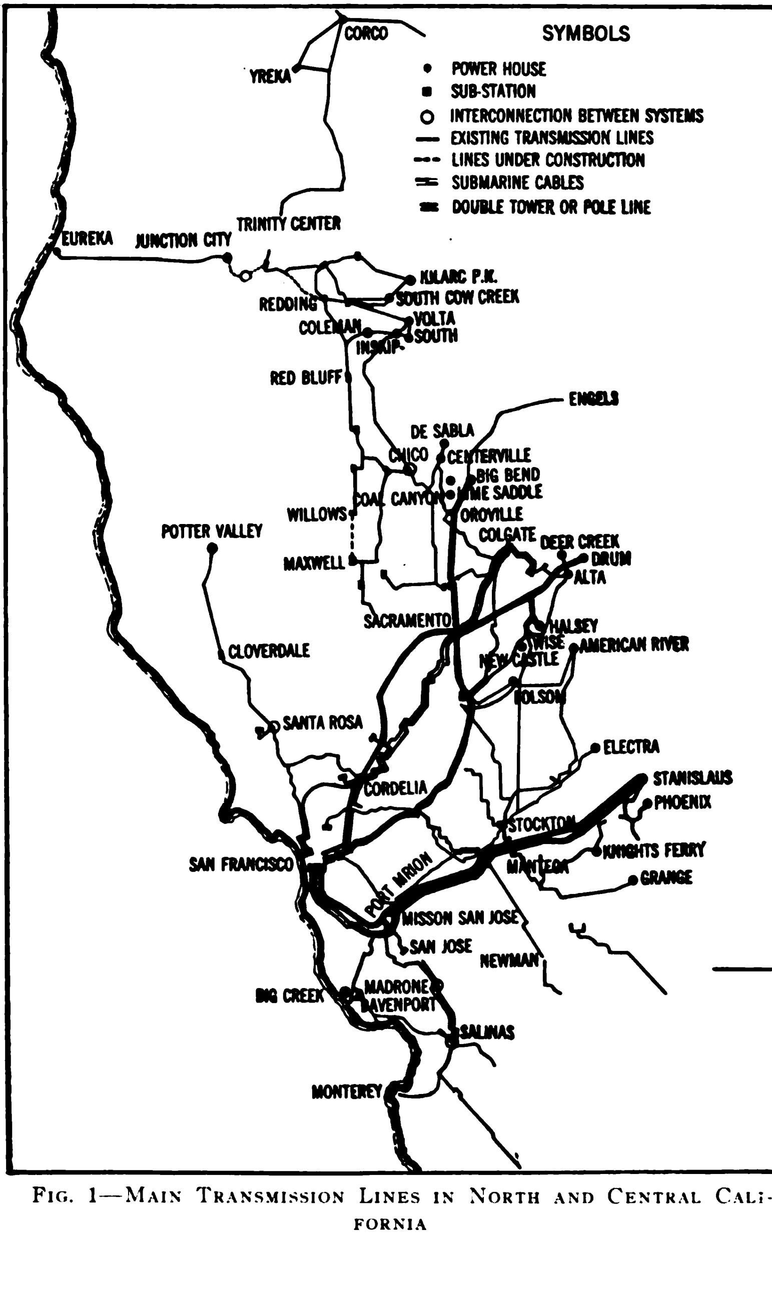


FIG. 1—MAIN TRANSMISSION LINES IN NORTH AND CENTRAL CALIFORNIA

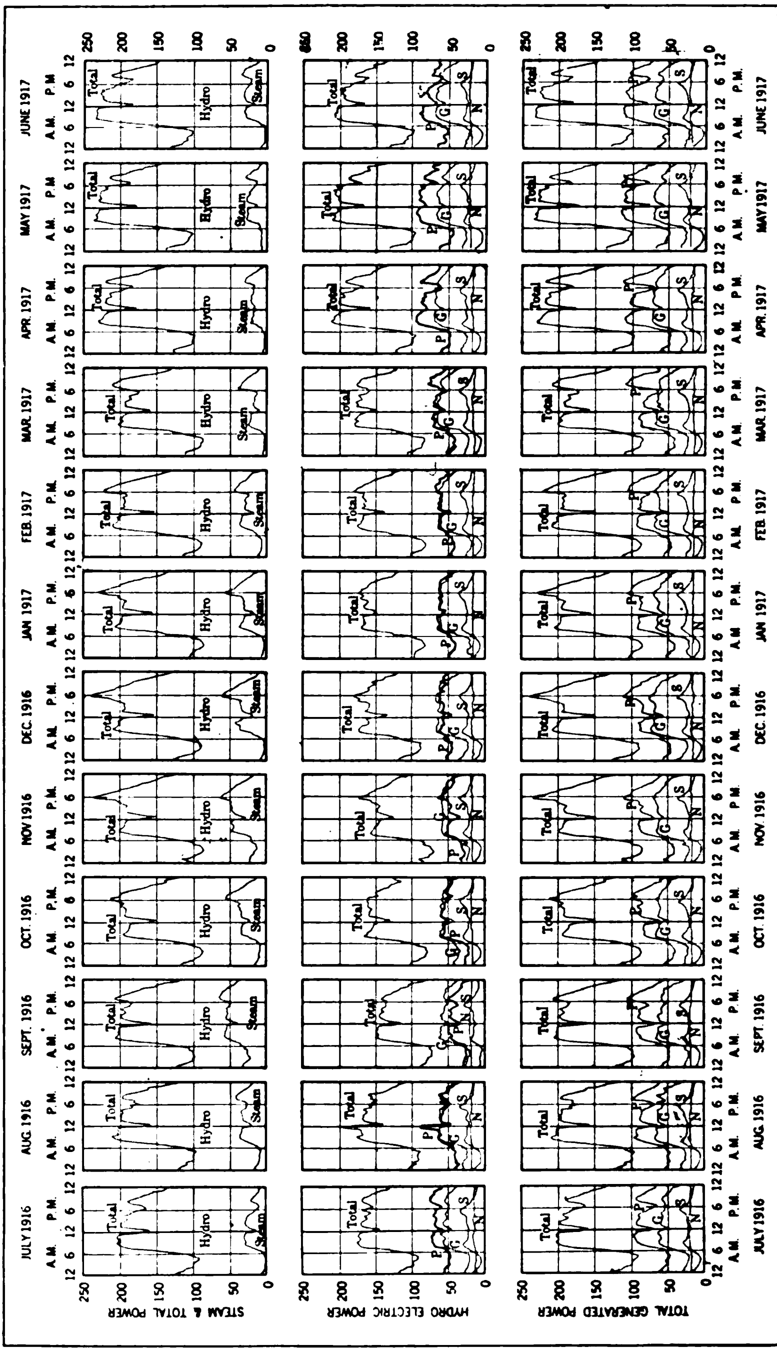


FIG. 2—PRIMARY POWER CHARACTERISTICS OF 1916–1917

Pacific Gas & Electric Company (P) Great Western Power Company (G) Sierra & San Francisco Power Co. (S) Northern California Power Co. (N)
NOTE:—This diagram is based on daily load curves of primary generated power that are typical of average conditions during each month in the several companies. All inter-Company exchange of power has been eliminated.

acter of the connected load is reflected in the typical daily load curves shown on Figs. 2 and 8.

The gravity of the situation confronting these utilities called for quick action, and the magnitude of the problem, together with the necessity for temporary relief, led to a consideration of the possibilities of increasing resources through a pooling

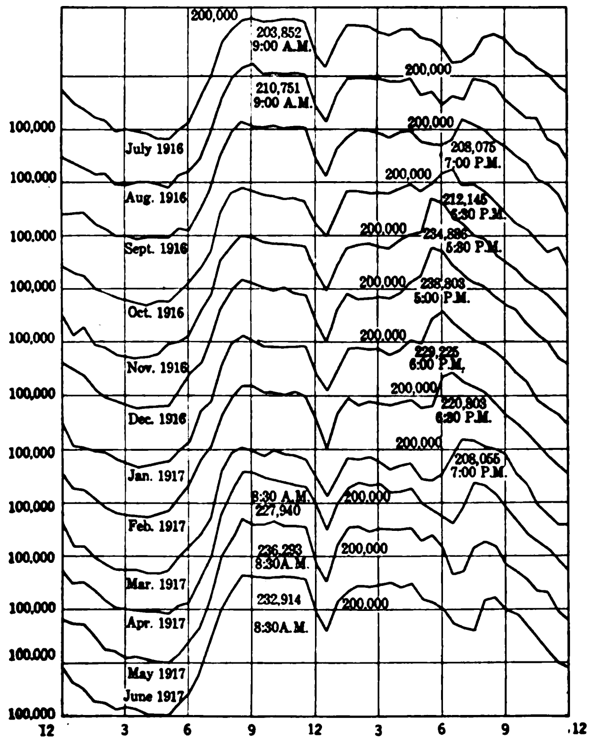


FIG. 3—PRIMARY POWER CHARACTERISTICS 1916–1917 OF COMBINED GENERATOR OUTPUT

NOTE:—This diagram is based on a combination of daily load curves that are typical of average conditions for each month in the several companies.

of facilities; and further to a consideration of the extent to which undeveloped resources could be quickly brought into use, with the prospect of a definite program of concerted development for the future that would enable the growing demands for electric power to be met in the most economical and efficient method possible. Some rather interesting results grew out of the investigation and their presentation follows.

TABLE NO. 1

PRIMARY POWER CHARACTERISTICS 1916–1917

COMBINES GENERATING SYSTEMS.

Pacific Gas and Electric Company
Great Western Power Company
Sierra and San Francisco Power Company
Northern California Power Company, Cons.

	System peaks (kw.)			Hydro peaks (kw.)			Average load (kw.)		Per cent load factor	
1916	Sum	Combined	Diversity	Sum	Combined	Diversity	System	Hydro	System	Hydro
July	218,182	203,852	14,330	190,025	175,632	14,393	160,612	145,712	78.8	83.0
August	217,431	210,751	6,680	207,011	195,456	11,555	161,904	137,929	76.7	70.5
September	229,685	208,075	21,610	163,420	152,280	11,140	163,496	117,975	78.6	77.5
October	218,765	212,145	6,620	173,005	166,046	6,959	157,358	131,104	74.1	79.0
November	235,486	234,886	600	177,186	173,286	3,900	161,104	126,696	68.7	73.1
December	242,203	238,803	3,400	183,623	178,773	4,850	166,608	141,725	69.8	79.4
1917										
January	229,855	229,255	600	177,837	171,995	5,842	161,996	139,162	70.8	81.0
February	230,238	220,803	9,435	184,950	180,153	4,797	164,008	141,746	74.3	78.3
March	231,105	208,055	23,050	186,820	178,945	7,875	158,279	144,175	76.2	80.7
April	238,957	227,940	11,017	216,782	211,112	5,670	174,146	161,700	76.5	76.7
May	247,033	236,293	10,740	220,018	209,533	10,485	181,087	165,579	76.6	79.0
June	241,364	232,914	8,450	214,919	206,804	8,115	180,646	162,517	77.6	78.5

1. Character and Distribution of Load

Daily load curves of the four systems were selected as typical of average load conditions during the week days of each month, and combined in such a manner as to show the effect of inter-connecting the systems. The load curves thus obtained for a unified system are shown in Fig. 2, and for comparative purposes the simultaneous load curves for each month are

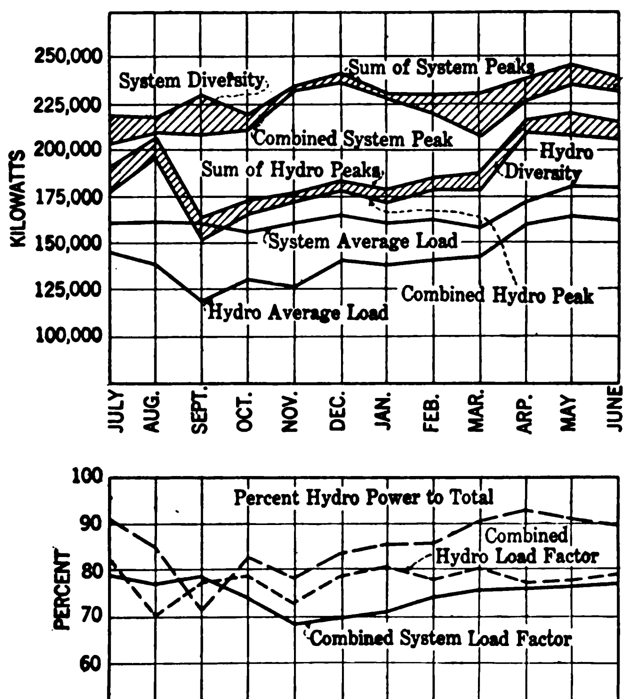


Fig. 4—Primary Power Characteristics 1916–1917 of Combined Generating Systems

superimposed in Fig. 3. The remarkable uniformity of load throughout the year, and the absence of sharp peaks except in two or three winter months, is apparent. During five months of the year the peaks occur in the morning, and the three periods of daily use—morning, afternoon and evening—are separate and distinct. The morning peaks during spring and summer are practically a combination of the industrial and irrigation loads, while the afternoon peaks in winter are the result of the coinci-

dence of the early lighting load, the street railway load and the late afternoon industrial demand. Considerable of the business represented is of high load factor, such as reclamation pumping, gold dredging operations, cement mills, mines, smelters, etc.

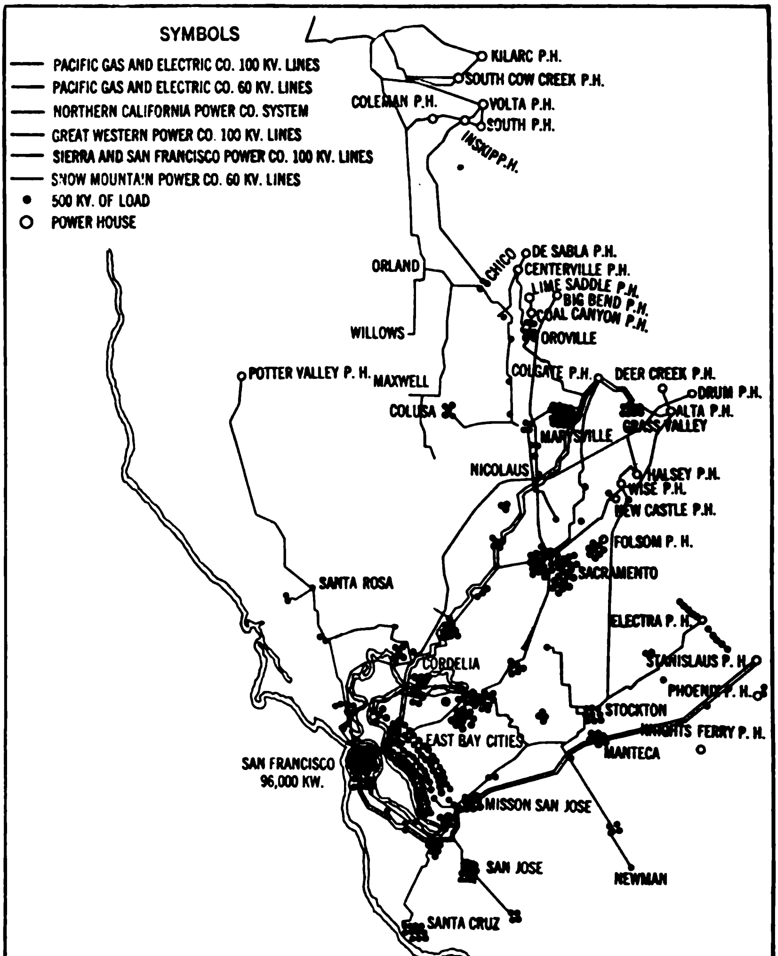


FIG. 5—DISTRIBUTION OF LOAD ON TRANSMISSION LINES

Fig. 2 indicates further the proportion of steam and hydroelectric energy necessary during the different months of the year to carry peak loads and offset the seasonal depletion of hydroelectric power.

The most important phase of this matter is the diversity of the systems. Table 1 and its graphical counterpart, Fig. 4, show both the system diversity and the diversity of the hydroelectric plants alone. The width of the bands in the upper part of this diagram is the measure of diversity. The over-all diversity reaches a maximum of 20,000 kw. during the spring and autumn, and disappears during the peak winter months, but averages between 9000 and 10,000 kw. throughout the year. It is also of interest to note that the diversity is high during the low water season when the hydroelectric plants are drawing on water storage to supplement reduced stream flow. There is also a uniform diversity of hydroelectric plant operation throughout the year of from 4000 to 15,000 kw. indicating that its utilization would assist in the reduction of the steam peaks to be carried.

Apart from the diversity of the load on these systems there is a very marked difference in the character of their water supply and in the forebay regulating features of the various plants. The hydroelectric plants located in the more northerly section have a much less seasonal variation of stream flow than the plants in the central and southern parts. Many plants lacking forebay capacity are limited in their utilization of normal daily stream flow, and spill water at times of load, but by interconection their loads could be increased and water could be held back in other plants of large fore bay capacity, during such periods of the day or night as load conditions permit, and render the water thus stored available for carrying higher hydroelectric peaks. A daily cycle involving load transfers of this kind would materially reduce the operations of the auxiliary steam plants.

The more efficient utilization of transmission facilities is intimately connected with the geographical distribution of the load and the routes traversed by main feeder lines. The distribution of the load as measured by peaks on the substations at points of ultimate consumption, is shown in Fig. 5. In this diagram the location of the load centers is clearly set forth, and the necessity for carrying large amounts of power in a general westerly and southwesterly direction toward the San Francisco Bay region is apparent. If a series of concentric circles be drawn with a center at San Francisco and spaced 25 miles apart, the loads on the main transmission substations in the several zones are as follows:

Zone	Miles from San Francisco	Substation peaks in kilowatts	Per cent
A	0– 25	157,500	53
B	26– 50	50,500	17
C	50– 75	25,000	9
D	75–100	34,000	11
E	100–125	23,000	8
F	125–150	5,000	2
		Total 295,000	100

The concentration of load in San Francisco and vicinity is evident, when it is noted that the peak demand of the City of San Francisco alone is 96,000 kw., or 32 per cent of the total. Within a radius of 25 miles of San Francisco, 53 per cent of the load, and within 50 miles of San Francisco, 70 per cent of the total demand on the transmission lines occurs. The necessity for large amounts of power being carried to San Francisco and the bay regions is in itself an important phase of this problem and particularly because the rate of increase of load in the bay region is much greater than in other portions of the territory served, due partly to the enlarged manufacturing and industrial activity, and the changing over of many plants from steam to electric power as a result of the high price of fuel oil, and also to the fact that the electric requirements of urban districts generally increase faster than those of rural districts.

Conditions in San Francisco itself are such that the full utilization of hydroelectric power cannot be advantageously effected unless additional line and transformer capacity is installed to connect San Francisco with the main transmission lines. This situation is peculiar in that the City is surrounded by water on three sides, and the only land route for transmission lines is from the south. Power from the north and east is brought in through cables under the Bay. This fact has practically compelled the location of auxiliary steam plants in the City, both to supply the load and to insure continuity of service. Under the most favorable operating conditions, a maximun of 65,500 kw. can be brought into San Francisco. This leaves 34,500 kw. of peak load that must be produced by steam for San Francisco alone, to say nothing of the deficiencies outside of San Francisco that must also be carried by steam at the plants located around the bay regions, in the event of line failures, for voltage regulation, or otherwise. The following table shows briefly the power situation in San Francisco with refer-

ence to the available power from without, the peak loads of the several companies serving, and the steam peak necessary to offset the transmission deficiency:

Company	Limiting factor of power supply	Maximum outside power available kw.	Estimated peak load of San Francisco kw.	Minimum steam generated power kw.	Excess capacity not available kw.
Pacific Gas and Electric Company	2 Bay Cables Martin Substa.	10,000 13,500			
		23,500	47,000	23,500	
Great Western Power Company	3 Bay Cables	12,000	23,000	11,000	
Sierra and San Francisco Power Company	Bay Shore Sub.	30,000	26,000		*4,000
Total		65,500	96,000	34,500	4,000

NOTE: *This excess capacity would be available with interconnection, and reduce the steam peak by an equal amount.

It is very evident that additional transmission line capacity into San Francisco from the south must be made available, or additional cables be laid from the north and east shores of the bay in order to make full use of hydroelectric power and decrease the amount of fuel oil consumed in auxiliary steam generating plants.

2. Use of Existing Facilities

From the standpoint of separate operation, all the hydroelectric generating facilities are being used to the greatest advantage by the individual companies in accordance with the character of their respective loads. More intensive utilization of the hydroelectric resources could be obtained by taking advantage of the diversity as set forth in the preceding paragraphs, and water now wasted can be used by effecting a daily cycle of load transfers from plants with large forebay regulating capacity to plants without storage by interconnecting.

Steam plant economies in operation can be effected by interconnecting the steam stations in the bay region, such that at least two out of five plants can be shut down during off-peak hours. If transmission facilities were enlarged into San Francisco, one plant might suffice for carrying steam peaks, leaving the others available for future load growth.

Transmission lines are, with a single exception, carrying far in excess of their contemplated capacity, thereby aggravating the problem of voltage regulation and requiring the production of steam energy for boosting voltage, in addition to carrying peak deficiencies. The Sierra and San Francisco Company's 104,000-volt line has about 12,000 kw. of excess capacity into San Francisco. The load on this system is largely a street railway load fed through 20,000 kw. of synchronous converter sets, so that an interconnection with this system affords not only much needed additional line capacity but has the further possibility of improvement in voltage regulation through the manipulation of its synchronous apparatus. All of the other trunk lines will require reinforcement at an early date.

TABLE NO. II

ACTUAL ENERGY GENERATED 1915–1917 ESTIMATED ENERGY REQUIREMENTS 1918

(Millions of Kilowatt Hours)

	1915	1916	1917*	Estimate 1918
January	91.07	105.47	115.80	133.95
February	82.84	96.25	105.65	123.38
March	96.93	103.27	116.93	134.95
April	93.79	102.98	118.36	136.76
May	97.00	114.56	130.48	148.95
June	102.55	113.25	128.64	146.76
July	106.24	114.87	131.65	149.95
August	106.28	117.83	134.71	149.95
September	103.78	114.16	130.52	146.76
October	108.57	115.76	132.34	147.95
November	105.63	113.44	129.69	146.76
December	104.71	117.03	133.79	149.95
Total	1,199.39	1,328.87	1,508.56	1,716.07
Per cent Increase over Preceding Year		10.8%	13.5%	13.8%

NOTE: *Estimated August–December 1917 inclusive.

3. Present and Future Requirements

The growth of energy output and peak loads on these systems for the years 1915, 1916 and 1917, with a forecast of load conditions for each month of the year 1918, are given in Tables No. II and III, and are shown graphically in Figs.6 and 7. The energy requirements of these systems have been increasing at a rate of from 10 per cent to 12 per cent per annum. For the

current year a production of over 1,700,000,000 kw-hr. is contemplated with a simultaneous peak load in November 1918

TABLE NO. III

ESTIMATED SIMULTANEOUS PEAK LOADS COMBINED SYSTEM OF FOUR COMPANIES

(Kilowatts)

	1915	1916	1917	Estimated 1918
January	173,000	200,000	229,300	267,000
February	166,000	193,000	220,800	263,500
March	172,000	183,000	208,000	251,8C0
April	171,000	188,000	227,900	264,500
May	169,000	200,000	236,300	274,800
June	182,000	202,000	232,900	277,000
July	182,000	203,800	224,000	269,800
August	186,000	210,800	235,000	276,600
September	185,000	208,100	232,000	286,900
October	197,000	212,100	241,000	283,700
November	212,000	234,900	261,000	312,000
December	201,000	238,800	257,000	303,500

of 312,000 kw. A more detailed analysis of the load conditions for the year 1918 is shown in the typical week day load curves for each month (Fig. 8) and is summarized in Table No. IV.

TABLE NO. IV

ESTIMATED 1918 LOAD CONDITIONS COMBINED SYSTEM OF THE FOUR COMPANIES.

WEEK DAY LOADS ONLY.

	Average load on systems	Peak load on systems	Load factor
January	191,908	267,000	.718
February	197,102	263,500	.748
March	193,344	251,800	.767
April	202,940	264,500	.767
May	213,459	274,800	.776
June	217,821	277,000	.786
July	214,896	269,800	.796
August	214,896	276,600	.776
September	225,592	286,900	.786
October	212,022	283,700	.747
November	217,821	312,000	.698
December	214,896	303,500	.708

The load demands which are certain to occur during the year 1918 would, under normal water-power conditions, absorb practically all the combined resources of the companies, but

with the prospect of a shortage of hydroelectric power, to which all present indications point, the situation is serious. The utilities have already interconnected their steam plants, and plans are being perfected for further joint operations. The Pacific Gas and Electric Company has ordered a 15,000 kw. steam turbine to be ready in the fall of 1918, and there is the

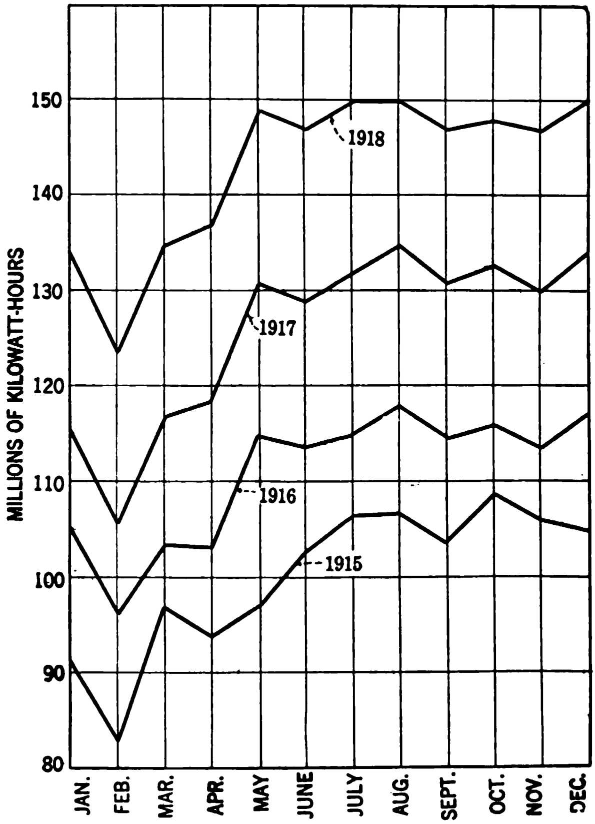


FIG. 6—ENERGY GENERATED 1915–1917—ESTIMATED REQUIREMENTS 1918

prospect of bringing an additional 8000 to 12,000 kw. of hydroelectric power from the northern end of the State, which is most attractive from the standpoint of cost and the relief it will offer from the increasing consumption of fuel oil in steam plants A thorough canvass is being made of all new hydraulic developments that can be quickly installed. The Railroad Commission

of California and the United States Fuel Administration are co-operating with the utilities in the solution of the many phases of this problem.

5. Prospective Development

With a continued growth such as has been experienced for several years past, the territory served by the four utilities herein considered will require additional facilities in the future

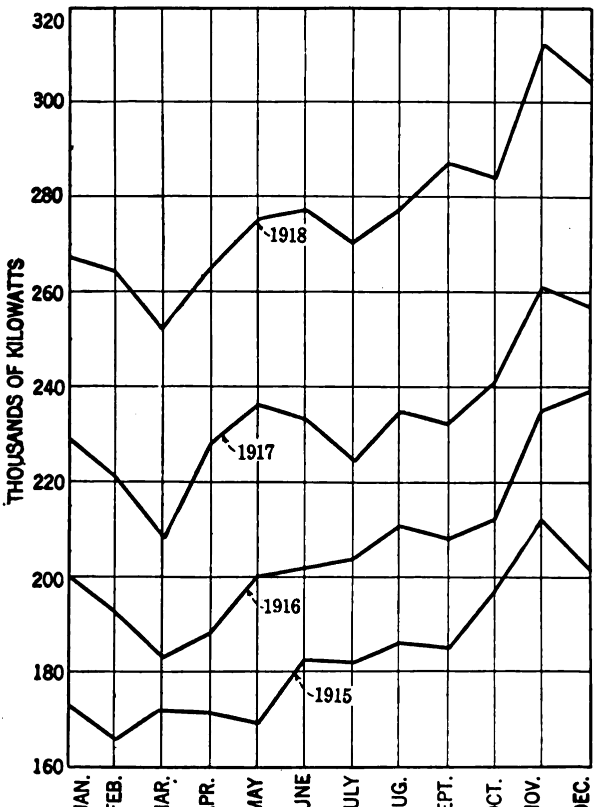


Fig. 7—Peak Loads 1915–1917—Estimated Peaks 1918

to provide for an expansion of at least 30,000 kw. per annum, with a corresponding expenditure of not less than $5,000,000 each year for power stations and transmission lines. Looking ahead five years to conditions that will in all probability exist in 1922, these systems must be ready to provide a peak of 450,000 kw., and an annual energy output of 2,400,000,000 kw-hr. (See Fig. 9).

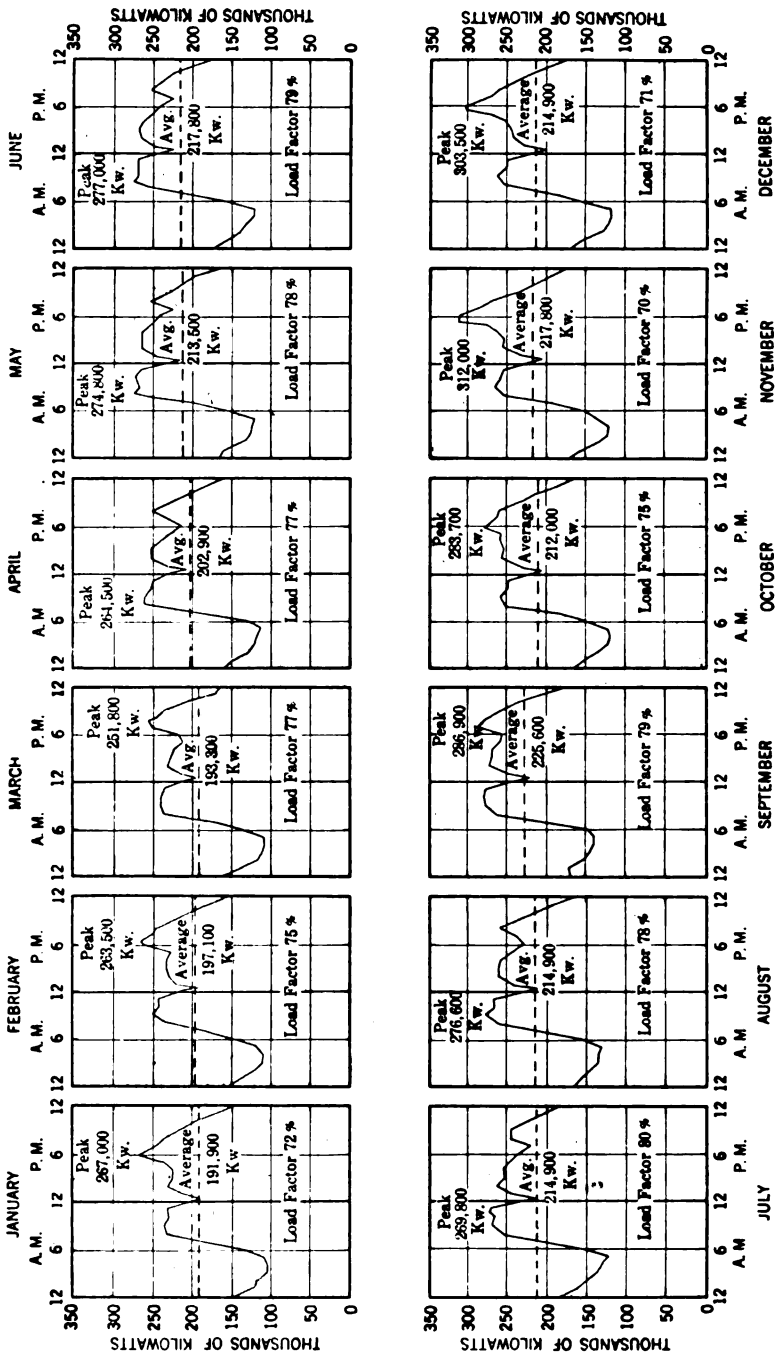


Fig. 8—Estimated Average Load Curves 1918 Showing Combined Load

These four companies have undeveloped hydroelectric resources aggregating 500,000 kw., most of which is susceptible of quick development. Much can be accomplished within three years to provide the capacity needed to keep pace with their growth. This is predicated, of course, upon the ability of the power interests and the Federal Government to agree upon terms that will release the water power on public lands, and

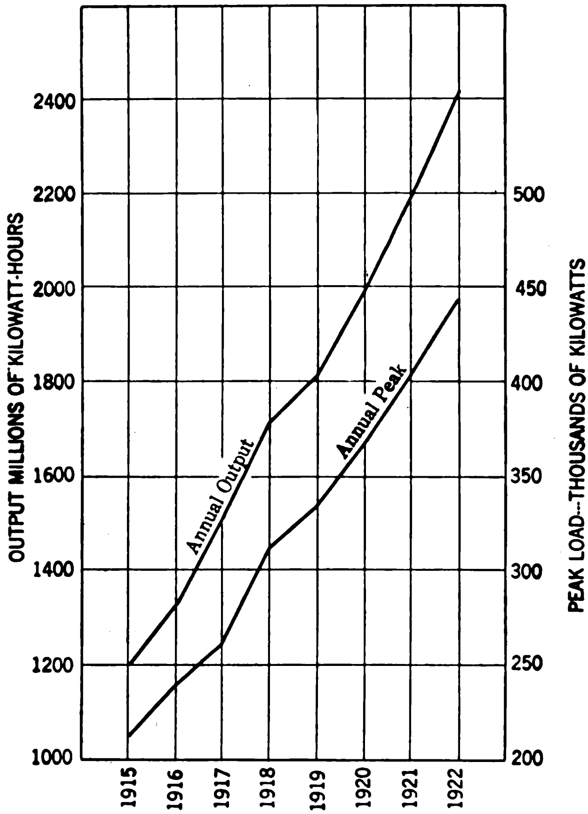


FIG. 9—ESTIMATED GROWTH IN PEAK AND OUTPUT 1915-1922

upon the response of capital to the call for funds for these projects.

The growth of the power requirements of the Pacific Coast States is far in excess of their oil, coal and other fuel resources. Some increase in the coal supply of the west is expected, but the oil resources of California have probably reached their maximum of production. The power of the future must therefore come

from the mountain streams. California, especially, stands at the threshold of an era of unprecedented electrical development.

THE FUNCTION OF THE PACIFIC GAS AND ELECTRIC CO. IN THE INTERCONNECTED OPERATION OF THE POWER COMPANIES OF CENTRAL AND NORTHERN CALIFORNIA

BY J. P. JOLLYMAN

FOR the purpose of this paper interconnection will be considered to mean a connection for the delivery or exchange of power between two power systems, each of which has power plants, transmission lines and loads of its own.

The system of the P. G. & E. Co. has been built up by the interconnection of a number of power systems originally planned to be more or less complete within themselves. The Folsom-Sacramento system was the first constructed. Later the Colgate power plant was built with a line into Sacramento. These two systems were interconnected at Sacramento. The Bay Counties Power Co. built transmission lines from Colgate into Oakland, and the Standard Electric Co. from Electra into Oakland and San Francisco. These two systems were interconnected at Oakland, where they were also connected to the system of the Oakland Gas Light and Heat Co. The Standard system was interconnected with that of the S. F. Gas and Electric Co. at San Francisco.

From these beginnings has grown the present system of the P. G. & E. Co. with 1386 miles of 60-kv. circuit; 120 miles of 110-kv. circuit; 13 hydroelectric plants with an installed capacity of 121,600 kw.; three steam plants with a steam turbine capacity of 69,250 kw. and 161 substations. The week-day average load is now about 110,000 kw. and the peak about 145,000 kw. The minimum load at 3:00 a.m. is about 60,000 kw.

To better show the relation of the system of the P. G. & E. Co. to those of the other companies with which it is interconnected, a diagrammatic map has been prepared. The lines tying the several systems together have been made the most prominent, the principal sources of power are shown, the most important load centers indicated, and the direction of the flow of power shown.

The 60-kv. line starting at Chico, and extending through Marysville, Davis, Cordelia, Oakland and Mission San Jose to San Francisco, is the 241-mile bus bar connecting together the Northern California Power Co., which ties directly in at Chico; the Snow Mountain Water and Power Co., which ties directly in

at Santa Rosa; the Great Western Power Co., which ties in through 100-kv. to 60-kv. transformers at Oakland; and the Sierra and San Francisco Power Co., which ties in at San Francisco on the 11-kv. side of the stepdown transformers.

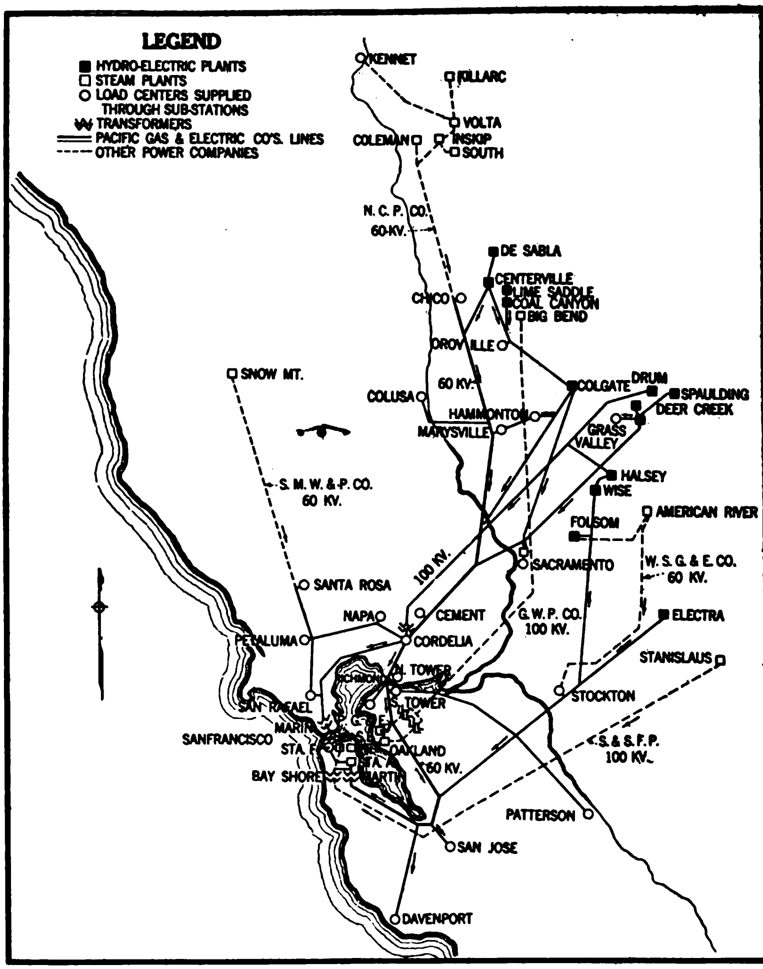


FIG. 1—MAP DIAGRAM SHOWING RELATION OF PACIFIC GAS & ELECTRIC CO. TO INTERCONNECTED POWER SYSTEMS

Important branches of this main bus are the Cordelia-Martin line over which power may be fed to San Francisco and the Halsey-Wise-Electra branch which feeds in at Mission San Jose and with which the Western States Gas and Electric is connected at Folsom.

It should not be inferred that this bus bar has a greater carrying capacity than any of the lines connected to it, as such is not the case. The distribution of the sources of power and of the loads is such that it is not called on to carry more than about 30,000 kw. at any point (out of Mission), whereas some of the lines connected to it carry twice this amount (G. W. P. 100 kv.).

In addition to serving as a busbar connecting the five companies named, these lines receive much of the output of the P. G. & E. Co. hydroelectric plants and feed a large number of substations.

The effect on the P. G. & Co. system of these interconnections is not greatly different than would be the case if power plants or loads of its own were located at the points where the interconnections are made. With the exception of the tie with the Sierra & S. F. the connections were originally made for the purpose of the delivery of power to the Pacific Co. The daily load factor on which this power is delivered is different in each case. In some cases it is substantially unity in others as low as 70 per cent.

In either case the control of the amount of load delivered is not difficult. The load is watched at the point of delivery and the company supplying the power advised from time to time of any changes required to insure the delivery of the correct amount of power.

In addition to the interconnections through the 60-kv. lines, important interconnections are made in San Francisco between the steam plants of the Sierra and S. F. Power Co., the Great Western Power Co., and the P. G. & E. Co.

The Pacific Company has a substation called Station F in the North Beach steam plant of the Sierra Co., originally built for supplying power to the Exposition. The substation bus is interconnected with the Sierra steam plant bus. Eleven kv. tie lines exist between the steam plant, called Station A, of the Pacific Co. and this substation; also between Station A and the 60-kv. system of the Pacific Co. through Martin station. A second connection from this substation and the 60-kv. network is had through the submarine cables crossing the Golden Gate to Marin substation.

The Sierra steam plant is tied to the transmission system with 11-kv. lines through Bay Shore. The great Western steam plant is tied to the transmission through submarine cables to Oakland.

It might appear that it would be possible to close all the lines centering at Station F and use it as a common connection point for both ends of the Pacific system, the Sierra system and that of the Great Western. In making a study of this question, some of the limitations appear which make impossible or undesirable the simultaneous interconnections of two systems at more than one point.

The voltage required to force current over a transmission line causes a difference in phase between the impressed and the received voltage. The amount of this difference in phase depends on the length of the line and the current transmitted. If two lines from a common source of power to a load center are of greatly different length and both are carrying separate loads at the receiving ends, the voltages at the receiving ends will be found to be out of phase. If the receiving ends are thrown together, the shorter line will assume the greater part of the load.

This condition applies to the two ends of the Pacific system which come together at Station F. Starting from Cordelia as a common point, the Marin line is 44 miles long while the line via Oakland to Martin is 110 miles long. The actual difference in phase when both lines are loaded is about 10 deg. It is found that when both ends are closed in, the Marin end delivers considerably more load than the Martin end, thus preventing the full use of both facilities and reducing the amount of power which can be delivered from the transmission system.

When running in parallel with 60-kv. system in Oakland, the voltage of the Great Western in San Francisco is out of phase with that of the Pacific Co. due to a difference in transformer connections; hence it is impossible to tie the two systems together a second time. The interconnection may be used by separating a part of either system from the network usually operated in parallel.

There are certain limitations on the possibilities of tying the Sierra system to that of either the Pacific Co. or the Great Western at North Beach, due to the fact that the ties are through 11-kv. lines, which are very long in proportion to the voltage or of small capacity in proportion to the capacities of the systems interconnected.

One of the difficulties which has appeared in the operation of existing interconnections is involved with the speed control of transmission systems and is the tendency to pull out of synchronism of two systems, each of which has considerable gen-

erating capacity and load of its own. This is especially the case where the interconnecting lines are long in proportion to their voltage, or are of small capacity in proportion to the capacities of the systems connected.

In a case of this kind trouble may be occasioned by a sudden change in load on either system. If the load on one system increases suddenly the additional energy must be drawn from the flywheel effect of all synchronously rotating apparatus through a reduction in speed. This reduction in speed causes the governors on the prime movers to act and supply more power. A portion of the energy supplied to the system on which the load has increased will come momentarily from the other system through a change in the load carried by the interconnecting line. Now such a change means a change in the phase displacement between the two ends of the interconnecting line; hence of the two systems. Such a change in phase between two systems causes a tendency to hunt, which will result in a dropping out of step unless the interconnecting line has sufficient capacity to transfer enough energy to hold the two systems together.

A rather simple mechanical analogy would be two motor trucks, each carrying load, and one supplying a part of the power required by the other through a rather small, somewhat elastic tow line. No trouble would occur until the load on either truck would suddenly increase, as might be occasioned by a wheel striking a rock, in which event the load on the tow line might be more than it could carry, or a surging between the two trucks might be set up which would finally break the tow line.

The greatest difficulty will occur when the load over the interconnection is subject to rapid reversals. If the load flows in one direction and can be held fairly steady, little trouble need be expected with interconnections having a reasonable margin of capacity.

To make the utmost use of all existing facilities, further interconnections between the power systems of California have been suggested as a means of taking advantage of the diversity of the hydroelectric sources of supply and of the loads on the several systems. Additional interconnections involve lines of considerable length. Some of the difficulties which must be guarded against have been mentioned.

The map shows that the general flow of power is from the north and east toward the Bay Districts. This is fundamentally due to the fact that the natural minimum stream flow of the

Sierra rivers is greater in the north than in the south. In addition the San Joaquin Valley and Southern California require the power that can be developed in the Sierras southeast of the Bay Districts.

The suggestion has been made that there would be important advantages in having a high capacity trunk line running through the Sacramento and San Joaquin valleys to which all the power companies could be connected and through which power could be exchanged. To provide the necessary capacity for this purpose, such a trunk line would have to be operated at high voltage and to safeguard service would have to be in duplicate with sectionalizing stations at intervals. The existing lines of the power companies do not fulfill the requirements of such a trunk even in part.

The engineering problems which would be encountered in the construction and operation of such a trunk are numerous and difficult. Let us assume that such a trunk would consist of two 150-kv., 60-cycle circuits from Kennet to Bakersfield, a distance of 420 miles. The charging current at full voltage would require 50,000 kv-a. per circuit or 100,000 kv-a. for both circuits, and this is much more than the capacity of any one hydroelectric plant in California.

It seems probable that further interconnections will be worked out more nearly along the lines that have been followed in the past. This has usually involved the construction of comparatively short and inexpensive interconnections in specific cases where an exchange of power was economically desirable.

CALIFORNIA-OREGON POWER CO. AND NORTHERN CALIFORNIA POWER CO.

BY WILLIAM M. SHEPARD

THE proposed interconnection of the California Oregon Power Company with the other power companies of California would be made through the system of the Northern California Power Company to the lines of the Pacific Gas and Electric Company near Colusa. The Northern California Power Company has already a connection with the system of the Pacific Gas and Electric Company at Chico, and delivers from 8000 to 10,000 kw. which is the limit of the available line capacity of both companies at this point.

Delivery near Colusa would be considerably nearer the center of load of the Pacific Gas and Electric Company, being

about 50 miles further south and west by way of transmission lines than Chico.

The connection between the California-Oregon Power Company and the Northern California Power Company will be made at Kennet, the California-Oregon Power Company extending its lines from Castella south a distance of about 32 miles and reinforcing its lines from Castella north to Copco, a distance of 63 miles, raising the voltage on this line from 60,000 volts to 66,000 volts and increasing the copper in the line from No. 2 to No. 00.

The California-Oregon Power Company will then deliver at Kennet approximately 8600 kw. which will release about 9100 kw. at the power houses of the Northern California Power Company, the Northern California Power Company transmitting about 35 miles north to Kennet at present and the load in this vicinity being between 8500 kw. and 9000 kw. with a line loss of approximately 6.5 per cent.

In order to transmit this power released at the plants of the Northern California Power Company south, it is proposed to increase the capacity of the 66-kv. line from Coleman to Willows Junction, a distance of 55 miles, and from Willows Junction to Hamilton, a distance of 10 miles, from No. 1 to No. 000 copper. From Willows Junction to the connection point with the Pacific Gas and Electric Company near Colusa there are two 66-kv. lines, one 48 miles of No. 4 and one 43 miles, 10 miles to be reinforced to No 000 to Hamilton, the balance being No 1.

The present lines have in the summer a combined load of about 4000 kw. This falls off in about 1000 to 1200 kw. in winter due to the irrigation load.

This reinforcement of line capacity would allow the delivery of the present summer load plus 8000 kw. near Colusa. The total line loss for 12,000 kw. delivered being about 1350 kw. The line loss over the present lines with the summer load is approximately 230 kw. making a net additional loss chargeable to the 8000-kw. delivered at Colusa of about 1120 kw. In other words the 9120 kw. released at the power houses of the Northern California Power Company by the delivery from the California-Oregon Power Company at Kennet of 8620 kw. will make available at Colusa 8000 kw.

The loss on the California-Oregon Power Company lines from Copco to Kennet will be about 880 kw. This means that

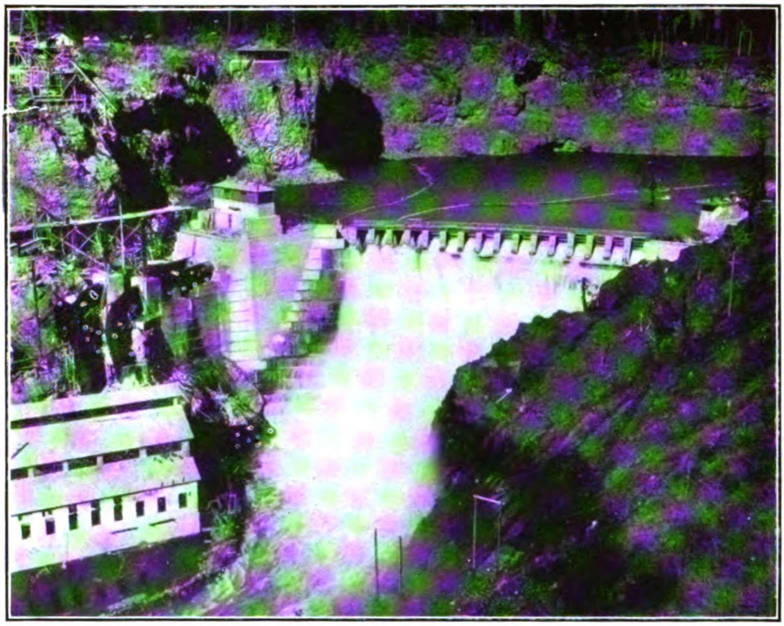


FIG. 1—POWER HOUSE AND DAM—COPCO POWER DEVELOPMENT

[SHEPARD]

FIG. 2—COPCO POWER DEVELOPMENT CALIFORNIA-OREGON POWER CO.
KLAMATH RIVER, CALIFORNIA

9500 kw. at Copco will deliver through the means just described 9000 kw. near Colusa into the Pacific Gas and Electric system the net transmission loss being about 16 per cent of the generated power.

This interconnection will make available a block of power 230 miles distant largely through the medium of existing facilities and with reasonable losses. The maximum actual distance of transmission is about 100 miles, from Coleman to the delivery point near Colusa.

It is also possible to deliver during the winter months when the irrigation load of the Northern California Power Company is off, an additional block of from 3000 kw. to 4000 kw. over these same lines. This additional power can be delivered during a period of seven months from October 1st to May 1st and could be used as needed by the Pacific Gas and Electric Company.

The Northern California Power Company has considerable excess generating capacity over its available water supply and for this reason is in an excellent position to control the voltage at Kennet, the power factor of the load taken by the California-Oregon Power Company and therefore the capacity of its lines into Kennet. As a matter of fact, 1100 kw. of generating capacity in the Northern California Power houses will give the same result, as a 2800-kv-a. synchronous condenser at Kennet and will allow the California-Oregon Power Company to deliver at above 95 per cent power factor.

The power factor of the load moving south from the Northern California Power Company's plants will also be high on account of the large amount of 66-kv. transmission line, there being about 185 miles of this line, the charging current of which will be available for raising the power factor of this load. It is assumed that the 8000 kw. will be delivered at Colusa at 85 per cent power factor, the local loads at 80 per cent power factor. On this basis the power factor at the plants of the Northern California Power Company could be in the neighborhood of 95 per cent.

The power from the California-Oregon Power Company system is made available by the recent installation of the first unit of the Copco power development on the Klamath River, near the Oregon Line.

The ultimate development at this site will consist of two units, each consisting of a 12,500-kv-a. generator driven by

an 18,600-h.p. double Francis turbine designed for operation under 125 feet head. Water is supplied to the present turbine through two steel pipes 10 feet in diameter, and 200 feet long.

The pipes are connected to the reservoir through a gate house structure of reinforced concrete, forming the west abutment for the top 40 feet of the dam. The center line of the pipes being some 27 feet below high water level in the reservoir, thus making available the upper 12 feet or approximately 9000 acre-feet of water for pondage. This represents a reserve of 750,000 kw-hr.

With the complete development and control of the river, 20,000 kw. will be obtainable at 75 per cent load factor, or 25,000 kw. at 60 per cent load factor throughout the year.

FUEL CONSERVATION BY INTERCONNECTIONS WITH THE SYSTEM OF THE SIERRA & SAN FRANCISCO POWER COMPANY

BY J. E. WOODBRIDGE

IN so far as it is of interest from the standpoint of interconnection, the system of the Sierra and San Francisco Power Company consists essentially of the following elements:

1. A hydroelectric plant in the Stanislaus canyon, with four 8500-kw. units operating under a head of 1500 feet, and a forebay having a capacity equivalent to approximately 300,000 kw-hr.

2. A two-circuit steel tower transmission line, operated nominally at 104,000 volts, with a length of 135 miles to San Francisco, each circuit of No. 00 wire having a capacity usually considered as 20,000 kw.

3. A steam power plant in San Francisco, containing three 9000-kw. generators.

4. A load of from 550,000 to 600,000 kw-hr. per day, with a peak of approximately 40,000 kw., a large portion of this load being street car service in San Francisco supplied through synchronous motor-generator sets.

In so far as the consumption of fuel oil is concerned, the following facts are of primary importance:

a. The system has little or no hydroelectric generated energy to spare, since the average daily load is about equal to the flume capacity.

b. Some fuel oil must be burned in the Company's San Francisco steam standby station even during flood water periods,

to take care of peaks greater than the capacity of the hydroelectric plant, and to insure reliability of service. This is on the assumption that the system is operated as an independent unit not interconnected with the systems of other companies.

c. While the company has two storage reservoirs in the mountains, with an aggregate capacity of approximately 30,000 acre-feet, this amount of storage is not sufficient to bring up the natural flow during the dry season to the power requirements, the deficiency in an average year with the present load being such as to call for the burning of fuel oil to the order of magnitude of 150,000 barrels per annum.

The reduction of oil consumption which might be accomplished by interconnection may be considered under three divisions:

First: Substitution of water power of one system for steam power of another;

Second: Substitution of steam power of one system for steam power of another;

Third: Use of forebay facilities of one company for indirectly absorbing spilled water of another;

Fourth: Use of spare transmission facilities of one company for bringing in hydroelectric power of another system.

With reference to the first, that is, the substitution of water power of one system for steam power of another, the three systems serving the San Francisco Bay region all have approximately the same flood and stored water season, so that no one has any material amount of excess hydroelectric energy available when another has a deficiency of energy from that source. Their daily load peaks are approximately simultaneous and all water power plants are loaded at the peak. For these reasons there is little opportunity for substitution of hydroelectric energy from one system for steam energy of another system.

With reference to the substitution of steam-generated energy of one system for energy generated from the same source on another system, this obviously offers two opportunities for fuel oil saving. First, the operation of more economical rather than less economical steam plants; second, the shutting down of one or more of several plants, reliance being placed on the others. The several large steam plants of the three companies operating in the San Francisco Bay district are very much alike in the matter of kilowatt hours per barrel of oil, since they are plants of similar magnitude containing units of similar size, 6000 to 15,000 kw. each. For this reason, there is little oppor-

tunity to save fuel oil by substituting steam-generated energy from one plant for that from another plant. There is, however, an opportunity for considerable saving by shutting down at least one of the plants during the high-water season, provided the plant so handled can be really shut down, that is, kept cold for any considerable length of time, that is, provided the other steam and water power plants can serve as a standby to the system whose steam plant is so treated.

In the matter of the use of forebay facilities, the equipment of the Sierra Company happens to be such that it can be used as a hydroelectric storage battery for a portion of the loads and power sources of its neighbors.

During the flood water season, the Great Western Power Company has available during the night hours of each day approximately 200,000 kw-hr. in excess of its requirements, although its peak load exceeds its hydroelectric generating capacity. The Pacific Gas and Electric Company requires more energy each day than can be developed in its own hydroelectric plants, plus energy at present purchased. Some fuel oil could be saved by the Pacific Gas and Electric Company and by the Great Western Power Company if energy which is available during the night on the system of the Great Western Power Company could be carried over and delivered to the Pacific Gas and Electric Company or the Great Western Power Company during their heavily loaded periods, as from 7 a. m. to 11 p. m. The Pacific Gas and Electric Company cannot absorb the energy available during the night due to the fact that its minimum hydroelectric output, without spilling water, is about equal to its minimum load. By the use of its forebay facilities, the Sierra and San Francisco Power Company can absorb approximately 60,000 kw-hr. each night, which the Pacific Gas and Electric Company and the Great Western Power Company can utilize during the day. This 60,000 kw-hr. can be transferred from the Great Western Power Company to the Sierra and San Francisco Power Company by means of existing interconnections between the three companies concerned.

Assuming a normal season, this surplus will be available and can be utilized about a hundred days each year. This results in the utilization of about 6,000,000 kw-hr. of hydroelectric energy per annum, or a saving of 30,000 barrels of fuel oil.

In the matter of the fourth possibility for the saving of fuel oil mentioned above, namely, the use of the transmission lines

of one system to carry hydroelectric energy of another from the Sierra Nevada district to the load district near San Francisco Bay, an analysis of the situation has shown that each of the three systems serving this district has in service or under construction sufficient line capacity for the transmission of all of its own hydroelectric power, for which reason no more energy could be developed in the water power plants by utilizing any part of the existing transmission facilities of one system for transmission of power of another system. While there are times that this is not the case, as when lines are out on account of trouble or for repairs, the cost of interconncting the systems to make the lines of one company transmit the energy of another, together with the dispatching and other operating difficulties involved, appear to more than outweigh the advantages that might be obtained, which as noted above, are not of great magnitude.

PROCEEDINGS

OF THE

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

INDEX TO VOLUME XXXVII

JANUARY 1 TO DECEMBER 31, 1918

INDEX TO SUBJECTS

INDEX OF AUTHORS